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Essays on International Effects of Monetary Policy

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Abstract

During recent decades the monetary policies of central banks have shown similar patterns - mostly led by the policy changes of large economies such as the US and the Eurozone. These large economies care little for other countries' monetary policies, while other economies have concerns about the fluctuations of international variables such as currency values and capital flows. This results in asymmetric relationships between the central bank policies across countries. The major central banks' quantitative policies also have significant effects on the other economies by way of changes to cross-border capital flows.

The first chapter presents (i) a theoretical framework that explains the asymmetric policy relationships and (ii) empirical evidence for those relationships. In the two-country model analysis, home welfare is approximated as a function of international variables. These variables also influence optimal home inflation and output. The optimal home policy rate is affected by the foreign policy rate, and the effect becomes stronger as home openness is greater. Therefore, when the degrees of openness are significantly different between countries, there can be an asymmetric policy rates relationship. The empirical analysis investigates the policy rate relationships between two large economies (the US and the Eurozone) and other economies. In the probit model analysis, 12 out of 14 countries have leader and follower relation-

ships with at least one of the US and the Eurozone. The VAR analyses indicate that 11 countries have one-way relationships with the large economies.

In the second chapter, the leader-follower policy relationship between central banks is investigated based on a small open economy model. The asymmetric relationship is strengthened by economic globalization. When the foreign policy rate is lowered, the home currency appreciates. This leads to a decrease in net exports via the expenditure switching effect, thus reducing home output. The lower import price reduces home inflation. In response to the changes in home output, inflation and the real exchange rate, the home central bank lowers its policy rate. The policy relationship becomes stronger when (i) the international assets holding cost is lower, (ii) home openness is higher, (iii) the home central bank adopts more aggressive inflation targeting, and (iv) the home monetary policy responds to currency value changes. The banking friction also strengthens the policy relationship.

The third chapter models the international effects of quantitative easing (QE) in terms of cross-border capital flows. A two-country model is set up with financial frictions. The foreign central bank conducts QE. As the amount being loaned to home agents by foreign banks increases, capital stock, investment and the asset price of the home economy rise, accompanied by a currency appreciation. The fall in the foreign interest rate lowers the home interest rate through a decline in the home capital return. The financial accelerator strengthens the foreign QE effects. The rise in the asset price boosts home borrowers' net worth, and eventually investment and capital stock increase more. The international effects of QE are stronger with (i) a greater assets adjustment cost in the foreign economy, (ii) a lower home investment adjustment cost, and (iii) a higher degree of home financial openness.

Declaration

The material contained in this thesis has not been submitted in support of an application for another degree or qualification in this or any other institution.

Acknowledgements

Dedicated to

My Parents and Jaeyeong

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Chapter 1

Asymmetric Policy Relationships between Central Banks: Theoretical and Empirical Evidence

1.1 Introduction

Over recent years, the policy decisions of many central banks have followed similar patterns, as illustrated by Figure 1.1. Indeed, the Taylor rule (Taylor, 1993) states that monetary policy responds to domestic output and inflation changes. However, according to Hofmann and Bogdanova (2012), the policy rates in many countries have been below the levels recommended by the Taylor rule for more than a decade. He and McCauley (2013), and Taylor (2013) indicate that the deviation from the traditional rule is related to the central banks' concerns about international variables, such as the exchange rate and the terms of trade.

For some large economies, however, the importance of external variables is relatively small due to their low levels of openness. Moreover, international variables
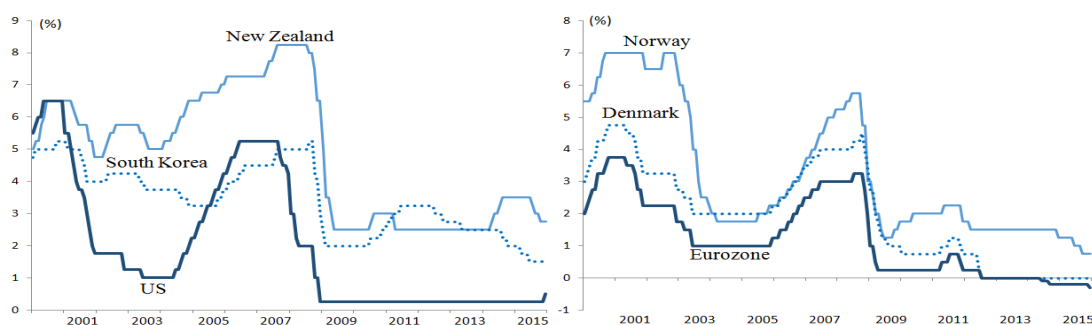
Figure 1.1: Central Bank Policy Rates during 2000-2015

are considerably affected by the policies of the large economies. This can possibly lead to asymmetric policy relationships, where only the large economies' monetary policies affect other economies' policies. In many countries, the policy rate adjustments have been driven by the policy changes of major central banks, such as the US Federal Reserve (Fed) and the European Central Bank (ECB)[1]. Takáts and Vela (2014) also find that the US monetary policy causes emerging economies' policies to deviate from what domestic factors suggest.

This chapter provides (i) a theoretical framework that explains the asymmetric policy relationships and (ii) empirical evidence supporting those relationships. In the theoretical analysis, an open economy model based on Clarida, Galí and Gertler (CGG, 2002) is presented. There are two countries: home and foreign.

The model analysis demonstrates that the home economy's welfare is a function of international variables, such as the foreign output gap and the terms of trade. Therefore, these external variables affect optimal inflation and the optimal output gap in the home economy. Moreover, the home policy rate is influenced by the foreign policy rate, and the foreign policy effects become more substantial as home openness is greater. Therefore, the policy rate relationship between two countries

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[1]In the early 2000s, many central banks lowered interest rates right after the rapid monetary expansion of the US Fed. There were similar patterns in the mid 2000s. Also, in response to the policy rate cut of the US in 2007:Q3 the UK and Canada lowered policy rates immediately. In 2008:Q4, many other central banks began expansionary policies.

can be asymmetric when the degrees of openness are significantly different from each other.

This contradicts CGG (2002), which argue that the central bank's policy problem in an open economy is isomorphic to the one in a closed economy. In CGG (2002), the foreign output gap is not regarded as a variable while approximating home welfare. However, in this two-country open economy model, the foreign output gap is a variable that affects the home economy. Thus, regardless of exogeneity of foreign variables, the foreign output gap should be considered in the home welfare function. Taking the foreign output gap as a variable, in this chapter, the home welfare is also affected by the foreign variable.

The result is in line with earlier New Keynesian studies on optimal open economy monetary policy. In the model, the intertemporal elasticity of substitution in consumption is not unity, while the intratemporal elasticity of substitution between home and foreign goods is unity. This is related to the non-negligible role of international variables in the home welfare function. Benigno and Benigno (2006) indicate that, the terms of trade effect on welfare disappears only when the multiplication of the intratemporal and intertemporal elasticity of substitution becomes unity. Gali and Monacelli (2005) also argue that stabilizing domestic variables is optimal when the intratemporal and intertemporal elasticity of substitution is both unity. According to Corsetti and Pensenti (2001), international variables' effects are determined by the relative size of the intratemporal and intertemporal elasticity of substitution. In De Paoli (2009), exchange rate targeting outperforms domestic variables targeting for a sufficiently large elasticity of intratemporal substitution.

The asymmetric degrees of bilateral openness can be observed between some large economies and other economies. Table 1.1 reports some selected countries' trade shares (%) with the US and the Eurozone (EZ)[2]. Many countries have strong

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[2]There are 19 countries in the Eurozone: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, the Netherlands, Portugal,

Table 1.1: Trade Shares (%) with the US and the Eurozone (EZ)

Home Country	for US	US for Home	Home Country	for EZ	EZ for Home
Australia	1.2	7.5	Chile	0.4	12.1
Canada	20.9	65.3	Czech	4.3	62.6
Chile	0.8	16.0	Malaysia	0.7	8.2
Indonesia	0.9	7.0	Norway	2.0	42.9
Malaysia	1.4	8.1	Sweden	3.3	45.6
South Korea	3.6	10.6	Switzerland	5.3	54.2
Switzerland	1.7	10.0	Thailand	0.6	6.8
Thailand	1.2	8.5	UK	11.9	46.7

Data: IMF eLibrary (Direction of Trade by Country, 2014)

trade links with these large economies. However, for the US or the Eurozone the trade share with an individual country is relatively small. This can be a plausible reason for asymmetric policy rate relationships between those economies.

Following the theoretical approach, the empirical analysis investigates the asymmetric relationships between policy rates. In this section, the policy relationships between two large economies (the US and the Eurozone) and 14 other economies are verified. Those 'other economies' are as follow: Australia, Canada, Chile, Czech, Denmark, Malaysia, Norway, India, Indonesia, South Africa, South Korea, Sweden, Thailand and UK. There are two methodologies in the empirical section. First, given that policy rates are not continuous and not frequently adjusted, ordered probit models are used, as in Bergin and Jordà (2004) and Scotti (2011). After this, the results of structural VAR (Vector Auto-regression) analyses are presented following the related literature, such as Kim (2001), and Sousa and Zaghini (2008).

In the probit model analysis, 12 out of 14 countries have leader and follower relationships with at least one of the US and the Eurozone. Four of these countries have asymmetric relationships with both large economies. The VAR analyses indicate that 11 countries have one-way relationships with the US or/and the Eurozone with respect to the short-term interest rates. Combining the results of these two

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Slovakia, Slovenia, and Spain.

kinds of tests, nine economies have asymmetric relationships with at least one of the large economies in both the probit and the VAR models analyses.

The empirical study in this chapter has novelty in some features. First, it analyses both home and foreign models investigating the policy relationships. Many previous empirical studies consider only one-way models, such as 'the US $\rightarrow$ other economies' (Bergin and Jordà, 2004; Kim, 2001). Secondly, the identification procedure of the structural VAR analysis is based on the theoretical framework in the present chapter. This kind of model-based approach is similar to Giordani (2004) and Sims and Zha (2006). Finally, the sample period reflects the recent changes in global circumstances - such as the start-up of the ECB (1999) and inflation targeting of many central banks[3]. Moreover, the data include the global financial crisis in the late 2000s, when many central banks abandoned the inward looking policy rule[4].

This chapter continues as follows. The relevant literature is illustrated in Section 1.2. A simple two-country model and the welfare analysis are presented in Section 1.3. Section 1.4 describes the empirical investigation of the asymmetric relationships between central banks. Section 1.5 makes conclusion.

1.2 Literature Review

There are plenty of studies that discuss the optimal monetary policy in an open economy. One of the earliest papers is Obstfeld and Rogoff (1995a), which indicate that targeting the real exchange rate is as problematic as a fixed nominal exchange rate regime. There also exist remarkable studies denying the benefits of stabilizing international variables. Some papers compare the welfare loss of each monetary pol-

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[3]Inflation targeting was firstly adopted by New Zealand in 1989. During the first half of 1990s five more countries, and during 1997-2002, 15 more countries adopted inflation targeting.

[4]Reserve Bank of New Zealand cut its policy rate from 8% to 5% during 2008:Q3-Q4 even though CPI inflation rose from 1.8% to 5.1% during Jul 2007-Sep 2008. Bank of Korea maintained its policy rate during Aug 2007-Jul 2008 while CPI inflation increased from 2.0% to 5.9%.

icy rule and conclude that domestic inflation targeting is optimal (Batini, Harrison and Millard, 2003; Galí and Monacelli, 2005; Kollmann, 2002). Clarida, Galí and Gertler (2002) also indicate that the form of the optimal open economy policy rule is identical to that of the optimal closed economy rule.

By contrast, De Paoli (2001) argues that the monetary policy needs to consider international variables, since the terms of trade directly affect domestic welfare. Ball (1998) explains an exchange rate channel of transmission of the monetary policy shock. Corsetti and Pesenti (2001, 2005) also emphasize the effect of the international relative price - influencing consumers' purchasing power. As discussed in Edwards (2006) and Pappa (2004), the reality is that most central banks consider currency value fluctuations while making policy decisions. Mohanty and Klau (2004) also find that real exchange rates perform significant roles in 11 out of 13 countries while central banks make policy decisions.

A stream of empirical literature investigates the monetary policy relationships between countries. Maćkowiak (2006) and Takáts and Vela (2014) find that the US monetary policy has significant effects on the policies of emerging economies. These results are similar to Gray (2013) which indicates that the US short-term rate plays a significant role in other countries' policy functions. Bergin and Jordà (2004) show European countries' responses to the US and German monetary policies before 1998. Moreover, Clarida, Galí and Gertler (1998) highlight the influence of German policy on the policies of the UK, France and Italy. In a recent study, Kucharčuková, Claeys and Vašíček (2014) identify the immediate policy changes of non-EU European countries following the ECB's policy decisions. However, Kim (2001) finds that the US monetary policy has no transmission effect on the policies of non-US G6 countries.

There are studies which focus on the relationship between the US Fed and the ECB. Some of them find a one-way relationship, indicating that the ECB has followed the policy of the US Fed (Ullrich, 2003; Belke and Gros, 2005; Belke and Cui,

2009). Taylor (2007) also estimates a significant coefficient of the US federal funds rate in the ECB's policy rate function. On the other hand, Scotti (2011) denies the leader-follower relationship between them.

There is a substantial body of literature which attempts to identify the plausible reasons for the asymmetric monetary policy relationships between countries. According to Taylor (2013) and He and McCauley (2013), central banks tend to cut policy rates to avert their currencies' appreciations when another central bank lowers its policy rate. Hofmann and Bogdanova (2012) and Rey (2015) emphasize the central bank's concern about the fluctuations of cross-border capital flows. Takáts and Vela (2014) suggest that stronger inflation targeting makes the policies of emerging economies more linked to the US policy.

Investigating the cross-border transmission channel of monetary shocks, various kinds of empirical methods are used. First, structural VAR models are adopted in many studies (Kim, 2001; Sousa and Zaghini, 2008; Kucharčuková, Claeys and Vašíček, 2014). In order to address the variables' instability problem, Judd and Rudebusch (1998) and Ulrich (2003) use VECM (vector error correction model) to examine Taylor-type policy rules. However, some papers such as Bergin and Jordà (2004) argue that VAR and VECM methodologies are not proper for the policy analysis since policy rates have unusual statistical properties: (i) a low frequency of variations and (ii) discrete amounts of changes. Therefore, some researches adopt ordered probit model to verify the interdependent relationships between monetary policies (Bergin and Jordà, 2004; Scotti, 2011).

1.3 Optimal Monetary Policy in Open Economies

1.3.1 Overview

In this section, a theoretical framework supporting the asymmetric policy rate relationships between countries is presented; a simple two-country model is established based on the New Keynesian framework of Clarida, Galí and Gertler (CGG, 2002). The model consists of two countries: home and foreign. Deriving the welfare function and the optimal monetary policy rule of each country, the reason for the leader and follower relationship is verified.

There are households, and final and intermediate goods producers in the home and the foreign economies. In each country, the central bank maximizes the sum of domestic households' welfare, choosing the optimal output gap and the inflation gap. The policy rate is determined endogenously. The mass of the home households is $1 - \alpha$ and that of the foreign households is α. The parameter α also denotes the shares of foreign produced goods in aggregate consumption in the home and the foreign countries; α represents the degree of openness of the home economy and $1 - \alpha$ denotes foreign openness. Assuming the law of one price (LOOP) and identical consumption baskets in both economies, the purchasing power parity holds.

1.3.2 Model Description

Households

There are representative households in the home and the foreign economies. Each household consumes a basket of home and foreign produced goods. Assuming that the combinations of home and foreign goods in the home and the foreign countries are identical, aggregate consumption per capita in the home (C_t) and the foreign

(C_t^*) economies are illustrated as follow:

$$C_t = C_{H,t}^{1-\alpha}C_{F,t}^{\alpha} \qquad\qquad C_t^* = C_{H,t}^{*1-\alpha}C_{F,t}^{*\alpha} \qquad\qquad (1.3.1)$$

where α represents the weight of the foreign goods in each consumption basket, which means the degree of openness of the home economy. $C_{H,t}$ and $C_{F,t}$ denote the home and the foreign goods in the home consumption basket, respectively. Likewise, $C_{H,t}^*$ and $C_{F,t}^*$ are the home and the foreign goods in foreign aggregate consumption. These variables are all in per capita terms. The aggregate prices in the home (P_t) and the foreign economies (P_t^*) are then

$$P_t = m^{-1}P_{H,t}^{1-\alpha}P_{F,t}^{\alpha} \qquad\qquad P_t^* = m^{-1}P_{H,t}^{*1-\alpha}P_{F,t}^{*\alpha} \qquad\qquad (1.3.2)$$

where $m = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. $P_{H,t}$ and $P_{F,t}$ denote the aggregate home currency prices of the home and the foreign goods in the home economy, respectively. Also, $P_{H,t}^*$ and $P_{F,t}^*$ represent the foreign currency prices of those goods in the foreign economy.

Defining $S_t = P_{F,t}/P_{H,t}$ as the terms of trade, the aggregate prices can be rewritten as follow:

$$P_t = m^{-1}P_{H,t}S_t^{\alpha} \qquad\qquad P_t^* = m^{-1}P_{F,t}^*S_t^{\alpha-1}. \qquad\qquad (1.3.3)$$

The optimal consumption allocations in the home and the foreign economies suggest

$$P_{H,t}C_{H,t} = (1-\alpha)P_tC_t \qquad\qquad P_{F,t}C_{F,t} = \alpha P_tC_t \qquad\qquad (1.3.4)$$

$$P_{H,t}^*C_{H,t}^* = (1-\alpha)P_t^*C_t^* \qquad\qquad P_{F,t}^*C_{F,t}^* = \alpha P_t^*C_t^*. \qquad\qquad (1.3.5)$$

Between the markets, the law of one price holds which means $P_{H,t} = \mathcal{E}_tP_{H,t}^*$ and $P_{F,t} = \mathcal{E}_tP_{F,t}^*$, where $\mathcal{E}_t$ denotes the nominal exchange rate.

Defining N_t as the representative home household's labour supply to the produc-

tion sector, the household in the home economy maximizes

$$\sum_{t=\tau}^{\infty} \beta^{t-\tau} E_\tau \left[U(C_t) - V(N_t) \right] \tag{1.3.6}$$

where

$$U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \qquad V(N_t) = \frac{N_t^{1+\phi}}{1+\phi}. \tag{1.3.7}$$

The flow budget constraint is given by

$$P_t C_t + E_t \left(Q_{t,t+1} B_{t+1} \right) = W_t N_t + B_t + D_t \tag{1.3.8}$$

where B_{t+1} denotes risk-free bonds purchased at time t. $Q_{t,t+1}$ is the nominal stochastic discount factor between t and $t+1$, and $E_t \left(Q_{t,t+1} \right) = R_t^{-1}$ where R_t is a risk-free gross nominal interest rate. W_t is the nominal wage and D_t is the dividends from home intermediate goods producers. Also, no-Ponzi scheme is assumed, which means $\lim_{t\to\infty} \prod_{s=1}^{t} R_s^{-1} B_{t+1} = 0$. The first-order conditions imply

$$\beta R_t E_t \left\{ (C_{t+1}/C_t)^{-\sigma} (P_t/P_{t+1}) \right\} = 1 \qquad \text{and} \qquad W_t/P_t = N_t^\phi C_t^\sigma, \tag{1.3.9}$$

and from the equation (1.3.3),

$$\beta R_t E_t \left\{ (C_{t+1}/C_t)^{-\sigma} (P_{H,t}/P_{H,t+1}) (S_t/S_{t+1})^\alpha \right\} = 1. \tag{1.3.10}$$

Define C_t^* and R_t^* as foreign consumption and the risk-free interest rate, respectively. Assuming symmetric households preference in the foreign economy[5],

$$\beta R_t^* E_t \left\{ (C_{t+1}^*/C_t^*)^{-\sigma} (P_{F,t}^*/P_{F,t+1}^*) (S_t/S_{t+1})^{\alpha-1} \right\} = 1. \tag{1.3.11}$$

---

[5]With symmetric utility maximization of foreign households,

$$\beta R_t^* E_t \left\{ (C_{t+1}^*/C_t^*)^{-\sigma} (P_t^*/P_{t+1}^*) \right\} = 1 \qquad \text{and} \qquad W_t^*/P_t^* = N_t^{*\phi} C_t^{*\sigma}.$$

Since state-contingent securities can be traded internationally, the home and the foreign $(Q^*_{t,t+1})$ stochastic discount factors have a relationship as follows: $Q^*_{t,t+1} = \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}Q_{t,t+1}$. With a suitable normalization of initial conditions, then $C_t = C^*_t$ every period.

Intermediate Goods Producers

The mass of home intermediate goods producers is unity. Each home intermediate good firm i produces a differentiated good $(1-\alpha)Y_t(i)$ with labour inputs $(1-\alpha)N^G_t(i)$. $Y_t(i)$ and $N^G_t(i)$ are in per capita terms, given the home households' mass $(1-\alpha)^6$. The production function is illustrated as:

$$Y_t(i) = A_t N^G_t(i). \tag{1.3.12}$$

where A_t denotes productivity.

Each home intermediate good producer receives a subsidy $\tau \in (0,1)$ of the wage bills. Thus, the real marginal cost of home intermediate goods production can be illustrated as:

$$MC_t = \frac{(1-\tau)(W_t/P_{H,t})}{A_t} = \frac{(1-\tau)(W_t/P_t)S^\alpha_t}{mA_t} \tag{1.3.13}$$

where MC_t is in units of home produced goods. From equation (1.3.9), the equation (1.3.13) can be rewritten by:

$$MC_t = (1-\tau)m^{-1}A_t^{-1}N_t^\phi C_t^\sigma S_t^\alpha. \tag{1.3.14}$$

Symmetrically, in the foreign economy

$$MC^*_t = \frac{(1-\tau^*)(W^*_t/P^*_{F,t})}{A^*_t} = \frac{(1-\tau^*)(W^*_t/P^*_t)S^{\alpha-1}_t}{mA^*_t} \tag{1.3.15}$$

---

[6]Since the individual home household's labour supply is identical (N_t) across households, labour market clearing suggests: $(1-\alpha)N_t = \int_0^1(1-\alpha)N^G_t(i)di$.

which can be rewritten by:

$$MC_t^* = (1 - \tau^*)m^{-1}A_t^{*-1}N_t^{*\phi}C_t^{*\sigma}S_t^{\alpha-1} \tag{1.3.16}$$

where MC_t^* and N_t^* denote the foreign marginal cost and the labour supply, respectively. The parameters ϕ and σ are common in both economies.

Following Calvo (1983), the home intermediate good producer can reset its price each period with a probability $1 - \theta$; with a probability θ it maintains the previous price. If the intermediate good producer i can reset the prices at t, it chooses the optimal price $P_{H,t}^o$ that maximizes

$$\sum_{\tau=0}^{\infty}\theta^\tau E_t\left\{Q_{t,t+\tau}Y_{t+\tau}(i)\left[P_{H,t}^o - P_{H,t+\tau}MC_{t+\tau}\right]\right\} \tag{1.3.17}$$

subject to the demand function which is presented later. $Q_{t,t+\tau}$ is the stochastic discount factor for the nominal cash flows at time $t + \tau$. Since all the producers are owned by the home households, $Q_{t,t+\tau} = \beta^\tau(C_{t+\tau}/C_t)^{-\sigma}(P_t/P_{t+\tau})$. The solution of the optimization problem is then as follows:

$$\sum_{\tau=0}^{\infty}\theta^\tau E_t\left\{Q_{t,t+\tau}Y_{t+\tau}(i)\left[P_{H,t}^o - (1 + \mu^P)P_{H,t+\tau}MC_{t+\tau}\right]\right\} = 0 \tag{1.3.18}$$

where $\mu^P(= 1/(\varepsilon - 1))$ represents the mark-up in the intermediate goods market, and ε denotes the elasticity of substitution between individual intermediate goods. By the law of large numbers, then the home price index of the home intermediate goods is given by

$$P_{H,t} = \left[\theta P_{H,t-1}^{1-\varepsilon} + (1 - \theta)(P_{H,t}^o)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \tag{1.3.19}$$

Price stickiness is symmetric in the foreign economy.

Final Good Producer

In the home economy, there exists a final good producer which produces the final good $(1-\alpha)Y_t$ following the technology as below:

$$Y_t = \left(\int_0^1 Y_t(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)} \tag{1.3.20}$$

where Y_t is aggregate home output per capita. Given the intermediate goods prices, cost minimization subject to the equation (1.3.20) yields the following demand function for the intermediate good i:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t \tag{1.3.21}$$

which is used in deriving the equation (1.3.18). The aggregate home price of the home intermediate goods $(P_{H,t})$ is

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\varepsilon} di \right)^{1/(1-\varepsilon)}. \tag{1.3.22}$$

In the foreign economy the final goods production and the demand functions are symmetric.

Equilibrium

The goods market clearing conditions in the home and the foreign economies are given by

$$(1-\alpha)Y_t = (1-\alpha)C_{H,t} + \alpha C_{H,t}^* \tag{1.3.23}$$

$$\alpha Y_t^* = (1-\alpha)C_{F,t} + \alpha C_{F,t}^*. \tag{1.3.24}$$

Y_t^* denotes per capita final goods production in the foreign economy, where the households' mass is α. Given the purchasing power parity and $C_t = C_t^*$, the equa-

tions (1.3.4) and (1.3.5) imply the zero trade balance in each country[7], which means

$$P_{H,t}Y_t = P_t C_t \qquad\qquad P_{F,t}^* Y_t^* = P_t^* C_t^*. \tag{1.3.25}$$

Combining the equations (1.3.3) and (1.3.25) yields

$$Y_t = m^{-1} C_t S_t^\alpha \qquad\qquad Y_t^* = m^{-1} C_t^* S_t^{\alpha-1} \tag{1.3.26}$$

and these lead to

$$S_t = Y_t / Y_t^*. \tag{1.3.27}$$

Also, from the equations (1.3.26) and (1.3.27), home and foreign consumption can be expressed as a function of home and foreign output as follows:

$$C_t = C_t^* = m Y_t^{1-\alpha} Y_t^{*\alpha}. \tag{1.3.28}$$

The home labour market clearing condition suggests

$$N_t = \int_0^1 N_t^G(i)di = \frac{Y_t}{A_t} \int_0^1 \frac{Y_t(i)}{Y_t} di = \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} di, \tag{1.3.29}$$

and defining $V_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} di$, the equation (1.3.29) is rewritten by

$$N_t = A_t^{-1} V_t Y_t. \tag{1.3.30}$$

Also, combining the equations (1.3.14), (1.3.27), (1.3.28) and (1.3.30) yields

$$MC_t = (1-\tau) m^{\sigma-1} A_t^{-(1+\phi)} Y_t^\kappa Y_t^{*\eta} V_t^\phi \tag{1.3.31}$$

---

[7]The equation (1.3.23) implies $(1-\alpha)P_{H,t}Y_t = (1-\alpha)P_{H,t}C_{H,t} + \alpha P_{H,t}C_{H,t}^*$. From (1.3.5) and $P_{H,t} = \mathcal{E}P_{H,t}^*$, this can be rewritten by $(1-\alpha)P_{H,t}Y_t = (1-\alpha)^2 P_t C_t + (1-\alpha)\alpha \mathcal{E}_t P_t^* C_t^*$. The purchasing power parity $(P_t = \mathcal{E}_t P_t^*)$ then suggests $P_{H,t}Y_t = P_t C_t$.

where $\kappa = (1 + \phi) + (\sigma - 1)(1 - \alpha)$ and $\eta = \alpha(\sigma - 1)$.

Assuming σ is larger than unity[8], as the home economy becomes more open (higher α) the effect of foreign output (η) on the home marginal cost increases, while that of home output (κ) becomes weaker. Similarly, from the equation (1.3.16) in the foreign economy

$$MC_t^* = (1 - \tau^*)m^{\sigma-1}A_t^{*-(1+\phi)}Y_t^{*\kappa^*}Y_t^{\eta^*}V_t^{*\phi} \tag{1.3.32}$$

where $\kappa^* = (1 + \phi) + (\sigma - 1)\alpha$ and $\eta^* = (1 - \alpha)(\sigma - 1)$.

When the price is flexible for the home intermediate goods ($P_{H,t}^o/P_{H,t} = 1$), the home real marginal cost is constant ($MC_t = MC_e$) and $MC_e = 1/(1+\mu^P)$. Flexible price output can be expressed as $Y_{e,t} = A_t N_{e,t}$ where $N_{e,t}$ denotes the flexible price labour input. From the equation (1.3.31) and $V_e = 1$, then,

$$Y_{e,t} = \left[\frac{m^{1-\sigma}A_t^{1+\phi}Y_t^{*-\eta}}{(1-\tau)(1+\mu^P)}\right]^{1/\kappa}. \tag{1.3.33}$$

Linearized Model

The equations of the home economy are linearized around the steady state. Define the log deviation form $\hat{X}_t = lnX_t - ln\bar{X}$, where $\bar{X}$ denotes the steady state value of X_t. The equations (1.3.10), (1.3.26) and (1.3.27) are then linearized as[9]

$$\hat{C}_t = E_t(\hat{C}_{t+1}) - \frac{1}{\sigma}\left[\hat{R}_t - E_t(\hat{\Pi}_{H,t+1}) - \alpha E_t(\Delta\hat{S}_{t+1})\right] \tag{1.3.34}$$

$$\hat{Y}_t = \hat{C}_t + \alpha\hat{S}_t \tag{1.3.35}$$

$$\hat{S}_t = \hat{Y}_t - \hat{Y}_t^* \tag{1.3.36}$$

---

[8]Most RBC literature assumes that the inverse of the intertemporal elasticity of substitution (σ) is larger than unity (Smets and Wouters, 2003).

[9]In the foreign economy, $\hat{C}_t^* = E_t\left(\hat{C}_{t+1}^*\right) - \sigma^{-1}[\hat{R}_t^* - E_t(\hat{\Pi}_{F,t+1}^*) + (1 - \alpha)E_t(\Delta\hat{S}_{t+1})]$ from the equation (1.3.11), and from (1.3.26) $\hat{C}_t^* = \hat{Y}_t^* + (1 - \alpha)\hat{S}_t$.

where $\Pi_{H,t+1} = P_{H,t+1}/P_{H,t}$.

Combining the optimal home price setting equations (1.3.18) and (1.3.19) yields

$$\hat{\Pi}_{H,t} = \delta \widehat{MC}_t + \beta E_t(\hat{\Pi}_{H,t+1}) \tag{1.3.37}$$

where $\delta = [(1-\theta)(1-\beta\theta)]/\theta$. From the equation (1.3.33), the log-deviation of the flexible price home output from the steady state can be represented by

$$\hat{Y}_{e,t} = \kappa^{-1}[(1+\phi)\hat{A}_t - \eta \hat{Y}_t^*]. \tag{1.3.38}$$

Defining $\tilde{Y}_t = \hat{Y}_t - \hat{Y}_{e,t}$ as the home output gap, from the equations (1.3.31) and (1.3.38) the following equation holds[10]:

$$\widehat{MC}_t = \kappa \left(\hat{Y}_t - \hat{Y}_{e,t} \right) = \kappa \tilde{Y}_t. \tag{1.3.39}$$

Also, combining the equations (1.3.34) and (1.3.35), and using $\hat{Y}_{e,t}$ formula,

$$\tilde{Y}_t = E_t(\tilde{Y}_{t+1}) - \sigma_0^{-1} \left[\hat{R}_t - E_t(\hat{\Pi}_{H,t+1}) - \hat{r}_{N,t} \right] \tag{1.3.40}$$

where $\sigma_0 = \sigma - \eta$ and $r_{N,t}$ represents the home natural real interest rate, where

$$\hat{r}_{N,t} = \sigma_0 E_t(\Delta \hat{Y}_{e,t+1}) + \eta E_t(\Delta \hat{Y}_{t+1}^*). \tag{1.3.41}$$

Combining (1.3.37) and (1.3.39) yields the following Phillips-type equation:

$$\hat{\Pi}_{H,t} = \lambda \tilde{Y}_t + \beta E_t(\hat{\Pi}_{H,t+1}) \qquad \text{where} \qquad \lambda = \delta\kappa. \tag{1.3.42}$$

---

[10]The equation (1.3.31) is linearized as $\widehat{MC}_t = -\left[(1+\phi)\hat{A}_t - \eta \hat{Y}_t^*\right] + \kappa \hat{Y}_t + \phi \hat{V}_t$ where $\hat{V}_t = 0$.

1.3.3 Open Economy Welfare and Optimal Policy

This section firstly clarifies the welfare functions of the home and the foreign economies. The output gap, the inflation gap, and the policy rate that maximize domestic welfare in both countries are then derived. Eventually, the reason for the asymmetric policy rate relationship is verified.

Open Economy Welfare Function

As in the non-cooperative equilibrium of Clarida, Galí and Gertler (CGG, 2002), each central bank maximizes domestic welfare. The steady state equilibrium of the home economy is illustrated first. Assuming that the foreign economy is at its steady state, the optimality condition of the home economy is illustrated by

$$V'(\bar{N})\bar{N} = (1-\alpha)U'(\bar{C})\bar{C}, \tag{1.3.43}$$

which is the result of maximizing $U(\bar{C}) - V(\bar{N})$ subject to $\bar{C} = m\bar{Y}^{1-\alpha}\bar{Y}^{*\alpha}$ and $\bar{Y} = \bar{N}$. Also, from the equation (1.3.14), at the steady state

$$\overline{MC} = \frac{1}{1+\mu^P} = (1-\tau)\bar{N}^\phi\bar{C}^\sigma m^{-1}\bar{S}^\alpha = (1-\tau)\frac{V'(\bar{N})}{U'(\bar{C})}m^{-1}\bar{S}^\alpha. \tag{1.3.44}$$

From the equations (1.3.26) and (1.3.43), the optimal subsidy (τ) satisfies[11]

$$(1-\tau)(1+\mu^P)(1-\alpha) = 1. \tag{1.3.45}$$

Using a Taylor series expansion of the variable X_t, the second-order approximation of the deviation from the flexible price equilibrium $(X_{e,t})$ can be represented by:

---

[11]Optimality in (1.3.43) implies $V'(\bar{N})/U'(\bar{C}) = (1-\alpha)\bar{C}/\bar{N}$, which implies $\left(1+\mu^P\right)^{-1} = (1-\tau)(1-\alpha)\bar{C}\bar{N}^{-1}m^{-1}\bar{S}^\alpha$. From (1.3.26), at the steady state $\bar{C}\bar{Y}^{-1}m^{-1}\bar{S}^\alpha = 1$, and $\bar{Y} = \bar{N}$.

$$\frac{X_t - X_{e,t}}{X_{e,t}} = \tilde{X}_t + \frac{1}{2}\tilde{X}_t^2 + o\left(\|a\|^3\right) \tag{1.3.46}$$

where $o\left(\|a\|^n\right)$ denotes the terms that are of order higher than $(n-1)$th and $\tilde{X}_t = \hat{X}_t - \hat{X}_{e,t}$. In this model, (i) $\hat{X}_t = lnX_t - ln\bar{X}$, (ii) $\hat{X}_{e,t} = lnX_{e,t} - ln\bar{X}$, and (iii) $\tilde{X}_t = lnX_t - lnX_{e,t}$.

Home welfare is represented by the sum of the home households' utilities given by the equation (1.3.6). Contrary to CGG (2002) which take foreign output (Y_t^*) parametrically, in this approximation Y_t^* is a variable that responds to the changes in other economic variables. Using a second-order approximation around the flexible price equilibrium, the home economy's welfare function can be derived[12] as

$$W_\tau^H = -(1-\alpha)\frac{\Lambda}{2}\sum_{t=\tau}^{\infty}\beta^{t-\tau}E_\tau\left[\hat{\Pi}_{H,t}^2 + \psi\tilde{Y}_t^2 + \frac{\psi}{\kappa}\Gamma\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}\right)\right] \tag{1.3.47}$$

where

$$\Gamma\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}\right) = \left(\frac{\alpha}{1-\alpha}\right)\tilde{Y}_t^*\left\{(\sigma-1)\left[\alpha\tilde{Y}_t^* + 2(1-\alpha)\tilde{Y}_t + 2\hat{C}_{e,t}\right] - 2\right\}, \tag{1.3.48}$$

$\psi = \delta\kappa/\varepsilon$ and $\Lambda = \varepsilon/\delta$. $\hat{C}_{e,t}$ represents the log deviation of flexible price consumption from the steady state level $(\hat{C}_{e,t} = lnC_{e,t} - ln\bar{C})$. Using $\tilde{S}_t = \tilde{Y}_t - \tilde{Y}_t^*$ from the equation (1.3.27), the home welfare function (1.3.47) can be rewritten by

$$W_\tau^H = -(1-\alpha)\frac{\Lambda}{2}\sum_{t=\tau}^{\infty}\beta^{t-\tau}E_\tau\left[\hat{\Pi}_{H,t}^2 + \psi\tilde{Y}_t^2 + \frac{\psi}{\kappa}\Gamma\left(\tilde{Y}_t, \tilde{S}_t, \hat{C}_{e,t}\right)\right] \tag{1.3.49}$$

where

$$\Gamma\left(\tilde{Y}_t, \tilde{S}_t, \hat{C}_{e,t}\right) = \left(\frac{\alpha}{1-\alpha}\right)\tilde{Y}_t^*\left\{(\sigma-1)\left[(2-\alpha)\tilde{Y}_t - \alpha\tilde{S}_t + 2\hat{C}_{e,t}\right] - 2\right\}. \tag{1.3.50}$$

The result of the approximation suggests that home welfare is a function of not only home variables but also international variables such as the foreign output

---

[12]The derivations of the equations (1.3.47) and (1.3.51) are illustrated in the appendix 1.A.1.

gap and the terms of trade. This contradicts the argument of CGG (2002) which indicate that, in a non-cooperative equilibrium, the central bank's policy problem is isomorphic to the one in a closed economy. Therefore, the optimal home policy needs to consider the effect of the foreign variable changes on home welfare.

It is noteworthy that the effect of the international variables on the home economy's welfare is greater as home openness is higher. As the parameter α increases, the coefficient $\frac{\alpha}{1-\alpha}$ becomes larger. The consumer's risk aversion parameter σ is assumed to be larger than unity.

The welfare function of the foreign economy can be represented by

$$W_\tau^F = -\alpha \frac{\Lambda}{2} \sum_{t=\tau}^\infty \beta^{t-\tau} E_\tau \left[\hat{\Pi}_{F,t}^{*2} + \psi^* \tilde{Y}_t^{*2} + \frac{\psi^*}{\kappa^*} \Gamma^* \left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}^* \right) \right] \qquad (1.3.51)$$

where

$$\Gamma^* \left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}^* \right) = \left(\frac{1-\alpha}{\alpha} \right) \tilde{Y}_t \left\{ (\sigma - 1) \left[(1-\alpha)\tilde{Y}_t + 2\alpha\tilde{Y}_t^* + 2\hat{C}_{e,t}^* \right] - 2 \right\}, \qquad (1.3.52)$$

with $\kappa^* = (1+\phi) + (\sigma - 1)\alpha$ and $\psi^* = \delta\kappa^*/\varepsilon$.

Open Economy Optimal Policy

The home central bank determines the optimal output gap and the inflation gap, maximizing domestic welfare subject to the Phillips-type equation (1.3.42). In the optimization problem, the expected values of the variables are taken as given. The first-order condition is then derived as follows[13]:

$$\lambda \hat{\Pi}_{H,t}^o + \psi \tilde{Y}_t^o + \frac{\psi}{\kappa} \alpha(\sigma - 1) \tilde{Y}_t^* = 0, \qquad (1.3.53)$$

---

[13]$\lambda = \delta\kappa$ where $\delta = \left[(1-\theta)(1-\beta\theta) \right]/\theta$. Also, $\kappa = (1+\phi) + (\sigma - 1)(1-\alpha)$ and $\psi = \delta\kappa/\varepsilon$.

which can be rewritten as follows:

$$\hat{\Pi}^o_{H,t} + \frac{1}{\varepsilon}\tilde{Y}^o_t + \frac{1}{\varepsilon\kappa}\alpha(\sigma-1)\tilde{Y}^*_t = 0 \qquad (1.3.54)$$

where $\hat{\Pi}^o_{H,t}$ and $\tilde{Y}^o_t$ denote the optimal inflation gap and the output gap, respectively. Since $\kappa = (1+\phi) + (\sigma-1)(1-\alpha) > 0$ assuming $\sigma > 1$, the coefficient of the foreign output gap is increasing in home openness (α). As α is higher, the home central bank should allow greater fluctuations of home inflation and output in response to a change in the foreign output gap. The optimal foreign policy rule is as follows:

$$\lambda^*\hat{\Pi}^{*o}_{F,t} + \psi^*\tilde{Y}^{*o}_t + \frac{\psi^*}{\kappa^*}(1-\alpha)(\sigma-1)\tilde{Y}_t = 0 \qquad (1.3.55)$$

which can be rewritten by

$$\hat{\Pi}^{*o}_{F,t} + \frac{1}{\varepsilon}\tilde{Y}^{*o}_t + \frac{1}{\varepsilon\kappa^*}(1-\alpha)(\sigma-1)\tilde{Y}_t = 0 \qquad (1.3.56)$$

where $\lambda^* = \delta\kappa^*$. Given $\kappa^* = (1+\phi) + (\sigma-1)\alpha$, the coefficient of the home output gap is increasing in foreign openness $(1-\alpha)$.

From (1.3.42) and (1.3.54), the optimal home inflation dynamics is as below:

$$\hat{\Pi}^o_{H,t} = \beta\left(\frac{1}{1+\lambda\varepsilon}\right)E_t\left(\hat{\Pi}_{H,t+1}\right) - \nu\left(\frac{1}{1+\lambda\varepsilon}\right)\tilde{Y}^*_t, \qquad (1.3.57)$$

where $\nu = \alpha\delta(\sigma-1)$. Solving forward yields the following reduced form solutions for the optimal inflation gap and the output gap in terms of the foreign output gap:

$$\hat{\Pi}^o_{H,t} = -\nu\left(\frac{1}{1+\lambda\varepsilon}\right)\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left(\frac{1}{1+\lambda\varepsilon}\right)^{\tau-t}E_t(\tilde{Y}^*_\tau) \qquad (1.3.58)$$

$$\tilde{Y}^o_t = -\varepsilon\hat{\Pi}^o_{H,t} - \frac{\alpha}{\kappa}(\sigma-1)\tilde{Y}^*_t \qquad (1.3.59)$$

Since α/κ is increasing in α, the effect of the foreign output gap on the optimal home

output gap is larger as home openness is higher. In the foreign economy, following a similar procedure and from $\nu^* = (1-\alpha)\delta(\sigma-1)$

$$\hat{\Pi}_{F,t}^{*o} = -\nu^* \left(\frac{1}{1+\lambda^*\varepsilon}\right) \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{1}{1+\lambda^*\varepsilon}\right)^{\tau-t} E_t(\tilde{Y}_\tau) \qquad (1.3.60)$$

$$\tilde{Y}_t^{*o} = -\varepsilon\hat{\Pi}_{F,t}^{*o} - \frac{1-\alpha}{\kappa^*}(\sigma-1)\tilde{Y}_t. \qquad (1.3.61)$$

In a closed economy, the welfare function includes only domestic variables. Without any exogenous shock, then the optimal output gap and the inflation gap would be zero. However, in the optimal policy rules represented by the equations (1.3.60) and (1.3.61), the optimal output gap and the inflation gap can be non-zero when international variables deviate from the flexible price levels.

The optimal home policy rule helps the monetary policy in stabilizing international variables. When the foreign output gap is expected to increase, the terms of trade are expected to decline following the equation $(1.3.27)$[14]. According to the policy rule (1.3.58), the home central bank targets a lower level of inflation, which corresponds to a higher output gap in the equation (1.3.59). The higher home output gap has upward pressure on S, and consequently the expected fall in the terms of trade would be mitigated.

Combining the equations (1.3.59) and (1.3.61), the non-cooperative equilibrium of this two-country model can be derived as follow:

$$\tilde{Y}_t^o = -\frac{\varepsilon}{\Omega}\left[\hat{\Pi}_{H,t}^o - \frac{\alpha}{\kappa}(\sigma-1)\hat{\Pi}_{F,t}^{*o}\right] \qquad (1.3.62)$$

$$\tilde{Y}_t^{*o} = -\frac{\varepsilon}{\Omega}\left[\hat{\Pi}_{F,t}^{*o} - \frac{1-\alpha}{\kappa^*}(\sigma-1)\hat{\Pi}_{H,t}^o\right] \qquad (1.3.63)$$

where $\Omega = 1 - \frac{\alpha(1-\alpha)(\sigma-1)^2}{\kappa\kappa^*}$ and $0 < \Omega < 1$[15].

---

[14]The anticipated rise in foreign production lowers the expected foreign goods price - putting downward pressure on $P_F/P_H \ (= S)$.

[15]$\kappa\kappa^* - \alpha(1-\alpha)(\sigma-1)^2 = \left[(1+\phi)^2 + (1+\phi)(\sigma-1) + \alpha(1-\alpha)(\sigma-1)^2\right] - \alpha(1-\alpha)(\sigma-1)^2 =$

Policy Rate Relationship

Without the Taylor-type monetary policy rule, the optimal home policy rate (R) is endogenously determined by the central bank such that both welfare maximization (1.3.54) and the consumer's efficiency condition (1.3.34) are satisfied. Firstly, the flexible price equation (1.3.38) suggests

$$E_t(\Delta \hat{Y}_{e,t+1}) = \kappa^{-1}[(1+\phi)E_t(\Delta \hat{A}_{t+1}) - \eta E_t(\Delta \hat{Y}_{t+1}^*)]. \tag{1.3.64}$$

From the equations (1.3.40) and (1.3.41), the equilibrium interest rate in the home economy can be rewritten by

$$\hat{R}_t = E_t\left(\hat{\Pi}_{H,t+1}\right) + \sigma_0\left[E_t\left(\tilde{Y}_{t+1}\right) - \tilde{Y}_{t+1}\right] + \eta\left(1 - \sigma_0\kappa^{-1}\right)E_t\left(\Delta \hat{Y}_{t+1}^*\right) + \sigma_0 d_t \tag{1.3.65}$$

where $d_t = \kappa^{-1}(1+\phi)[E_t(\hat{A}_{t+1}) - \hat{A}_t]$, $\eta = \alpha(\sigma-1)$ and $\sigma_0 = \sigma - \eta$. Correspondingly in the foreign economy,

$$\hat{R}_t^* = E_t\left(\hat{\Pi}_{H,t+1}^*\right) + \sigma_0^*\left[E_t\left(\tilde{Y}_{t+1}^*\right) - \tilde{Y}_t^*\right] + \eta^*\left(1 - \sigma_0^*\kappa^{*-1}\right)E_t\left(\Delta \hat{Y}_{t+1}\right) + \sigma_0^* d_t^* \tag{1.3.66}$$

where $d_t^* = \kappa^{*-1}(1+\phi)[E_t(\hat{A}_{t+1}^*) - \hat{A}_t^*]$, $\eta^* = (1-\alpha)(\sigma-1)$ and $\sigma_0^* = \sigma - \eta^*$. Also, using the equations (1.3.34) and (1.3.35), and from the corresponding equations of the foreign economy, the home and the foreign consumers' efficiency conditions can be rewritten by

$$E_t\left(\Delta \hat{Y}_{t+1}\right) = \sigma^{-1}\left[\hat{R}_t - E_t\left(\hat{\Pi}_{H,t+1}\right) + \eta E_t\left(\widehat{\Delta S}_{t+1}\right)\right] \tag{1.3.67}$$

$$E_t\left(\Delta \hat{Y}_{t+1}^*\right) = \sigma^{-1}\left[\hat{R}_t^* - E_t\left(\hat{\Pi}_{F,t+1}^*\right) - \eta^* E_t\left(\widehat{\Delta S}_{t+1}\right)\right]. \tag{1.3.68}$$

Define $\hat{R}_t^o$ and $\hat{R}_t^{*o}$ as the optimal policy rates in the home and the foreign

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$(1+\phi)(\sigma+\phi) > 0$

economies, respectively. These optimal rates depend on the optimal output gaps determined by the central banks ($\tilde{Y}_t^o$ and $\tilde{Y}_t^{*o}$). Combining the equations (1.3.65), (1.3.66), (1.3.67) and (1.3.68) then yields

$$
\begin{aligned}
\hat{R}_t^o = E_t(\hat{\Pi}_{H,t+1}) \quad &+ \quad \sigma_0 \left[E_t(\tilde{Y}_{t+1}) - \tilde{Y}_t^o \right] \\
&+ \quad \rho \left[\hat{R}_t^* - E_t(\hat{\Pi}_{t+1}^*) - \eta^* E_t(\widehat{\Delta S}_{t+1}) \right] + \sigma_0 d_t \quad (1.3.69) \\
\hat{R}_t^{*o} = E_t(\hat{\Pi}_{H,t+1}^*) \quad &+ \quad \sigma_0^* \left[E_t(\tilde{Y}_{t+1}^*) - \tilde{Y}_t^{*o} \right] \\
&+ \quad \rho^* \left[\hat{R}_t - E_t(\hat{\Pi}_{t+1}) + \eta E_t(\widehat{\Delta S}_{t+1}) \right] + \sigma_0^* d_t^* \quad (1.3.70)
\end{aligned}
$$

where

$$
\rho = \eta(1 - \sigma_0 \kappa^{-1})\sigma^{-1} \qquad \rho^* = \eta^*(1 - \sigma_0^* \kappa^{*-1})\sigma^{-1}, \qquad (1.3.71)
$$

and alternatively,

$$
\rho = \phi \left(\frac{\sigma - 1}{\sigma} \right) \left[\frac{\alpha}{\alpha + \phi + \sigma(1 - \alpha)} \right] \qquad \rho^* = \phi \left(\frac{\sigma - 1}{\sigma} \right) \left[\frac{1 - \alpha}{1 - \alpha + \phi + \alpha\sigma} \right].
$$
$$
(1.3.72)
$$

The coefficients ρ and ρ^* are all positive assuming $\sigma > 1$, and $0 < \rho < 1$ and $0 < \rho^* < 1$; the home (foreign) optimal interest rate is positively affected by the foreign (home) interest rate. The coefficient ρ is increasing in α, and ρ^* is decreasing in α. This implies that the optimal home (foreign) interest rate is more significantly affected by the foreign (home) rate as the degree of openness is higher.

The analysis in this section suggests that the different level of openness of each country can lead to asymmetric relationships between the optimal interest rates. The asymmetry of openness can be also significant in terms of the bilateral trade relationship - especially between a small and a large economies. For instance, although the US is an important trade partner for an emerging market country, for the US the trade share with the emerging country would be fairly small (Table 1.1). In this case, the leader and follower relationship can be observed between the policy

rates of the US and the emerging economy.

When the degree of home openness is extremely high and that of the foreign economy is close to zero (i.e. $\alpha \to 1$), there would be only a one-way relationship between interest rates; only the home policy rate is affected by the foreign policy. Woodford (2007) explains that, as one country's openness approaches to unity in a two-country model, the simulation results reflect the small open economy case. When $\alpha \to 1$ in this model, the equations in (1.3.72) imply

$$\rho \simeq \frac{\phi}{1+\phi} \frac{\sigma-1}{\sigma} \qquad \text{and} \qquad \rho^* \simeq 0. \tag{1.3.73}$$

Also, as α approaches to unity $\sigma_0^* \to \sigma$ and $\eta^* \to 0$[16]. Ignoring the terms d_t and d_t^*, the optimal policy rates in the home and the foreign economies are illustrated by

$$\hat{R}_t^o \simeq E_t(\hat{\Pi}_{H,t+1}) + \left[E_t(\tilde{Y}_{t+1}) - \tilde{Y}_t^o \right] + \rho_0 \left[\hat{R}_t^* - E_t(\hat{\Pi}_{t+1}^*) \right] \tag{1.3.74}$$

$$\hat{R}_t^{*o} \simeq E_t(\hat{\Pi}_{H,t+1}^*) + \sigma \left[E_t(\tilde{Y}_{t+1}^*) - \tilde{Y}_t^{*o} \right] \tag{1.3.75}$$

where $\rho_0 = \frac{\phi}{1+\phi} \frac{\sigma-1}{\sigma}$ and $0 < \rho_0 < 1$ assuming $\sigma > 1$. In this $\alpha \to 1$ case, only the home policy rate is affected by the foreign rate. Furthermore, from the equation (1.3.56), the foreign optimal output gap ($\tilde{Y}_t^{*o}$) in the equation (1.3.75) is not affected by the home output gap ($\tilde{Y}_t$) when $\alpha \to 1$.

Finally, combining the home and foreign best response functions (1.3.69) and (1.3.70), the equilibrium home policy rate can be derived as follows:

$$\begin{aligned} \hat{R}_t^o = {} & E_t(\hat{\Pi}_{H,t+1}) + \frac{\sigma_0}{1-\rho\rho^*} \left[E_t(\tilde{Y}_{t+1}) - \tilde{Y}_t^o \right] \\ & + \frac{\rho}{1-\rho\rho^*} \left\{ \sigma_0^* \left[E_t(\tilde{Y}_{t+1}^*) - \tilde{Y}_t^{*o} \right] - (\eta^* - \rho^*\eta) E_t(\widehat{\Delta S}_{t+1}) \right\} + \varrho_t \end{aligned} \tag{1.3.76}$$

where $\varrho_t = \left(\sigma_0 d_t + \rho \sigma_0^* d_t^* \right) / \left(1 - \rho \rho^* \right)$.

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[16]$\eta^* = (1-\alpha)(\sigma-1)$ and $\sigma_0^* = \sigma - \eta^*$.

1.3.4   Implication

The welfare analysis suggests that the home economy's welfare is also affected by the foreign output gap and the terms of trade, which contradicts CGG (2002). This is because the model in this chapter takes the foreign output gap as a variable while CGG (2002) does not. Therefore, the optimal monetary policy rule in an open economy is not isomorphic to the one in a closed economy. Optimal monetary policy in open economies should respond to not only domestic variables, but also international variables such as the foreign output gap and the terms of trade. This implies that stabilizing domestic variables is not enough to maximize domestic welfare. The effect of the international variables on home welfare is greater as the degree of home openness is higher.

The optimal home (foreign) policy rate is positively affected by the foreign (home) interest rate, and the effect becomes stronger as openness of the home (foreign) economy is higher. Therefore, when the degrees of bilateral trade openness of a small and a large economy are significantly different from each other, the policy rates relationship between these two economies would be asymmetric.

1.4   Empirical Evidence: Asymmetric Policy Relationship

1.4.1   Overview

Following the theoretical approach that clarifies the reason for asymmetric policy rate relationships, in this section those relationships are empirically evidenced by data analyses. There are two large *foreign economies* in the empirical models: the US and the Eurozone. This section then investigates the monetary policy relationships between these large economies and 14 other economies - *home economies*: Australia, Canada, Chile, Czech, Denmark, India, Indonesia, South Africa, South

Korea, Malaysia, Norway, Sweden, Thailand, and UK. Using both the home and the foreign model equations, the effect of the foreign (home) policy rate changes on the home (foreign) policy rate is verified.

The empirical study adopts two methodologies. First, given that policy rates are not continuous and also not frequently adjusted, ordered probit models are used as in Bergin and Jordà (2004) and Scotti (2011). Following this, as in the studies of Kim (2001), Sousa and Zaghini (2008) and Kucharčuková, Claeys and Vašíček (2014), structural VAR analyses are carried out using the short-term interest rates. The structural scheme is based on the two-country model of Section 1.3.

1.4.2   Data Description

For the empirical analysis, monthly data from 2001:M2 to 2015:M12 are used[17]. The data include the output gap, the inflation rate, the short-term interest rate, and the monetary policy rate. Since monthly output data are not provided, as in Kim (2001), industrial production (IP) data are used - replacing the output gap data. For the inflation rate, the period to period inflation data are used. In the probit model analysis, CPI inflation data are used, while the VAR analysis uses PPI inflation data. This is because the structural VAR identification is based on the model of Section 1.3. Also, net inflation (π) is used instead of the gross inflation rate (Π) in the probit model analysis. Most of the short-term interest rates (i) are interbank rates, and some of them are either treasury bill yields or money market interest rates. For US dollar ($\mathcal{E}_D$) and euro ($\mathcal{E}_E$) exchange rates, the nominal values of the foreign currencies (US dollar or euro) denominated by home currencies are used. All the data are provided by Datastream[18].

Detecting the order of integration of the data, Augmented Dickey-Fuller (ADF)

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[17]Given the length of the available data, the sample period of Malaysia is 2005:M7-2015:M12, and for India the sample period is 2005:M4-2015:M12.

[18]Data sources are reported in the appendix 1.A.3.

tests are carried out. Difference data are used for some of the data having unit roots: short-term interest rates and nominal exchange rates. The appendix 1.A.3 illustrates the ADF test results for the level and the difference data. Some of industrial production data are transformed via the seasonal adjustment (SA) if those data are not adjusted; X-12 ARIMA[19] method is used, which was developed by the U.S. Census Bureau (Findley *et al.*, 1998)[20].

1.4.3 Ordered Probit Model

Model Description

The probit model is useful when dependent variables are discrete; it adopts latent continuous variables which replace the original ones. Furthermore, the ordered probit model can also take the magnitude of the discrete variables into account. There are extant studies which use the ordered probit model to investigate the factors that affect policy rate changes (Bergin and Jordà, 2004; Hamilton and Jordà, 2002; Scotti, 2011). Including foreign interest rates in the ordered probit model equations, they estimate the influence of the foreign monetary policy on domestic policy rate changes.

The conventional amounts of policy rate changes are not identical across countries. Therefore, in this model, the magnitudes of the policy rate changes are classified into five categories: (i) strong tightening ($30\text{bp} \leq \Delta R$), (ii) normal tightening ($0\text{bp} < \Delta R < 30\text{bp}$), (iii) no change ($\Delta R = 0\text{bp}$), (iv) normal expansion ($-30\text{bp} < \Delta R < 0\text{bp}$), and (v) strong expansion ($\Delta R \leq -30\text{bp}$). Bergin and Jordà (2004) and Scotti (2011) use similar classification.

The observed discrete policy rate changes are then transformed into a series of ordered variables, $z_t \in \{-2, -1, 0, 1, 2\}$. In this series, '2' represents *strong tighten-*

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[19]Autoregressive Integrated Moving Average

[20]Details of the adjustments are illustrated in the appendix 1.A.4.

ing, '-2' is for *strong expansion*, and '0' means *no policy change*[21]. As long as the series of the numbers consistently reflects the actual order (ranking) of the original dependent variables, any nominal numbers in the series are appropriate for the analysis. It is then hypothesized that the discrete policy rate change (z_t) is related to the continuous latent variable z_t^* according to

$$
z_t = f(z_t^*) = \begin{cases}
-2 \ (= s^1) & \text{if } z_t^* \in \{c_0(= -\infty), c_1\} \\
-1 \ (= s^2) & \text{if } z_t^* \in \{c_1, c_2\} \\
\vdots \\
2 \ (= s^5) & \text{if } z_t^* \in \{c_4, c_5(= \infty)\}
\end{cases}
\tag{1.4.1}
$$

where $c_0 < c_1 < c_2 < c_3 < c_4 < c_5$. Given the decision making process of the central bank, the latent variable z_t^* is assumed to be determined by economic variables such as inflation and output. These variables are then represented by a vector X_t in the following equation with coefficients (β) and error terms (ε_t):

$$
z_t^* = X_t' \beta + \varepsilon_t.
\tag{1.4.2}
$$

The model establishment depends on the assumption of the probability distribution of the error term ε_t. The ordered probit model is based on the assumption of normality of ε_t, while the logit model assumes a logistic distribution. The probability of each policy stance change (s^j) is then given by

$$
Pr(z_t = s^j) = Pr(c^{j-1} < z_t^* \leq c^j) = F(c_j - X_t'\beta) - F(c_{j-1} - X_t'\beta),
\tag{1.4.3}
$$

where $F(\cdot)$ is the standard normal cumulative distribution function and $j \in \{1, 2, 3, 4, 5\}$.

---

[21]Regarding the federal funds rate target in the US, Hamilton and Jordà (2002) use a series of $\{\cdots, -0.50, -0.25, 0, 0.25, 0.50, \cdots\}$. In Bergin and Jordà (2004) the ordered variables are $\{-0.50, -0.25, 0, 0.25, 0.50\}$, and Scotti (2011) uses $\{-50, -25, 0, 25, 50\}$.

Define $l(z_t; \beta, c)$ as the log-likelihood function of observing z_t conditional on β and $c \; (= \{c_1, \cdots, c_4\})$. The solution of the maximum likelihood estimation of the parameters is then illustrated as follows:

$$\{\hat{\beta}, \hat{c}_1, \cdots, \hat{c}_4\} = \underset{\{\beta, c_1, \cdots, c_4\}}{argmax} \sum_{t=1}^{T} l(z_t; \beta, c). \tag{1.4.4}$$

The cross-border effect of the monetary policy is represented by the role of the other country's short-term rate (i) in the domestic policy decision. The estimated equations based on a standard Taylor-type rule incorporate interest rate smoothing. As in Clarida, Galí and Gertler (1998) and Taylor (2013)[22], the foreign (home) rate is included in the home (foreign) rate equation. Since current period data are not observable while making policy decisions, the policy decision responds to the expected current period values of the output gap and domestic inflation[23]. The analysis then tests the significance of the coefficients of the other economy's interest rates in Taylor-type rule equations. In order to avert the unit root problem, difference variables are used for short-term rate data[24]. The ordered probit model (1.4.2) is then illustrated as follow:

$$z_t^{H*} \;=\; \beta_1 i_{t-1}^{H} + \beta_2 E\left(\pi_t^{H} \mid \Omega_t^{H}\right) + \beta_3 E\left(\widetilde{Y}_t^{H} \mid \Omega_t^{H}\right) + \beta_4 i_{t-1}^{F} + \varepsilon_t^{H} \quad (1.4.5)$$

$$z_t^{F*} \;=\; \beta_1 i_{t-1}^{F} + \beta_2 E\left(\pi_t^{F} \mid \Omega_t^{F}\right) + \beta_3 E\left(\widetilde{Y}_t^{H} \mid \Omega_t^{F}\right) + \beta_4 i_{t-1}^{H} + \varepsilon_t^{F} \quad (1.4.6)$$

where superscript F is for foreign and H is for home variables. Ω_t^{H} and Ω_t^{F} represent the available information sets at time t, which consist of the $t-1$ and $t-2$ data of CPI inflation, PPI inflation, the output gap and the nominal exchange rate. Under

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[22]Taylor (2013) illustrates a home policy rate rule, $i = z + \alpha i^*$, where i and i^* denote the home and the US rates, respectively. z represents domestic factors such as inflation and GDP, and $0 < \alpha < 1$.

[23]This is similar to Clarida, Galí and Gertler (1998), where the central bank responds to the expected value of the current period output gap, based on an available information set.

[24]The appendix 1.A.3 reports the unit root test results.

the rational expectations hypothesis, the equations (1.4.5) and (1.4.6) are estimated using two-step procedures. The estimation procedures and rationality test results are illustrated in the appendix 1.A.5.

Results of the Probit Model Analysis

The significance of the coefficient β_4 indicates the influence of the previous foreign (home) policy changes on the home (foreign) policy rate decision. If β_4 in the home policy equation is significant and it is not in the foreign country's equation, it is categorized as an one-way relationship, namely the 'leader and follower relationship' between central banks. This kind of approach is similar to Ullrich (2003) which verifies the asymmetric relationship between the US Fed and the ECB using VECM type Taylor rule tests.

Table 1.2 illustrates the results of the probit model analyses which indicate each *home* economy's policy relationship with the US - the *foreign* economy. The foreign rate coefficients (β_4) of eight countries' models are significant. Seven of these economies have one-way relationships: Australia, Chile, Czech, India, South Korea, Malaysia, and the UK. Marginal probability effects (MPE)[25] indicate that when the interest rate of the US is raised by 30bp, the probability of the *home* rate increase rises by 10-34% in these economies. However, the US monetary policy is not affected by the short-term rate changes of these economies. Canada reveals a two-way relationships, because the economies of the US and Canada are highly integrated. Six other economies do not have any significant policy relationships with the US: Denmark, Indonesia, Norway, South Africa, Sweden, and Thailand.

Regarding the relationships with the Eurozone, 11 *home* economies' policy deci-

---

[25]Marginal probability effects (MPE) are calculated as

$$MPE_{j,i} = \frac{\delta Pr\left[z_t = s^j \mid X\right]}{\delta x_i} = \left[f\left(c_{j-1} - X_t'\beta\right) - f\left(c_j - X_t'\beta\right)\right]\beta_i,$$

where $f\left(\cdot\right)$ is the standard normal distribution function, following Boes and Winkelmann (2006).

Table 1.2: The Policy Relationships with the US (Ordered probit model)

Relationship	Home Country	Foreign → Home			Home → Foreign		
		β_4	z-statistic	MPE*	β_4	z-statistic	MPE*
One-way (7)	Australia	3.78	1.96	0.16	-1.24	-0.66	-
	Chile	5.32	2.88	0.28	0.22	0.41	-
	Czech	4.13	2.09	0.10	-1.23	-0.37	-
	India	6.72	3.08	0.34	0.89	0.85	-
	South Korea	4.50	2.27	0.14	4.13	1.68	-
	Malaysia	7.17	2.13	0.14	8.33	1.91	-
	UK	9.50	4.75	0.21	1.53	0.69	-
Two-way (1)	Canada	5.45	2.79	0.21	6.46	2.35	0.23
None (6)	Denmark	1.60	0.79	-	5.41	1.58	-
	Indonesia	1.75	0.85	-	0.25	1.76	-
	Norway	1.49	0.86	-	-0.11	-0.06	-
	South Africa	0.75	0.35	-	-1.65	-1.01	-
	Sweden	0.68	0.38	-	-0.60	-0.27	-
	Thailand	1.64	1.02	-	1.35	0.72	-

* Probability increase of raising policy rates when the other country raises its rate by 30bp

sions are affected by the *foreign* short-term rate changes (Table 1.3). Among them, nine countries exhibit leader and follower relationships: Chile, Denmark, India, Indonesia, Malaysia, Norway, South Africa, Sweden, and the UK. Marginal probability effects (MPE) indicate that when the interest rate of the Eurozone is raised by 30bp, the probability of the *home* rate increase rises by 5-67% in these economies. Norway reveals the highest probability increase (67%). Australia and Canada have two-way relationships, possibly because the ECB considers some of advanced central banks' decisions. Czech, South Korea and Thailand do not have any relationships with the Eurozone.

Combining these two test results, 12 out of 14 countries have leader and follower relationships with at least one of the large *foreign* economies. Four of these countries have the asymmetric relationships with both large economies: Chile, India, Malaysia, and the UK. Only one country, Thailand, does not have any significant relationship with the US and the Eurozone.

Table 1.3: The Policy Relationships with the Eurozone (Ordered probit model)

Relationship	Home Country	Foreign → Home			Home → Foreign		
		β_4	z-statistic	MPE*	β_4	z-statistic	MPE*
	Chile	6.68	2.28	0.35	-0.35	-0.64	-
	Denmark	13.89	2.64	0.36	1.56	0.32	-
	India	9.69	2.98	0.48	0.64	0.56	-
	Indonesia	6.11	2.41	0.24	-0.11	-0.71	-
One-way (9)	Malaysia	9.24	2.23	0.25	-1.19	-0.26	-
	Norway	14.00	3.99	0.67	3.33	1.51	-
	South Africa	7.28	2.56	0.05	1.66	1.01	-
	Sweden	8.61	2.53	0.48	5.93	1.82	-
	UK	16.14	3.05	0.40	-7.04	-1.78	-
Two-way (2)	Australia	5.26	1.97	0.24	5.13	2.70	0.12
	Canada	7.89	2.72	0.30	5.71	2.13	0.14
	Czech	4.57	1.56	-	-6.12	-1.56	-
None (3)	South Korea	7.21	1.65	-	-1.18	-0.39	-
	Thailand	4.53	1.65	-	0.68	0.29	-

* Probability increase of raising policy rates when the other country raises its rate by 30bp

1.4.4   Vector Auto-regression (VAR) Model

Model Description

This section proposes an open economy structural VAR model to analyse the leader and follower relationship between central banks. Consider an n-dimensional vector Z_t, which is approximated by a vector auto-regression of finite order p. The reduced form equation is given by

$$Z_t = B_1 Z_{t-1} + B_2 Z_{t-2} + \cdots + B_p Z_{t-p} + u_t \qquad (1.4.7)$$

where u_t denotes the reduced form disturbance and $E(u_t u_t') = \Sigma$.

Regarding the choice of the vector Z_t, there are some indicative examples in literature (Christiano, Eichenbaum and Evans, 1996; Kim,2000; Sousa and Zaghini, 2008). For instance, investigating the spill-over effect of a foreign monetary policy shock, Kim (2001) establishes a vector Z_t that consists of industrial production, the

CPI, M2, commodity prices, the exchange rate, and the US federal fund rate (*foreign interest rate*). Analysing the foreign liquidity effect on the Euro area economy, Sousa and Zaghini (2008) use foreign liquidity, real GDP, the CPI, M3, the short-term interest rate, and the exchange rate.

The structural VAR (SVAR) scheme indicates the relationship between the reduced form disturbance (u_t) and the structural shock (ε_t) as follows:

$$A_0 u_t = \varepsilon_t \tag{1.4.8}$$

where $E(\varepsilon_t \varepsilon_t') = \Lambda$ which is diagonal. This means that the shocks are mutually uncorrelated. The structural matrix A_0 can be calculated by:

$$A_0 \Sigma A_0' = \Lambda. \tag{1.4.9}$$

Identifying the structural VAR scheme, there can be some restrictions on contemporaneous transmission of shocks. One example is a recursive structure where the structural matrix A_0 is a lower triangular matrix, as proposed by Sims (1980). Another method is to assume a non-recursive structure by imposing individual restrictions on the elements of the matrix A_0. Many studies such as Kim (2001) and Sousa and Zaghini (2008) use this method analysing international transmission of a monetary policy shock; this chapter also adopts a non-recursive structure of VAR.

The impulse responses to an economic shock can be calculated by the structural VAR scheme. Suppose there is a shock on the j-th element in the error term. In the first period then the impulse response of Z_t would be

$$\Gamma_j(1) = A_0^{-1} e_j \tag{1.4.10}$$

where e_j is an n-dimensional vector where the j-th element is unity and the other

elements are all zero. In the second and third periods, the impulse responses become

$$\Gamma_j(2) = B_1 A_0^{-1} e_j \tag{1.4.11}$$

$$\Gamma_j(3) = B_1 B_1 A_0^{-1} e_j + B_2 A_0^{-1} e_j. \tag{1.4.12}$$

Identification

Since the monetary policy relationships across countries depend on the international transmission channel of shocks, the identification of the structural matrix A_0 is important in this analysis. The identification in this section is based on the model framework in Section 1.3. This kind of model-based identification is similar to Giordani (2004) which uses the simple New-Keynesian framework of Svensson (1997). Sims and Zha (2006) also use a model-based identification method. In this section, the central bank conducts the monetary policy following a Taylor-type rule instead of maximizing domestic welfare. Also, various shocks are added to the equation system in order to verify the structural matrix A_0.

First, from the linearized equations (1.3.34) and (1.3.35), the dynamic IS equation can be rewritten by:

$$\hat{Y}_{t+1} = \hat{Y}_t + \frac{1}{\sigma}\left[\hat{R}_t - \hat{\Pi}_{H,t+1}\right] + \frac{\alpha}{\sigma}(\sigma-1)\widehat{\Delta S}_{t+1} + \varepsilon_{y,t+1} \tag{1.4.13}$$

where $\varepsilon_{y,t+1}$ denotes the home output shock at $t+1$. From the definition of the terms of trade $S_t = \mathcal{E}_t P_{F,t}^*/P_{H,t}$,

$$\widehat{\Delta S}_{t+1} = \widehat{\Delta \mathcal{E}}_{t+1} - \hat{\Pi}_{H,t+1} + t.i.p, \tag{1.4.14}$$

where $t.i.p$ represents *terms independent of policy* that includes the foreign goods inflation rate in the foreign market ($\hat{\Pi}_{F,t+1}^*$). Also, the uncovered interest rate parity (UIP) holds as follows:

$$\widehat{\Delta \mathcal{E}}_{t+1} = \hat{R}_t - \hat{R}_t^* + \varepsilon_{x,t+1} \tag{1.4.15}$$

where $\varepsilon_{x,t+1}$ denotes the UIP shock, which is also illustrated in Kollmann (2002). From $\hat{Y}_t = \tilde{Y}_t + t.i.p^{26}$, combining the equations (1.4.13), (1.4.14) and (1.4.15) leads to the equation as below:

$$\tilde{Y}_{t+1} = \tilde{Y}_t + \Omega_1 \left[\hat{R}_t - \hat{\Pi}_{H,t+1}\right] - \frac{\alpha}{\sigma}(\sigma - 1)\hat{R}_t^* + \varepsilon_{y,t+1} + \frac{\alpha}{\sigma}(\sigma - 1)\varepsilon_{x,t+1} + t.i.p. \quad (1.4.16)$$

where $\Omega_1 = [1 + \alpha(\sigma - 1)]/\sigma$.

The equation (1.3.42) indicates the New Keynesian Phillips curve which can be rewritten by

$$\hat{\Pi}_{H,t+1} = -\frac{\lambda}{\beta}\hat{Y}_t + \frac{1}{\beta}\hat{\Pi}_{H,t} + \varepsilon_{\pi,t+1} \quad (1.4.17)$$

where $\varepsilon_{\pi,t+1}$ represents the inflation shock. Combining the equation (1.4.17) with the equation (1.4.16) then yields

$$
\begin{aligned}
\tilde{Y}_{t+1} &= \left(1 + \frac{\lambda}{\beta}\Omega_1\right)\tilde{Y}_t - \frac{1}{\beta}\Omega_1\hat{\Pi}_{H,t} + \Omega_1\hat{R}_t - \frac{\alpha}{\sigma}(\sigma - 1)\hat{R}_t^* \\
&\quad - \Omega_1\varepsilon_{\pi,t+1} + \varepsilon_{y,t+1} + \frac{\alpha}{\sigma}(\sigma - 1)\varepsilon_{x,t+1} + t.i.p.
\end{aligned}
\quad (1.4.18)
$$

Based on the home welfare approximation in the equation (1.3.49), the home central bank's Taylor-type policy rule includes the fluctuation of the terms of trade (ΔS_t). At time $t + 1$, the home monetary policy rule is given by

$$\hat{R}_{t+1} = \phi_\pi\hat{\Pi}_{H,t+1} + \phi_y\tilde{Y}_{t+1} + \phi_s\widehat{\Delta S}_{t+1} + \varepsilon_{R,t+1} \quad (1.4.19)$$

where $\varepsilon_{R,t+1}$ is the home policy rate shock at $t+1$. Combining the equation (1.4.19) with the equations (1.4.15), (1.4.17) and (1.4.18) then yields

$$\hat{R}_{t+1} = \Omega_2\tilde{Y}_t + \Omega_3\hat{\Pi}_{H,t} + (\phi_y\Omega_1 + \phi_s)\hat{R}_t - \Omega_4\hat{R}_t^*$$

$$+ (\phi_\pi\beta - \phi_y\Omega_1)\varepsilon_{\pi,t+1} + \phi_y\varepsilon_{y,t+1} + \varepsilon_{R,t+1} + \Omega_4\varepsilon_{x,t+1} + t.i.p \quad (1.4.20)$$

---

[26]In the equation $\hat{Y}_t = \tilde{Y}_t + \hat{Y}_{e,t}$, the flexible price output gap $(\hat{Y}_{e,t})$ is not controlled by the policy. In this model, $\hat{Y}_t = lnY_t - ln\bar{Y}$, $\hat{Y}_{e,t} = lnY_{e,t} - ln\bar{Y}$, and $\tilde{Y}_t = lnY_t - lnY_{e,t}$.

where

$$\Omega_2 = \phi_y \left(1 + \frac{\lambda}{\beta} \Omega_1 \right) - \phi_\pi \lambda \qquad (1.4.21)$$

$$\Omega_3 = \phi_\pi - \frac{\phi_y}{\beta} \Omega_1 \qquad (1.4.22)$$

$$\Omega_4 = \phi_y \frac{\alpha}{\sigma} (\sigma - 1) + \phi_s. \qquad (1.4.23)$$

Openness of the foreign economy is assumed to be very low, and thus the foreign monetary policy rule does not include the terms of trade. At time $t + 1$, with the foreign policy rate shock $(\varepsilon^*_{R,t+1})$, the foreign policy rule is given by:

$$\hat{R}^*_{t+1} = \phi^*_\pi \hat{\Pi}^*_{F,t+1} + \phi^*_y \tilde{Y}^*_{t+1} + \varepsilon^*_{R,t+1}. \qquad (1.4.24)$$

Plugging the foreign New Keynesian Phillips curve[27] into the equation (1.4.24),

$$\hat{R}^*_{t+1} = \frac{1}{\beta} \hat{R}^*_t + \varepsilon^*_{R,t+1} + t.i.p. \qquad (1.4.25)$$

where $t.i.p$ includes the foreign output gap.

The equations based on the modified CGG (2002) model in this section are used for deriving a structural VAR scheme (A_0) - the relationship between structural shocks and the reduced form disturbances. There are five structural shocks; home output, inflation, the UIP, and the home and the foreign monetary policy shocks. Ignoring all $t.i.p$, the five key equations (1.4.15), (1.4.17), (1.4.18), (1.4.20) and (1.4.25) provide the following VAR representation:

$$Z_{t+1} = BZ_t + D\varepsilon_{t+1} \qquad (1.4.26)$$

---

[27]$\hat{\Pi}^*_{F,t+1} = -\frac{\lambda^*}{\beta} \tilde{Y}^*_t + \frac{1}{\beta} \hat{\Pi}^*_{F,t}$ where $\lambda^* = \beta \kappa^* = \beta[\sigma - \alpha(\sigma - 1) + \phi]$

where $Z_t = [\hat{R}_t^*, \hat{\Pi}_{H,t}, \tilde{Y}_t, \hat{R}_t, \widehat{\Delta\mathcal{E}_t}]'$, $\varepsilon_{t+1} = [\varepsilon_{R,t+1}^*, \varepsilon_{\pi,t+1}, \varepsilon_{y,t+1}, \varepsilon_{R,t+1}, \varepsilon_{x,t+1}]'$, and

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -\Omega_1 & 1 & 0 & \frac{\alpha}{\sigma}(\sigma-1) \\ 0 & \phi_\pi\beta - \phi_y\Omega_1 & \phi_y & 1 & \Omega_4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1.4.27}$$

Given that $E(\varepsilon_{t+1}\varepsilon_{t+1}')$ is a diagonal matrix, D corresponds to the inverse of the structural matrix A_0. With the zero-element restrictions, A_0 has a form of

$$A_0 = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & 0 & a_{35} \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix}. \tag{1.4.28}$$

The first row in the matrix A_0 implies the exogeneity of the foreign policy rate shock. The second row means that, on the home inflation rate, there is no contemporary effect of the financial sector disturbances such as the exchange rate, home and foreign interest rates shocks. Contrary to Sousa and Zaghini (2008), in this model there is a contemporary effect of the nominal exchange rate change on the output gap, which is represented by the third row. This is because the home output gap is also a function of the terms of trade which is affected by the nominal exchange rate changes. Following Sims and Zha (2006) and Kim (2001), it is assumed that the monetary policy does not respond to the inflation rate and the output shocks contemporaneously, since current period data are not observable (*delayed information*). Thus, in the forth row, there is an additional constraint, $\alpha_{42} = \alpha_{43} = 0$. The last row illustrates the exogenous UIP shock on the nominal exchange rate.

Table 1.4: Short-term Rate Relationships with the US

Countries	US→ Others	Others →US	Result	Countries	US→ Others	Others →US	Result
Australia	×	×	-	Malaysia	O	×	One-way
Canada	O	×	One-way	Norway	O	×	One-way
Chile	×	×	-	South Africa	×	×	-
Czech	O	×	One-way	South Korea	O	×	One-way
Denmark	O	×	One-way	Sweden	×	×	-
India	O	×	One-way	Thailand	O	×	One-way
Indonesia	×	×	-	UK	O	×	One-way

Note: Within 6 months, movements in the same direction at 5% level of significance

Results of the Impulse Response Analysis

In this SVAR analysis, the relationships between one of the large economies (the US and the Eurozone) and 14 other economies are investigated. Following the equation (1.4.26), the vector Z consists of five elements: (i) the foreign gross short-term rate (R^*), (ii) the home PPI inflation rate (Π_H)[28], (iii) home industrial production (Y), (iv) the home gross short-term rate (R) and (v) the nominal exchange rate change $(\Delta\mathcal{E})$. In order to avoid the unit root problem, difference variables are used for the short-term rate and the output gap. The lag (p) of each country's autoregressive equation is determined based on Akaike's Information Criterion (AIC).

Table 1.4 illustrates the relationships between 14 other economies' short-term interest rates and the US rate. When there is a positive one-standard deviation (SD) shock on the US short-term rate, in 9 countries, short-term rates rise within 6 months at the 5 percent level of significance. On the other hand, the US interest rate does not respond to other countries' shocks. Thus, 9 out of 14 countries have one-way (leader and follower) relationships with the US policy changes: Canada, Czech, Denmark, Inida, Malaysia, Norway, South Korea, Thailand, and the UK.

---

[28]For Australia, Chile, India and South Africa, the CPI data are used. The PPI data of Chile are from Jan 2009 and the South African data are from Jan 2012. Australia provides only the quarterly PPI data.

Table 1.5: Short-term Rate Relationships with the Eurozone (EZ)

Countries	EZ→ Others	Others →EZ	Result	Countries	EZ→ Others	Others →EZ	Result
Australia	O	O	Two-way	Malaysia	O	×	One-way
Canada	O	O	Two -way	Norway	O	O	Two -way
Chile	O	×	One-way	South Africa	O	×	One-way
Czech	O	×	One-way	South Korea	O	×	One-way
Denmark	O	×	One-way	Sweden	O	O	Two -way
India	O	×	-	Thailand	O	×	One-way
Indonesia	×	×	-	UK	×	×	-

Note: Within 6 months, movements in the same direction at 5% level of significance

The other five economies do not exhibit any significant policy relationships with the US: Australia, Chile, Indonesia, South Africa, and Sweden. Figure 1.2 presents individual impulse responses.

When the Eurozone short-term rate rises by one SD, 12 other economies' short-term rates significantly respond to the shock (Table 1.5). Among them, 8 countries have one-way relationships with the Eurozone: Chile, Czech, Denmark, India, Malaysia, South Africa, South Korea and Thailand. Compared to the relationships with the US Fed, the ECB considers more advanced central banks' policy decisions: Australia, Canada, Norway and Sweden. Indonesia and the UK do not exhibit any relationship. It is noteworthy that the UK short-term rate is affected by the US rate, but not by the Eurozone rate. Figure 1.3 illustrates the individual response of each country.

Combining the results of the VAR analyses suggests that 11 out of 14 countries have leader and follower relationships with at least one of the large economies - with respect to the short-term interest rate. Among these economies, six countries have one-way relationships with both large economies: Czech, Denmark, India, Malaysia, South Korea, and Thailand. Only one country, Indonesia, does not have any relationship with them.

Figure 1.2: Impulse Responses to Foreign and Home Policy Shocks with the US
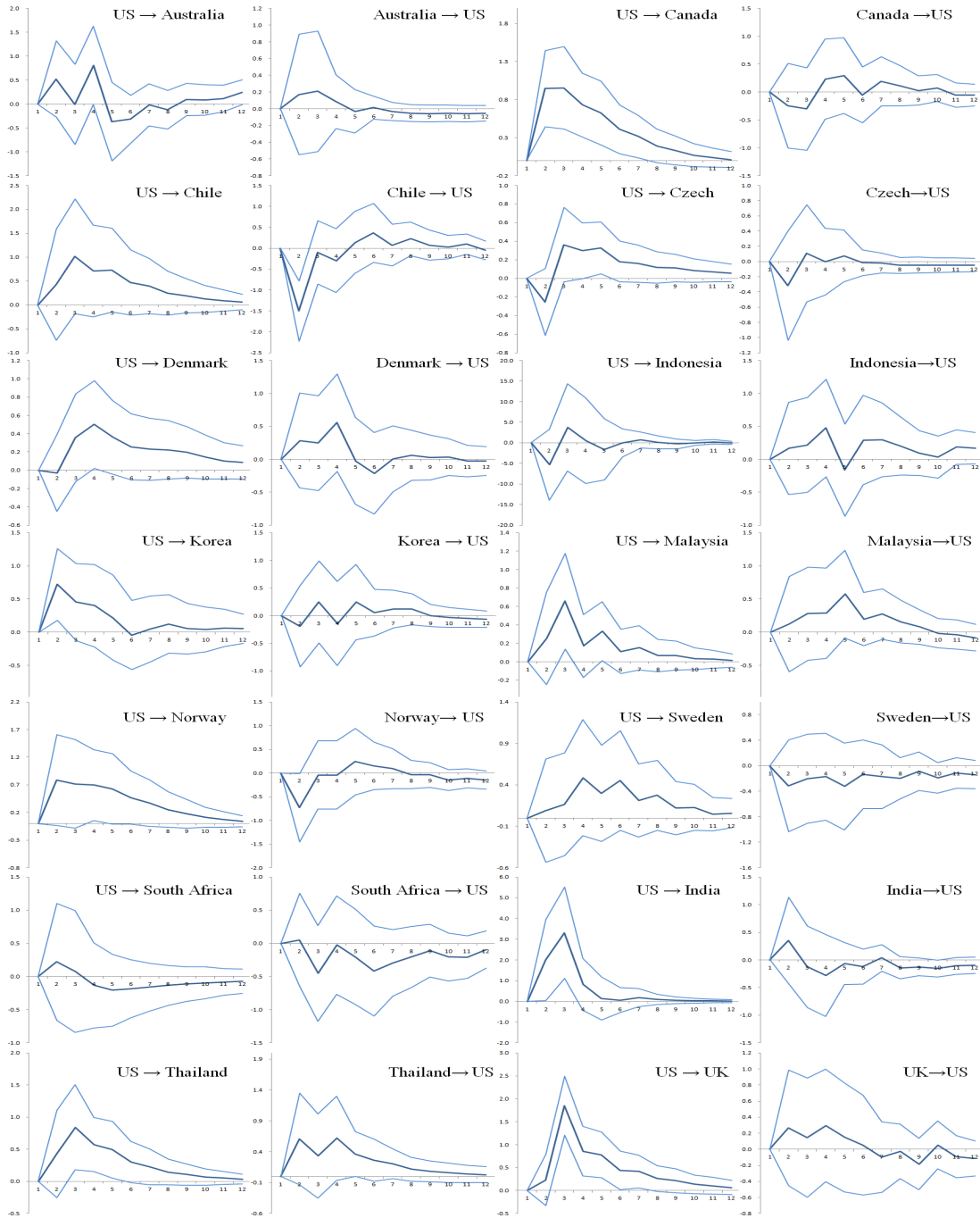
Figure 1.3: Impulse Responses to Foreign and Home Policy Shocks with the Eurozone
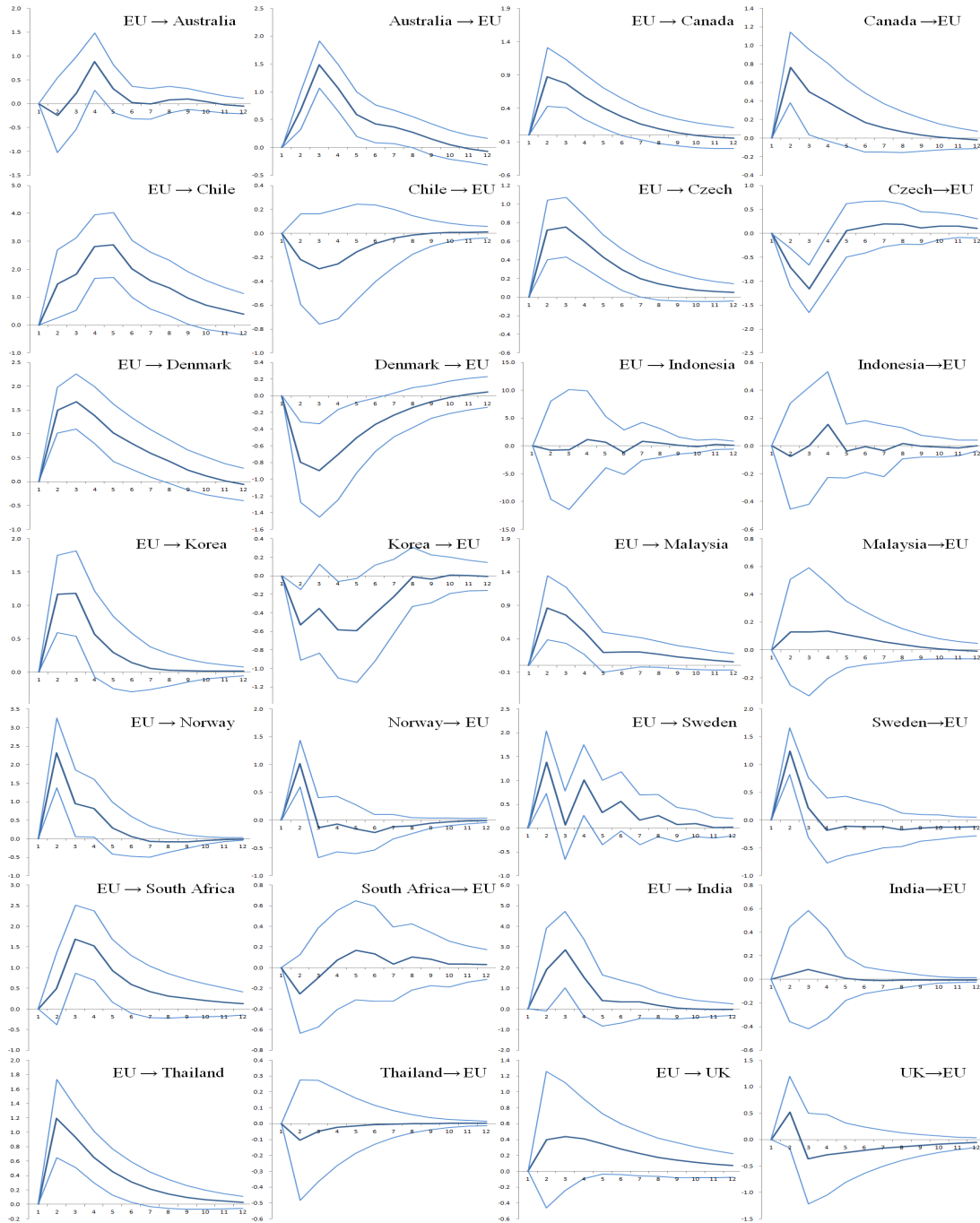
1.5   Conclusion

Many central banks' policy decisions are affected by the policy changes of major central banks, such as the US Fed and the ECB. In many cases, the relationships between central banks look asymmetric, which can be described as the leader and follower relationship. This chapter presents a simple model that can explain the reason for the asymmetric monetary policy relationships and provides supporting empirical evidence.

The theoretical analysis based on a two-country model suggests that the home economy's welfare is a function of international variables, such as the foreign output gap and the terms of trade. Therefore, optimal inflation and the output gap are influenced by these external variables. Moreover, the home policy rate is affected by the foreign policy rate, and the foreign policy effects become stronger as home openness is greater. As a consequence, there can be an asymmetric policy rate relationship between two countries when the degrees of openness are significantly different from each other.

The empirical study uncovers the asymmetric policy rate relationships based on data analyses. In the probit model analyses, 12 out of 14 economies exhibit leader and follower relationships with at least one of the US and the Eurozone. The VAR analyses indicate that 11 countries have one-way policy relationships with the US or/and the Eurozone. Combining the results of these two kinds of tests, nine economies have asymmetric relationships with at least one of the large economies in both the probit and the VAR models analyses.

1.A   Appendix

1.A.1   Derivation of the Welfare Function

Home Economy's Welfare

Defining $X_{e,t}$ as the flexible price equilibrium value of X_t, $\tilde{X}_t$ denotes the log deviation from the flexible price equilibrium ($\tilde{X}_t = lnX_t - lnX_{e,t}$). Also, $\hat{X}_t$ denotes the log deviation of the variable X_t from the steady state $\bar{X}_t$ ($\hat{X}_t = lnX_t - ln\bar{X}$). The deviation of the flexible price equilibrium from the steady state is then $\hat{X}_{e,t} = lnX_{e,t} - ln\bar{X}$.

From the second-order approximation, the utility of consumption $U(C_t)$ around the flexible price equilibrium ($C_{e,t}$) can be approximated by

$$U(C_t) = U(C_{e,t}) + U'(C_{e,t})(C_t - C_{e,t}) + \frac{1}{2}U''(C_{e,t})(C_t - C_{e,t})^2 + o\left(\|a\|^3\right). \quad (1.A.1)$$

Using the equation (1.3.46), then (1.A.1) can be rewritten by

$$U(C_t) = U(C_{e,t}) + U'(C_{e,t})C_{e,t}\left[\tilde{C}_t + \frac{1}{2}(1-\sigma)\tilde{C}_t^2\right] + o\left(\|a\|^3\right). \quad (1.A.2)$$

Contrary to Clarida, Galí and Gertler (2002), the foreign output is taken into account while approximating welfare. From the equation (1.3.28) $\tilde{C}_t = (1-\alpha)\tilde{Y}_t + \alpha\tilde{Y}_t^*$, and accordingly (1.A.2) can be rewritten by

$$U(C_t) = U(C_{e,t}) + U'(C_{e,t})C_{e,t}\Psi\left(\tilde{Y}_t, \tilde{Y}_t^*\right) + U(C_{e,t}) + o\left(\|a\|^3\right) \quad (1.A.3)$$

where

$$\Psi\left(\tilde{Y}_t, \tilde{Y}_t^*\right) = (1-\alpha)\tilde{Y}_t + \alpha\tilde{Y}_t^* + \frac{1}{2}(1-\sigma)\left[(1-\alpha)^2\tilde{Y}_t + 2\alpha(1-\alpha)\tilde{Y}_t\tilde{Y}_t^* + \alpha^2\tilde{Y}_t^{*2}\right].$$

Also, the linearization of $U'(C_{e,t})C_{e,t}$ around the steady state suggests

$$
\begin{aligned}
U'(C_{e,t})C_{e,t} &= U'(\bar{C})\bar{C} + \left[U''(\bar{C})\bar{C} + U'(\bar{C})\right]\bar{C}\left(\frac{C_{e,t} - \bar{C}}{\bar{C}}\right) + o\left(\|a\|^3\right) \\
&= U'(\bar{C})\bar{C}\left[1 + (1-\sigma)\hat{C}_{e,t}\right] + o\left(\|a\|^3\right).
\end{aligned}
$$

At the flexible price equilibrium, the real marginal cost $(MC_{e,t})$ is $1/(1+\mu^P)$ which is a constant. Therefore, the equation (1.3.14) leads to $\hat{A}_{e,t} = \sigma\hat{C}_{e,t} + \phi\hat{N}_{e,t} + \alpha\hat{S}_{e,t}$. Also, the equations (1.3.12) and (1.3.26) imiply $\hat{Y}_{e,t} = \hat{A}_{e,t} + \hat{N}_{e,t} = \hat{C}_{e,t} + \alpha\hat{S}_{e,t}$. From these, the following equation holds:

$$
(1-\sigma)\hat{C}_{e,t} = (1+\phi)\hat{N}_{e,t} \tag{1.A.4}
$$

and $U'(C_{e,t})C_{e,t}$ can be re-approximated as

$$
U'(C_{e,t})C_{e,t} = U'(\bar{C})\bar{C}\left[1 + (1+\phi)\hat{N}_{e,t}\right] + o\left(\|a\|^3\right). \tag{1.A.5}
$$

Plugging (1.A.5) into (1.A.3) then yields

$$
U(C_t) = U'(\bar{C})\bar{C}\left[1 + (1+\phi)\hat{N}_{e,t}\right]\Psi\left(\tilde{Y}_t, \tilde{Y}_t^*\right) + t.i.p + o\left(\|a\|^3\right). \tag{1.A.6}
$$

Following this, the approximation of $U(C_t)$ can be derived as follows:

$$
U(C_t) = (1-\alpha)U'(\bar{C})\bar{C}\left[\tilde{Y}_t + \frac{1}{2}(1-\sigma)(1-\alpha)\tilde{Y}_t^2 + (1+\phi)\tilde{N}_{e,t}\tilde{Y}_t - \frac{1}{2}\Gamma\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}\right)\right] + t.i.p + o\left(\|a\|^3\right) \tag{1.A.7}
$$

where

$$
\Gamma\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}\right) = \left(\frac{\alpha}{1-\alpha}\right)\tilde{Y}_t^*\left\{(\sigma-1)\left[\alpha\tilde{Y}_t^* + 2\left((1-\alpha)\tilde{Y}_t + \tilde{C}_{e,t}\right)\right] - 2\right\}. \tag{1.A.8}
$$

In a similar way, the disutility of the labour supply $(V(N_t))$ can be approximated around the flexible price equilibrium by

$$
V(N_t) = V(N_{e,t}) + V'(N_{e,t})N_{e,t}\left[\tilde{N}_t + \frac{1}{2}(1+\phi)\tilde{N}_t^2\right] + o\left(\|a\|^3\right), \tag{1.A.9}
$$

and $V'(N_{e,t})N_{e,t}$ can be linearized as

$$V'(N_{e,t})N_{e,t} = V(\bar{N})\bar{N} + V'(\bar{N})\bar{N}\left[1 + (1+\phi)\hat{N}_{e,t}\right] + o\left(\|a\|^3\right). \tag{1.A.10}$$

Plugging (1.A.10) into (1.A.9) then yields

$$
\begin{aligned}
V(N_t) &= V'(\bar{N})\bar{N}\left[1 + (1+\phi)\hat{N}_{e,t}\right]\left[\tilde{N}_t + \frac{1}{2}(1+\phi)\tilde{N}_t^2\right] + t.i.p + o\left(\|a\|^3\right)\\
&= V'(\bar{N})\bar{N}\left[\tilde{N}_t + \frac{1}{2}(1+\phi)\tilde{N}_t^2 + (1+\phi)\hat{N}_{e,t}\tilde{N}_t\right] + t.i.p + o\left(\|a\|^3\right).
\end{aligned}
$$

From the equation (1.3.30), $\tilde{N}_t = \tilde{Y}_t + v_t$ where $v_t = lnV_t$. This implies $\hat{N}_{e,t}\tilde{N}_t = \hat{N}_{e,t}\tilde{Y}_t - \tilde{N}_{e,t}v_t = \hat{N}_{e,t}\tilde{Y}_t + o\left(\|a\|^3\right)$. From this, the approximated labour disutility can be rewritten by:

$$V(N_t) = V'(\bar{N})\bar{N}\left[\left(\tilde{Y}_t + v_t\right) + \frac{1}{2}(1+\phi)\tilde{Y}_t^2 + (1+\phi)\hat{N}_{e,t}\tilde{Y}_t\right] + t.i.p + o\left(\|a\|^3\right). \tag{1.A.11}$$

Lemma 1.A.1. *Define $\rho_t \left(= \int_0^1 \left(lnP_{H,t}(i) - lnP_{H,t}\right)^2 di\right)$ as the cross-sectional dispersion of prices. Up to a second-order approximation, then $v_t \simeq \frac{\varepsilon}{2}\rho_t$.*

Proof. See the appendix 1.A.2. □

From the equation (1.3.43) and the Lemma 1.A.1, the equation (1.A.11) implies

$$V(N_t) = (1-\alpha)U'(\bar{C})\bar{C}\left[\tilde{Y}_t + \frac{\varepsilon}{2}\rho_t + \frac{(1+\phi)}{2}\tilde{Y}_t^2 + (1+\phi)\hat{N}_{e,t}\tilde{Y}_t\right] + t.i.p + o\left(\|a\|^3\right). \tag{1.A.12}$$

Subtracting (1.A.12) from (1.A.7) then yields

$$\frac{U(C_t) - V(N_t)}{U(\bar{C})\bar{C}} = -(1-\alpha)\left[\frac{\varepsilon}{2}\rho_t + \frac{1}{2}\kappa\tilde{Y}_t^2 + \frac{1}{2}\Gamma\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}\right)\right]. \tag{1.A.13}$$

where $\kappa = (1+\phi) + (\sigma - 1)(1-\alpha) > 0$.

Defining W_τ^H as the approximated value of home welfare $\sum\limits_{t=\tau}^\infty \beta^{t-\tau}\left(\frac{U(C_t)-V(N_t)}{U(\bar{C})\bar{C}}\right)$,

$$W_\tau^H = -(1-\alpha)\frac{\Lambda}{2}\sum_{t=\tau}^\infty \beta^{t-\tau}\left\{\delta\rho_t + \psi\tilde{Y}_t^2 + \frac{\psi}{\kappa}\Gamma\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}\right)\right\} \tag{1.A.14}$$

where $\delta = (1-\theta)(1-\beta\theta)/\theta$, $\Lambda = \varepsilon/\delta$ and $\psi = \delta\kappa/\varepsilon$.

Lemma 1.A.2. *Define* $\rho_t = \int_0^1 \left(lnP_{H,t}(i) - lnP_{H,t}\right)^2 di$. *Then,* $\delta\sum_{t=0}^{\infty}\beta^t\rho_t = \sum_{t=0}^{\infty}\beta^t\hat{\Pi}_{H,t}^2$ *where* $\delta = (1-\theta)(1-\beta\theta)/\theta$.

Proof. See the appendix 1.A.2. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

From the Lemma 1.A.2, the welfare function (1.A.14) can be rewritten by

$$W_\tau^H = -(1-\alpha)\frac{\Lambda}{2}\sum_{t=\tau}^{\infty}\beta^{t-\tau}\left\{\hat{\Pi}_{H,t}^2 + \psi\tilde{Y}_t^2 + \frac{\psi}{\kappa}\Gamma\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}\right)\right\}.$$

Foreign Economy's Welfare

From the equations (1.3.28) and (1.A.2), $\tilde{C}_t^* = (1-\alpha)\tilde{Y}_t + \alpha\tilde{Y}_t^*$ and

$$U(C_t^*) = U(C_{e,t}^*) + U'(C_{e,t}^*)C_{e,t}^*\Psi\left(\tilde{Y}_t, \tilde{Y}_t^*\right) + U(C_{e,t}^*) + o\left(\|a\|^3\right). \qquad (1.A.15)$$

Also, as in the home economy case,

$$U'(C_{e,t}^*)C_{e,t}^* = U'(\bar{C}^*)\bar{C}^*\left[1 + (1+\phi)\hat{N}_{e,t}^*\right] + o\left(\|a\|^3\right). \qquad (1.A.16)$$

Plugging (1.A.16) into (1.A.15) then yields

$$U(C_t^*) = U'(\bar{C}^*)\bar{C}^*\left[1 + (1+\phi)\hat{N}_{e,t}^*\right]\Psi\left(\tilde{Y}_t, \tilde{Y}_t^*\right) + t.i.p + o\left(\|a\|^3\right). \qquad (1.A.17)$$

The equation (1.A.17) can be rewritten by:

$$U(C_t^*) = \alpha U'(\bar{C}^*)\bar{C}^*\left[\tilde{Y}_t^* + \frac{1}{2}(1-\sigma)\alpha\tilde{Y}_t^{*2} + (1+\phi)\hat{N}_{e,t}^*\tilde{Y}_t^* - \frac{1}{2}\Gamma^*\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}^*\right)\right] + t.i.p + o\left(\|a\|^3\right) \qquad (1.A.18)$$

where

$$\Gamma^*\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}\right) = \left(\frac{1-\alpha}{\alpha}\right)\tilde{Y}_t\left\{(\sigma-1)\left[(1-\alpha)\tilde{Y}_t + 2\left(\alpha\tilde{Y}_t^* + \tilde{C}_{e,t}^*\right)\right] - 2\right\}. \qquad (1.A.19)$$

Similarly, around the flexible price equilibrium the disutility of the labour supply

$(V(N_t^*)$ is approximated by

$$V(N_t^*) = V'(\bar{N}^*)\bar{N}^* \left[\tilde{N}_t^* + \frac{1}{2}(1+\phi)\tilde{N}_t^{*2} + (1+\phi)\hat{N}_{e,t}^*\tilde{N}_t^*\right] + t.i.p + o\left(\|a\|^3\right) \quad (1.A.20)$$

and from $\tilde{N}_t^* = \tilde{Y}_t^* + v_t^*$ where $v_t^* = lnV_t^*$ and $V_t^* = \int_0^1 \left(\frac{P_{F,t}^*(i)}{P_{F,t}^*}\right)^{-\varepsilon} di$,

$$V(N_t^*) = V'(\bar{N}^*)\bar{N}^* \left[\left(\tilde{Y}_t^* + v_t^*\right) + \frac{1}{2}(1+\phi)\tilde{Y}_t^{*2} + (1+\phi)\hat{N}_{e,t}^*\tilde{Y}_t^*\right] + t.i.p + o\left(\|a\|^3\right).$$
$$(1.A.21)$$

Defining $\rho_t^* = \int_0^1 \left(lnP_{F,t}^*(i) - lnP_{F,t}^*\right)^2 di$, the Lemma 1.A.1 of the foreign economy implies $v_t^* \simeq \frac{\varepsilon}{2}\rho_t^*$. The equation (1.A.21) can be rewritten by:

$$V(N_t^*) = U'(\bar{C}^*)\bar{C}^* \left[\tilde{Y}_t^* + \frac{\varepsilon}{2}\rho_t^* + \frac{(1+\phi)}{2}\tilde{Y}_t^{*2} + (1+\phi)\hat{N}_{e,t}^*\tilde{Y}_t^*\right] + t.i.p + o\left(\|a\|^3\right).$$
$$(1.A.22)$$

Subtracting (1.A.22) from (1.A.18) then yields,

$$\frac{U(C_t^*) - V(N_t^*)}{U(\bar{C}^*)\bar{C}^*} = -\alpha \left[\frac{\varepsilon}{2}\rho_t^* + \frac{1}{2}\kappa^*\tilde{Y}_t^{*2} + \frac{1}{2}\Gamma^*\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}^*\right)\right], \quad (1.A.23)$$

where $\kappa^* = (1+\phi) + (\sigma - 1)\alpha > 0$.

Defining W_τ^F as the approximated value of foreign welfare $\sum_{t=\tau}^\infty \beta^{t-\tau} \left(\frac{U(C_t^*) - V(N_t^*)}{U(\bar{C}^*)\bar{C}^*}\right)$,

$$W_\tau^F = -\alpha\frac{\Lambda}{2}\sum_{t=\tau}^\infty \beta^{t-\tau} \left\{\delta\rho_t^* + \psi^*\tilde{Y}_t^{*2} + \frac{\psi^*}{\kappa^*}\Gamma^*\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}^*\right)\right\} \quad (1.A.24)$$

where $\delta = (1-\theta)(1-\beta\theta)/\theta$, $\Lambda = \varepsilon/\delta$ and $\psi^* = \delta\kappa^*/\varepsilon$. From the lemma 1.A.2, then the welfare function of the foreign economy can be illustrated as

$$W_\tau^F = -\alpha\frac{\Lambda}{2}\sum_{t=\tau}^\infty \beta^{t-\tau} \left\{\hat{\Pi}_{F,t}^{*2} + \psi^*\tilde{Y}_t^{*2} + \frac{\psi^*}{\kappa^*}\Gamma^*\left(\tilde{Y}_t, \tilde{Y}_t^*, \hat{C}_{e,t}^*\right)\right\}. \quad (1.A.25)$$

1.A.2 Proofs of Lemma 1.A.1 and Lemma 1.A.2

Lemma 1.A.1

The proof of the Lemma 1.A.1 follows Galí and Monacelli (2005). Defining $\hat{\hat{P}}_{H,t}(i) = lnP_{H,t}(i) - lnP_{H,t}$,

$$
\begin{aligned}
\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon} &= exp\left[(1-\varepsilon)\hat{\hat{P}}_{H,t}(i)\right] \\
&= 1 + (1-\varepsilon)\hat{\hat{P}}_{H,t}(i) + \frac{(1-\varepsilon)^2}{2}\hat{\hat{P}}_{H,t}(i)^2 + o\left(\|a\|^3\right).
\end{aligned}
$$

Also, from the definition of $P_{H,t}$, $1 = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon} di$. Therefore,

$$
\int_0^1 \hat{\hat{P}}_{H,t}(i)di = \frac{\varepsilon-1}{2}\int_0^1 \hat{P}_{H,t}^2 di. \tag{1.A.26}
$$

Furthermore, by a second-order approximation,

$$
\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} = 1 - \varepsilon\hat{\hat{P}}_{H,t}(i) + \frac{\varepsilon^2}{2}\hat{\hat{P}}_{H,t}(i)^2 + o\left(\|a\|^3\right) \tag{1.A.27}
$$

and combining (1.A.26) and (1.A.27) yields

$$
\begin{aligned}
\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di &= 1 - \frac{\varepsilon(\varepsilon-1)}{2}\int_0^1 \hat{P}_{H,t}^2 di + \frac{\varepsilon^2}{2}\int_0^1 \hat{\hat{P}}_{H,t}(i)^2 di + o\left(\|a\|^3\right) \\
&= 1 + \frac{\varepsilon}{2}\int_0^1 \hat{\hat{P}}_{H,t}(i)^2 di.
\end{aligned}
$$

From this, the followign equation holds:

$$
ln\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di = v_t \simeq \frac{\varepsilon}{2}\int_0^1 \left(lnP_{H,t}(i) - lnP_{H,t}\right)^2 di. \tag{1.A.28}
$$

Lemma 1.A.2

The proof of the Lemma 1.A.2 follows Woodford (2001). From Calvo (1983) pricing, with an optimal price $P_{H,t}^0$

$$ln\Pi_{H,t} = lnP_{H,t} - lnP_{H,t-1} = (1-\theta)\left(lnP_{H,t}^0 - lnP_{H,t-1}\right) \tag{1.A.29}$$

and with a zero steady state inflation rate $ln\Pi_{H,t} = \hat{\Pi}_{H,t}$.

Defining $\rho_t = \int_0^1 \left(lnP_{H,t}(i) - lnP_{H,t}\right)^2 di$,

$$\rho_t = var_i\left(lnP_{H,t}(i) - lnP_{H,t}\right) = var_i\left(lnP_{H,t}(i) - lnP_{H,t-1}\right) \tag{1.A.30}$$

and ρ_t can be rewritten by

$$
\begin{aligned}
\rho_t &= E_i\left[\left(lnP_{H,t}(i) - lnP_{H,t-1}\right)^2\right] - \left[E_i\left(lnP_{H,t}(i) - lnP_{H,t-1}\right)\right]^2 \\
&= \theta E_i\left(lnP_{H,t-1}(i) - lnP_{H,t-1}\right)^2 + (1-\theta)\left(lnP_{H,t}^0 - lnP_{H,t-1}\right)^2 - \left(lnP_{H,t} - lnP_{H,t-1}\right)^2.
\end{aligned}
$$

From (1.A.29),

$$\rho_t = \theta\rho_{t-1} + \frac{\theta}{1-\theta}\hat{\Pi}_{H,t}^2 \tag{1.A.31}$$

and solving backward yields

$$\rho_t = \theta^t\rho_0 + \frac{\theta}{1-\theta}\sum_{s=0}^t \theta^{t-s}\hat{\Pi}_{H,s}^2. \tag{1.A.32}$$

Since

$$
\begin{aligned}
\sum_{t=0}^\infty \beta^t\left(\sum_{s=0}^t \theta^{t-s}\hat{\Pi}_{H,s}^2\right) &= \left(\hat{\Pi}_{H,0}^2 + \beta\theta\hat{\Pi}_{H,0}^2 + \cdots\right) + \beta\left(\hat{\Pi}_{H,1}^2 + \beta\theta\hat{\Pi}_{H,1}^2 + \cdots\right) + \cdots \\
&= \frac{1}{1-\beta\theta}\sum_{t=0}^\infty \beta^t\hat{\Pi}_{H,t}^2,
\end{aligned}
$$

ignoring ρ_0, eventually the following equation holds:

$$\sum_{t=0}^\infty \beta^t\rho_t = \frac{\theta}{(1-\theta)(1-\beta\theta)}\sum_{t=0}^\infty \beta^t\hat{\Pi}_{H,t}^2 = \frac{1}{\delta}\sum_{t=0}^\infty \beta^t\hat{\Pi}_{H,t}^2. \tag{1.A.33}$$

1.A.3   Data Sources, Test Results and Lag Selection

Table 1.6: Datastream Codes for VAR and Probit Model Analyses

Country	CPI	PPI	IP	Short rate
Australia	AUCCPIP%E	-	AUCIND..G	S32523
Canada	CNCCPIP%E	CNPROPRCF	CNIPTOT.D	CNI60B..
China	CHCCPIP%E	-	-	CHYINTER
Chile	CLCCPIP%E	-	CLIPMAN.H	CLI60...
Czech	CZCCPIP%E	CZPROPRCF	CZCIND..G	CZINTER3
Denmark	DKCCPIP%E	DKPROPRCF	DKCIND..G	DKESSFON
India	INCCPIP%E	-	INCIND..G	INGBILL
Indonesia	IDCCPIP%E	IDPROPRCF	IDCIND..G	IDINTER3
South Korea	KOCCPIP%E	KOPROPRCF	KOCIND..G	KODPNNCD
Malaysia	MYCCPIP%E	MYPROPRCF	MYIPTOT.H	MYINTER3
New Zealand	NZCCPIP%E	-	-	NZINTER3
Norway	NWCCPIP%E	NWPROPRCF	NWCIND..G	NWINTER3
South Africa	SACCPIP%E	-	SACIND..G	SAMIR076R
Sweden	SDCCPIP%E	SDPROPRCF	SDCIND..G	SDINTER3
Thailand	THCCPIP%E	THPROPRCF	THCIND..G	THIBK30D
UK	UKCCPIP%E	UKPROPRCF	UKCIND..G	UKAAMIJ.R
US	USCCPIP%E	USPROPRCE	USCIND..G	USGBILL3
EU	EMEBCPALE	EKPROPRCF	EKIPTOT.G	EMINTER3

Country	Policy rate	ExR(USD)	ExR(Euro)
Australia	AUPRATE.	AUXRUSD.	BDAUDEURM
Canada	CNPRATE.	CNXRUSD.	BDCADEURN
China	CHPRATE.	-	-
Chile	CLPRATE.	CLXRUSD.	CLXREUR.
Czech	CZPRATE.	CZXRUSD.	CZXREUR.
Denmark	DKPRATE.	DKXRUSD.	DKXREUR.
India	INPRATE.	INXRUSD.	INXREUR.
Indonesia	IDPRATE.	INXRUSD.	BDIDREURC
South Korea	KOPRATE.	KOXRUSD.	BDKRWEURC
Malaysia	MYPRATE.	MYXRUSD.	BDMYREURC
New Zealand	NZPRATE.	-	-
Norway	NWPRATE.	NWXRUSD.	BDNOKEURC
South Africa	SAPRATE.	SAXRUSD.	BDZAREURA
Sweden	SDPRATE.	SDXRUSD.	BDSEKEURC
Thailand	THPRATE.	THXRUSD.	BDTHBEURC
UK	UKPRATE.	UKXRUSD.	S96458
US	USPRATE.	-	-
EU	BDSU0200A	-	-

Table 1.7: Unit Root Test Results: t-statistics of ADF Tests

Country	ΔR	IP	$\Delta \mathcal{E}_D$	$\Delta \mathcal{E}_E$	Π_{CP}	Π_{PP}
Australia	-10.06	-6.22	-9.12	-13.08	-3.93	-
Canada	-4.75	-4.27	-9.66	-11.58	-12.44	-9.89
Chile	-4.82	-4.03	-10.66	-11.12	-4.99	-
Czech	-7.81	-2.73	-10.55	-10.99	-6.98	-8.26
Denmark	-5.64	-3.23	-10.14	-12.28	-12.06	-8.56
India	-10.48	-3.54	-9.91	-9.91	-11.03	-
Indonesia	-14.78	-12.42	-10.20	-14.23	-11.08	-10.17
Korea	-8.21	-4.49	-14.19	-13.93	-11.24	-7.13
Malaysia	-6.61	-4.41	-12.29	-12.99	-9.49	-6.24
Norway	-5.96	-9.56	-13.23	-14.10	-9.81	-13.13
South Africa	-8.07	-3.85	-9.82	-14.35	-3.12	-
Sweden	-5.61	-3.13	-12.75	-14.59	-10.94	-12.76
Thailand	-6.81	-5.32	-12.02	-13.99	-10.09	-8.90
UK	-8.15	-4.18	-10.10	-11.36	-10.02	-7.34
US	-2.55	-4.56	-	-	-9.35	-9.41
Eurozone (EZ)	-5.74	-4.27	-	-	-9.67	-5.30

* 10% significance level is 2.57

Table 1.8: Time Lag (p) Selection at the SVAR Analysis: Based on AIC

Model	p	Model	p	Model	p	Model	p
US → Australia	4	Australia →US	2	EZ → Australia	3	Australia →EZ	2
US → Canada	2	Canada→US	4	EZ → Canada	1	Canada→EZ	2
US → Chile	2	Chile→US	5	EZ → Chile	3	Chile→EZ	2
US → Czech	2	Czech→US	2	EZ → Czech	1	Czech→EZ	4
US → Denmark	3	Denmark→US	5	EZ → Denmark	3	Denmark→EZ	2
US → India	2	India→US	3	EZ → India	2	India→EZ	2
US → Indonesia	3	Indonesia→US	5	EZ → Indonesia	3	Indonesia→EZ	3
US → Korea	4	Korea→US	4	EZ → Korea	2	Korea→EZ	4
US → Malaysia	2	Malaysia→US	4	EZ → Malaysia	2	Malaysia→EZ	2
US → Norway	2	Norway→US	4	EZ → Norway	2	Norway→EZ	2
US → South Africa	2	South Africa→US	5	EZ → South Africa	2	South Africa→EZ	4
US → Sweden	3	Sweden→US	4	EZ → Sweden	3	Sweden→EZ	3
US → Thailand	2	Thailand→US	3	EZ → Thailand	1	Thailand→EZ	1
US → UK	2	UK→US	5	EZ → UK	1	UK→EZ	2

1.A.4 Seasonal Adjustment using X-12 ARIMA

In the X-12 ARIMA method, the Regression-ARIMA model takes the form of $y_t = \sum_i \beta_i x_{i,t} + z_t$. y_t is an initially adjusted time series[29] and $x_{i,t}$ is a regressor, such as calendar effects and additive outliers. β_i is the coefficient for $x_{i,t}$, and z_t follows an ARIMA process as below:

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D z_t = \theta(B)\Theta(B^s)a_t \qquad (1.A.34)$$

where B is a backshift operator ($Bz_t = z_{t-1}$), and s is the seasonal period which is 4 for quarterly data in this study. $\phi(B)$ is the nonseasonal autoregressive (AR) operator ($\phi(B) = (1-\phi_1 B - \cdots - \phi_p B^p)$), and $\Phi(B^s)$ is the seasonal operator ($\Phi(B^s) = (1-\Phi_1 B^s - \cdots - \Phi_P B^{sP})$). Also, $\theta(B)$ ($= (1-\theta_1 B - \cdots - \theta_q B^q)$) and $\Theta(B^s)$ ($= (1-\Theta_1 B^s - \cdots - \Theta_Q B^{sQ})$) are nonseasonal and seasonal moving average (MA) operators, respectively. d and D represent the nonseasonal and the seasonal differencing orders. a_t denotes a white noise term. The optimal $(p,d,q)(P,D,Q)$ for each country's data is then selected according to the Akaike Information Criterion (AIC) statistic (Table 1.9).

X-12 ARIMA then performs seasonal adjustment using the X-11 method which decomposes the adjusted original data (Y_t) into a trend (T_t), a seasonal component (S_t), and an irregular component (I_t). Since the original data are in a percentage deviation form, additive decompositions are used in this study[30], which implies $Y_t = T_t + S_t + I_t$. The initial estimate of the trend ($T_t^{(1)}$) can be estimated by a moving average (MA) method, and the detrended time series ($SI_t^{(1)}$) can be obtained by $SI_t^{(1)} = Y_t - T_t^{(1)}$. The initial seasonal MA series of the detrended data can be then obtained, which is denoted by $\hat{S}_t^{(1)}$[31]. Defining the initial seasonal component $S_t^{(1)} = \hat{S}_t - (\text{MA of } \hat{S}_t)$, the preliminary seasonally adjusted series ($A_t^{(1)}$) can be obtained from $A_t^{(1)} = Y_t - S_t^{(1)}$. After this initial estimation,

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[29]$y_t = ln(Y_t/D_t)$ where Y_t is the original data and D_t is an optional adjustment term for known external effects.

[30]For series of positive values such as sales and exports, a multiplicative decomposition is used, where the form is $Y_t = T_t S_t I_t$.

[31]Following Findley, et. al. (1998), for the quarterly data of this study the seasonal (centered) MA of $SI_t^{(1)}$ can be obtained by

$$\hat{S}_t^{(1)} = \tfrac{1}{9}SI_{t-8}^{(1)} + \tfrac{2}{9}SI_{t-4}^{(1)} + \tfrac{3}{9}SI_t^{(1)} + \tfrac{2}{9}SI_{t+4}^{(1)} + \tfrac{1}{9}SI_{t+8}^{(1)}.$$

Table 1.9: Identification of ARIMA Model Orders

(PPI)

Country	$(p,d,q)(P,D,Q)$	Country	$(p,d,q)(P,D,Q)$
Canada	(0 1 1)(0 1 1)	Norway	(2 1 2)(0 1 1)
Czech	(2 1 0)(0 1 1)	South Korea	(2 1 0)(0 1 1)
Denmark	(2 1 0)(0 1 1)	Sweden	(0 1 1)(0 1 1)
Indonesia	(0 1 1)(0 1 1)	Thailand	(2 1 2)(0 1 1)
Malaysia	(2 1 0)(0 1 1)	UK	(2 1 0)(0 1 1)

* Note: CPI data are used for Australia, Chile, India, and South Africa

(IP)

Country	$(p,d,q)(P,D,Q)$	Country	$(p,d,q)(P,D,Q)$
Chile	(2 1 0)(0 1 1)	Malaysia	(2 1 2)(0 1 1)

* Note: Only two countries data are not seasonally adjusted for industrial production.

the process above is repeated.

1.A.5   Two-step Procedures of Ordered Probit Analyses

In the equations (1.4.5) and (1.4.6), central banks respond to the expected current period values of the output gap and domestic inflation. Since the expected values are different from realized ones, in the probit model analysis, two-stage methods are used: 1) constructing the expected values of the output gap and inflation (CPI) under the rational expectations (RE) hypothesis[32], and 2) estimating the probit model. This is similar to the two-stage estimation in McCallum (1976) and Fair (1993), except that in the second stage the ordered probit model is estimated. Since the second step is illustrated in Section 1.4.3, in this appendix the RE models estimation and RE test results are presented.

The central bank forms its expectation based on an available information set (Ω_t). In this model, the information set available at time t is assumed to consist of the $t-1$ and $t-2$ data of CPI inflation (π^{CP}), PPI inflation (π^{PP}), and the output gap, and the $t-1$ data of the nominal exchange rate change ($\Delta\mathcal{E}$). The model for expectations establishment is given by:

$$
\begin{aligned}
x_t \;=\; & \gamma_0 + \gamma_1 \pi_{t-1}^{CP} + \gamma_2 \pi_{t-2}^{CP} + \gamma_3 \tilde{Y}_{t-1} + \gamma_4 \tilde{Y}_{t-2} \\
& + \gamma_5 \pi_{t-1}^{PP} + \gamma_6 \pi_{t-2}^{PP} + \gamma_7 \Delta\mathcal{E}_{t-1}^{USD} + \gamma_8 \Delta\mathcal{E}_{t-1}^{Euro} + \nu_t
\end{aligned}
\tag{1.A.35}
$$

where x_t is the realized value of either π_t^{CP} or $\tilde{Y}_t$, and ν_t is a prediction error. Using only the variables with significant coefficients, expectations are constructed ($\hat{x}_t = x_t^e$), where x_t^e denotes the predicted value of the output gap or inflation. The estimation results are illustrated by Table 1.10.

Next, RE tests are conducted. Following Lovell (1986), in this appendix, the tests check whether all the variables in the information set are uncorrelated with the forecast error. In the test, the forecast errors ($x_t - x_t^e$) are regressed on the entire information set. For *strong rationality*, all the coefficients should not differ significantly from zero, and Table 1.11 indicates that RE assumptions are satisfied, where $H_0 : \gamma_0 = 0, \cdots, \gamma_8 = 0$.

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[32]In this chapter, for *rationality*, it is required that *the prediction error must be uncorrelated with the entire set of information that is available to the forecaster at the time the prediction is made* (Lovell, 1986, p.113).

Table 1.10: Rational Expectation Estimation Results: significant coefficients

(Inflation)	c	π^{CP}_{t-1}	π^{CP}_{t-2}	$\tilde{Y}_{t-1}$	$\tilde{Y}_{t-2}$	π^{PP}_{t-1}	π^{CP}_{t-2}	$\mathcal{E}^{USD}_{t-1}$	$\mathcal{E}^{Euro}_{t-1}$
US	0.135	0.371	-0.243			0.073			
EZ	0.102	0.305		0.009					
Australia	0.060	0.721		-0.011					
Canada	0.164		-0.166			0.138		-0.031	
Chile	0.150	0.446			0.017				
Czech	0.104	0.175	0.184						
Denmark	0.125					0.074	0.066		0.415
India	0.473	0.172							
Indonesia	0.457	0.238							
S.Korea	0.237		-0.213			0.199			
Malaysia	0.128	0.259			0.016	0.082			
Norway	0.132	0.350	-0.206						
S.Africa	0.254	0.444		0.021				0.010	
Sweden	0.080	0.197		0.009				-0.016	
Thailand	0.136	0.207				0.062			
UK	0.133					0.342			
(Output gap)	c	π^{CP}_{t-1}	π^{CP}_{t-2}	$\tilde{Y}_{t-1}$	$\tilde{Y}_{t-2}$	π^{PP}_{t-1}	π^{CP}_{t-2}	$\mathcal{E}^{USD}_{t-1}$	$\mathcal{E}^{Euro}_{t-1}$
US	-0.112	0.505		1.093	-0.154				
EZ	-0.253	1.613		0.915					
Australia	-0.070		0.312	1.618	-0.728				
Canada				0.903			0.282		
Chile		1.585		0.178	0.253			-1.355	-0.158
Czech				0.630	0.235	0.842			
Denmark				0.495	0.240				
India				0.409	0.311				
Indonesia	0.500		-1.403		0.162		0.434		
S.Korea		-1.231		0.734		1.299		-0.079	
Malaysia				0.387	0.195	0.234	0.456		
Norway				0.354					
S.Africa				0.588	0.224				
Sweden	-0.492	2.364	2.276	0.250	0.392				
Thailand	-0.651	2.174		0.682			0.489	-0.343	
UK				0.700	0.160			-0.064	

Note: Coefficients that are significant at the 10 percent level

Table 1.11: Rational Expectation Test Results: t-values of coefficients

(Inflation)	c	π^{CP}_{t-1}	π^{CP}_{t-2}	$\tilde{Y}_{t-1}$	$\tilde{Y}_{t-2}$	π^{PP}_{t-1}	π^{CP}_{t-2}	$\mathcal{E}^{USD}_{t-1}$	$\mathcal{E}^{Euro}_{t-1}$
US	0.641	0.064	-1.289	0.817	-0.519	-0.247	1.428	-	-
EZ	0.195	-1.174	0.295	0.669	-1.018	1.538	-0.122	-	-
Australia	0.568	1.285	-1.770	-0.197	0.337	-	-	-0.882	1.140
Canada	0.917	-1.621	-0.198	1.374	-0.969	0.679	-0.302	-0.777	0.708
Chile	0.476	-0.057	-1.106	0.599	0.140	-	-	0.329	1.411
Czech	0.836	-1.149	-1.011	0.814	0.483	0.995	0.228	0.009	-0.534
Denmark	-0.611	0.426	0.899	1.281	-1.034	-0.309	-0.458	-0.907	0.023
India	1.477	-0.711	-0.117	-1.405	0.858	-	-	-1.777	1.794
Indonesia	0.260	-0.249	-1.172	-0.245	0.505	0.606	0.886	1.297	-0.803
S.Korea	-0.391	0.881	-0.390	-0.140	-0.079	-0.669	0.803	1.215	-0.284
Malaysia	0.433	0.203	-1.572	-0.612	0.510	-0.028	1.282	0.508	0.025
Norway	-0.133	-0.888	0.137	0.248	0.815	1.910	0.561	-1.283	0.287
S.Africa	-0.580	-0.689	1.437	-0.488	0.406	-	-	-0.104	0.239
Sweden	-0.315	-0.225	0.689	-0.088	-0.112	0.770	-0.369	-0.164	0.282
Thailand	-0.216	-0.174	0.797	-0.055	0.616	-0.326	0.096	0.828	1.054
UK	-0.406	0.166	0.306	0.187	-0.542	-0.831	1.090	-1.155	-0.389
(Output gap)	c	π^{CP}_{t-1}	π^{CP}_{t-2}	$\tilde{Y}_{t-1}$	$\tilde{Y}_{t-2}$	π^{PP}_{t-1}	π^{CP}_{t-2}	$\mathcal{E}^{USD}_{t-1}$	$\mathcal{E}^{Euro}_{t-1}$
US	0.327	-1.609	0.672	-0.539	0.520	1.884	-0.086	-	-
EZ	0.821	-1.320	-0.883	-0.783	0.471	1.788	1.076	-	-
Australia	0.144	0.062	-0.218	-0.121	0.223	-	-	-1.666	0.868
Canada	-0.351	-0.013	-0.872	0.808	-0.832	-0.127	1.378	-1.334	0.565
Chile	-1.006	0.114	0.686	-0.213	-0.221	-	-	0.264	0.147
Czech	-1.310	-0.365	1.139	0.001	-0.218	0.163	0.385	0.260	0.447
Denmark	-1.735	1.145	0.886	-0.342	0.133	0.913	0.755	0.553	-0.604
India	-0.794	1.188	-0.046	-0.172	0.233	-	-	-0.888	1.008
Indonesia	0.285	-0.680	0.165	0.578	-0.319	-0.197	0.172	0.027	0.689
S.Korea	0.516	-0.483	-0.164	0.238	-0.481	-0.211	0.730	-0.510	0.849
Malaysia	-1.274	0.611	0.934	-0.314	-0.095	0.081	-0.398	0.110	-1.265
Norway	0.189	-1.107	0.345	0.449	-0.528	0.971	-0.188	0.946	-1.556
S.Africa	-1.581	1.470	0.329	-0.090	-0.451	-	-	-0.425	1.462
Sweden	0.031	-0.294	0.080	-0.011	0.061	0.407	-0.131	0.129	-1.154
Thailand	-0.226	-0.186	0.429	0.834	-1.250	0.743	-0.499	0.260	-1.228
UK	-0.621	0.214	-0.510	-0.398	-0.027	0.836	0.746	0.434	-0.769

Chapter 2

Globalization and the Leader-Follower Relationship between Monetary Policies

2.1 Introduction

The monetary policy of the US Federal Reserve (Fed) has widespread international effects on the monetary policy decisions of other central banks. Since the beginning of the financial crisis in the late 2000s, many central banks sequentially lowered interest rates following the policy changes of the US. The initial policy rate cut of the US was in 2007:Q3. Canada and the UK then followed it in the next quarter. In 2008:Q3, Australia and New Zealand started to lower their policy rates, and in 2008:Q4, the Eurozone and many other economies began expansionary policies which included both advanced and emerging economies, such as Indonesia, South Korea, Malaysia, Norway, Poland, South Africa, Sweden, Thailand, and others.

Figure 2.1 indicates that this was not the first time that central banks had followed the policy decisions of the major central banks. In the early 2000s, many central banks lowered interest rates immediately after the rapid monetary expansion
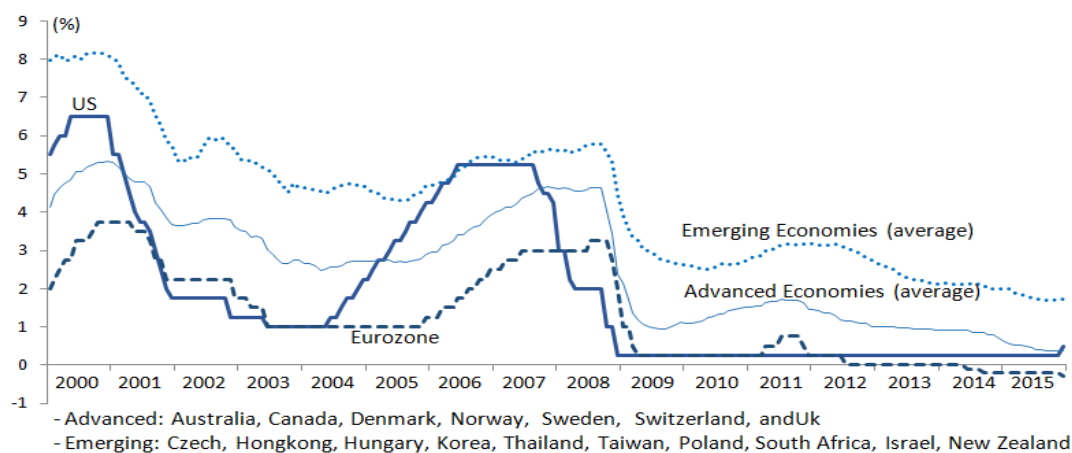
Figure 2.1: Policy Rates since 2000: US, Eurozone, and other economies

of the US in response to the IT bubble collapse and the 9/11 incident. In the mid-2000s, triggered by the policy change of the US again, central banks raised their policy rates. The recent policy changes of central banks during the financial crisis were also stimulated by the policy decisions of the Eurozone, which suffered from a fiscal crisis. Since the co-movements of central banks' policy rates were mostly led by the US or the Eurozone policy changes, their relationships have been in the form of the leader-follower relationship.

This chapter investigates the reason for the asymmetric relationship between policy rates, with a focus on the effects of economic globalization. A small open economy dynamic stochastic general equilibrium (DSGE) model is established. There are two economies: home and foreign. The home economy is a small open economy, and the foreign economy represents the rest of the world.

The model analysis indicates that, when the foreign policy rate is lowered, the home central bank also cuts its policy rate. Provided that there is a large economy in reality with a fairly low degree of openness as in this small open economy model, the relationship between policy rates will be asymmetric. The simulation results also reveal the factors that strengthen the leader-follower policy relationship. Globalization in financial markets, economic integration, and the central bank's enhanced

role in inflation targeting make the policy relationship stronger.

When the foreign policy rate is lowered, the home currency appreciates following the modified uncovered interest rate parity (UIP) in this model. Consequently, the relative export price rises and the relative import price decreases. These price changes then reduce the demand for home produced goods, which leads to a fall in exports and an increase in imports (expenditure switching effect). The decline in net exports makes home output fall. Moreover, the drop in the import price leads to an initial decline in the home inflation rate. Following a modified Taylor rule[1], the home central bank lowers the policy rate.

The sensitivity analyses suggest that the positive correlation between the home and the foreign policy rates is stronger when (i) the international assets holding cost facing the home economy is lower, (ii) openness of the home country is higher, (iii) the home central bank is more active in inflation targeting, and (iv) the home monetary policy responds to the real exchange rate change. The trend towards globalization in financial markets and economic integration magnifies the fluctuations in home output, inflation, and the home currency value. This leads to a more aggressive policy reaction of the home central bank.

The banking friction also has a significant role in determining the policy rates relationship. A novel feature of the model in this chapter is the extension of the banking friction framework of Gertler and Karadi (GK, 2011) to an open economy model[2]. In this respect, the model differs from other studies based on the framework of Bernanke, Gertler and Gilchrist (1999), which focus on the agency problem in the non-financial sector. In this chapter, the banking friction strengthens the co-movements of the home and the foreign policy rates. The financial accelerator

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[1]Under the small open economy assumption in this chapter, the home monetary policy responds to the changes in home output, inflation, and the real exchange rate, as in Adolfson *et al.* (2007).

[2]Similarly Dedola, Karadi and Lombardo (2013) extend the banking friction model of GK (2011) to an open economy framework, focusing on the international spillovers of the real economic shocks.

amplifies the increase in investment and the fall in net exports, thus resulting in a further decrease in the home agents' foreign assets holding. As the home currency becomes more appreciated following the modified UIP, home inflation remains lower. Eventually, the home policy rate becomes closer to the foreign policy rate.

This chapter is organized as follows. In the following section, the related literature is discussed. In Section 2.3, a small open economy model is illustrated. Section 2.4 presents estimation and calibration results. Impulse responses and variance decomposition results are analysed in Section 2.5. Section 2.6 then concludes.

2.2 Literature Review

There are numerous empirical investigations of the policy relationship between central banks. Maćkowiak (2006) indicates that the US monetary policy affects the short-term rates of other countries, and Bergin and Jordà (2004) show the European countries' significant responses to the US and German monetary policies before 1998. Clarida, Galí and Gertler (1998) also investigate the influence of German policy on the policies of the UK, France and Italy during the period spanning 1979-1993. In a recent study, Kucharčuková, Claeys and Vašíček (2014) identify the immediate policy changes of non-EU European countries following the policy of the European Central Bank (ECB).

Cross-border monetary policy relationships are related to the optimal policy in open economies. Indeed, there exist influential studies that explore the optimal monetary policy rules in open economy models. For instance, Ball (1998), Corsetti and Pesenti (2005) and De Paoli (2009) conclude that the optimal policy needs to focus on reducing the volatility of the exchange rate as well as domestic variables - such as output and the inflation rate. On the other hand, Galí and Monacelli (2005) and Batini, Harrison and Millard (2003) argue that domestic inflation targeting is optimal even in an open economy.

There is a substantial body of literature that analyse the issue of global monetary policy interdependence within a cooperative framework (Benigno and Benigno, 2006; Pappa, 2004). However, as Dominquez (1996) indicates, policy makers normally try to maximize domestic welfare at the expense of global welfare and there is no incentive to boost foreign welfare. Furthermore, for some large economies, the degrees of openness are too low to achieve gains from the policy cooperation. Coenen *et al.* (2010) also indicate that for the US the gains from the monetary policy coordination are small due to its low degree of openness.

A stream of literature expands the financial accelerator framework of Bernanke, Gertler and Gilchrist (BGG, 1999) to the open economy environment (Davis and Huang, 2011; Faia, 2007; Gertler, Gilchrist and Natalucci, 2007; Kolasa and Lombardo, 2014; Unsal, 2013). These studies focus on the agency problem in the non-financial sector, rather than that in the banking sector. Even though Bruno and Shin (2013) and Hwang (2012) analyse banking frictions in open economies, the anlaysis of Bruno and Shin (2013) is not based on a DSGE framework and Hwang (2012) assumes a lending cost which is not caused by the agency problem. Only a few studies, such as Dedola, Karadi and Lombardo (2013), incorporate the banking friction model of Gertler and Karadi (2011) into open economy frameworks.

2.3 A Small Open Economy Model

2.3.1 Model Description

The theoretical framework consists of a general equilibrium small open economy model. There are two economies: home and foreign. The foreign economy can be interpreted as the rest of the world. International variables, such as foreign inflation and the foreign interest rate, are exogenously given. In the home economy, households can purchase both home and foreign assets by holding deposits. As in Benigno (2009), home assets cannot be traded in international markets, since the

home currency is not a global currency.

There are seven types of agents in the home economy: households, financial intermediaries, the central bank, the government, capital producers, and final and intermediate goods producers. Intermediate goods are produced with capital and labour inputs. Final goods are produced by combining home intermediate goods and imported foreign intermediate goods. These final goods are purchased for consumption by households, investment by capital producers, and government spending. Households provide labour to the intermediate goods producers - receiving wages. They also get the dividends (cash flows) from goods/capital producers and financial intermediaries. The government imposes income taxes on the households, and the central bank sets the nominal risk free interest rate.

Home financial intermediaries obtain funds from household deposits. They purchase claims on intermediate goods producers - transferring funds from the households to the producers. As in Gertler and Karadi (2011), the financial intermediaries face borrowing constraints due to the agency problem. They can divert a fraction of funds during each period.

2.3.2 Intermediate Goods Producers

Each home intermediate good firm i produces a differentiated good $Y_t(i)$ with a Cobb-Douglas technology. $K_t(i)$ and $L_t(i)$ denote the amounts of capital stock and labour that are used for production. The variable A_t is the level of productivity which is common to all firms. The production function is as below:

$$Y_t(i) = A_t K_t(i)^\psi L_t(i)^{1-\psi}. \tag{2.3.1}$$

Intermediate goods firms borrow funds from home financial intermediaries by issuing claims (S_t) to them. They purchase capital stock (K_{t+1}) from the capital producer for production next period. As in Gertler and Karadi (2011), the number
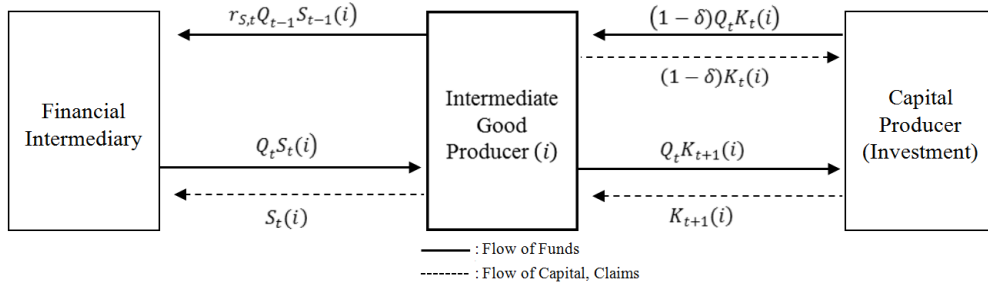
Figure 2.2: Flow of Funds after Production; Intermediate Good Producer (i)

of the claims issued by the firm i is equivalent to the amount of capital stock it purchases $(S_t(i) = K_{t+1}(i))$ and the price of one unit of the claim equals the capital price. Thus, given a relative price of capital Q_t in units of final goods, the following equation holds:

$$Q_t S_t(i) = Q_t K_{t+1}(i). \tag{2.3.2}$$

After production in period t, the intermediate good firm i pays back $r_{S,t}Q_{t-1}S_{t-1}(i)$ to the financial intermediaries, where $r_{S,t}$ denotes the gross real return of each claim. In order to repay $r_{S,t}Q_{t-1}S_{t-1}(i)$, the firm i resells the used and depreciated capital, $(1-\delta)K_t$, to the capital producer with the price of Q_t. The parameter δ denotes the depreciation ratio. Since $S_{t-1}(i) = K_t(i)$, the real cost of using $K_t(i)$ in production is illustrated by $[r_{S,t}Q_{t-1} - (1-\delta)Q_t]K_t(i)$. Define the user cost of one unit of capital as $r_{K,t} = r_{S,t}Q_{t-1} - (1-\delta)Q_t$. Given the real wage (W_t), then the total real cost of production becomes $r_{K,t}K_t(i) + W_tL_t(i)$. Figure 2.2 illustrates the flows of funds regarding debt repayment and the capital purchase.

Since all firms face same input prices with an identical technology, the real marginal cost (MC_t) is the same across firms. Given the constant-returns-to-scale technology, the optimality condition implies:

$$\frac{L_t(i)}{K_t(i)} = \frac{1-\psi}{\psi}\frac{r_{K,t}}{W_t} \tag{2.3.3}$$

$$MC_t = \frac{1}{A_t\psi^{\psi}(1-\psi)^{1-\psi}}r_{K,t}^{\psi}W_t^{1-\psi}. \tag{2.3.4}$$

The intermediate good of the firm i is either purchased in the home economy or exported abroad: $Y_t(i) = Y_{H,t}(i) + Y_{H,t}^*(i)$. $Y_{H,t}(i)$ denotes the amount sold in the home market and $Y_{H,t}^*(i)$ is the amount purchased in the foreign market (home exports)[3]. The demand function for an individual intermediate good is determined by cost minimization of the final goods producers in the home and the foreign economies. Define $P_{H,t}(i)$ and $P_{H,t}^*(i)$ as the prices of home produced goods in the home and the foreign economies, respectively. The demand function for the good i in each economy is then illustrated as:

$$Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t} \qquad\qquad Y_{H,t}^*(i) = \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*}\right)^{-\varepsilon} Y_{H,t}^* \qquad\qquad (2.3.5)$$

where ε denotes the elasticity of substitution among individual intermediate goods in both home and foreign markets. $Y_{H,t}$ and $Y_{H,t}^*$ are the aggregate demands for home goods in those two economies. $P_{H,t}$ and $P_{H,t}^*$ denote the aggregate prices. The aggregate demands and prices follow the aggregator form of Dixit and Stiglitz (1977):

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad\qquad P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$$

$$Y_{H,t}^* = \left[\int_0^1 Y_{H,t}^*(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad\qquad P_{H,t}^* = \left[\int_0^1 P_{H,t}^*(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$$

Symmetrically, the demand functions for the foreign intermediate good j in the home and the foreign economies are

$$Y_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon} Y_{F,t} \qquad\qquad Y_{F,t}^*(j) = \left(\frac{P_{F,t}^*(j)}{P_{F,t}^*}\right)^{-\varepsilon} Y_{F,t}^* \qquad\qquad (2.3.6)$$

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[3]For quantity variables (inputs and outputs), the subscript H or F denotes the country of production. An asterisk indicates foreign consumption/use, while the lack of an asterisk indicates home consumption/use. Prices denominated in foreign currency are indicated with an asterisk.

where $P_{F,t}(j)$ and $P^*_{F,t}(j)$ denote the prices of the foreign intermediate good j in the home and the foreign economies, respectively. $P^*_{F,t}(j)$ is denominated in foreign currency. The aggregators are as follow:

$$Y_{F,t} = \left[\int_0^1 Y_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{1-\varepsilon}} \qquad\qquad P_{F,t} = \left[\int_0^1 P_{F,t}(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}$$

$$Y^*_{F,t} = \left[\int_0^1 Y^*_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{1-\varepsilon}} \qquad\qquad P^*_{F,t} = \left[\int_0^1 P^*_{F,t}(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}.$$

Following a classical view of the New Open Economy Macroeconomics literature (i.e. Obstfeld and Rogoff, 1995b), home firms set export prices in home currency, which means producer currency pricing (PCP). Firms choose identical prices for both domestically purchased goods and exported goods. Also, as in Obstfeld and Rogoff (1995b), there are no trade costs or trade barriers. Assuming demand elasticities are constant and identical across borders, the law of one price (LOOP) holds (Corsetti, Dedola and Leduc, 2010). This implies $P_{H,t}(i) = \mathcal{E}_t P^*_{H,t}(i)$, where $\mathcal{E}_t$ denotes the nominal exchange rate in units of home currency per unit of foreign currency. With the LOOP, the aggregate price indices for domestically purchased goods ($P_{H,t}$) and exported goods ($P^*_{H,t}$) have a relationship as below:

$$P_{H,t} = \mathcal{E}_t P^*_{H,t}. \tag{2.3.7}$$

Assuming symmetric price aggregation in the foreign economy, $P_{F,t}(j) = \mathcal{E}_t P^*_{F,t}(j)$ for the foreign intermediate good j and

$$P_{F,t} = \mathcal{E}_t P^*_{F,t}. \tag{2.3.8}$$

Since the home intermediate good firm i sells its products in both home and foreign markets, its revenue in period t is the sum of the revenue from each market. The real cash flow at t is

$$\frac{P_{H,t}(i)}{P_t}Y_{H,t}(i) + \frac{\mathcal{E}_t P_{H,t}^*(i)}{P_t}Y_{H,t}^*(i) - r_{K,t}K_t(i) - W_tL_t(i). \tag{2.3.9}$$

Defining $\Phi\left(Y_{H,t}(i) + Y_{H,t}^*(i)\right)$ as the nominal cost of producing $Y_{H,t}(i) + Y_{H,t}^*(i)$, the nominal cash flow at time t can be expressed as

$$P_{H,t}(i)Y_{H,t}(i) + \mathcal{E}_t P_{H,t}^*(i)Y_{H,t}^*(i) - \Phi(Y_{H,t}(i) + Y_{H,t}^*(i)). \tag{2.3.10}$$

Following Calvo (1983), in the home market an individual intermediate good producer can adjust its price with a probability $1-\xi$ each period. As in Yun (1996), when it cannot optimally change the price, its home price rises at the steady state home inflation rate ($\bar{\Pi}$). The steady state inflation of the home and the foreign economies are assumed to be the same ($\bar{\Pi} = \bar{\Pi}^*$). Define $\tilde{P}_{H,t}$ and $\tilde{P}_{H,t}^*$ as the optimized home and foreign prices of home produced goods at time t. Also, define $P_{H,t+\tau|t}$ and $P_{H,t+\tau|t}^*$ as the prices τ periods later, if no further optimization has taken place. Since the LOOP holds, the price of home produced goods in the foreign economy is indexed to not only the steady state inflation rate ($\bar{\Pi}^*$), but also the inverse of the nominal exchange rate change[4]. Then

$$P_{H,t+\tau|t} = \bar{\Pi}^\tau \tilde{P}_{H,t} \qquad \text{and} \qquad P_{H,t+\tau|t}^* = \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+\tau}}\right)\bar{\Pi}^{*\tau}\tilde{P}_{H,t}^*. \tag{2.3.11}$$

The price stickiness parameter (ξ) and the elasticity of substitution among intermediate goods (ε) are identical in both economies. For the firm whose last price reset was at time t, the income from the exports at time $t+\tau$ would be $\mathcal{E}_{t+\tau}P_{H,t+\tau|t}^*Y_{H,t+\tau|t}^*$ in home currency, which is equivalent to $\bar{\Pi}^{*\tau}\mathcal{E}_t\tilde{P}_{H,t}^*Y_{H,t+\tau|t}^*$ following the equation (2.3.11). The home producer who has a chance to optimize its price then maximizes

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[4]Assuming the LOOP and $\bar{\Pi} = \bar{\Pi}^*$, $P_{H,t+\tau|t} = \bar{\Pi}^\tau \tilde{P}_{H,t} = \bar{\Pi}^{*\tau}\mathcal{E}_t\tilde{P}_{H,t}^*$. From $P_{H,t+\tau|t} = \mathcal{E}_{t+\tau}P_{H,t+\tau|t}^*$, $\bar{\Pi}^{*\tau}(\mathcal{E}_t/\mathcal{E}_{t+\tau})\tilde{P}_{H,t}^* = P_{H,t+\tau|t}^*$.

$$\sum_{\tau=0}^{\infty} \beta^\tau \xi^\tau E_t \left\{ D_{t,t+\tau} \left[\bar{\Pi}^\tau \tilde{P}_{H,t} \left(\frac{P_{H,t+\tau|t}}{P_{H,t+\tau}} \right)^{-\varepsilon} Y_{H,t+\tau} + \bar{\Pi}^{*\tau} \mathcal{E}_t \tilde{P}_{H,t}^* \left(\frac{P_{H,t+\tau|t}^*}{P_{H,t+\tau}^*} \right)^{-\varepsilon} Y_{H,t+\tau}^* - \Phi(Y_{t+\tau|t}) \right] \right\}$$

$$(2.3.12)$$

where $\beta^\tau D_{t,t+\tau}(= \beta^\tau \Lambda_{t,t+\tau} \frac{P_t}{P_{t+\tau}})$ is the stochastic discount factor for nominal payoffs. The real discount factor $\Lambda_{t,t+\tau}$ will be defined later. $Y_{t+\tau|t}$ denotes output at $t + \tau$ for a firm whose last price reset was at time t, which is the sum of the domestically sold goods $(Y_{H,t+\tau|t})$ and the exported home goods $(Y_{H,t+\tau|t}^*)$.

Given the LOOP, $P_{H,t+\tau|t}/P_{H,t+\tau} = P_{H,t+\tau|t}^*/P_{H,t+\tau}^*$ and $\tilde{P}_{H,t} = \mathcal{E}_t \tilde{P}_{H,t}^*$. Since $\bar{\Pi} = \bar{\Pi}^*$, the optimization problem can be rewritten by:

$$\max_{\tilde{P}_{H,t}} \sum_{\tau=0}^{\infty} \beta^\tau \xi^\tau E_t \left\{ D_{t,t+\tau} \left[\bar{\Pi}^\tau \tilde{P}_{H,t} \left(\frac{P_{H,t+\tau|t}}{P_{H,t+\tau}} \right)^{-\varepsilon} \left(Y_{H,t+\tau} + Y_{H,t+\tau}^* \right) - \Phi(Y_{t+\tau|t}) \right] \right\}$$

$$(2.3.13)$$

which yields the following optimal price equation[5]:

$$\frac{\tilde{P}_{H,t}}{P_t} = \frac{\frac{\varepsilon}{\varepsilon-1} \sum_{\tau=0}^{\infty} \tilde{\beta}^\tau E_t \left(D_{t,t+\tau} \bar{\Pi}^{-\varepsilon\tau} \Pi_{H,t,t+\tau}^\varepsilon \Pi_{t,t+\tau} Y_{t+\tau} V_{t+\tau} MC_{t+\tau} \right)}{\sum_{\tau=0}^{\infty} \tilde{\beta}^\tau E_t \left(D_{t,t+\tau} \bar{\Pi}^{(1-\varepsilon)\tau} \Pi_{H,t,t+\tau}^\varepsilon Y_{t+\tau} V_{t+\tau} \right)}$$

$$(2.3.14)$$

where $MC_{t+\tau}$ is the real marginal cost at $t+\tau$. $\tilde{\beta} = \beta\xi$ and $V_{t+\tau} = \left[\int_0^1 \left(\frac{P_{H,t+\tau}(i)}{P_{H,t+\tau}} \right)^{-\varepsilon} di \right]^{-1}$. $\Pi_{t,t+\tau}$ $(= P_{t+\tau}/P_t)$ is cumulative inflation in the home country and $\Pi_{H,t,t+\tau}$ $(= P_{H,t+\tau}/P_{H,t})$ is cumulative inflation of home produced goods.

As in Yun (1996), the aggregate price index $P_{H,t}$ evolves over time according to the recursive form below:

$$P_{H,t} = \left[\xi \left(P_{H,t-1} \bar{\Pi} \right)^{1-\varepsilon} + (1-\xi) \tilde{P}_{H,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

$$(2.3.15)$$

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[5]The appendix 2.A.3 provides the details of the derivation of the equation (2.3.14).

which can be rewritten by

$$\frac{P_{H,t}}{P_t} = \left[\xi \left(\frac{\bar{\Pi}}{\Pi_t} \right)^{1-\varepsilon} \left(\frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\varepsilon} + (1-\xi) \left(\frac{\tilde{P}_{H,t}}{P_t} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{2.3.16}$$

2.3.3 Final Goods Producers

Final goods producing firms in the home and the foreign markets produce final goods Z_t and Z_t^* by combining home and foreign intermediate goods. The final goods production functions are given by:

$$Z_t = \left[\alpha^{\frac{1}{\theta}} Y_{H,t}^{\frac{\theta-1}{\theta}} + (1-\alpha)^{\frac{1}{\theta}} Y_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{2.3.17}$$

$$Z_t^* = \left[\alpha^{*\frac{1}{\theta}} Y_{F,t}^{*\frac{\theta-1}{\theta}} + (1-\alpha^*)^{\frac{1}{\theta}} Y_{H,t}^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{2.3.18}$$

where the parameter θ denotes the intratemporal elasticity of substitution between home and foreign intermediate goods, which is identical in the home and the foreign economies. $\alpha \in (0,1)$ and $\alpha^* \in (0,1)$ represent the long-run weights of domestic goods in the home and the foreign economies, respectively. The amount of foreign final goods production (Z_t^*) is exogenously given.

For the home and the foreign (imported) intermediate goods in the home economy, cost minimization by the home final goods producers yields the following demand equations:

$$Y_{H,t} = \alpha \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} Z_t \tag{2.3.19}$$

$$Y_{F,t} = (1-\alpha) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} Z_t, \tag{2.3.20}$$

with a price index

$$P_t = \left[\alpha P_{H,t}^{1-\theta} + (1-\alpha)P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{2.3.21}$$

Dividing both sides of the equation (2.3.21) by P_{t-1} yields

$$\Pi_t = \left[\alpha\left(\Pi_{H,t}\frac{P_{H,t-1}}{P_{t-1}}\right)^{1-\theta} + (1-\alpha)\left(\Pi_{F,t}\frac{P_{F,t-1}}{P_{t-1}}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{2.3.22}$$

where $\Pi_{H,t} = P_{H,t}/P_{H,t-1}$ and $\Pi_{F,t} = P_{F,t}/P_{F,t-1}$. Also, by the definition

$$\Pi_{H,t} = \frac{P_{H,t}/P_t}{P_{H,t-1}/P_{t-1}}\Pi_t \qquad \text{and} \qquad \Pi_{F,t} = \frac{P_{F,t}/P_t}{P_{F,t-1}/P_{t-1}}\Pi_t. \tag{2.3.23}$$

Similarly in the foreign economy (with superscript *),

$$Y_{F,t}^* = \alpha^*\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\theta} Z_t^* \tag{2.3.24}$$

$$Y_{H,t}^* = (1-\alpha^*)\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\theta} Z_t^* \tag{2.3.25}$$

with a price index

$$P_t^* = \left[\alpha^* P_{F,t}^{*1-\theta} + (1-\alpha^*)P_{H,t}^{*1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{2.3.26}$$

Dividing both sides of the equation (2.3.26) by P_{t-1}^* leads to

$$\Pi_t^* = \left[\alpha^*\left(\Pi_{F,t}^*\frac{P_{F,t-1}^*}{P_{t-1}^*}\right)^{1-\theta} + (1-\alpha^*)\left(\Pi_{F,t}^*\frac{P_{F,t-1}^*}{P_{t-1}^*}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{2.3.27}$$

where $\Pi_t^* = P_t^*/P_{t-1}^*$, $\Pi_{F,t}^* = P_{F,t}^*/P_{F,t-1}^*$ and $\Pi_{H,t}^* = P_{H,t}^*/P_{H,t-1}^*$. Also,

$$\Pi_{F,t}^* = \frac{P_{F,t}^*/P_t^*}{P_{F,t-1}^*/P_{t-1}^*}\Pi_t^* \qquad \text{and} \qquad \Pi_{H,t}^* = \frac{P_{H,t}^*/P_t^*}{P_{H,t-1}^*/P_{t-1}^*}\Pi_t^*. \tag{2.3.28}$$

The foreign inflation rate is assumed to be at its long-run level ($\Pi_t^* = \bar{\Pi}^*$).

Given the LOOP ($\mathcal{E}_t P_{H,t}^* = P_{H,t}$ and $\mathcal{E}_t P_{F,t}^* = P_{F,t}$) and the definition of the real

exchange rate ($\mathcal{E}_{R,t} = \mathcal{E}_t P_t^*/P_t$), the intermediate goods' relative prices in the home and the foreign economies have the relationships as follow:

$$\frac{P_{H,t}^*}{P_t^*} = \frac{P_{H,t}}{P_t}\mathcal{E}_{R,t}^{-1} \tag{2.3.29}$$

$$\frac{P_{F,t}^*}{P_t^*} = \frac{P_{F,t}}{P_t}\mathcal{E}_{R,t}^{-1}. \tag{2.3.30}$$

Combining the equations (2.3.25) and (2.3.29), the aggregate demand for imported goods in the foreign economy ($Y_{H,t}^*$, home exports) can be rewritten as

$$Y_{H,t}^* = (1 - \alpha^*)\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\mathcal{E}_{R,t}^{\theta}Z_t^*. \tag{2.3.31}$$

Since the weights of home and foreign intermediate goods in final goods production are different across economies, the real exchange rate is not necessarily unity in the short-run (Corsetti, Dedola and Leduc, 2010). Defining $\Delta\mathcal{E}_t = \mathcal{E}_t/\mathcal{E}_{t-1}$, $\mathcal{E}_{R,t}$ evolves following the process as below:

$$\mathcal{E}_{R,t} = \bar{\Pi}^*\Delta\mathcal{E}_t\Pi_t^{-1}\mathcal{E}_{R,t-1}. \tag{2.3.32}$$

2.3.4 Households

There is a continuum of identical households in the home economy, indicated by $h \in (0,1)$. As in Gertler and Karadi (2011), there are two types of members in each household: workers and bankers. At any moment a fraction $(1 - f)$ of the members are workers, and the fraction f are bankers who are running financial intermediaries. Workers can consume and deposit money at the home and the foreign financial intermediaries. Households (Workers) supply labour to the intermediate goods firms and receive wages. Also, they purchase home and foreign assets (deposits). The household h has a preference over consumption and labour supply as follows:

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left[\frac{C_t(h)^{1-\sigma}}{1-\sigma} - \frac{L_t^s(h)^{1+\chi}}{1+\chi} \right] \tag{2.3.33}$$

where $C_t(h)$ and $L_t^s(h)$ denote individual levels of consumption and labour supply at time t, respectively. σ represents the coefficient of relative risk aversion of households or the reciprocal of the intertemporal elasticity of substitutions, and χ is the inverse of the elasticity of labour supply.

The household h faces a nominal flow budget constraint as below:

$$P_t C_t(h) + R_t^{-1} B_{H,t}^d(h) + \Gamma(b_{F,t}^d) R_t^{*-1} \mathcal{E}_t B_{F,t}^d(h) =$$
$$P_t(1-m)W_t L_t^s(h) + D_t(h) + B_{H,t-1}^d(h) + \mathcal{E}_t B_{F,t-1}^d(h) + P_t \Omega_t(h)$$

where P_t is the overall price level. R_t and R_t^* are home and foreign nominal risk-free interest rates determined by central banks. $R_t^{-1} B_{H,t}^d(h)$ and $R_t^{*-1} \mathcal{E}_t B_{F,t}^d(h)$ denote the nominal amounts of deposits in the home and the foreign financial intermediaries, respectively. $B_{F,t}^d(h)$ is denominated in foreign currency, and all deposits are for one period. $m \in (0,1)$ is an income tax ratio, and $D_t(h)$ is the sum of the dividends from intermediate goods and capital producing firms - owned by households. $\Omega_t(h)$ is the real net transfer from the financial intermediary sector, which will be explained later. In real terms, the budget constraint can be illustrated as follows:

$$C_t(h) + \left(\frac{R_t}{\Pi_{t+1}} \right)^{-1} b_{H,t}^d(h) + \Gamma(b_{F,t}^d) \left(\frac{R_t^*}{\Pi_{t+1}} \right)^{-1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} b_{F,t}^d(h) =$$
$$(1-m)W_t L_t^s(h) + D_t(h) + b_{H,t-1}^d(h) + b_{F,t-1}^d(h) + \Omega_t(h)$$

where $b_{H,t}^d(h)$ and $b_{F,t}^d(h)$ denote the real amounts of home and foreign deposits, respectively. Thus, $b_{H,t}^d(h) = B_{H,t}^d(h)/P_{t+1}$ and $b_{F,t}^d(h) = \mathcal{E}_{t+1} B_{F,t}^d(h)/P_{t+1}$. Defining $r_s = R_s/\Pi_{s+1}$ and $r_s^* = R_s^*/\Pi_{s+1}$, $\lim_{t\to\infty} \prod_{s=1}^{t} r_s^{-1} b_{H,t}^d(h) = 0$ and $\lim_{t\to\infty} \prod_{s=1}^{t} r_s^{*-1} b_{F,t}^d(h) = 0$ (no-Ponzi scheme).

As in Benigno (2001), and Basu and Thoenissen (2011), each household bears a foreign assets (deposits) holding cost, which is reflected in the function $\Gamma(\cdot)$. This function is multiplied to the original price of foreign deposits. This cost helps to

determine a well-defined steady state assets portfolio. The foreign assets holding cost increases as the home economy's total amount of foreign deposits holding $(b_{F,t}^d)$ rises; $\Gamma(b_{F,t}^d)$ is increasing in $b_{F,t}^d$. Thus, each household regards this cost as given when choosing an optimal consumption and the foreign assets holding combination. The foreign assets holding cost function $(\Gamma(b_{F,t}^d))$ is illustrated as

$$\Gamma(b_{F,t}^d) = \left[1 - \mu_T \left(\frac{b_{F,t}^d}{\bar{b}_F^d} - 1\right)\right]^{-1}. \tag{2.3.34}$$

Defining $\lambda_{M,t}$ as the Lagrange multiplier associated with the flow budget constraint, the first-order conditions facing the household h are:

$$\lambda_{M,t} = C_t(h)^{-\sigma} \tag{2.3.35}$$

$$\lambda_{M,t}(1-m)W_t = L_t^s(h)^{\chi} \tag{2.3.36}$$

$$\beta R_t E_t \left[\left(\frac{C_{t+1}(h)}{C_t(h)}\right)^{-\sigma} \frac{1}{\Pi_{t+1}}\right] = 1 \tag{2.3.37}$$

$$\beta R_t^* \Gamma(b_{F,t}^d)^{-1} E_t \left[\left(\frac{C_{t+1}(h)}{C_t(h)}\right)^{-\sigma} \frac{1}{\Pi_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right] = 1. \tag{2.3.38}$$

Combining the equations (2.3.37) and (2.3.38) yields the following modified uncovered interest rate parity (UIP) condition:

$$R_t = R_t^* \left[1 - \mu_T \left(\frac{b_{F,t}^d}{\bar{b}_F^d} - 1\right)\right] E_t \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right). \tag{2.3.39}$$

In the producer's optimization problem (2.3.12), $\Lambda_{t,t+\tau}$ is determined by the household's perceived intertemporal rate of substitution in consumption as follows:

$$\Lambda_{t,t+\tau} = \left(\frac{C_{t+\tau}(h)}{C_t(h)}\right)^{-\sigma}. \tag{2.3.40}$$

2.3.5 Financial Intermediaries

Financial intermediaries are modelled as in Gertler and Karadi (2011). There is a continuum of financial intermediaries indexed by $j \in (0, 1)$ in the home lending market. Each intermediary obtains funds from household deposits. Using the funds and its own net worth, it holds claims on intermediate goods producers. The nominal balance sheet of an individual financial intermediary j can be written as:

$$P_t Q_t S_t(j) = P_t N_t(j) + R_t^{-1} B_{H,t}(j) \qquad (2.3.41)$$

where $N_t(j)$ is the amount of the intermediary j's net worth in real terms. Q_t is the relative price of each claim which is identical across the financial intermediaries. $R_t^{-1} B_{H,t}(j)$ denotes the amount of funds borrowed from households (deposits). Defining $b_{H,t}(j) = B_{H,t}(j)/P_{t+1}$, in real terms

$$Q_t S_t(j) = N_t(j) + r_t^{-1} b_{H,t}(j) \qquad (2.3.42)$$

where

$$r_t = \frac{R_t}{\Pi_{t+1}}. \qquad (2.3.43)$$

Given the real gross return from the intermediate goods firms for each claim ($r_{S,t}$), the real profit at each period is accumulated as net worth:

$$N_t(j) = r_{S,t} Q_{t-1} S_{t-1}(j) - b_{H,t-1}(j). \qquad (2.3.44)$$

Combining (2.3.42) and (2.3.44) yields the following law of motion of net worth:

$$N_t(j) = (r_{S,t} - r_{t-1}) Q_{t-1} S_{t-1}(j) + r_{t-1} N_{t-1}(j) \qquad (2.3.45)$$

where $r_{S,t} - r_{t-1}$ represents the *excess return* on the claims.

The probability that a banker continues its business next period is ζ. The number

of bankers exiting from the financial intermediary sector is assumed to be the same as the number of new bankers each period. The exiting bankers bring final net worth back to the households; the financial intermediary j then maximizes the expected final net worth $(V_t^E(j))$ which can be expressed by

$$V_t^E(j) = E_t \sum_{\tau=t}^{\infty} (1-\zeta)\zeta^{\tau-t}\beta^{\tau+1-t}\Lambda_{t,\tau+1}N_{\tau+1}(j) \tag{2.3.46}$$

where $N_{\tau+1}(j) = (r_{S,\tau+1} - r_\tau) Q_\tau S_\tau(j) + r_\tau N_\tau(j)$. When $E_t(r_{S,\tau+1} - r_\tau)$ is positive[6] and there is no other constraint, the financial intermediary would increase its assets indefinitely. However, a moral hazard problem sets a limit on borrowing; at each period, the banker j can divert a fraction (λ) of its available funds. The banker j then exits from the banking sector with $\lambda Q_t S_t(j)$. However, in this case the banker sacrifices the expected value of the business $(V_t^E(j))$. Therefore, an incentive constraint must be satisfied in order for the depositors to be willing to supply funds to the banking sector as below:

$$V_t^E(j) \geq \lambda Q_t S_t(j). \tag{2.3.47}$$

The expected value of the banking business $(V_t^E(j))$ can be expressed by a recursive form as follows:

$$V_t^E(j) = \upsilon_t Q_t S_t(j) + \eta_t N_t(j) \tag{2.3.48}$$

where

$$\upsilon_t = E_t\left[(1-\zeta)\,\beta\Lambda_{t,t+1}(r_{S,t+1} - r_t) + \beta\Lambda_{t,t+1}\zeta x_{t+1}\upsilon_{t+1}\right] \tag{2.3.49}$$

$$\eta_t = E_t\left[(1-\zeta) + \beta\Lambda_{t,t+1}\zeta h_{t+1}\eta_{t+1}\right]. \tag{2.3.50}$$

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[6]With imperfect capital markets, the excess return can be positive due to the limits to arbitrage imposed by banking frictions (Gertler and Karadi, 2011).

v_t is the expected discounted marginal gain of expanding assets $Q_t S_t(j)$ by one unit, holding $N_t(j)$ constant; η_t indicates the expected discounted value of having one additional unit of $N_t(j)$ while holding $S_t(j)$ constant. x_t is the gross growth rate of assets, and h_t denotes the gross growth rate of net worth at time t.

$$x_t = Q_t S_t(j)/Q_{t-1} S_{t-1}(j) \qquad h_t = N_t(j)/N_{t-1}(j). \tag{2.3.51}$$

From the equation (2.3.48), the incentive constraint (2.3.47) can be rewritten by:

$$v_t Q_t S_t(j) + \eta_t N_t(j) \geq \lambda Q_t S_t(j).$$

If $\lambda \leq v_t$, the incentive constraint is not binding since the value of the banking business is always larger than the gain from diverting funds. As in Gertler and Karadi (2011), with reasonable parameters that lead to $0 < v_t < \lambda$, the constraint is assumed to bind in the equilibrium of this model. In consequence, the amount of the financial intermediary j's available funds depends positively on its net worth:

$$Q_t S_t(j) = \phi_t N_t(j) \qquad \text{where} \qquad \phi_t = \frac{\eta_t}{\lambda - v_t}. \tag{2.3.52}$$

The variable ϕ_t implies the leverage ratio of the intermediary j, which is determined such that the benefit of diverting funds is balanced by the opportunity cost.

Using the leverage ratio, the evolution of net worth of the intermediary j in (2.3.45) can be rewritten by:

$$N_t(j) = \left[(r_{S,t} - r_{t-1}) \phi_{t-1} + r_{t-1} \right] N_{t-1}(j). \tag{2.3.53}$$

From the equations (2.3.51), (2.3.52) and (2.3.53), then

$$
\begin{aligned}
h_t &= (r_{S,t} - r_{t-1}) \phi_{t-1} + r_{t-1} & \text{(2.3.54)} \\
x_t &= \frac{\phi_t}{\phi_{t-1}} \left(\frac{N_t(j)}{N_{t-1}(j)} \right) = \frac{\phi_t}{\phi_{t-1}} h_t. & \text{(2.3.55)}
\end{aligned}
$$

Since the leverage ratio (ϕ_t) does not depend on individual factors, from the equation (2.3.52), the aggregate demand for claims is determined by:

$$Q_t S_t = \phi_t N_t \qquad (2.3.56)$$

where N_t denotes aggregate net worth after new bankers enter into the banking sector. Therefore, N_t can be illustrated as the sum of existing net worth $(N_{e,t})$ and net worth of the new bankers $(N_{n,t})$ as below:

$$N_t = N_{e,t} + N_{n,t} \qquad (2.3.57)$$

Given the survival ratio ζ, from the equation (2.3.44) existing net worth can be illustrated by:

$$N_{e,t} = \zeta \left(r_{S,t} Q_{t-1} S_{t-1} - b_{H,t-1} \right) \qquad (2.3.58)$$

where $S_{t-1} = \int_0^1 S_{t-1}(j)dj$ and $b_{H,t-1} = \int_0^1 b_{H,t-1}(j)dj$. The amount of exiting bankers' net worth is then

$$N_{x,t} = (1 - \zeta) \left(r_{S,t} Q_{t-1} S_{t-1} - b_{H,t-1} \right). \qquad (2.3.59)$$

When the exiting bankers transfer terminal net worth $(N_{x,t})$ to the households, each household needs to pay income tax to the government. In order to avoid double taxation, no tax is levied on financial intermediaries when they receive the return from the claims. Given the income tax ratio m, then the actual amount of net worth transferred to the households is $(1 - m)N_{x,t}$.

The amount of new bankers' net worth $(N_{n,t})$ is assumed to be a fraction $\frac{\omega}{1-\zeta}$ of the total amount of the exiting bankers' assets, $(1-\zeta)Q_t S_{t-1}$. From this, net worth of the new bankers transferred from the households can be expressed as

$$N_{n,t} = \omega Q_t S_{t-1}. \qquad (2.3.60)$$

Aggregating individual balance sheets of (2.3.42) across all intermediaries yields $Q_t S_t = N_t + r_t^{-1} b_{H,t}$. From the equations (2.3.57) and (2.3.58), then the following equation holds:

$$Q_t S_t - r_t^{-1} b_{H,t} = \zeta \left(r_{S,t} Q_{t-1} S_{t-1} - b_{H,t-1} \right) + \omega Q_t S_{t-1}. \tag{2.3.61}$$

From the equation (2.3.53), the equation (2.3.57) can be also rewritten by:

$$N_t = \zeta \left[\left(r_{S,t} - r_{t-1} \right) \phi_{t-1} + r_{t-1} \right] N_{t-1} + \omega Q_t S_{t-1}. \tag{2.3.62}$$

2.3.6 Capital Producing Firm

After intermediate goods production at time t, a representative home capital producing firm purchases $(1 - \delta) K_t$ units of used capital from the home intermediate goods firms given the relative capital price Q_t. It produces new capital stock (K_{t+1}) via investment (I_t). The process of capital accumulation is then given by:

$$K_{t+1} = (1 - \delta) K_t + I_t. \tag{2.3.63}$$

After investment, the capital producing firm sells new capital stock (K_{t+1}) to the intermediate goods producers with the price Q_t. Therefore, from this capital trade the capital producing firm can get the real cash flow $Q_t I_t$.

For one unit of investment, the capital producer purchases $[1 + g(A_{I,t} I_t / I_{t-1})]$ of final goods. The function $g(\cdot)$ represents an investment adjustment cost and $A_{I,t}$ is an investment adjustment cost shock with an expected value of unity. The capital producing firm then solves

$$\max_{I_t} \sum_{t=\tau}^{\infty} \beta^{t-\tau} E_\tau \left\{ \Lambda_{\tau,t} \left[Q_t I_t - I_t - g\left(\frac{A_{I,t} I_t}{I_{t-1}} \right) I_t \right] \right\}.$$

The first-order condition is illustrated by

$$Q_t = 1 + g\left(\frac{A_{I,t}I_t}{I_{t-1}}\right) + \frac{A_{I,t}I_t}{I_{t-1}}g'\left(\frac{A_{I,t}I_t}{I_{t-1}}\right) - \beta E_t\left\{A_{I,t+1}\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 g'\left(\frac{A_{I,t+1}I_{t+1}}{I_t}\right)\right\} \tag{2.3.64}$$

where the investment adjustment cost is given as

$$g\left(\frac{I_t}{I_{t-1}}\right) = \frac{\mu_I}{2}\left(\frac{A_{I,t}I_t}{I_{t-1}} - 1\right)^2. \tag{2.3.65}$$

2.3.7 Government

The home government purchases G_t of final goods, and the government spending is financed by taxing labour and exiting net worth from the banking sector ($N_{x,t}$). With the income tax ratio m, the government budget constraint is

$$m(W_t L_t + N_{x,t}) = G_t. \tag{2.3.66}$$

Plugging the equation (2.3.59) into (2.3.66) then yields

$$m\left[W_t L_t + (1-\zeta)(r_{S,t}Q_{t-1}S_{t-1} - b_{H,t-1})\right] = G_t. \tag{2.3.67}$$

2.3.8 Central Bank

The home central bank adjusts the gross nominal risk-free rate (R_t, policy rate). Under the small open economy assumption, the home central bank considers not only domestic output and inflation, but also international variables[7]. Following Adolfson *et al.* (2007), then the home central bank in this model responds to the real exchange rate changes. Indeed, Hofmann and Bogdanova (2012) find that the policy rates in many countries have been below the levels suggested by the standard

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[7]A similar model with an inward-looking Taylor rule is analysed in Basu, Lee and Reinhorn (2015), which also reveals the asymmetric relationship between policy rates.

Taylor rule for more than a decade. According to He and McCauley (2013), and Taylor (2013), the deviation from the traditional rule was caused by the central banks' concerns about international variables.

The desired level of the policy rate is determined by a modified Taylor rule, and the central bank takes gradual steps toward the desired policy rate - policy rate smoothing. With a monetary policy smoothing parameter $\rho_R \in (0,1)$, the home policy rate rule is determined as follows:

$$R_t = \kappa R_{t-1}^{\rho_R} \left(\Pi_t^{\gamma_P} Y_t^{\gamma_Y} \mathcal{E}_{R,t}^{\gamma_E} \right)^{1-\rho_R} \mu_t \qquad (2.3.68)$$

where Y_t denotes aggregate home output ($\int_0^1 Y_t(i)di$), and κ is a scale parameter. The parameters γ_P, γ_Y, and γ_E represent the policy weights on the inflation rate, output, and the real exchange rate, respectively. μ_t denotes a policy shock with an expected value of one. On the other hand, the foreign policy rate (R_t^*) is exogenously given, following the small open economy assumption.

2.3.9 Market Clearing

The international asset market clearing condition is given by:

$$[1 - \Gamma(b_{F,t})]^{-1} R_t^{*-1} \mathcal{E}_t B_{F,t} - \mathcal{E}_t B_{F,t-1} = P_t N X_t \qquad (2.3.69)$$

where NX_t denotes net exports in real terms, and by definition $P_t N X_t = \mathcal{E}_t P_{H,t}^* Y_{H,t}^* - P_{F,t} Y_{F,t}$. Given the LOOP ($\mathcal{E}_t P_{H,t}^* = P_{H,t}$), the equation (2.3.69) can be rewritten as

$$[1 - \Gamma(b_{F,t})]^{-1} R_t^{*-1} \Delta \mathcal{E}_{t+1}^{-1} \Pi_{t+1} b_{F,t} - b_{F,t-1} = \frac{P_{H,t}}{P_t} Y_{H,t}^* - \frac{P_{F,t}}{P_t} Y_{F,t} \qquad (2.3.70)$$

where $b_{F,t-1} = \mathcal{E}_t B_{F,t-1}/P_t$ and $\Delta \mathcal{E}_{t+1} = \mathcal{E}_{t+1}/\mathcal{E}_t$.

The amount of the total intermediate goods produced in the home economy is equal to the sum of domestically purchased intermediate goods ($Y_{H,t}$) and exported

intermediate goods ($Y_{H,t}^*$). From the equation (2.3.5), then

$$Y_t = Y_{H,t} \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} di + Y_{H,t}^* \int_0^1 \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{-\varepsilon} di \qquad (2.3.71)$$

where

$$Y_t = \int_0^1 Y_t(i) di \qquad \text{and} \qquad Y_t(i) = Y_{H,t}(i) + Y_{H,t}^*(i).$$

Since $\frac{P_{H,t}(i)}{P_{H,t}} = \frac{P_{H,t}^*(i)}{P_{H,t}^*}$ given the LOOP, the equation (2.3.71) can be rewritten by:

$$Y_t = \left(Y_{H,t} + Y_{H,t}^* \right) V_t^{-1} \qquad \text{where} \qquad V_t = \left[\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} di \right]^{-1}. \qquad (2.3.72)$$

The amount of home final goods production (Z_t) is equivalent to the sum of households' consumption, government spending, investment, and the investment adjustment cost, which is represented by:

$$C_t + I_t + G_t + g \left(\frac{I_t}{I_{t-1}} \right) I_t = Z_t. \qquad (2.3.73)$$

The capital and the labour markets clear. The intermediate goods producers' entire demand for capital stock is equal to the sum of the equity claims purchased by the financial intermediaries. Also, the sum of the producers' labour demands is equivalent to the total labour supply of the households. These are illustrated by:

$$K_t = \int_0^1 K_t(i) di = \int_0^1 S_{t-1}(j) di \qquad (2.3.74)$$

$$L_t = \int_0^1 L_t(i) di = \int_0^1 L_t^s(h) dh \qquad (2.3.75)$$

where K_t and L_t denote aggregate capital stock and labour, respectively.

The sum of the individual deposit holding of each household h is same to the sum of the individual debt of each financial intermediary j. Therefore, the home

lending market clears as follows:

$$B_{H,t} = \int_0^1 B_{H,t}^d(h)dh = \int_0^1 B_{H,t}(j)dj, \tag{2.3.76}$$

and this can be rewritten by $b_{H,t} = \int_0^1 b_{H,t}^d(h)dh = \int_0^1 b_{H,t}(j)dj$.

2.3.10 Exogenous Variables

Home technology (A_t), the investment adjustment cost $(A_{I,t})$, the home policy rate shock (μ_t), the foreign policy rate (R_t^*) and foreign final goods production (Z_t^*) follow

$$A_t = \bar{A}^{1-\rho_A} A_{t-1}^{\rho_A} \varepsilon_{A,t} \tag{2.3.77}$$

$$A_{I,t} = \bar{A}_I^{1-\rho_{AI}} A_{t-1}^{\rho_{AI}} \varepsilon_{AI,t} \tag{2.3.78}$$

$$\mu_t = \bar{\mu}^{1-\rho_m} \mu_{t-1}^{\rho_m} \varepsilon_{m,t} \tag{2.3.79}$$

$$R_t^* = \bar{R}^{*1-\rho_{mf}} R_{t-1}^{*\rho_{mf}} \varepsilon_{m,t}^* \tag{2.3.80}$$

$$Z_t^* = \bar{Z}^{*1-\rho_Z} Z_{t-1}^{*\rho_Z} \varepsilon_{Z,t}^* \tag{2.3.81}$$

where variables with bars represent the steady state values. The expected values of $\varepsilon_{A,t}$, $\varepsilon_{AI,t}$, $\varepsilon_{m,t}$, $\varepsilon_{m,t}^*$ and $\varepsilon_{Z,t}^*$ are all unity, and for the coefficients, $\rho_A, \rho_{AI}, \rho_m, \rho_{mf}, \rho_Z \in (0,1)$.

2.3.11 National Income Identity

From the aggregate households' budget constraint and the asset market clearing condition (2.3.69),

$$C_t + \left(r_t^{-1}b_{H,t} - b_{H,t-1}\right) + NX_t = (1-m)W_tL_t + D_t + \Omega_t \qquad (2.3.82)$$

where D_t is the sum of the aggregate dividends from intermediate goods producers ($D_{G,t}$) and the capital producer ($D_{C,t}$). $D_{G,t}$ and $D_{C,t}$ can be illustrated as follow:

$$
\begin{aligned}
D_{G,t} &= \left(\frac{P_{H,t}}{P_t}\right) Y_{H,t} + \left(\frac{\mathcal{E}_t P_{H,t}^*}{P_t}\right) Y_{H,t}^* - \left[r_{S,t}Q_{t-1} - (1-\delta)Q_t\right]K_t - W_tL_t \\
&= \left(\frac{P_{H,t}}{P_t}\right)\left(Y_{H,t} + Y_{H,t}^*\right) - r_{S,t}Q_{t-1}S_{t-1} + Q_tK_{t+1} - Q_tI_t - W_tL_t \quad (2.3.83)\\
D_{C,t} &= Q_tK_{t+1} - (1-\delta)Q_tK_t - \left(1 + g\left(\frac{I_t}{I_{t-1}}\right)\right)I_t \\
&= Q_tI_t - \left(1 + g\left(\frac{I_t}{I_{t-1}}\right)\right)I_t \qquad (2.3.84)
\end{aligned}
$$

Also, given Ω_t[8], combining the equations (2.3.57), (2.3.58), (2.3.59) and (2.3.62) yields

$$r_{S,t}Q_{t-1}S_{t-1} - Q_tS_t = \left(b_{H,t-1} - r_t^{-1}b_{H,t}\right) + \Omega_t + mN_{x,t}. \qquad (2.3.85)$$

From the equation (2.3.85) and $K_{t+1} = S_t$, the real dividends from intermediate goods producers can be rewritten as follows:

$$D_{G,t} = \left(\frac{P_{H,t}}{P_t}\right)\left(Y_{H,t} + Y_{H,t}^*\right) + \left(r_t^{-1}b_{H,t} - b_{H,t-1}\right) - \left(\Omega_t + mN_{x,t}\right) - Q_tI_t - W_tL_t. \qquad (2.3.86)$$

Plugging the equations (2.3.84) and (2.3.86) into the households' aggregate budget constraint (2.3.82) and using government budget constraint, the national income

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[8]Ω_t is the net transfer from banking sector to the households, which is
$\Omega_t = (1-m)N_{x,t} - \omega Q_tS_{t-1} = (1-\zeta)\left(r_{S,t}Q_{t-1}S_{t-1} - b_{H,t-1}\right) - \omega Q_tS_{t-1} - mN_{x,t}.$

identity is derived as follows:

$$C_t + I_t + G_t + NX_t + g\left(I_t/I_{t-1}\right)I_t = \left(\frac{P_{H,t}}{P_t}\right)\left(Y_{H,t} + Y_{H,t}^*\right) \qquad (2.3.87)$$

where $Y_{H,t} + Y_{H,t}^* = Y_t V_t$.

2.3.12 Equilibrium

The equilibrium consists of 42 equations and 42 variables. Home intermediate goods producers' optimization is illustrated by the equations (2.3.1), (2.3.3), (2.3.4), (2.3.14) and (2.3.16). Final goods demands and equilibrium conditions are indicated by (2.3.19), (2.3.20), (2.3.22), (2.3.23) and (2.3.27)-(2.3.32). Home households' utility maximization is represented by the equations (2.3.36), (2.3.37), (2.3.39) and (2.3.40). In the financial intermediary sector, the equilibrium consists of the equations (2.3.43), (2.3.49), (2.3.50), (2.3.52), (2.3.54)-(2.3.56), (2.3.61), and (2.3.62). Capital producers' optimization and the capital stock evolution are represented by the equations (2.3.63) and (2.3.64). Government balanced budget and the monetary policy are given by (2.3.67) and (2.3.68). Market clearing conditions are illustrated by (2.3.70), (2.3.71) and (2.3.73). There are five exogenous shocks, represented by (2.3.77)-(2.3.81)[9].

Given $S_t = K_{t+1}$, and assuming $b_{H,t} = b_{H,t}^d$ and $L_t = L_t^d$, corresponding variables are as follow: C, I, Q, G, Z, Y, K, L, MC, W, R, R^*, r, r_S, Λ, b_H, b_F, Y_H, Y_F, Y_H^*, $\frac{P_H}{P}$, $\frac{P_F}{P}$, $\frac{\tilde{P_H}}{P}$, $\frac{P_H^*}{P^*}$, $\frac{P_F^*}{P^*}$, Π, Π_H, Π_F, Π_H^*, Π_F^*, x, h, ϕ, ν, N, $\mathcal{E}_R$, $\Delta\mathcal{E}$, A_I, A, η, μ, Z^*.

2.3.13 Solution Method

The solution method of the model starts with computing the well-defined steady state. The steady state values of variables are found analytically and illustrated

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[9]Appendix 2.A.1 and 2.A.2 outline the model's steady state and the log-linearized equation system.

in the appendix 2.A.1. Having computed the steady state, equations are approximated around the steady state, using log-linearization. The linearized equations are reported in the appendix 2.A.2. These equations are also represented by the following state space form:

$$AE(x_{t+1}) = Bx_t + Cz_t$$

where x is the vector of endogenous variables and z denotes the vector of exogenous variables. Following the method of Klein (2000), firstly a generalized Shur decomposition to matrices A and B is conducted. This yields $QAZ = S$ and $QBZ = T$, where S and T are upper triangular matrices. Using block matrices of Q, Z, S and T, eventually reduced form solutions are derived. Given a shock on the vector z, then the impulse response simulation results are computed.

In the Baeyesian estimation, a Markov Chain Monte Carlo (MCMC) simulation is used, with the Metropolis-Hastings (MH) algorithm. It is a process of finding the desired posterior distributions of parameters using a large number of iterations. In the MH algorithm, given a current state variable, a new draw is accepted only when it increases the posterior density (Fernández-Villaverde, 2010). The detailed method is illustrated in the appendix 2.A.5.

2.4 Parameters Calibration and Estimation

2.4.1 Data

In this section, both calibration and a Bayesian estimation are used to set up the parameters and the steady state values. For model validation, South Korea is used as the test bed for the following reasons. First, openness of the economy (0.515) is close to the world average (0.463). Also, the size of GDP is proper for a small open economy model (around 1.7% of the global GDP), and the central bank implements

an independent inflation targeting monetary policy. Finally, the relevant real and financial data are available.

Given that there are five shocks, five series of quarterly data are used in the Bayesian estimation: (i) consumption, (ii) employment (labour), (iii) the policy rate[10], (iv) investment, and (v) the real exchange rate. The sample period ranges from 1982:Q1 to 2014:Q4. Since the model analysis is based on log-linearized equations, all the variables are transformed into the percentage deviations from the long-run levels.

Calibrating the other parameters and steady state values, mostly the quarterly data during 1999:Q1-2014:Q4 are used. For long-run inflation, quarterly changes of CPI are used and the steady state excess return is derived from the lending-deposit rate spreads. Deriving the steady state level of openness of the home economy, 'imports/GDP' and 'exports/GDP' data are used. For the financial intermediary sector, the aggregate balance sheet of domestic banks during 2008-2013 are used. Data sources are reported in the appendix 2.A.4.

2.4.2 Calibration

In extant studies such as Galí and Monacelli (2005), and Christoffel, Coenen and Warne (2008), openness $(1 - \alpha)$ is calculated by the 'import/GDP' ratios. However, in this model the degree of openness is defined as the steady state level of 'import/final goods production' (Y_F/Z), which can be derived from the 'import/GDP' and the 'export/GDP' ratio[11]. From the data, openness of South Korea is 0.515 in 2013[12] which was below 0.3 in 1990s. Also, given that 'export/GDP' of South Korea

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[10]Since the policy rate does not have usual statistical property, short-term (3 month) interest rate data are used.

[11]National income identity implies $1 = (C + I + G)/Y + (Y_H^* - Y_F)/Y$ at the steady state. The value of Y_F/Z can be derived from $1 = Z/Y + Y_H^*/Y - Y_F/Y$.

[12]Using this way, various levels of openness by country (2013) can be observed such as the US 0.160, Netherlands 0.810, Malaysia 0.798, and Czech Republic 0.758.

is 0.54 and its output is 1.7% of global output in 2013, openness of the rest of the world $(1 - \alpha^*)$ is calibrated as 0.01.

In the financial intermediary sector, the quarterly excess return from funding and lending is derived as 41 basis point (bp) based on the long-run lending-deposit spread, which is higher than 25bp in Gertler and Karadi (2011). Also, the long-run leverage is calculated from the ratio of 'loans to business/net equity' in the aggregate balance sheet of South Korean banks, which yields $\bar{\phi} = 4.51$. The calibrated leverage is higher than that (4.00) in Gertler and Karadi (2011). The fraction of the possible funds diversion (λ) is related to the deposit holder's expected loss when a financial intermediary is at a state of bankruptcy. From the ratio of recovery during the financial crisis bailout since 1998, calibration suggests $\lambda = 0.270$[13], which is lower than 0.381 of Gertler and Karadi (2013). The corresponding survival ratio of an individual banker (ζ) is 0.899. This means that bankers return final net worth $(N_{x,t})$ from the financial intermediary sector to the households every 9.9 quarters (equal to 1/(1-0.899)) on average.

Using the CPI, the long-run quarterly inflation rate $(\bar{\Pi})$ is calibrated as 1.0067 which means the annual inflation rate is 2.70% at the steady state. Using the data of the deposit rate, the long-run nominal interest rate $(\bar{R})$ is determined at 1.0107, which yields the corresponding discount rate $(\beta = 0.996)$ and the real interest rate $(\bar{r} = 1.004)$. In the monetary policy rule, parameters are calibrated such that the estimated path of the policy rate fits the actual data[14]; $\rho_R = 0.9$, $\gamma_P = 1.1$ and $\gamma_Y = 0.3$. The annual depreciation ratio of capital is 10% $(\delta = 0.025)$ and the price stickiness parameter (ξ) is 0.75. The tax rate (m) is 0.20 such that 'government spending/GDP ratio' in the model fits the average ratio of it during

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[13]Among the total amount of South Korean banks bailout regarding the financial crisis in 1997-1998, 44.6% has not been recovered during 1998-2014. Given that 60.6% of total loans are not insured by Korea Deposit Insurance Corporation (KDIC) (2014), the expected loss when an asset (deposit) is default can be calibrated as 27.0%.

[14]Comparison between the estimated path and the actual data of the policy rate is illustrated in the appendix 2.A.7.

Table 2.1: Calibrated Values of Parameters

parameter	value	parameter	value	parameter	value
α	0.485	ζ	0.899	m	0.20
α^*	0.990	ω	0.018	ρ_A	0.85
δ	0.025	λ	0.270	ρ_{AI}	0.85
β	0.996	γ_P	1.100	ρ_m	0.85
$\tilde{\beta}$	0.824	γ_Y	0.300	ρ_{mf}	0.85
ϕ	4.510	ρ_R	0.900	ρ_{zf}	0.85

the sample period (14.7%). Using the long-run levels of net worth of the financial intermediary sector, the leverage, and net foreign assets (NFA), the steady state ratio of b_H/b_F is determined at 1.98. For exogenous variables, the coefficient parameters, $\rho_A, \rho_{AI}, \rho_m, \rho_{mf}$ and ρ_Z, are 0.85 following Kolasa and Lombardo (2014). Table 2.1 summarizes the calibrated values of the baseline parameters.

2.4.3   Bayesian Estimation

A Bayesian estimation is performed for the second moments of the five exogenous shocks and for eight parameters which are not calibrated[15]. These parameters are: (i) the share of the capital income in production (ψ), (ii) consumer's preference parameters (σ and χ), (iii) elasticities of substitution between intermediate goods (ε), and between home and foreign intermediate goods (θ), (iv) the investment adjustment cost (μ_I), (v) the international assets holding cost (μ_T), and (vi) the coefficient of the real exchange rate in the monetary policy rule (γ_E).

For the Bayesian approach, the Metropolis-Hastings algorithm (MH) is used with 50,000 draws. The first three columns of Table 2.2 provide the assumptions regarding the prior distributions of the parameters and the exogenous shocks. The share of the capital income (ψ) is assumed to have a beta distribution with a mean 0.33. Following Christoffel, Coenen and Warne (2008), the prior mean of the elasticity of

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[15]The Bayesian estimation method is summarized in the appendix 2.A.5.

Table 2.2: Parameter Estimates using Bayesian Approaches

	Prior Distribution			Posterior Distribution				
	Type	Mean	St.error	Mode	Median	Mean	5%	95%
ψ	beta	0.33	0.02	0.369	0.362	0.361	0.334	0.390
σ	normal	1.50	0.30	3.034	2.865	2.864	2.501	3.226
χ	normal	1.00	0.20	1.304	1.484	1.486	1.224	1.760
ε	gamma	6.00	1.00	5.986	5.446	5.498	3.944	7.152
θ	gamma	1.50	0.30	2.179	2.378	2.389	1.995	2.760
μ_I	gamma	4.00	0.50	3.856	3.526	3.550	2.926	4.237
μ_T	gamma	0.20	0.03	0.257	0.242	0.244	0.192	0.296
γ_E	gamma	0.05	0.02	0.035	0.044	0.046	0.018	0.073
ε_A	inv.gamma	0.10	-	0.124	0.125	0.126	0.106	0.147
ε_{AI}	inv.gamma	0.10	-	0.233	0.246	0.248	0.206	0.292
ε_m	inv.gamma	0.10	-	0.012	0.013	0.013	0.012	0.014
ε_{mf}	inv.gamma	0.10	-	0.064	0.059	0.060	0.048	0.071
ε_{mf}	inv.gamma	0.10	-	0.196	0.204	0.206	0.169	0.244

substitution between home and foreign goods (θ) is 1.5 and that of the investment adjustment cost (μ_I) is 4 with gamma distributions. As in Kollmann (2002), the prior mean of ε is 6 with a gamma distribution. Household preference parameters σ and χ have means 1.5 and 1.0 with normal distributions. The prior mean of the international assets holding cost (μ_T) is 0.2. The policy rule coefficient of the real exchange rate (γ_E) has a prior mean 0.05. All the standard errors of the shocks are assumed to have inverse gamma distributions.

The result of the parameters estimation reveals that all the estimated values of the parameters are significantly different from zero. The posterior mean of ψ is 0.36 during the sample period which means that capital has a share more than the conventional level, 0.33. The elasticity of substitution between home intermediate goods is lower ($\varepsilon = 5.50$) than the prior mean, and the elasticity of substitution between home and foreign goods (θ) is 2.39. The posterior mean of the investment adjustment cost parameter (μ_I) is 3.55, and for the international holding cost parameter (μ_T), it is 0.24. The posterior mean of the real exchange rate coefficient (γ_E) is 0.05. Regarding consumer's preference, both σ and χ have means higher

than unity, 2.86 and 1.49, respectively[16].

2.5 Impulse Responses: Foreign Policy Rate Shock

2.5.1 Bayesian Impulse Response Analysis

In this section, the impulse responses of home economic variables to the foreign policy rate shock are presented. Figure 2.3 reports the Bayesian impulse responses with respect to a negative foreign policy rate shock. The figure illustrates the mean responses (solid line) with the 5 and 95 per cent confidence bands (dotted line).

The lower foreign rate induces home households to decrease their foreign assets holdings. Moreover, the nominal exchange rate initially declines following the modified UIP condition[17]; thus, the real exchange rate also falls (home currency appreciation). Consequently, the relative price of home produced goods rises, and this lowers exports, while boosting imports (expenditure switching effect). The fall in net exports then reduces home output. On the other hand, home inflation declines, as the drop in import price inflation (Π_F) drags it down. The decline in the import price is caused by the home currency appreciation. Following the policy rule which responds to home output, inflation and the real exchange rate, the home central bank lowers its policy rate. This reveals the leader-follower relationship between the home and the foreign policy rates when there is a policy rate cut in the foreign economy.

As the amount of imports increases, final goods production (Z) rises, thus boosting investment. The unanticipated increase in investment raises the capital price (Q), and as a result the balance sheet of the entrepreneur sector expands; the amount of net worth increases and the excess return on claims ($E(r_S) - r$) falls. This raises

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[16]The appendix 2.A.6 illustrates the prior and the posterior distributions.

[17]In the modified UIP condition (2.3.39), a negative shock on R_t^* puts downward pressure on $b_{F,t}^d$ and $\mathcal{E}_t$.
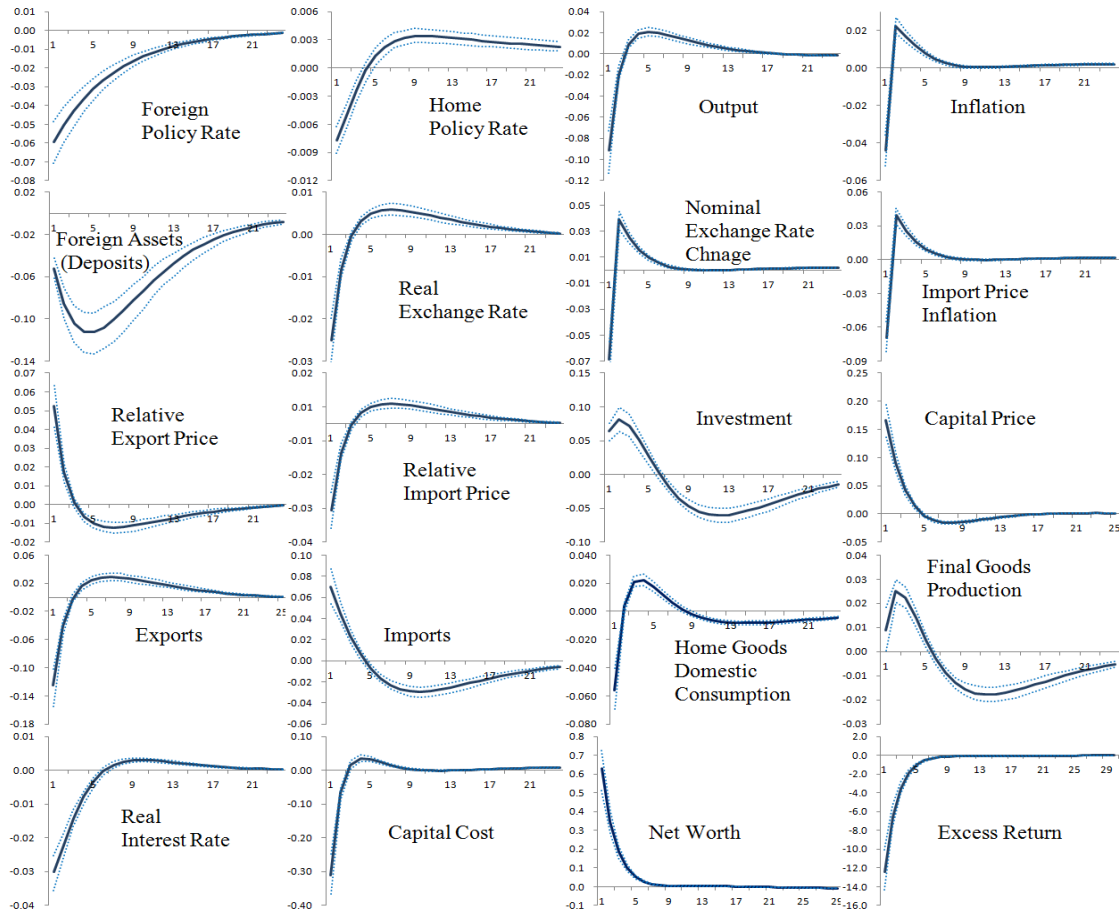
Figure 2.3: Responses to Foreign Policy Shock: Baseline Model

investment and the capital price further (financial accelerator). The higher level of Q lowers the intermediate goods producers' capital hiring costs[18].

2.5.2   Comparative Statics of Impulse Responses

In this section, the effects of a parameter value change or model modification on the monetary policy relationship are investigated. All other parameters are fixed at the calibrated or estimated levels. There is a negative 1%p foreign policy rate shock.

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[18]The producer's capital hiring cost is $r_{K,t} = r_{S,t}Q_{t-1} - (1 - \delta)Q_t$.
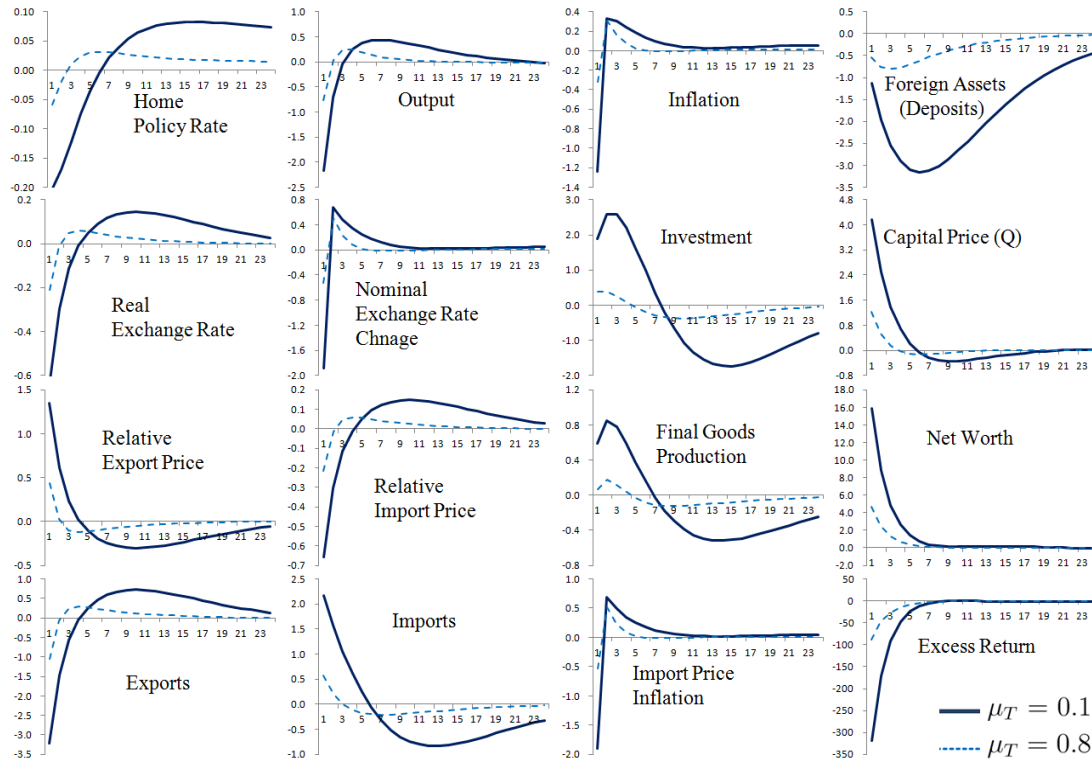
Figure 2.4: Responses to Foreign Policy Shock: Different Foreign Assets Holding Costs

Effect of a Change in the International Assets Holding Cost

The level of the international assets holding cost is represented by the parameter μ_T in the equation (2.3.34). The effect of a change in this cost on the impulse responses to a foreign rate shock is illustrated by Figure 2.4. The result indicates that the lower international assets holding cost ($\mu_T = 0.1$) makes the home policy rate follow the foreign rate more aggressively than the higher cost case ($\mu_T = 0.8$).

When the international assets holding cost is lower, the home agents' foreign assets holding decreases more in response to a negative foreign interest rate shock. Moreover, the nominal and the real exchange rates decline more significantly; the relative import and export prices also change more. This leads to a stronger expenditure switching effect - a greater decrease in net exports. Import price inflation also falls rapidly following the nominal exchange rate change. As a result, home output and inflation decline more sharply. Following the monetary policy rule, the

home central bank responds by cutting the policy rate more. This implies that the co-movements of policy rates become more significant as the financial markets are more globalized and international investment becomes easier.

Effect of a Change in Openness

The impulse responses in two different openness environments are compared in Figure 2.5. When the degree of home openness is higher ($\alpha = 0.2$), a fall in net exports yields a sharp decline in home output - inducing a stronger expenditure switching effect. Even though the exports and imports changes are smaller in terms of the deviation from the steady state, the larger portion of trade in the home economy leads to a greater decline in output than the other case with lower openness ($\alpha = 0.8$).

When the amount of of imports is larger, downward pressure of the fall in the import price on the home inflation rate becomes stronger. Therefore, home inflation declines more with higher openness. Combined with the fall in output, this leads to a more aggressive home policy rate cut. Higher openness strengthens the correlation between the home and the foreign policy rates when there is a foreign policy shock.

With higher openness, the rise in imports yields a further increase in final goods production; this raises investment, and the capital price rises more. Thus, net worth rises and the excess return drops more significantly. As trade volume is larger, home agents' foreign assets holding decreases more in response to the fall in net exports. This dampens the decline in the nominal and the real exchange rates[19].

Inflation Targeting and Policy Response to Real Exchange Rate

Since inflation targeting was initially adopted by New Zealand in 1989, many central banks have established the inflation-targeting frameworks[20]. Given that still many

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[19]In the modified UIP (2.3.39), a further decrease in $b_{F,t}^d$ puts upward pressure on $\mathcal{E}_t$.

[20]In the first half of 1990s, Canada, Israel, UK, Australia and Sweden joined the inflation-targeting regime. During 1997-2002, 15 more countries adopted inflation-targeting: Czech, Poland,
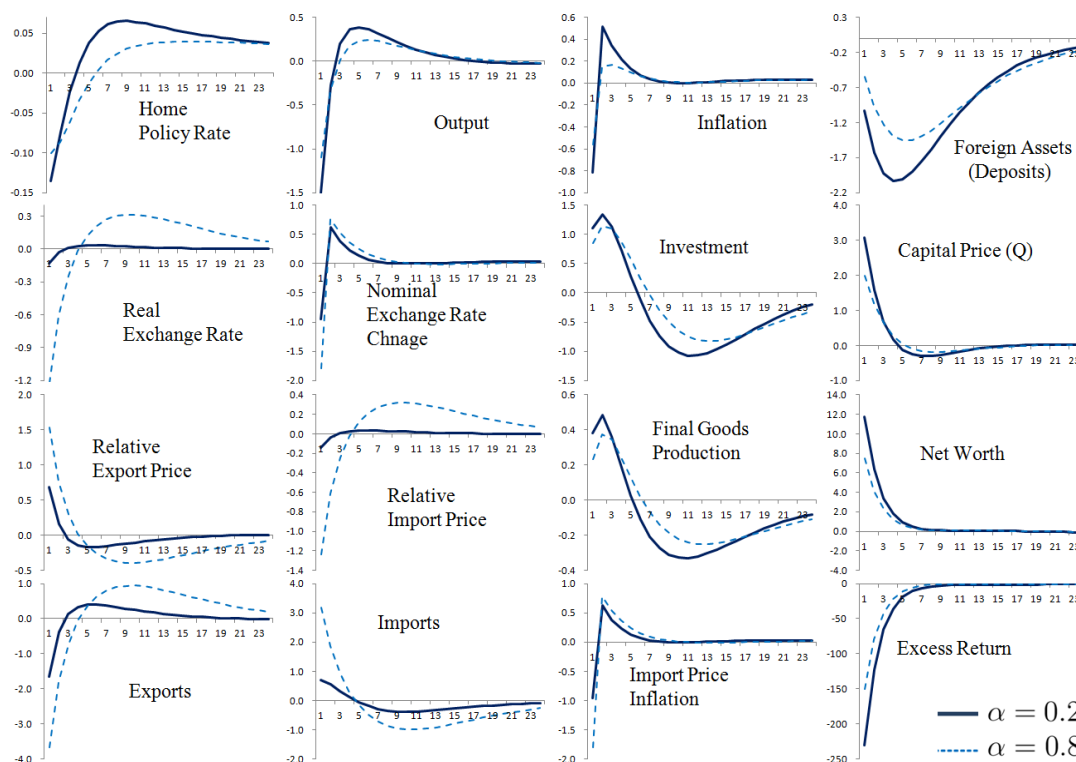
Figure 2.5: Responses to Foreign Policy Shock: Different Openness

central banks are in the process of adopting inflation-targeting regimes, it is meaningful to investigate the effect of such a policy framework on the policy relationship.

The sensitivity analysis is performed with different degrees of inflation targeting, as illustrated by Figure 2.6 (a). The aggressiveness of targeting is measured by the parameter γ_P in the policy rate rule equation with two different values: (i) strong targeting with $\gamma_P = 2.5$ and (ii) weak targeting with $\gamma_P = 1.2$. In response to a negative foreign rate shock, the home central bank with strong inflation targeting cuts its policy rate more aggressively than the case of weaker inflation targeting.

As the financial markets become more globalized and individual economies are integrated with each other, central banks have more serious concerns about the fluctuations in international variables. The parameter estimation ($\gamma_E > 0$) suggests

South Korea, Brazil, Chile, Colombia, South Africa, Thailand, Hungary, Iceland, Mexico, Norway, Ghana, Peru and Philippine (Hammond, 2012)

(a) Different Levels of Inflation Targeting

(b) Modified and Standard Taylor Rules
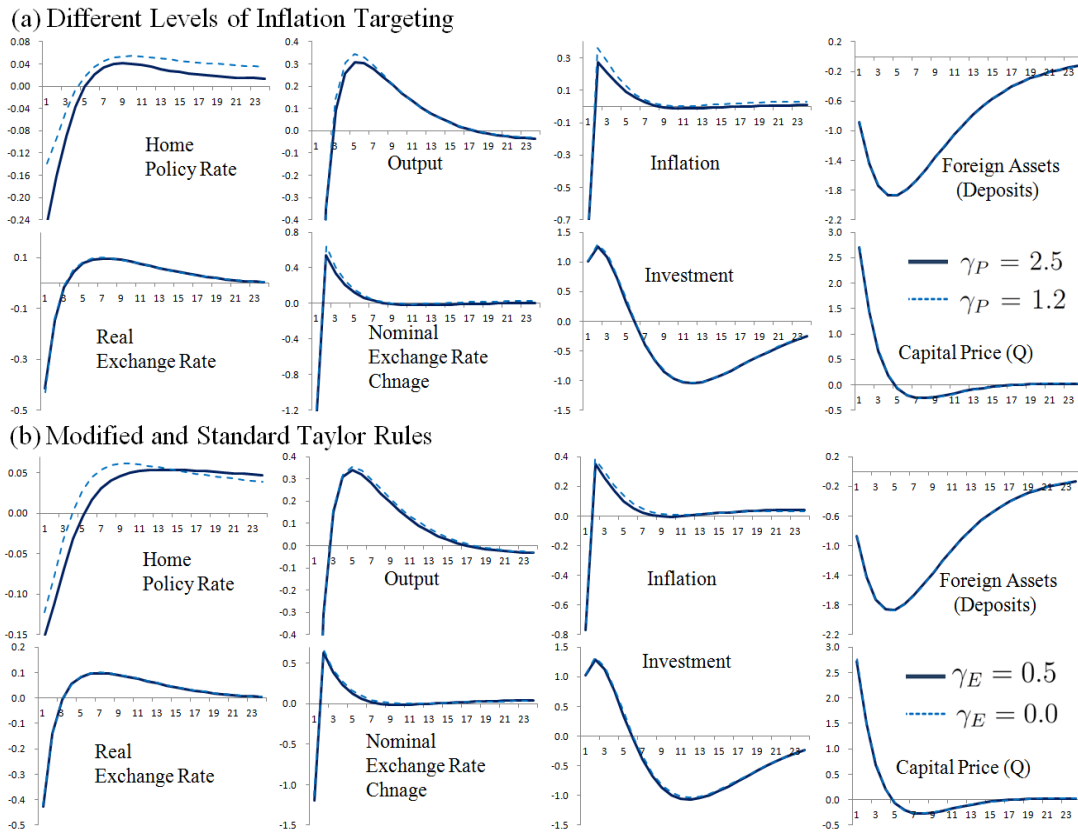
Figure 2.6: Effect of Aggressive Inflation Targeting and Modified Taylor Rule

that the home central bank lowers the policy rate when its currency appreciates. Figure 2.6 (b) describes the role of the policy response to the real exchange rate in the policy relationship. Compared to the case with the standard Taylor rule ($\gamma_E = 0$), the home central bank cuts its policy rate more when it responds to the real exchange rate as well ($\gamma_E = 0.5$). This result implies that the concern about the currency appreciation can lead to a deviation of the policy rate from the standard Taylor rule, as indicated by Taylor (2013), and He and McCauley (2013).

Effect of the Financial Friction

The financial friction strengthens the co-movements of the home and the foreign policy rates. This is related to the financial accelerator, which amplifies the movements of investment and the asset price through the changes in entrepreneurs' net
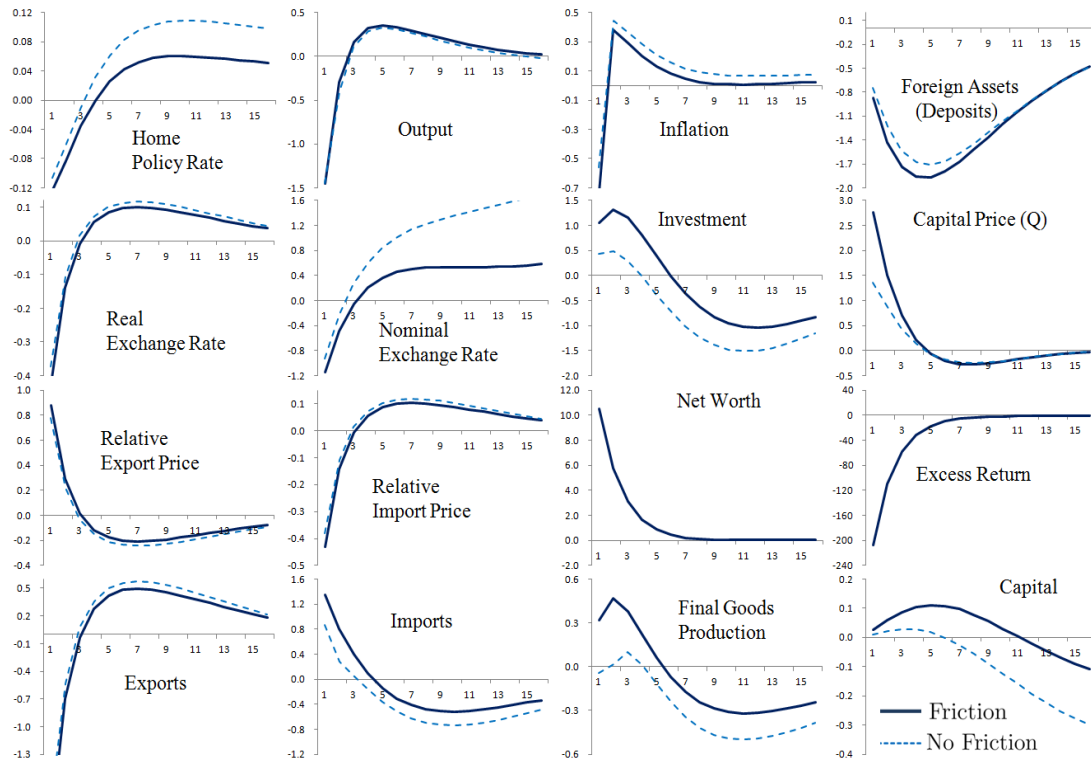
Figure 2.7: Responses to Foreign Policy Shock: Effects of Financial Friction

worth and the excess return on the claims $(E_t (r_{S,t+1}) - r_t)$. Two cases are compared in Figure 2.7: *with* and *without* the financial friction.

Without the friction, financial intermediaries simply intermediate funds without the agency problem[21], and the excess return remains zero $(E_t (r_{S,t+1}) = r_t)$. With the friction, the foreign policy rate cut lowers the home excess return, as the unanticipated increase in investment and the following rise in the asset price boost entrepreneurs' balance sheets; the expansion of net worth forces down the excess return. The fall in the excess return then lowers the cost of capital and raises capital demand more. This leads to further increases in investment and the asset price (financial accelerator).

The additional increase in investment requires more final goods (Z) in the home

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[21]Without the financial friction, the financial intermediaries are assumed to have no net worth, which means $Q_t S_t = r_t^{-1} b_{H,t}$.

economy, which yields a further decline in exports and a rise in imports - a larger drop in net exports. The international asset market clearing condition leads to a more significant fall in the foreign assets holding (b_F^d). The modified UIP suggests that, given the decline in R^*, the greater fall in b_F^d puts downward pressure on the exchange rate changes $(\Delta \mathcal{E})^{22}$, and as a result the nominal and the real exchange rates remain lower with the financial friction. As the higher home currency value lowers the relative import price, home inflation remains below the level without the financial friction; the home central bank keeps its policy rate lower. This makes the home policy rate closer to the foreign policy rate.

2.5.3 Variance Decomposition

Table 2.3 presents the variance decomposition of the home interest rate with respect to all five exogenous shocks: home technology (ε_A), the investment adjustment cost (ε_{AI}), the home policy rate (ε_m), the foreign policy rate (ε_m^*) and the foreign aggregate demand (ε_Z^*). In the estimated baseline model with the financial friction, the home monetary policy shock accounts for the largest fraction of the fluctuations in the home policy rate (58.5%). The productivity shock (30.9%) is next to it.

The foreign monetary policy shock (ε_m^*) explains 2.4% of the home policy rate fluctuations in the estimated baseline model with the friction. In the case of the lower international assets holding cost, it increases to 6.7%. With greater openness, the contribution (2.4%) is larger than the case of lower openness. More aggressive inflation targeting (6.9%) raise the contribution of the foreign monetary policy shock. When the home monetary policy responds to the real exchange rate, the foreign rate's contribution also rises to 2.8%. However, the foreign policy effect in the no friction case (5.5%) is larger, since the effect of the productivity shock is greater

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[22]The modified UIP condition is given by $R_t = R_t^* \left[1 - \mu_T \left(b_{F,t}^d / \bar{b}_F^d - 1\right)\right] E_t \left(\mathcal{E}_{t+1}/\mathcal{E}_t\right)$ (2.3.39). As $b_{F,t}^d$ becomes lower, the nominal exchange rate increase ($\Delta \mathcal{E}$) is smaller. The nominal exchange rate in the figure 2.7 is derived from $\hat{\mathcal{E}}_t = \sum_{\tau=1}^{t} \widehat{\Delta \mathcal{E}_\tau}$ since $\mathcal{E}_t$ is not stationary.

Table 2.3: Forecast Error Variance Decomposition (%) of Home Policy Rate (R)

Case		Parameter	ε_A	ε_{AI}	ε_m	ε_m^*	ε_Z^*
International	High	$\mu_T = 0.8$	30.8	6.8	60.5	0.4	1.5
Transaction Cost	Low	$\mu_T = 0.1$	30.1	6.4	55.2	6.7	1.6
Openness	Low	$1 - \alpha = 0.2$	36.7	7.8	53.6	1.7	0.2
	High	$1 - \alpha = 0.8$	25.0	5.0	63.0	2.4	4.6
Inflation Targeting	Weak	$\gamma_P = 1.2$	29.3	5.0	62.0	2.4	1.3
	Strong	$\gamma_P = 2.5$	23.6	1.0	68.0	6.9	0.5
Taylor Rule	Standard	$\gamma_E = 0.0$	30.1	6.3	59.6	2.4	1.7
	Modified	$\gamma_E = 0.5$	38.2	8.7	48.8	2.8	1.5
Financial Friction	None		27.1	8.3	58.0	5.5	1.0
	Baseline		30.9	6.5	58.5	2.4	1.6

with the financial friction.

2.6 Conclusion

During recent decades, the monetary policies of central banks in many countries have shown significant co-movements. Indeed, those co-movements were mostly led by the policy changes of major central banks, such as the US Fed and the ECB. As a result, the leader-follower relationships between central bank policies have been observed in many cases.

In the model analysis of this chapter, the leader-follower relationship between the home and the foreign policy rates is caused by the changes in the home and foreign goods' relative prices. When the foreign policy rate is lowered, the fall in home output and the drop in inflation make the home central bank cut its policy rate. The real currency appreciation also contributes to the home central bank's following behaviour. If one country is a large economy in reality with a very low degree of openness as in this small open economy model, there will be an asymmetric relationship between policy rates.

The model simulations suggest that the leader-follower policy relationship becomes stronger when (i) the home agents' international assets holding cost is lower,

(ii) openness of the home economy is higher, (iii) the home central bank adopts more aggressive inflation targeting, and (iv) the home monetary policy responds to the real exchange rate change. The first two results and the last one suggest that financial market globalization and economic integration lead to a stronger policy rates relationship. Also, as more central banks adopt inflation targeting, the policy rates co-movements would be more significant in the global economy. Finally, the banking friction strengthens the policy rates relationship through the financial accelerator, which raises investment further and lowers home inflation.

2.A Appendix

2.A.1 Steady State

$$\bar{\Pi} = 1.0067 (= \bar{\Pi}^*)$$

$$\bar{\Lambda} = \bar{Q} = 1$$

$$\bar{A} = \bar{A}_I = 1$$

$$\left(\overline{\frac{P_{F,t}^*}{P_t^*}}\right) = \left(\overline{\frac{P_{H,t}^*}{P_t^*}}\right) = 1$$

$$\bar{\Delta}_{\mathcal{E}} = 1$$

$$\bar{r}_S = \bar{r} + \bar{r}_P$$

$$\overline{MC} = \frac{\varepsilon - 1}{\varepsilon}$$

$$\frac{\bar{L}}{\bar{K}} = \frac{1-\psi}{\psi}\bar{r}_K\bar{W}^{-1}$$

$$\bar{Y} = \left(\frac{\bar{L}}{\bar{K}}\right)^{1-\psi}\bar{K}$$

$$\bar{I} = \delta\bar{K}$$

$$\bar{F} = \frac{1}{1-\beta\xi}$$

$$\bar{Y} = \bar{Y}_H + \bar{Y}_H^*$$

$$\bar{b}_H = (\phi - 1)\bar{N}\bar{r}$$

$$\bar{b}_H/\bar{b}_F = 1.98$$

$$\bar{G} = m[\bar{W}\bar{L} + (1-\zeta)(\bar{r}_S\bar{K} - \bar{b}_H)]$$

$$\bar{x} = \bar{h}$$

$$\bar{v} = \frac{(1-\zeta)\beta\bar{r}_P}{1-\beta\zeta\bar{x}}$$

$$\bar{r} = \beta^{-1}$$

$$\bar{R} = \bar{\Pi}\beta^{-1} = \bar{R}^*$$

$$\bar{\mathcal{E}}_R = \bar{\mu} = 1$$

$$\left(\overline{\frac{P_{H,t}}{P_t}}\right) = \left(\overline{\frac{P_{F,t}}{P_t}}\right) = \left(\overline{\frac{\tilde{P}_{H,t}}{P_t}}\right) = 1$$

$$\bar{r}_P = 0.0041$$

$$\bar{r}_K = \bar{r}_S - (1-\delta)$$

$$\bar{W} = \left[\frac{1}{\psi^\psi(1-\psi)^{1-\psi}}\bar{r}_K^\psi\overline{MC}^{-1}\right]^{\frac{1}{\psi-1}}$$

$$\bar{\phi} = 4.51$$

$$\bar{K} = \bar{\phi}\bar{N}$$

$$\bar{L} = \frac{\bar{L}}{\bar{K}}\bar{K}$$

$$\bar{C} = \left(\bar{L}^{-\varphi}\bar{W}\right)^{\frac{1}{\sigma}}$$

$$\bar{Z} = \bar{C} + \bar{I} + \bar{G}$$

$$\bar{Y}_H = \alpha\bar{Z}$$

$$\bar{Y}_F = (1-\alpha)\bar{Z}$$

$$\bar{b}_F = \left(\bar{R}^{*-1}\bar{\Pi} - 1\right)^{-1}\left(\bar{Y}_H^* - \bar{Y}_F\right)$$

$$\bar{h} = \bar{\phi}\bar{r}_P + \bar{r}$$

$$\bar{\eta} = \frac{1-\zeta}{1-\beta\zeta\bar{h}}$$

2.A.2 Log-linearized Equations

$$\hat{Y}_t = \hat{A}_t + \psi\hat{K}_t + (1-\psi)\hat{L}_t \tag{2.A.1}$$

$$\widehat{MC}_t = -\hat{A}_t + \psi\left(\frac{1}{\bar{r}_s - (1-\delta)}\right)\left[\bar{r}_s\left(\hat{r}_{s,t} + \hat{Q}_{t-1}\right) - (1-\delta)\hat{Q}_t\right] + (1-\psi)\hat{W}_t \tag{2.A.2}$$

$$\hat{L}_t - \hat{K}_t = \left(\frac{1}{\bar{r}_s - (1-\delta)}\right)\left[\bar{r}_s\left(\hat{r}_{s,t} + \hat{Q}_{t-1}\right) - (1-\delta)\hat{Q}_t\right] - \hat{W}_t \tag{2.A.3}$$

$$\left(\frac{\widehat{\tilde{P}_{H,t}}}{P_t}\right) = \bar{F}^{-1}\widehat{MC}_t + \left(1 - \bar{F}^{-1}\right)E_t\left(\frac{\widehat{\tilde{P}_{H,t+1}}}{P_{t+1}} + \hat{\Pi}_{t+1}\right) \tag{2.A.4}$$

$$\left(\frac{\widehat{P_{H,t}}}{P_t}\right) = \xi\left[\left(\frac{\widehat{P_{H,t-1}}}{P_{t-1}}\right) - \hat{\Pi}_t\right] + (1-\xi)\left(\frac{\widehat{\tilde{P}_{H,t}}}{P_t}\right) \tag{2.A.5}$$

$$\hat{Y}_{H,t} = -\theta\left(\frac{\widehat{P_{H,t}}}{P_t}\right) + \hat{Z}_t \tag{2.A.6}$$

$$\hat{Y}_{F,t} = -\theta\left(\frac{\widehat{P_{F,t}}}{P_t}\right) + \hat{Z}_t \tag{2.A.7}$$

$$\hat{\Pi}_t = \alpha\left[\left(\frac{\widehat{P_{H,t-1}}}{P_{t-1}}\right) + \hat{\Pi}_{H,t}\right] + (1-\alpha)\left[\left(\frac{\widehat{P_{F,t-1}}}{P_{t-1}}\right) + \hat{\Pi}_{F,t}\right] \tag{2.A.8}$$

$$0 = \alpha^*\left[\left(\frac{\widehat{P_{F,t-1}^*}}{P_{t-1}^*}\right) + \hat{\Pi}_{F,t}^*\right] + (1-\alpha^*)\left[\left(\frac{\widehat{P_{H,t-1}^*}}{P_{t-1}^*}\right) + \hat{\Pi}_{H,t}^*\right] \tag{2.A.9}$$

$$\left(\frac{\widehat{P_{F,t}}}{P_t}\right) = \left(\frac{\widehat{P_{F,t}^*}}{P_t^*}\right) + \hat{\mathcal{E}}_{R,t} \tag{2.A.10}$$

$$\left(\frac{\widehat{P_{H,t}^*}}{P_t^*}\right) = \left(\frac{\widehat{P_{H,t}}}{P_t}\right) - \hat{\mathcal{E}}_{R,t} \tag{2.A.11}$$

$$\hat{\Pi}_{H,t} = \left(\frac{\widehat{P_{H,t}}}{P_t}\right) - \left(\frac{\widehat{P_{H,t-1}}}{P_{t-1}}\right) + \hat{\Pi}_t \tag{2.A.12}$$

$$\hat{\Pi}_{F,t} = \left(\frac{\widehat{P_{F,t}}}{P_t}\right) - \left(\frac{\widehat{P_{F,t-1}}}{P_{t-1}}\right) + \hat{\Pi}_t \tag{2.A.13}$$

$$\hat{\Pi}_{F,t}^* = \left(\frac{\widehat{P_{F,t}^*}}{P_t^*}\right) - \left(\frac{\widehat{P_{F,t-1}^*}}{P_{t-1}^*}\right) \tag{2.A.14}$$

$$\hat{\Pi}_{H,t}^* = \left(\frac{\widehat{P_{H,t}^*}}{P_t^*}\right) - \left(\frac{\widehat{P_{H,t-1}^*}}{P_{t-1}^*}\right) \tag{2.A.15}$$

$$\hat{Y}_{H,t}^* = -\theta\left(\frac{\widehat{P_{H,t}}}{P_t}\right) + \theta\hat{\mathcal{E}}_{R,t} + \hat{Z}_t^* \tag{2.A.16}$$

$$\hat{\mathcal{E}}_{R,t} = \widehat{\Delta\mathcal{E}}_t - \hat{\Pi}_t + \hat{\mathcal{E}}_{R,t-1} \tag{2.A.17}$$

$$\hat{W}_t = \sigma\hat{C}_t + \chi\hat{L}_t \tag{2.A.18}$$

$$\hat{C}_t = E_t\left(\hat{C}_{t+1}\right) + \sigma^{-1}\left[E_t\left(\hat{\Pi}_{t+1}\right) - \hat{R}_t\right] \tag{2.A.19}$$

$$\hat{R}_t = \hat{R}_t^* + E_t\left(\widehat{\Delta\mathcal{E}}_{t+1}\right) - \mu_T\hat{b}_{F,t} \tag{2.A.20}$$

$$\hat{\Lambda}_{t,t+1} = \sigma(\hat{C}_t - \hat{C}_{t+1}) \tag{2.A.21}$$

$$\hat{r}_t = \hat{R}_t - \hat{\Pi}_{t+1} \tag{2.A.22}$$

$$\bar{v}\hat{v}_t = (1-\zeta)\,\beta\left(\bar{r}_P\hat{\Lambda}_{t,t+1} + \bar{r}_S\hat{r}_{S,t+1} - \bar{r}\hat{r}_t\right) + \beta\zeta\bar{x}\bar{v}\left(\hat{\Lambda}_{t,t+1} + \hat{x}_{t+1} + \hat{v}_{t+1}\right) \tag{2.A.23}$$

$$\hat{x}_t = \hat{h}_t + \hat{\phi}_t - \hat{\phi}_{t-1} \tag{2.A.24}$$

$$\bar{h}\hat{h}_t = \bar{\phi}\left[\left(\bar{r}_S\hat{r}_{S,t} - \bar{r}\hat{r}_{t-1}\right) + \left(\bar{r}_S - \bar{r}\right)\hat{\phi}_{t-1}\right] + \bar{r}\hat{r}_{t-1} \tag{2.A.25}$$

$$\hat{\eta}_t = \beta\zeta\bar{h}\left(\hat{\Lambda}_{t,t+1} + \hat{h}_{t+1} + \hat{\eta}_{t+1}\right) \tag{2.A.26}$$

$$\hat{Q}_t + \hat{K}_{t+1} = \hat{\phi}_t + \hat{N}_t \tag{2.A.27}$$

$$\hat{\phi}_t = \frac{1}{\lambda - \bar{v}}\left(\frac{\bar{\eta}}{\bar{\phi}}\hat{\eta}_t + \bar{v}\hat{v}_t\right) \tag{2.A.28}$$

$$\hat{N}_t = \frac{\omega\bar{K}}{\bar{N}}\left(\hat{Q}_t + \hat{K}_t\right) + \zeta\bar{h}\left(\hat{h}_t + \hat{N}_{t-1}\right) \tag{2.A.29}$$

$$\hat{K}_{t+1} = (1-\delta)\,\hat{K}_t + \frac{\bar{I}}{\bar{K}}\hat{I}_t \tag{2.A.30}$$

$$\hat{Q}_t = \mu_I\left(\hat{I}_t - \hat{I}_{t-1} + \hat{A}_{I,t}\right) - \beta\mu_I\left(\hat{I}_{t+1} - \hat{I}_t + \hat{A}_{I,t+1}\right) \tag{2.A.31}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left(\gamma_P\hat{\Pi}_t + \gamma_Y\hat{Y}_t\right) + \hat{\mu}_t \tag{2.A.32}$$

$$\hat{Y}_t = \frac{\bar{Y}_H}{\bar{Y}}\hat{Y}_{H,t} + \frac{\bar{Y}_M^*}{\bar{Y}}\hat{Y}_{H,t}^* \tag{2.A.33}$$

$$\hat{Z}_t = \frac{\bar{C}}{\bar{Z}}\hat{C}_t + \frac{\bar{I}}{\bar{Z}}\hat{I}_t + \frac{\bar{G}}{\bar{Z}}\hat{G}_t \tag{2.A.34}$$

$$\hat{A}_t = \rho_A\hat{A}_{t-1} + \hat{\varepsilon}_{A,t} \tag{2.A.35}$$

$$\hat{A}_{I,t} = \rho_{AI}\hat{A}_{I,t-1} + \hat{\varepsilon}_{AI,t} \tag{2.A.36}$$

$$\hat{\mu}_t = \rho_M\hat{\mu}_{t-1} + \hat{\varepsilon}_{M,t} \tag{2.A.37}$$

$$\hat{R}_t^* = \rho_{MF}\hat{R}_{t-1}^* + \hat{\varepsilon}_{M,t}^* \tag{2.A.38}$$

$$\hat{Z}_t^* = \rho_Z\hat{Z}_{t-1}^* + \hat{\varepsilon}_{Z,t}^* \tag{2.A.39}$$

$$(1-\omega)\,\hat{Q}_t + \hat{K}_{t+1} = \zeta\bar{r}_S\left(\hat{r}_{S,t} + \hat{Q}_{t-1} + \hat{K}_t\right) - \frac{\bar{b}_H}{\bar{K}}\left(\zeta\hat{b}_{H,t-1} - \frac{\hat{b}_{H,t} - \hat{r}_t}{\bar{r}}\right) + \omega\hat{K}_t \tag{2.A.40}$$

$$\bar{G}\hat{G}_t = m\,(1-\zeta)\left[\bar{r}_S\bar{K}\left(\hat{r}_{S,t} + \hat{Q}_{t-1} + \hat{K}_t\right) - \bar{b}_H\hat{b}_{H,t-1}\right] + m\bar{W}\bar{L}\left(\hat{W}_t + \hat{L}_t\right) \tag{2.A.41}$$

$$\bar{Y}_H^*\left[\left(\widehat{\frac{P_{H,t}}{P_t}}\right) + \hat{Y}_{H,t}^*\right] - \bar{Y}_F\left[\left(\widehat{\frac{P_{F,t}}{P_t}}\right) + \hat{Y}_{F,t}\right] = \left(\bar{Y}_H^* - \bar{Y}_F + \bar{b}_F\right)\left[(1+\mu_T)\,\hat{b}_{F,t} - \hat{R}_t^* - \widehat{\Delta\mathcal{E}}_{t+1} + \hat{\Pi}_{t+1}\right] - \bar{b}_F\hat{b}_{F,t-1} \tag{2.A.42}$$

2.A.3 Derivation of the Sticky Price Dynamics

Optimal Relative Price

Combining the equations (2.3.13) and (2.3.72), the optimization problem of the home intermediate good producer is given by:

$$\max_{\tilde{P}_{H,t}} \sum_{\tau=0}^{\infty} \beta^\tau \xi^\tau E_t \left\{ D_{t,t+\tau} \left[\bar{\Pi}^\tau \tilde{P}_{H,t} \left(\frac{P_{H,t+\tau|t}}{P_{H,t+\tau}} \right)^{-\varepsilon} Y_{t+\tau} V_{t+\tau} - \Phi(Y_{H,t+\tau|t} + Y^*_{H,t+\tau|t}) \right] \right\}. \quad (2.A.43)$$

From the demand functions of home purchased goods and exported goods, the equation (2.3.5), and the law of one price ($\mathcal{E}_{t+\tau} P^*_{H,t+\tau} = P_{H,t+\tau}$),

$$\frac{\partial Y_{H,t+\tau|t}}{\partial \tilde{P}_{H,t}} = -\varepsilon \left(\frac{\bar{\Pi}^\tau}{\Pi_{H,t,t+\tau}} \right)^{-\varepsilon} \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\varepsilon} \frac{Y_{H,t+\tau}}{\tilde{P}_{H,t}} \quad (2.A.44)$$

$$\frac{\partial Y^*_{H,t+\tau|t}}{\partial \tilde{P}_{H,t}} = -\varepsilon \left(\frac{\bar{\Pi}^\tau}{\Pi_{H,t,t+\tau}} \right)^{-\varepsilon} \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\varepsilon} \frac{Y^*_{H,t+\tau}}{\tilde{P}_{H,t}} \quad (2.A.45)$$

where $\Pi_{H,t,t+\tau} = P_{H,t+\tau}/P_{H,t}$. Also, $P_{H,t+\tau|t} = \bar{\Pi}^\tau \tilde{P}_{H,t}$ and $P_{H,t+\tau} = \Pi_{H,t,t+\tau} P_{H,t}$. Using the equations (2.A.44) and (2.A.45), the first-order condition is rewritten by:

$$\sum_{\tau=0}^{\infty} \beta^\tau \xi^\tau E_t \left\{ D_{t,t+\tau} \left(\frac{\bar{\Pi}^\tau}{\Pi_{H,t,t+\tau}} \right)^{-\varepsilon} \left[(1-\varepsilon)\bar{\Pi}^\tau Y_{t+\tau} V_{t+\tau} + \varepsilon P_{t+\tau} MC_{t+\tau} \left(\frac{Y_{H,t+\tau}}{\tilde{P}_{H,t}} + \frac{Y^*_{H,t+\tau}}{\tilde{P}_{H,t}} \right) \right] \right\} = 0. \quad (2.A.46)$$

Given that $Y_{t+\tau} V_{t+\tau} = Y_{H,t+\tau} + Y^*_{H,t+\tau}$, the equation (2.3.14) is then derived from

$$\sum_{\tau=0}^{\infty} \beta^\tau \xi^\tau E_t \left\{ D_{t,t+\tau} Y_{t+\tau} V_{t+\tau} \left(\frac{\bar{\Pi}^\tau}{\Pi_{H,t,t+\tau}} \right)^{-\varepsilon} \left[\bar{\Pi}^\tau \left(\frac{\tilde{P}_{H,t}}{P_t} \right) - \frac{\varepsilon}{\varepsilon-1} \Pi_{t,t+\tau} MC_{t+\tau} \right] \right\} = 0. \quad (2.A.47)$$

Dynamics of Optimal Relative Price

The optimality condition (2.3.14) can be rewritten by:

$$\frac{\tilde{P}_{H,t}}{P_t} \sum_{\tau=0}^{\infty} \tilde{\beta}^\tau E_t \left(D_{t,t+\tau} \bar{\Pi}^{(1-\varepsilon)\tau} \Pi^\varepsilon_{H,t,t+\tau} Y_{t+\tau} V_{t+\tau} \right) = \frac{\varepsilon}{\varepsilon-1} \sum_{\tau=0}^{\infty} \tilde{\beta}^\tau E_t \left(D_{t,t+\tau} \bar{\Pi}^{-\varepsilon\tau} \Pi^\varepsilon_{H,t,t+\tau} \Pi_{t,t+\tau} Y_{t+\tau} V_{t+\tau} MC_{t+\tau} \right). \quad (2.A.48)$$

Without the expectation term, at $t+1$ the optimal price would satisfy

$$\frac{\tilde{P}_{H,t+1}}{P_{t+1}} \sum_{\tau=0}^{\infty} \tilde{\beta}^\tau \left(D_{t+1,t+1+\tau} \bar{\Pi}^{(1-\varepsilon)\tau} \Pi^\varepsilon_{H,t+1,t+1+\tau} Y_{t+1+\tau} V_{t+1+\tau} \right)$$

$$= \frac{\varepsilon}{\varepsilon-1} \sum_{\tau=0}^{\infty} \tilde{\beta}^\tau \left(D_{t+1,t+1+\tau} \bar{\Pi}^{-\varepsilon\tau} \Pi^\varepsilon_{H,t+1,t+1+\tau} \Pi_{t+1,t+1+\tau} Y_{t+1+\tau} V_{t+1+\tau} MC_{t+1+\tau} \right) \quad (2.A.49)$$

By the definition, $D_{t,t+1}D_{t+1,t+\tau} = D_{t,t+\tau}$. Multiplying $\tilde{\beta}D_{t,t+1}\bar{\Pi}^{1-\varepsilon}\Pi^{\varepsilon}_{H,t,t+1}$ to the left hand side of the equation (2.A.49) yields

$$\frac{\tilde{P}_{H,t+1}}{P_{t+1}}\left(\tilde{\beta}D_{t,t+1}\bar{\Pi}^{1-\varepsilon}\Pi^{\varepsilon}_{H,t,t+1}Y_{t+1}V_{t+1} + \tilde{\beta}^2 D_{t,t+2}\bar{\Pi}^{2(1-\varepsilon)}\Pi^{\varepsilon}_{t,t+2}Y_{t+2}V_{t+2} + \cdots\right), \qquad (2.A.50)$$

which can be rewritten by:

$$\frac{\tilde{P}_{H,t+1}}{P_{t+1}}\sum_{\tau=0}^{\infty}\left(\tilde{\beta}^{\tau}D_{t,t+\tau}\bar{\Pi}^{(1-\varepsilon)\tau}\Pi^{\varepsilon}_{H,t,t+\tau}Y_{t+\tau}V_{t+\tau}\right) - \frac{\tilde{P}_{H,t+1}}{P_{t+1}}Y_t V_t. \qquad (2.A.51)$$

In a similar way, multiplying $\tilde{\beta}D_{t,t+1}\bar{\Pi}^{1-\varepsilon}\Pi^{\varepsilon}_{H,t,t+1}$ to the right hand side of the equation (2.A.49) yields

$$\frac{\varepsilon}{\varepsilon-1}\frac{\bar{\Pi}}{\Pi_{t,t+1}}\left[\sum_{\tau=0}^{\infty}\tilde{\beta}^{\tau}\left(D_{t,t+\tau}\bar{\Pi}^{-\epsilon\tau}\Pi^{\varepsilon}_{H,t,t+\tau}\Pi_{t,t+\tau}Y_{t+\tau}V_{t+\tau}MC_{t+\tau}\right) - MC_t Y_t V_t\right]. \qquad (2.A.52)$$

From the equality of (2.A.51) and (2.A.52), the following equation holds:

$$\frac{\tilde{P}_{H,t+1}}{P_{t+1}}\sum_{\tau=0}^{\infty}\left(\tilde{\beta}^{\tau}D_{t,t+\tau}\bar{\Pi}^{(1-\varepsilon)\tau}\Pi^{\varepsilon}_{H,t,t+\tau}Y_{t+\tau}V_{t+\tau}\right) - \frac{\tilde{P}_{H,t+1}}{P_{t+1}}Y_t V_t$$

$$= \frac{\varepsilon}{\varepsilon-1}\left(\frac{\bar{\Pi}}{\Pi_{t,t+1}}\right)\sum_{\tau=0}^{\infty}\tilde{\beta}^{\tau}\left(D_{t,t+\tau}\bar{\Pi}^{-\epsilon\tau}\Pi^{\varepsilon}_{H,t,t+\tau}\Pi_{t,t+\tau}Y_{t+\tau}V_{t+\tau}MC_{t+\tau}\right) - \frac{\varepsilon}{\varepsilon-1}\left(\frac{\bar{\Pi}}{\Pi_{t,t+1}}\right)MC_t Y_t V_t. \qquad (2.A.53)$$

From the equation (2.A.48), multiplying $\left[\sum\limits_{\tau=0}^{\infty}\left(\tilde{\beta}^{\tau}D_{t,t+\tau}\bar{\Pi}^{(1-\varepsilon)\tau}\Pi^{\varepsilon}_{H,t,t+\tau}Y_{t+\tau}V_{t+\tau}\right)\right]^{-1}$ to both sides of (2.A.53) yields

$$\frac{\tilde{P}_{H,t+1}}{P_{t+1}}\left\{1 - Y_t V_t \left[\sum_{\tau=0}^{\infty}\left(\tilde{\beta}^{\tau}D_{t,t+\tau}\bar{\Pi}^{(1-\varepsilon)\tau}\Pi^{\varepsilon}_{H,t,t+\tau}Y_{t+\tau}V_{t+\tau}\right)\right]^{-1}\right\}$$

$$= \left(\frac{\bar{\Pi}}{\Pi_{t,t+1}}\right)\frac{\tilde{P}_{H,t}}{P_t} - \frac{\varepsilon}{\varepsilon-1}\left(\frac{\bar{\Pi}}{\Pi_{t,t+1}}\right)MC_t Y_t V_t \left[\sum_{\tau=0}^{\infty}\tilde{\beta}^{\tau}\left(D_{t,t+\tau}\bar{\Pi}^{(1-\varepsilon)\tau}\Pi^{\varepsilon}_{H,t,t+\tau}Y_{t+\tau}V_{t+\tau}\right)\right]^{-1}. \qquad (2.A.54)$$

Defining $F_t = Y_t^{-1}V_t^{-1}\sum\limits_{\tau=0}^{\infty}\tilde{\beta}^{\tau}\left(D_{t,t+\tau}\bar{\Pi}^{(1-\varepsilon)\tau}\Pi^{\varepsilon}_{H,t,t+\tau}Y_{t+\tau}V_{t+\tau}\right)$, the equation (2.A.54) can be rewritten as follows:

$$\frac{\tilde{P}_{H,t}}{P_t} = F_t^{-1}\frac{\varepsilon}{\varepsilon-1}MC_t + \left(1 - F_t^{-1}\right)\left(\frac{\Pi_{t,t+1}}{\bar{\Pi}}\right)\left(\frac{\tilde{P}_{H,t+1}}{P_{t+1}}\right). \qquad (2.A.55)$$

This recursive form can be represented by a log-linearized equation as below:

$$\left(\widehat{\frac{\tilde{P}_{H,t}}{P_t}}\right) = \bar{F}^{-1}\widehat{MC}_t + \left(1 - \bar{F}^{-1}\right)\left(\widehat{\frac{\tilde{P}_{H,t+1}}{P_{t+1}}} + \hat{\Pi}_{t,t+1}\right). \qquad (2.A.56)$$

From the definition of the variable F_t,

$$F_{t+1} = Y_{t+1}^{-1} V_{t+1}^{-1} \sum_{\tau=0}^{\infty} \tilde{\beta}^\tau \left(D_{t+1,t+1+\tau} \bar{\Pi}^{(1-\varepsilon)\tau} \Pi_{H,t+1,t+1+\tau}^{\varepsilon} Y_{t+1+\tau} V_{t+1+\tau} \right) \tag{2.A.57}$$

and multiplying $\tilde{\beta} Y_t^{-1} V_t^{-1} Y_{t+1} V_{t+1} D_{t,t+1} \bar{\Pi}^{1-\varepsilon} \Pi_{H,t,t+1}^{\varepsilon}$ to both sides of (2.A.57) yields

$$\left(\tilde{\beta} Y_t^{-1} V_t^{-1} Y_{t+1} \bar{\Pi}^{1-\varepsilon} \Pi_{H,t,t+1}^{\varepsilon} D_{t,t+1} \right) F_{t+1} = Y_t^{-1} V_t^{-1} \sum_{\tau=1}^{\infty} \tilde{\beta}^\tau \left(D_{t,t+\tau} \bar{\Pi}^{(1-\varepsilon)\tau} \Pi_{H,t,t+\tau}^{\varepsilon} Y_{t+\tau} V_{t+\tau} \right). \tag{2.A.58}$$

Subtracting (2.A.58) from F_t yields

$$Y_t^{-1} V_t^{-1} \left[\sum_{\tau=0}^{\infty} \tilde{\beta}^\tau \left(D_{t,t+\tau} \bar{\Pi}^{(1-\varepsilon)\tau} \Pi_{H,t,t+\tau}^{\varepsilon} Y_{t+\tau} V_{t+\tau} \right) - \sum_{\tau=1}^{\infty} \tilde{\beta}^\tau \left(D_{t,t+\tau} \bar{\Pi}^{(1-\varepsilon)\tau} \Pi_{H,t,t+\tau}^{\varepsilon} Y_{t+\tau} V_{t+\tau} \right) \right] = 1$$

$$F_t - \tilde{\beta} Y_t^{-1} V_t^{-1} \bar{\Pi}^{1-\varepsilon} E_t \left(D_{t,t+1} \Pi_{H,t,t+1}^{\varepsilon} Y_{t+1} V_{t+1} F_{t+1} \right) = 1$$

which means the steady state value of F_t $(= \bar{F})$ can be derived as

$$\bar{F} = \frac{1}{1 - \tilde{\beta}} = \frac{1}{1 - \beta\xi}. \tag{2.A.59}$$

2.A.4 Data Source

Table 2.4: List of Data Sources

Data	Source (code)
Output (GDP)	Datastream: KOGDP...D
Inflation (CPI)	Bank of Korea
Real Exchange Rate	Datastream: KOQCC011H
Investment	Bank of Korea
Government Spending	Datastream: KOCNGOV.D
Consumption	Datastream: KOCNPER.D
Import/Output	The World Bank Data
Export/Output	The World Bank Data
World GDP	The World Bank Data
Net Foreign Assets	The World Bank Data
Certificate Deposit rate	Datastream: KODPNNCD
Loan-Deposit Spread	The World Bank Data
Bank Balance Sheet	Financial Supervisory Service (FSS Korea)
Bailout and Recovery	Financial Services Commission (FSC Korea)

2.A.5 Bayesian Estimation Method

The Bayesian approach in this section aims to produce the posterior densities of parameters using prior information, data and simulations. Define y as the value of data and θ as the parameter estimated. Following the Bayes' theorem[23], the posterior distribution $\pi(\theta \mid y)$ can be illustrated by

$$\pi(\theta \mid y) = \frac{f(y \mid \theta)\pi(\theta)}{f(y)} \tag{2.A.60}$$

where $\pi(\theta)$ denotes the prior distribution of the parameter. $f(y)$ is the probability density function of y, and $f(y \mid \theta)$ denotes the likelihood function. Since $f(y)$ is independent of θ, it is convenient to express the posterior distribution as follows:

$$\pi(\theta \mid y) \propto f(y \mid \theta)\pi(\theta). \tag{2.A.61}$$

The Markov Chain Monte Carlo (MCMC) simulation in the Bayesian estimation is based on the Markov process where the transition probabilities between different values in the state space depend only on the current state of the random variable[24]. Consider a stochastic process X_t in a continuous state space and a transition kernel $p(\theta_0, \theta_1) = P(X_{t+1} = \theta_1 \mid X_t = \theta_0)$. An invariant density $\pi^*(\cdot)$ for the kernel $p(\theta_0, \theta_1)$ is a density that satisfies

$$\pi^*(\theta_1) = \int_R \pi^*(\theta_0)p(\theta_0, \theta_1)d\theta_0. \tag{2.A.62}$$

Within this stationary distribution $\pi^*(\cdot)$, the probability values are independent of the actual starting value.

The invariant density equals to the desired posterior distribution with some required conditions of the Markov chain. The chain needs to be (i) irreducible, (ii) aperiodic, and (iii) positive recurrent[25]. These properties make it possible to find the transition kernel

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[23]$P(A \mid B) = P(B \mid A)P(A)/P(B)$ where $P(A)$ denotes the probability of an event A, and $P(A \mid B)$ is the probability of A given that B is true.

[24]In the MCMC algorithm, each draw slightly depends on the previous one.

[25]A Markov chain is aperiodic if for all states the period of returning to the state is one, and it is irreducible if the process can reach any other states with positive probabilities starting from a state. Also, the chain is positive recurrent if the expected return time to a state is finite. The chain is then ergodic.

using an MCMC algorithm with a large number of iterations: the Metropolis-Hastings (MH) algorithm (Greenberg, 2008). The MH algorithm is useful, since it does not require the full set of conditional distributions, contrary to the Gibbs algorithm[26].

In order to find a kernel $p(\theta_0, \theta_1)$ that has $\pi^*(\cdot)$ as its invariant distribution, firstly define a reversible kernel, $q(\cdot, \cdot)$, which satisfies

$$\pi^o(\theta_0)q(\theta_0, \theta_1) = \pi^o(\theta_1)q(\theta_1, \theta_0). \tag{2.A.63}$$

From the definition of $q(\cdot, \cdot)$ then,

$$\int_A \int_R \pi^o(\theta_0)q(\theta_0, \theta_1) \, d\theta_0 d\theta_1 = \int_A \int_R \pi^o(\theta_1)q(\theta_1, \theta_0) \, d\theta_0 d\theta_1 = \int_A \pi^o(\theta_1)d\theta_1 \tag{2.A.64}$$

which suggests that $\pi^o(\cdot)$ is an invariant distribution for the kernel $q(\cdot, \cdot)$. When $\pi(\theta_0)q(\theta_0, \theta_1) > \pi(\theta_1)q(\theta_1, \theta_0)$, it is suggested that the probability of $\theta_0 \to \theta_1$ is too high. In this case the transition density can be reduced by introducing a probability $\alpha(\theta_0, \theta_1) < 1$ (Chib and Greenberg, 1995). The probability then satisfies

$$\pi(\theta_0)q(\theta_0, \theta_1)\alpha(\theta_0, \theta_1) = \pi(\theta_1)q(\theta_1, \theta_0). \tag{2.A.65}$$

From the equation (3.4.6), in practice $\alpha(\theta_0, \theta_1)$ can be derived by

$$\alpha(\theta_0, \theta_1) = \begin{cases} \min\left\{\frac{\pi(\theta_1)q(\theta_1, \theta_0)}{\pi(\theta_0)q(\theta_0, \theta_1)}, 1\right\}, & \text{if } \pi(\theta_0)q(\theta_0, \theta_1) \neq 0 \\ 0, & \text{otherwise.} \end{cases} \tag{2.A.66}$$

In the MH algorithm, given a current state variable a new draw is accepted only when it increases the posterior density (Fernández-Villaverde, 2010). Otherwise it is accepted only proportionally. The steps of the MH algorithm are then illustrated as below:

1. Given θ_0 generate θ_1^* from $q(\theta_1, \theta_0)$, and yield u from $U(0, 1)$. Solve the model with

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[26]The invariant distribution for the Gibbs kernel $p(x, y)$ is derived from $\int p(x, y)\pi(x)dx = \int \pi(y_1 \mid x_1)\pi(y_2 \mid y_1)\pi(x_1, x_2)dx_1 dx_2 = \pi(y_2 \mid y_1)\int \pi(y_1 \mid x_2)\pi(x_2)dx_2 = \pi(y_2 \mid y_1)\pi(y_1) = \pi(y)$. $\pi(\cdot \mid \cdot)$ is a conditional distribution and $\pi(\cdot, \cdot)$ is a joint distribution.

θ_1^*. Evaluate $p(y \mid \theta_1^*)$ and $\pi(\theta_1^*)$ then with likelihood functions.

2. If $\pi(\theta_0 \mid y)q(\theta_0, \theta_1^*) < \pi(\theta_1^* \mid y)q(\theta_1^*, \theta_0)$, accept the new draw, since θ_1^* increases the posterior density assuming the symmetric transition kernel which implies $q(\theta_0, \theta_1^*) = q(\theta_1^*, \theta_0)$. If $\pi(\theta_0 \mid y)q(\theta_0, \theta_1^*) > \pi(\theta_1^* \mid y)q(\theta_1^*, \theta_0)$, accept it with the probability $\alpha(\theta_0, \theta_1^*) < 1$. This implies, accept if

$$u \le \alpha(\theta_0, \theta_1^*) = \min\left\{ \frac{\pi(\theta_1^* \mid y)q(\theta_1^*, \theta_0)}{\pi(\theta_0 \mid y)q(\theta_0, \theta_1^*)}, 1 \right\} = \min\left\{ \frac{p(y \mid \theta_1^*)\pi(\theta_1^*)q(\theta_1^*, \theta_0)}{p(y \mid \theta_0)\pi(\theta_0)q(\theta_0, \theta_1^*)}, 1 \right\}$$
(2.A.67)

where $\alpha(\theta_0, \theta_1^*)$ represents the acceptance ratio.

3. If accepted, $\theta_1 = \theta_1^*$, and otherwise $\theta_1 = \theta_0$. Eventually, the invariant density $\pi(\theta \mid y)$ is constructed.

Choosing the proposal density $q(\cdot, \cdot)$, two kernels are frequently used: the random-walk kernel and the independence kernel[27] (Greenberg, 2008).

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[27]The random-walk kernel uses a process $\theta_1 = \theta_0 + \omega$ where ω is a random variable. Since the kernel is symmetric which means $q(\theta_0, \theta_1) = q(\theta_1, \theta_0)$, the acceptance ratio satisfies $\alpha = \pi(\theta_1)/\pi(\theta_0)$. The independence kernel assumes $q(\theta_0, \theta_1) = q(\theta_1)$ which means the density is independent of the current state.

2.A.6 Bayesian Estimation Results

(a) Prior and Posterior Distributions of Estimated Parameters
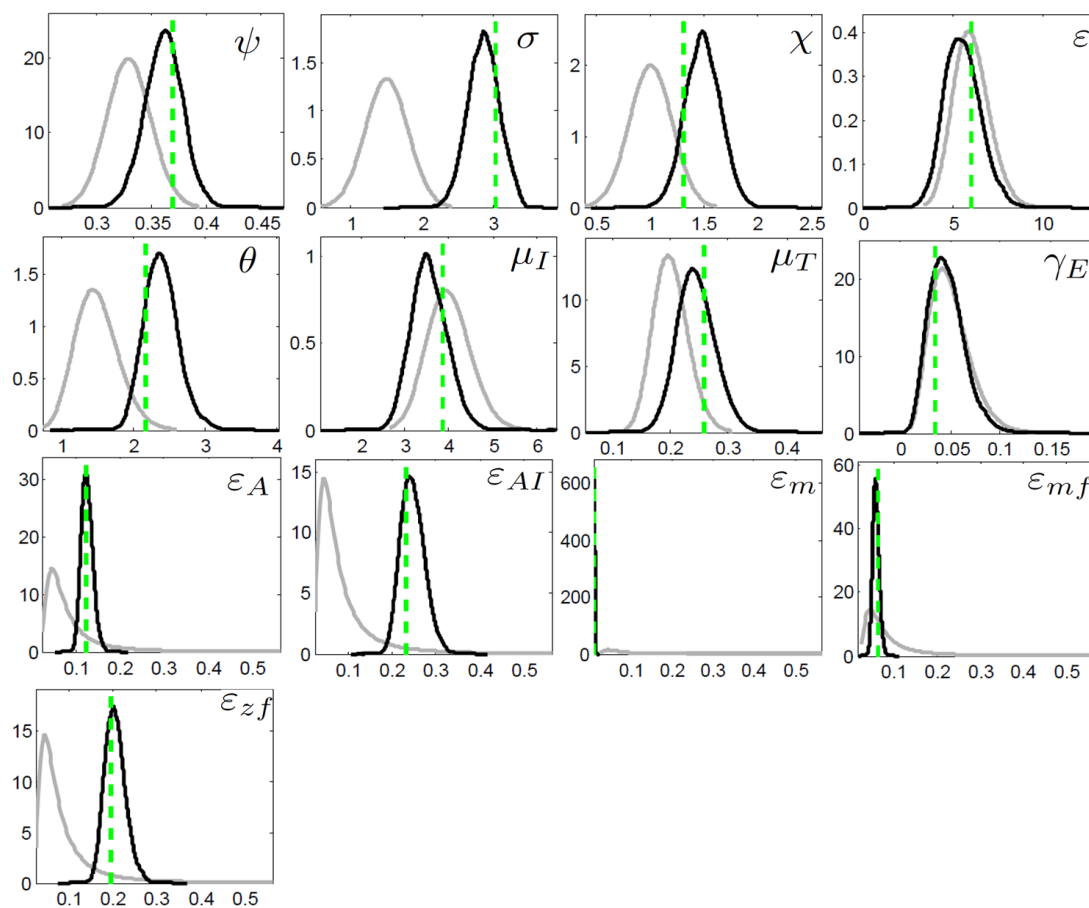
(b) Identification Results
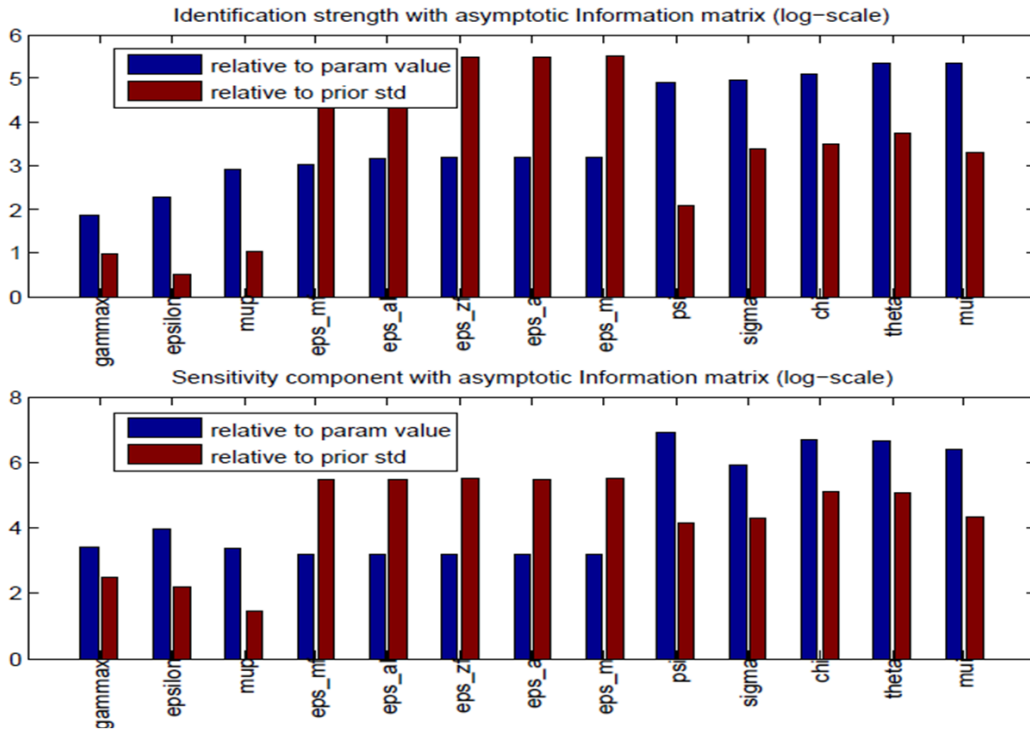
(b) Multivariate MH Convergence Diagnosis
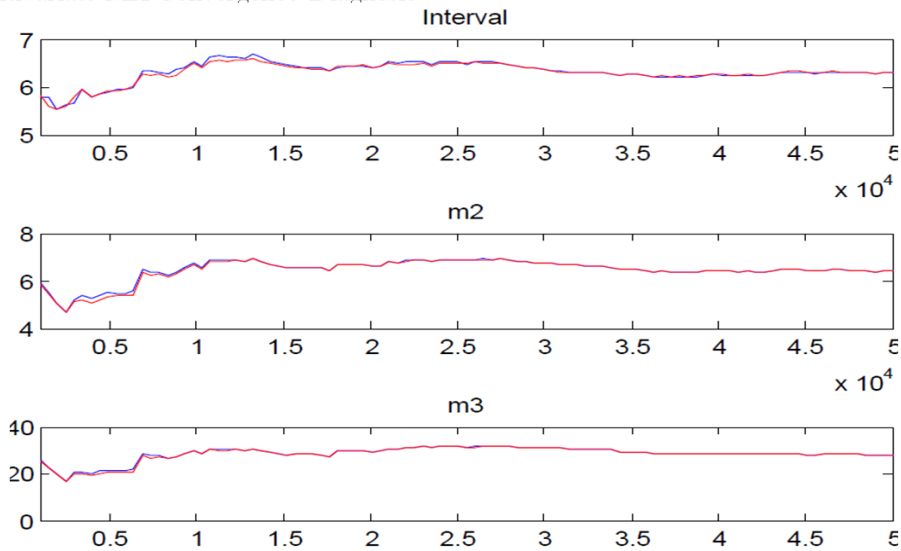
2.A.7   Comparison between Implied Interest Rate and Data
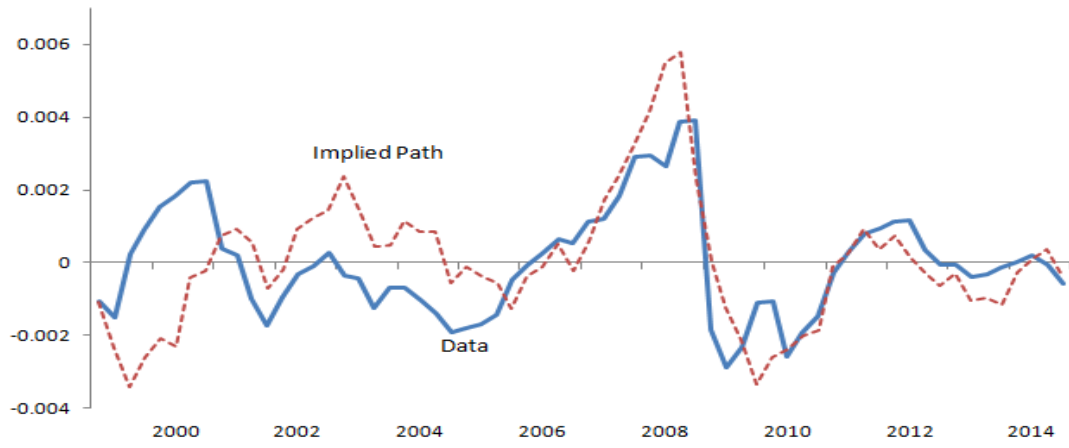
Chapter 3

Cross-border Capital Flows and International Effects of Quantitative Easing

3.1 Introduction

Since the economic crisis in the late 2000s, the unconventional monetary policy conducted by major central banks, namely quantitative easing (QE)[1], has been one of the most significant factors influencing international capital flows. Most empirical studies find that QE of those advanced economies stimulated the capital flows into other economies, lowering interest rates and raising asset prices in those economies - accompanied by currency appreciations (IMF, 2013a; Chen *et al.*, 2012). Moore *et al.* (2013) indicate that the capital flows were driven by the higher risk-adjusted returns in other economies, because QE lowered domestic interest rates. According to the portfolio balance channel, the decline in the domestic interest rate caused by

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[1]The Bank of Japan and the US Federal Reserve expanded balance sheets from 2001 and 2008, respectively. The Bank of England and the European Central Bank began QE in 2009. In Switzerland and Sweden, the balance sheet policies were adopted in 2013 and 2015, respectively.
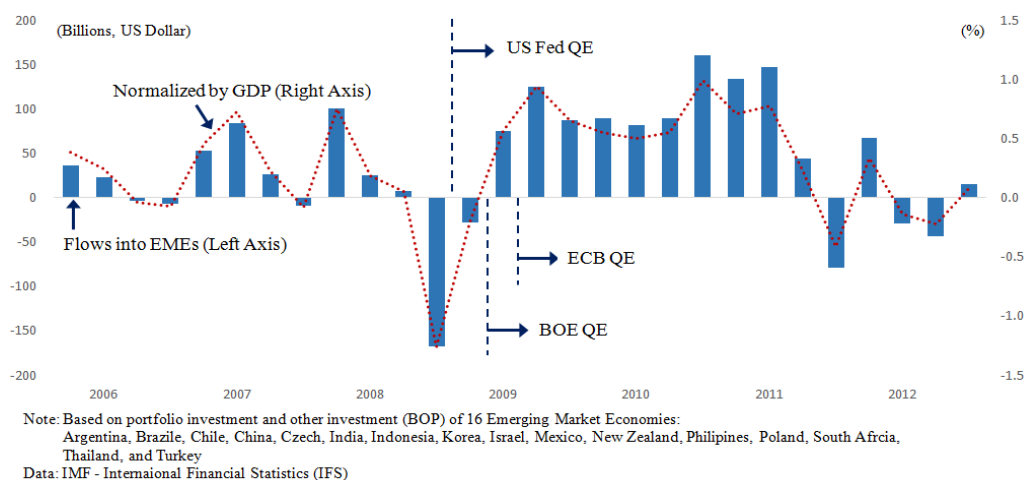
Figure 3.1: QE of Major Central Banks and Capital Flows into EMEs

QE boosts demands for other economies' assets. This also lowers interest rates and raises asset prices in those economies (Lavigne, Sarker and Vasishtha, 2014)[2].

Many empirical studies focus on the capital flows into emerging market economies (EMEs) (Cho and Rhee, 2013; IMF, 2013a). The capital flows into EMEs around the QE periods are illustrated by Figure 3.1. Emerging economies are heavily reliant on external debts (Table 3.1), and the financial crises in those economies have been closely related to capital flows (Gourinchas and Obstfeld, 2012). In this respect, investigating the cross-border effects of QE is meaningful, since the spillovers affect international capital flows and financial stability in the global economy.

This chapter models the international spill-over effects of QE, filling in the gap between empirical findings and theoretical approaches. The analysis focuses on cross-border capital flows - international lending and borrowing. The model simulations suggest that foreign QE enlarges the home economy's foreign borrowing, investment, capital stock, and the asset price - lowering interest rates in both economies; this raises home loans as well, and the home currency appreciates. These results

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[2]Lavigne, Sarker and Vasishtha (2014) explain three more channels of QE spillovers. First, the home currency appreciation caused by foreign QE negatively affects home exports (exchange rate channel). However, the increase in the foreign aggregate demand boosts home exports (trade flow channel). The foreign QE commitment also stimulates the capital flows (signalling channel).

Table 3.1: Portfolio and Other Investment Position/GDP Ratio (%, 2014)

Country	Ratio	Country	Ratio	Country	Ratio	Country	Ratio
Argentina	-20.7	Czech	-10.1	Israel	-2.4	Poland	-46.0
Brazile	-27.4	India	-27.6	Mexico	1.4	South Africa	-23.6
Chile	15.1	Indonesia	-34.0	New Zealand	-38.7	Thailand	-29.2
China	-5.6	Korea	-25.3	Philipines	-35.0	Turkey	-53.9

Note: Net assets (Assets-Liabilities). Central banks' external assets positions are not included.

Data: IMF - International Financial Statistics (IFS) and World Economic Outlook (WEO)

are in good agreement with the findings of empirical studies, such as Fratzscher, Lo Duca and Straub (2013), and He and McCauley (2013).

A two-country open economy dynamic stochastic general equilibrium (DSGE) model is established, with the home and the foreign economies. The home economy represents an emerging market economy, and the foreign economy indicates an advanced economy where the central bank conducts QE. The foreign central bank purchases foreign bank securities implementing QE. In the lending market, each home borrower (entrepreneur) can use either home or foreign loans - purchasing home capital stock. This helps to verify the link between the home and the foreign interest rates. Following the financial friction framework of Bernanke, Gertler and Gilchrist (BGG, 1999), agency problems exist not only between home agents, but also between home borrowers and foreign lenders.

The impulse response analysis suggests that foreign QE firstly expands foreign banks' liabilities, thus boosting assets (loans). As a consequence, the amount of foreign banks' overseas loans - home agents' foreign borrowing - increases; funds flow into the home economy. This raises home capital stock, investment, and the asset price. In the foreign economy, as foreign agents' borrowing rises, capital stock increases; the foreign borrowers' capital return declines, and this lowers the foreign banks' return on loans. Under the banks' no arbitrage condition, then the foreign banks' funding rate declines. As the foreign nominal interest rate falls sharply, the home currency appreciates following a modified uncovered interest rate parity.

Foreign QE lowers the home interest rate through the change in the home capital return. The drop in the foreign banks' funding rate leads to a fall in the required return on foreign banks' cross-border lending. Since the amount of home borrowers' debt repayment depends on the capital return, the home gross capital return declines. On the other hand, the fall in the home capital return lowers the home banks' income from domestic loans. Under the banks' no arbitrage condition, then the home banks' funding rate also declines.

In summary, when the foreign central bank conducts QE, the increase in the assets demand and the fall in the interest rate in the foreign economy cause funds to flow into the home economy, lowering the home interest rate. Therefore, the effects of foreign QE through capital flows in this model represent the portfolio balance channel. On the other hand, foreign QE also affects the home economy through the trade flow channel. Since the increase in foreign capital stock boosts the foreign economy, the foreign aggregate demand rises. This raises home exports, output and employment. As the wage and the marginal cost increase, home inflation rises.

The financial friction magnifies the spill-over effects of foreign QE through the financial accelerator. The unanticipated rise in the home asset price enlarges home borrowers' net worth - lowering the external finance premium as in BGG (1999). As a result, investment rises more, and this leads to more significant increases in capital stock, loans and output in the home economy.

The comparative analysis provides further implications. At first, a higher assets adjustment cost in the foreign economy increases the strength of the international effects of QE. As the foreign deposit rate declines due to QE, foreign households reduce their deposits. However, facing a higher assets adjustment cost, the decline in deposits becomes more sluggish. Combined with the liquidity injection of QE, this leads to a greater rise in foreign banks' funding; foreign banks' loans to home borrowers thus increase more. This implies that, as the foreign financial market suffers from low liquidity, the cross-border effects of foreign QE become more sig-

nificant. The analysis also suggests that a lower home investment adjustment cost yields stronger effects of foreign QE, since the propagation is through the investment and capital stock changes. Finally, a higher degree of home financial openness leads to stronger international effects of foreign QE, as home investment and capital stock are more substantially affected by the cross-border flows of funds.

The chapter continues as follows. Section 3.2 presents the relative literature. The model is illustrated in Section 3.3, and Section 3.4 describes the results of calibration and the estimation. The estimation procedure uses Bayesian approaches based on the data of South Korea and the US. In Section 3.5, impulse response analyses are presented. Finally, Section 3.6 concludes.

3.2   Literature Review

There are recent empirical studies that verify the international effects of QE (Cho and Rhee, 2013; Fratzscher, Lo Duca and Straub, 2013; He and McCauley, 2013; Lavigne, Sarker and Vasishtha, 2014; IMF, 2013a; Chen *et al.*, 2012; Lim, Mohapatra and Stocker, 2014; Park, Ramayandi and Shin, 2014). These papers find that the quantitative policies of advanced economies stimulated the capital flows into other economies, raising asset prices and inducing currency appreciations. Chen *et al.* (2012) indicate that the spillover effects of QE were stronger in emerging economies than in advanced economies. However, Ahmed and Zlate (2013) find no significant effect of QE on the capital flows into emerging market economies. IMF (2013b) argues that the cross-border effect of QE is remarkable when it restores domestic market stability.

Some studies focus on the effects of QE lowering interest rates and market risks in other economies. Neely (2012), Bauer and Neely (2013), and Bayoumi and Bui (2011) argue that QE of the large economies lowered interest rates in other countries. Moore *et al.* (2013) also find the falls in the emerging economies' bond yields.

Moreover, Krishnamurthy and Vissing-Jorgensen (2011) and Chen *et al.* (2012) highlight the cross-border effects on the credit risk.

Despite the profusion of empirical research on QE, there has been little attempt to investigate the international effects of QE theoretically. Most QE papers are based on closed economy frameworks (Cúrdia and Woodford, 2010; Gertler and Karadi, 2011; Gertler and Kiyotaki, 2011). Only a few studies analyse the cross-border effects of QE. For instance, Dedola, Karadi and Lombardo (2013) investigate the role of an endogenous QE policy rule in the propagation of other shocks. Gieck (2014) focuses on the effect of QE on the exchange rate and analyses different scenarios, such as coordinated and non-coordinated QE policies between countries.

This chapter is different from other open economy financial friction papers, since the model assumes both domestic and cross-border borrowing. Indeed, many open economy papers using the financial friction framework of Bernanke, Gertler and Gilchrist (1999) consider only domestic funding (Davis and Huang, 2011; Dedola and Lombardo, 2012; Hirakata and Kurozumi, 2013; Faia, 2007; Gertler, Gilchrist and Natalucci, 2007; Kolasa and Lombardo, 2014). Some other studies assume that home agents' borrowing is only from the foreign economy (Akinci, 2014; Cúrdia, 2009; Elekdag, Justiniano and Tchakarov, 2005). Only a few literature such as Unsal (2013) considers both domestic and cross-border borrowing, where a fraction of home agents borrow funds from the foreign economy.

Many studies indicate the enhanced role of international capital flows in the business cycle. Goldfajn and Valdés (1997) and Goldberg (2009) argue that the capital flows between countries magnify the fluctuations of economic variables. Bruno and Shin (2013) also indicate that the cross-border effect of the foreign monetary policy shock is amplified by the changes in foreign borrowing. According to Zhou (2008), a decrease in foreign loans reduces home employment and production.

3.3 The Model

3.3.1 Model Description

The baseline framework of the model is a two-country open economy version of Bernanke, Gertler and Gilchrist (BGG, 1999). The model also adopts a New Keynesian dynamic stochastic general equilibrium (DSGE) framework with price stickiness and monopolistic competition. There are two economies: home and foreign. Figure 3.2 illustrates the flows of goods and funds in the home and the foreign economies.

In the home economy, there are eight types of economic agents: households, entrepreneurs, banks, final and intermediate goods producers, a capital producer, the central bank and the government. Households consume, supply funds to home banks, and hold home government bonds. Yet they cannot purchase foreign assets nor borrow from the foreign economy due to lack of expertise as in Aoki, Benigno and Kiyotaki (2016). The foreign exchange market is only accessed by goods producers, entrepreneurs and the government[3], because only licensed institutions can trade foreign currencies. Intermediate goods firms use households' labour and capital inputs. Final goods producers combine home and foreign intermediate goods. Entrepreneurs borrow funds from home and foreign banks to purchase home capital stock and lend it to home intermediate goods producers - receiving rental incomes.

A fraction of home entrepreneurs seek funding from foreign banks due to a low level of home financial development; at the steady state, home banks' domestic funding is far less than the total loan demands from home entrepreneurs[4]. Foreign loans are denominated in foreign currency[5]. As in Unsal (2013), there are two

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[3]This assumption is similar to Cúrdia (2009) and Montoro and Ortiz (2012) where households do not participate in the foreign exchange market. Carlson, Dahl and Osler (2008) also indicate that currencies are traded in wholesale markets where most of participants are large institutions.

[4]In many emerging market economies, the amount of debts to foreign economies is larger than that of external assets (Table 3.1).

[5]As indicated by Elekdag, Justiniano and Tchakarov (2005), and Gourinchas and Obstfeld (2012), in emerging market economies foreign credit is typically denominated in foreign currency.
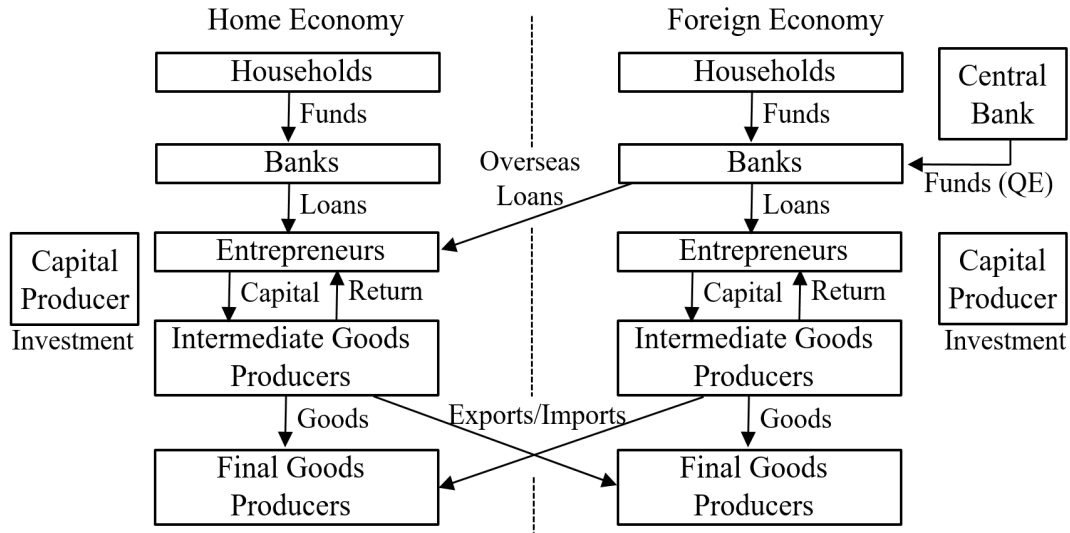
Figure 3.2: Home and Foreign Economies: Goods and Funds Flows

groups of entrepreneurs: group I and group J. The entrepreneurs in the group I borrow funds from home banks, and group J entrepreneurs use foreign loans. As in BGG (1999), each entrepreneur is exposed to an idiosyncratic risk that affects the realized capital return. Since the disturbance is not observed by home and foreign banks, there are agency problems between borrowers and lenders. In case of an entrepreneur's default, the lender needs to pay an auditing cost in order to verify the entrepreneur's realized income.

The basic framework of the foreign economy is similar to that of the home economy - with eight types of economic agents. For simplicity, there is no agency problem between foreign banks and entrepreneurs. Also, competitiveness is assumed among foreign banks and capital producers, and intermediate goods producers set prices in a flexible way. Implementing QE, the foreign central bank injects liquidity into the banking sector by purchasing the securities issued by foreign banks.

3.3.2 Households

In the home economy, there is a continuum of identical households indicated by $h \in (0, 1)$. Also, there is a continuum of households indicated by $f \in (0, \tau)$ in the foreign

economy, where τ denotes the mass of the foreign households. Households consume and provide the production sector with labour. The home household h and the foreign household f maximize the following lifetime utility functions, respectively:

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left[\frac{C_t(h)^{1-\sigma}}{1-\sigma} - \frac{N_t^s(h)^{1+\chi}}{1+\chi} \right] \qquad E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left[\frac{C_t^*(f)^{1-\sigma}}{1-\sigma} - \frac{N_t^{s*}(f)^{1+\chi}}{1+\chi} \right]$$
$$(3.3.1)$$

where $C_t(h)$ $(C_t^*(f))$ and $N_t^s(h)$ $(N_t^{s*}(f))$ denote consumption and labour supply of the home (foreign) household h (f) at time t in real terms, respectively. The parameter σ represents the coefficient of relative risk aversion of households or the reciprocal of the intertemporal elasticity of substitution. χ denotes the inverse of the elasticity of the labour supply.

Households receive wages and earn incomes from ownership of goods and capital producers. During each period, the home and the foreign households consume, deposit, and purchase government bonds. They do not hold international assets. The home household h faces a real budget constraint as below:

$$\begin{aligned} C_t(h) \quad + \quad & R_t^{-1} B_t(h) + R_{D,t}^{-1} D_t(h) + \Psi\left(D_t(h)\right) \\ = \quad & W_t N_t^s(h) + D_{t-1}(h) + B_{t-1}(h) + \Omega_t(h) + \Omega_t^E + \kappa_t^A \end{aligned} \qquad (3.3.2)$$

and the real budget constraint of the foreign household f is given by:

$$\begin{aligned} C_t^*(f) \quad + \quad & R_t^{*-1} B_t^*(f) + R_{D,t}^{*-1} D_t^*(f) + \Psi^*\left(D_t^*(f)\right) + T^* + \Omega_t^{CB*} \\ = \quad & W_t^* N_t^{s*}(f) + D_{t-1}^*(f) + B_{t-1}^*(f) + \Omega_t^*(f) + \kappa_t^{A*}, \end{aligned} \qquad (3.3.3)$$

where $B_t(h)$ $(B_t^*(f))$ and $D_t(h)$ $(D_t^*(f))$ denote the government bonds holding and deposits of the home (foreign) household h (f), respectively. R_t (R_t^*) and $R_{D,t}$ $(R_{D,t}^*)$ denote the real gross interest rate on home (foreign) government bonds and the home (foreign) banks' funding rate (deposit rate), respectively. W_t and W_t^* denote real

wages, and $\Omega_t(h)$ $(\Omega_t^*(f))$ indicates the profits from home (foreign) intermediate goods and capital producers in real terms. Ω_t^E is net transfer from the home entrepreneur sector[6]. Ω_t^{CB*} denotes the net transfer to the foreign central bank, which supports the unconventional policy. When $\Omega_t^{CB*} > 0$, the foreign households finance the central bank's assets increase. T^* denotes the tax in the foreign economy, which is fixed every period and supports foreign government spending. For simplicity, there is no government spending in the home economy. Also, no-Ponzi scheme is assumed, which means $\lim\limits_{\tau\to\infty}\prod\limits_{t=1}^{\tau}R_t^{-1}B_\tau(h)$, $\lim\limits_{\tau\to\infty}\prod\limits_{t=1}^{\tau}R_{D,t}^{-1}D_\tau(h) = 0$, $\lim\limits_{\tau\to\infty}\prod\limits_{t=1}^{\tau}R_t^{*-1}B_\tau^*(f) = 0$ and $\lim\limits_{\tau\to\infty}\prod\limits_{t=1}^{\tau}R_{D,t}^{*-1}D_\tau^*(f) = 0$.

As in Ghironi, Lee and Rebucci (2007), the home and the foreign households h and f pay assets transaction costs $\Psi(\cdot)$ and $\Psi^*(\cdot)$ when their deposit holdings deviate from the steady states, $\bar{D}$ and $\bar{D}^*$, respectively. These costs help determining the home and the foreign households' well-defined steady state assets portfolios without taking second-order approximations. The cost functions are given by:

$$\Psi(D_t(h)) = \frac{\mu_A}{2}\left(D_t(h) - \bar{D}\right)^2 \tag{3.3.4}$$

$$\Psi^*(D_t^*(f)) = \frac{\mu_A^*}{2}\left(D_t^*(f) - \bar{D}^*\right)^2. \tag{3.3.5}$$

The parameters μ_A and μ_A^* represent the levels of the home and the foreign assets transaction costs. King (2016) indicates that portfolio investors can sell their assets with a very small reduction in the price when the financial market is liquid. Without enough market liquidity, the reduction would be larger - represented by higher μ_A or μ_A^*; each agent needs to pay a greater cost of finding the counterparts of the trade. As the international assets transaction cost in Christoffel, Coenen and Warne (2008), the average assets transaction costs are rebated in a lump-sum manner, being indicated by κ_t^A and κ_t^{A*}.

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[6]There is no net worth in the foreign entrepreneur sector, which is to be explained later.

The first-order conditions facing the home household h are:

$$\frac{N_t^s(h)^\chi}{C_t(h)^{-\sigma}} = W_t \qquad (3.3.6)$$

$$\beta R_t E_t \left(\frac{C_{t+1}(h)}{C_t(h)}\right)^{-\sigma} = 1 \qquad (3.3.7)$$

$$\beta \left[R_{D,t}^{-1} + \mu_A \left(D_t(h) - \bar{D}\right)\right]^{-1} E_t \left(\frac{C_{t+1}(h)}{C_t(h)}\right)^{-\sigma} = 1, \qquad (3.3.8)$$

and the optimization of the foreign household f implies:

$$\frac{N_t^{s*}(f)^\chi}{C_t^*(f)^{-\sigma}} = W_t^* \qquad (3.3.9)$$

$$\beta R_t^* E_t \left(\frac{C_{t+1}^*(f)}{C_t^*(f)}\right)^{-\sigma} = 1 \qquad (3.3.10)$$

$$\beta \left[R_{D,t}^{*-1} + \mu_A^* \left(D_t^*(f) - \bar{D}^*\right)\right]^{-1} E_t \left(\frac{C_{t+1}^*(f)}{C_t^*(f)}\right)^{-\sigma} = 1. \qquad (3.3.11)$$

3.3.3 Home Entrepreneurs

Home entrepreneurs borrow funds from home and foreign banks. Using the funds, they purchase capital stock from the home capital producer. The entrepreneurs then lend capital stock to intermediate goods producers and receive rental incomes. The baseline framework follows Bernanke, Gertler and Gilchrist (1999).

Home and Foreign Loans, and Idiosyncratic Risk

Due to a low level of financial development, the amount of home banks' total funding is far less than the entire home entrepreneurs' loan demands at the steady state. Therefore, some of the home entrepreneurs use foreign loans. Following Unsal (2013), there are two groups of entrepreneurs: group I and group J. The entrepreneurs in the group I borrow funds from home banks, and the others in the group J are funded by foreign banks. Foreign loans are denominated in foreign currency, and thus the group J entrepreneurs are licensed by the home government to participate

in the foreign exchange market.

The total mass of the home entrepreneur sector is unity. The share of the group J entrepreneurs borrowing funds from foreign banks at time t is $\gamma_{t+1} \in (0, 1)$. Unsal (2013) assumes identical sizes of those two types of entrepreneurs. In this model, the steady state value of γ_{t+1} ($= \bar{\gamma}$) is fixed - reflecting the long-run level of demand and supply of loans in the home economy. In the short-run, γ_{t+1} can deviate from the steady state, as the government can adjust the mass of group J entrepreneurs[7].

In order to finance capital stock ($K_{t+1}(i)$) at time t, the entrepreneur $i \in (0, 1 - \gamma_{t+1})$ in the group I uses home bank loans in addition to its own net worth ($NW_t(i)$). Likewise the entrepreneur $j \in (1 - \gamma_{t+1}, 1)$ in the group J borrows funds from foreign banks. The price of one unit of capital is Q_t in terms of home final goods. Defining $L_t(i)$ and $L_t^F(j)$ as the real amounts of home and foreign (cross-border) loans, respectively,

$$L_t(i) + NW_t(i) \;=\; Q_t K_{t+1}(i) \tag{3.3.12}$$

$$S_{R,t} L_t^F(j) + NW_t(j) \;=\; Q_t K_{t+1}(j) \tag{3.3.13}$$

where $L_t^F(j)$ is denominated in units of foreign final goods[8]. $S_{R,t}$ denotes the real exchange rate which is defined as $S_{R,t} = S_t P_t^*/P_t$. S_t is the nominal exchange rate - the value of the foreign currency denominated in home currency. P_t^* and P_t are foreign and home final goods aggregate prices. The average capital stock purchased in each group can be illustrated as:

$$K_{t+1}^I = \frac{1}{1 - \gamma_{t+1}} \int_0^{1-\gamma_{t+1}} K_{t+1}(i)di \qquad K_{t+1}^J = \frac{1}{\gamma_{t+1}} \int_{1-\gamma_{t+1}}^1 K_{t+1}(j)dj, \tag{3.3.14}$$

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[7]This assumption is not fundamentally different from that of fixed γ; even when the ratio γ is fixed, in the short-run the ratio of 'foreign loans/home loans' deviates from the steady state in response to shocks.

[8]In home currency, the nominal amount of foreign loans can be expressed by $S_t P_t^* L_t^F(j) = P_t (Q_t K_{t+1}(j) - NW_t(j))$.
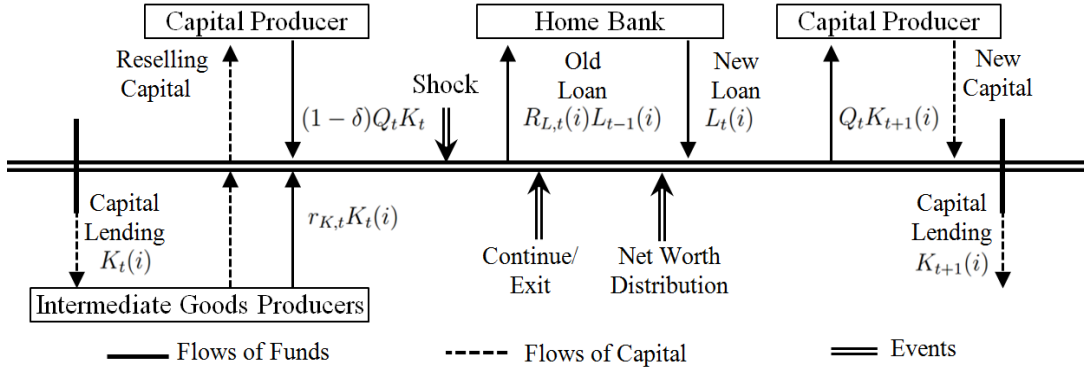
Figure 3.3: Entrepreneur i's Business at time t: Flows of Funds and Capital

and the average home and foreign loans are

$$L_t = \frac{1}{1-\gamma_{t+1}} \int_0^{1-\gamma_{t+1}} L_t(i)di \qquad L_t^F = \frac{1}{\gamma_{t+1}} \int_{1-\gamma_{t+1}}^1 L_t^F(j)dj. \qquad (3.3.15)$$

The aggregate capital stock (K_{t+1}) can be illustrated as follows:

$$K_{t+1} = (1-\gamma_{t+1})K_{t+1}^I + \gamma_{t+1}K_{t+1}^J. \qquad (3.3.16)$$

From capital lent last period ($K_t(i)$ and $K_t(j)$), entrepreneurs i and j receive rental incomes from intermediate goods producers. The flows of funds and capital for the entrepreneur i are illustrated by Figure 3.3[9]. The rental rate of one unit of capital $(r_{K,t})$ is common to all entrepreneurs. During production, capital stock depreciates by the rate of δ. After production, entrepreneurs resell depreciated capital stock to the capital producer with the price Q_t. Eventually the gross return on one unit of capital (R_t^K) is then illustrated as follows:

$$R_t^K = \frac{r_{K,t} + (1-\delta)Q_t}{Q_{t-1}}. \qquad (3.3.17)$$

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[9]The entrepreneur j borrows funds from foreign banks. All other funds and capital flows are the same.

When the gross capital return (R_t^K) is realized, each entrepreneur experiences an idiosyncratic shock. For the entrepreneurs i and j, these are represented by idiosyncratic disturbances $\omega_t(i)$ and $\omega_t(j)$ - random variables, and i.i.d across time and across entrepreneurs. Those variables have continuous log-normal density functions, $f(\omega_t(i))$ and $f(\omega_t(j))$. The expected values are all unity, $E\{\omega_t(i)\} = 1$ and $E\{\omega_t(j)\} = 1$. For the entrepreneurs i and j, then the realized gross capital incomes at time t become

$$\omega_t(i)R_t^K Q_{t-1}K_t(i) \qquad \text{and} \qquad \omega_t(j)R_t^K Q_{t-1}K_t(j). \tag{3.3.18}$$

Agency Problem and Loan Contracts

As in BGG (1999), given the expected gross capital return, the entrepreneur i in the group I chooses $K_{t+1}(i)$ at time t prior to the realization of the idiosyncratic shock $(t+1)$. The optimal contract is then characterized by a cut-off value $\omega_{t+1}^I(i) \in (0,1)$ and a non-default real gross interest rates $R_{L,t+1}(i)$. When the realized shock $\omega_{t+1}(i)$ is below the threshold value $\omega_{t+1}^I(i)$, the entrepreneur i becomes bankrupt. Before the realization of $\omega_{t+1}(i)$, the gross capital income is $R_{t+1}^K Q_t K_{t+1}(i)$ at time $t+1$, and the amount of debt repayment is $R_{L,t+1}(i)L_t(i)$. Therefore, when $\omega_{t+1}(i) < \omega_{t+1}^I(i)$, $R_{L,t+1}(i)L_t(i)$ is not payable: $\omega_{t+1}(i)R_{t+1}^K Q_t K_{t+1}(i) < R_{L,t+1}(i)L_t(i)$. When $\omega_{t+1}(i) \geq \omega_{t+1}^I(i)$, it pays $R_{L,t+1}(i)$. The gross capital income is equivalent to debt repayment when $\omega_{t+1}(i) = \omega_{t+1}^I(i)$, which is illustrated as below:

$$\omega_{t+1}^I(i)R_{t+1}^K Q_t K_{t+1}(i) = R_{L,t+1}(i)L_t(i). \tag{3.3.19}$$

For the entrepreneur j in the group J, similarly, in terms of home final goods

$$\omega_{t+1}^J(j)R_{t+1}^K Q_t K_{t+1}(j) = R_{L,t+1}^F(j)L_t^F(j)S_{R,t+1}. \tag{3.3.20}$$

In case of default, home (foreign) banks can take all the remaining income of the

entrepreneur, $\omega_{t+1}(i)R_{t+1}^K Q_t K_{t+1}(i)$ $(\omega_{t+1}(j)R_{t+1}^K Q_t K_{t+1}(j))$. However, the realized idiosyncratic disturbances are not observed by home and foreign banks, and there are auditing (monitoring) costs for the verification, μ^I and μ^J for one unit of the income (costly state verification). The levels of these costs are identical in this model ($\mu^I = \mu^J = \mu$). Home and foreign banks purchase home final goods in order to conduct the audits.

For the entrepreneur i, the amount of expected debt repayment to home banks at $t+1$ is determined by two cases: (i) default ($0 \leq \omega_{t+1}(i) < \omega_{t+1}^I(i)$) and (ii) non-default ($\omega_{t+1}^I(i) \leq \omega_{t+1}(i)$). Defining $\Xi\left(\omega_{t+1}^I(i)\right)$ as the amount of expected repayment of the entrepreneur i,

$$\Xi\left(\omega_{t+1}^I(i)\right) = R_{t+1}^K Q_t K_{t+1}(i) \int_0^{\omega_{t+1}^I(i)} \omega f(\omega)d\omega + R_{L,t+1}L_t(i)\int_{\omega_{t+1}^I(i)}^\infty f(\omega)d\omega.$$
(3.3.21)

Likewise the entrepreneur j's expected debt repayment to foreign banks is

$$\Xi\left(\omega_{t+1}^J(j)\right) = R_{t+1}^K Q_t K_{t+1}(j) \int_0^{\omega_{t+1}^J(j)} \omega f(\omega)d\omega + R_{L,t+1}^F L_t^F(j)S_{R,t+1}\int_{\omega_{t+1}^J(j)}^\infty f(\omega)d\omega$$
(3.3.22)

in terms of home final goods. Define $\Gamma(\omega_{t+1}^I(i))$ and $\Gamma(\omega_{t+1}^J(j))$ as

$$\Gamma\left(\omega_{t+1}^I(i)\right) = \int_0^{\omega_{t+1}^I(i)} \omega f(\omega)d\omega + \omega_{t+1}^I(i)\int_{\omega_{t+1}^I(i)}^\infty f(\omega)d\omega \qquad (3.3.23)$$

$$\Gamma\left(\omega_{t+1}^J(j)\right) = \int_0^{\omega_{t+1}^J(j)} \omega f(\omega)d\omega + \omega_{t+1}^J(j)\int_{\omega_{t+1}^J(j)}^\infty f(\omega)d\omega. \qquad (3.3.24)$$

From the equations (3.3.19) and (3.3.20), then the amounts of expected debt repayment of the entrepreneurs i and j can be rewritten by

$$\Xi\left(\omega_{t+1}^I(i)\right) = R_{t+1}^K Q_t K_{t+1}(i)\Gamma\left(\omega_{t+1}^I(i)\right) \qquad \Xi\left(\omega_{t+1}^J(j)\right) = R_{t+1}^K Q_t K_{t+1}(j)\Gamma\left(\omega_{t+1}^J(j)\right)$$
(3.3.25)

where $\Gamma\left(\omega_{t+1}^I(i)\right)$ and $\Gamma\left(\omega_{t+1}^J(j)\right)$ represent home and foreign banks' income shares

in one unit of the capital income with respect to the loans to i and j, respectively.

In case of default, the expected remaining capital incomes of the entrepreneurs i and j would be

$$R_{t+1}^K Q_t K_{t+1}(i) \int_0^{\omega_{t+1}^I(i)} \omega f(\omega) d\omega \qquad \text{and} \qquad R_{t+1}^K Q_t K_{t+1}(j) \int_0^{\omega_{t+1}^J(j)} \omega f(\omega) d\omega \tag{3.3.26}$$

in terms of home final goods, respectively. Define then

$$M\left(\omega_{t+1}^I(i)\right) = \int_0^{\omega_{t+1}^I(i)} \omega f(\omega) d\omega \qquad \text{and} \qquad M\left(\omega_{t+1}^J(j)\right) = \int_0^{\omega_{t+1}^J(j)} \omega f(\omega) d\omega. \tag{3.3.27}$$

In order to verify the remaining incomes, home and foreign banks pay

$$\mu^I R_{t+1}^K Q_t K_{t+1}(i) M\left(\omega_{t+1}^I(i)\right) \qquad \text{and} \qquad \mu^J R_{t+1}^K Q_t K_{t+1}(j) M\left(\omega_{t+1}^J(j)\right) \tag{3.3.28}$$

as the monitoring costs, respectively.

The home and the cross-border lending markets are competitive, and thus the expected profits are zero across lenders. Given the home (R_D) and the foreign banks' funding rates (R_D^*), from (3.3.25) and (3.3.28), following equations hold:

$$E_t\left(R_{t+1}^K\right) Q_t K_{t+1}(i) \left[\Gamma\left(\omega_{t+1}^I(i)\right) - \mu^I M\left(\omega_{t+1}^I(i)\right)\right] = R_{D,t} L_t(i) \tag{3.3.29}$$

$$E_t\left(R_{t+1}^K\right) Q_t K_{t+1}(j) \left[\Gamma\left(\omega_{t+1}^J(j)\right) - \mu^J M\left(\omega_{t+1}^J(j)\right)\right] = R_{D,t}^* E_t\left(S_{R,t+1}\right) L_t^F(j). \tag{3.3.30}$$

Due to competitiveness of the home and the cross-border lending markets, loan contracts are made such that entrepreneurs' profits are maximized. Given the home and the foreign banks' incomes illustrated by (3.3.25), the income shares of the entrepreneurs i and j in one unit of the capital return are $1 - \Gamma\left(\omega_{t+1}^I(i)\right)$ and $1 - \Gamma\left(\omega_{t+1}^J(j)\right)$, respectively. Given the banks' expected zero profit conditions (3.3.29) and (3.3.30), the loan contracts specify $\left\{\omega_{t+1}^I(i), K_{t+1}(i)\right\}$ and $\left\{\omega_{t+1}^J(j), K_{t+1}(j)\right\}$

that solve the following optimization problems:

$$\max_{\left\{K_{t+1}(i),\omega_{t+1}^I(i)\right\}} \left[1-\Gamma\left(\omega_{t+1}^I(i)\right)\right] E_t\left(R_{t+1}^K\right)Q_tK_{t+1}(i) \qquad (3.3.31)$$

$$\max_{\left\{K_{t+1}(j),\omega_{t+1}^J(j)\right\}} \left[1-\Gamma\left(\omega_{t+1}^J(j)\right)\right] E_t\left(R_{t+1}^K\right)Q_tK_{t+1}(j). \qquad (3.3.32)$$

Assume solutions are interior[10], and $\omega^I f(\omega^I)/\left(1-F(\omega^I)\right)$ and $\omega^J f(\omega^J)/\left(1-F(\omega^J)\right)$ are both increasing in ω^I and ω^J, respectively (regularity condition). First-order conditions then imply[11]

$$\frac{E_t\left(R_{t+1}^K\right)}{R_{D,t}} = \left\{\frac{\left[1-\Gamma\left(\omega_{t+1}^I(i)\right)\right]\left[\Gamma'\left(\omega_{t+1}^I(i)\right)-\mu^I M'\left(\omega_{t+1}^I(i)\right)\right]}{\Gamma'\left(\omega_{t+1}^I(i)\right)} + \left[\Gamma\left(\omega_{t+1}^I(i)\right)-\mu^I M\left(\omega_{t+1}^I(i)\right)\right]\right\}^{-1}$$
$$(3.3.33)$$

$$\frac{E_t\left(R_{t+1}^K\right)}{R_{D,t}^*} = \frac{E_t\left(S_{R,t+1}\right)}{S_{R,t}}\left\{\frac{\left[1-\Gamma\left(\omega_{t+1}^J(j)\right)\right]\left[\Gamma'\left(\omega_{t+1}^J(j)\right)-\mu^J M'\left(\omega_{t+1}^J(j)\right)\right]}{\Gamma'\left(\omega_{t+1}^J(j)\right)} + \left[\Gamma\left(\omega_{t+1}^J(j)\right)-\mu^J M\left(\omega_{t+1}^J(j)\right)\right]\right\}^{-1}$$
$$(3.3.34)$$

which determine the optimal threshold values of $\omega_{t+1}^I(i)$ and $\omega_{t+1}^J(j)$ given $E_t\left(R_{t+1}^K\right)/R_{D,t}$ and $E_t\left(R_{t+1}^K\right)/R_{D,t}^*$. As can be seen from the first-order conditions, $E_t\left(R_{t+1}^K\right) = R_{D,t}$ and $E_t\left(R_{t+1}^K\right) = R_{D,t}^*\frac{E_t\left(S_{R,t+1}\right)}{S_{R,t}}$ when the monitoring costs are zero ($\mu^I = \mu^J = 0$). Since R^K, R_D, R_D^* and S_R are common across entrepreneurs, the first-order conditions suggest that $\omega_{t+1}^I(i)$ and $\omega_{t+1}^J(j)$ are identical within each group (I and J). Defining ω_{t+1}^I and ω_{t+1}^J as the common cut-off values in each group, the equations (3.3.33) and (3.3.34) are then rewritten as

$$\frac{E_t\left(R_{t+1}^K\right)}{R_{D,t}} = \rho_I(\omega_{t+1}^I) \qquad \frac{E_t\left(R_{t+1}^K\right)}{R_{D,t}^*} = \frac{E_t\left(S_{R,t+1}\right)}{S_{R,t}}\rho_J(\omega_{t+1}^J), \qquad (3.3.35)$$

where $\rho_I'(\omega_{t+1}^I) > 0$ and $\rho_J'(\omega_{t+1}^J) > 0$ following the regularity condition[12].

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[10]As in BGG (1999), $E_t\left(R_{t+1}^K\right)/R_{D,t} < 1/\left(\Gamma(\omega_{G,t+1}^I)-\mu M(\omega_{G,t+1}^I)\right)$ and $E_t\left(R_{t+1}^K\right)/R_{D,t}^* < 1/\left(\Gamma(\omega_{G,t+1}^J)-\mu M(\omega_{G,t+1}^J)\right)$ are assumed. $\omega_{G,t+1}^I$ and $\omega_{G,t+1}^I$ are lenders' global optimal values such that $\Gamma'\left(\omega_{G,t+1}^I\right)-\mu^I M'\left(\omega_{G,t+1}^I\right) = 0$ and $\Gamma'\left(\omega_{G,t+1}^J\right)-\mu^J M'\left(\omega_{G,t+1}^J\right) = 0$

[11]The derivations of the first-order conditions are illustrated in the appendix 3.A.3.

[12]Proofs are illustrated in BGG (1999). Also, any monotonically increasing transformation of the normal distribution satisfies this condition (BGG, 1999). In this chapter, ω^I and ω^J are

The first-order conditions also imply that R_D and R_D^* are linked by a modified uncovered interest rate parity (UIP) through the common home capital return (R^K). Combining the two equations in (3.3.35) and using $S_{R,t} = S_t P_t^*/P_t$,

$$R_{D,t} = R_{D,t}^* E_t \left(\frac{\Pi_{t+1}^*}{\Pi_{t+1}} \frac{S_{t+1}}{S_t} \right) \frac{\rho_J(\omega_{t+1}^J)}{\rho_I(\omega_{t+1}^I)} \qquad (3.3.36)$$

where

$$E_t \left(\frac{\Pi_{t+1}^*}{\Pi_{t+1}} \frac{S_{t+1}}{S_t} \right) = \frac{E_t \left(S_{R,t+1} \right)}{S_{R,t}}. \qquad (3.3.37)$$

Defining then R_{ND} and R_{ND}^* as the home and the foreign nominal deposit rates,

$$R_{ND,t} = R_{D,t} E_t \left(\Pi_{t+1} \right) \qquad R_{ND,t}^* = R_{D,t}^* E_t \left(\Pi_{t+1}^* \right). \qquad (3.3.38)$$

From the equations (3.3.12) and (3.3.13), the equations (3.3.29) and (3.3.30) are then transformed into[13]

$$Q_t K_{t+1}(i) = \phi_t^I NW_t(i) \qquad Q_t K_{t+1}(j) = \phi_t^J NW_t(j) \qquad (3.3.39)$$

where

$$\begin{aligned} \phi_t^I &= \left\{ 1 - \rho_I(\omega_{t+1}^I) \left[\Gamma \left(\omega_{t+1}^I \right) - \mu^I M \left(\omega_{t+1}^I \right) \right] \right\}^{-1} \\ \phi_t^J &= \left\{ 1 - \rho_I(\omega_{t+1}^J) \left[\Gamma \left(\omega_{t+1}^J \right) - \mu^J M \left(\omega_{t+1}^J \right) \right] \right\}^{-1}. \end{aligned} \qquad (3.3.40)$$

The variables ϕ_t^I and ϕ_t^J denote the leverage ratios of group I and J, respectively. Since $\Gamma' \left(\omega_{t+1}^I \right) - \mu^I M' \left(\omega_{t+1}^I \right) > 0$ and $\Gamma' \left(\omega_{t+1}^J \right) - \mu^J M' \left(\omega_{t+1}^J \right) > 0$ with the interior solutions assumption, the leverages are increasing in ω_{t+1}^I and ω_{t+1}^J. Therefore, ϕ_t^I and ϕ_t^J are increasing in $E_t \left(R_{t+1}^K \right)/R_{D,t}$ and $E_t \left(R_{t+1}^K \right)/R_{D,t}^*$, respectively[14].

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assumed to have log-normal distributions.

[13] The derivation of the equation (3.3.39) is illustrated in the appendix 3.A.3.

[14] ω_{t+1}^I and ω_{t+1}^J are increasing in $E_t \left(R_{t+1}^K \right)/R_{D,t}$ and $E_t \left(R_{t+1}^K \right)/R_{D,t}^*$ in (3.3.35), respectively.

When the expected income of foreign borrowing increases (decreases) due to an external shock, more (less) home entrepreneurs would like to borrow funds from the foreign economy. This happens when the foreign interest rate drops and thus home agents' foreign debts repayment is reduced. Without any constraint, however, the number of the group J entrepreneurs (γ_{t+1}) would be indeterminate in response to the shock. Solving this problem, it is assumed that the number of foreign loans users is controlled by the home government in the short-run. Since home agents would get foreign funds only if they can participate in the FX market, the government control is through FX trade authorization[15]. The home government maximizes the sum of the total entrepreneurs' expected income at $t+1$ by choosing γ_{t+1} at time t, facing an additional social cost. When γ_{t+1} deviates from the steady state ($\bar{\gamma}$), there is a cost of setting up temporary funding channels, such as local branches. This cost is paid by home households, reducing the amount of net transfer to them at time $t+1$. The optimization problem of the government is then illustrated by:

$$\max_{\{\gamma_{t+1}\}} \left\{ (1-\gamma_{t+1}) \left[1 - \Gamma\left(\omega_{t+1}^I\right)\right] K_{t+1}^I + \gamma_{t+1} \left[1 - \Gamma\left(\omega_{t+1}^J\right)\right] K_{t+1}^J \right\} E_t\left(R_{t+1}^K\right) Q_t - \frac{\mu_\gamma}{2} \left(\gamma_{t+1} - \bar{\gamma}\right)^2$$

where μ_γ denotes the entrepreneurs reallocation cost parameter. The first-order condition implies:

$$\mu_\gamma \left(\gamma_{t+1} - \bar{\gamma}\right) = \left\{ \left[1 - \Gamma\left(\omega_{t+1}^J\right)\right] K_{t+1}^J - \left[1 - \Gamma\left(\omega_{t+1}^I\right)\right] K_{t+1}^I \right\} E_t\left(R_{t+1}^K\right) Q_t.$$

$$(3.3.41)$$

The equation (3.3.41) implies that the number of the group J entrepreneurs (γ_{t+1}) declines when each foreign bank's share of income ($\Gamma\left(\omega_{t+1}^J\right)$) increases. The fluctuation in capital flows from the foreign economy is affected by not only the average foreign borrowing (L^F) variations but also the size of γ_{t+1} changes.

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[15]For instance, FX traders are authorized by the FCA (Financial Conduct Authority) in the UK. In South Korea, the Ministry of Finance imposes the requirements for participating in the FX markets. In Thailand, only the agents authorized by the central bank can trade foreign currencies.

Net Worth and External Finance Premium

After paying debts, home entrepreneurs accumulate their profits as net worth. An individual entrepreneur then continues its business with a probability ζ and exits from the entrepreneur sector with a probability $1 - \zeta$. Exiting entrepreneurs $(1 - \zeta)$ return their net worth (NW_X) to households. Simultaneously, the same number of new entrepreneurs begin their business with new net worth (NW_N), which is financially invested by the households. Continuing and new entrepreneurs are then able to borrow funds from banks to purchase new capital stock.

After debt repayment at time t, the group I and the group J entrepreneurs' average profits are $R_t^K Q_{t-1} K_t^I \left[1 - \Gamma \left(\omega_t^I \right) \right]$ and $R_t^K Q_{t-1} K_t^J \left[1 - \Gamma \left(\omega_t^J \right) \right]$, respectively. Given the survival ratio (ζ) and the mass of the group J entrepreneurs at $t - 1$ (γ_t), at the moment of borrowing, the amount of aggregate net worth (NW_t) is the sum of continuing entrepreneurs' net worth and new net worth as follows[16]:

$$NW_t = \zeta R_t^K Q_{t-1} \left\{ (1 - \gamma_t) \left[1 - \Gamma \left(\omega_t^I \right) \right] K_t^I + \gamma_t \left[1 - \Gamma \left(\omega_t^J \right) \right] K_t^J \right\} + NW_N.$$
(3.3.42)

For simplicity, after the exits and the new entrepreneurs' entrances, the total net worth (NW_t) is identically distributed to the entrepreneurs. Entrepreneurs still have incentives to maximize profits, since they return net worth to households with a probability $1 - \zeta$ in the next period. Given the unitary mass of the home entrepreneurs,

$$NW_t(i) = NW_t(j) = NW_t.$$
(3.3.43)

Since the capital return and the capital price are common, the entrepreneurs are identical within groups while making loan contracts - before the realization of indi-

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[16]Exiting net worth is given by

$$NW_{X,t} = (1 - \zeta) R_t^K Q_{t-1} \left\{ (1 - \gamma_t) \left[1 - \Gamma \left(\omega_t^I \right) \right] K_t^I + \gamma_t \left[1 - \Gamma \left(\omega_t^J \right) \right] K_t^J \right\}.$$

vidual shocks. The identical contracts within each group lead to $K_{t+1}(i) = K_{t+1}^I$, $K_{t+1}(j) = K_{t+1}^J$, $L_t(i) = L_t$ and $L_t^F(j) = L_t^F$. Defining $R_{L,t+1}$ and $R_{L,t+1}^F$ as the common non-default loan rates in group I and J, respectively, $R_{L,t+1}(i) = R_{L,t+1}$ and $R_{L,t+1}^F(j) = R_{L,t+1}^F$. From the equations (3.3.16) and (3.3.39), then the following equation holds:

$$Q_t K_{t+1} = \phi_t NW_t \tag{3.3.44}$$

where ϕ is the aggregate leverage ratio, which satisfies $\phi_t = (1 - \gamma_{t+1})\phi_t^I + \gamma_{t+1}\phi_t^J$.

The entrepreneurs reallocation cost is paid when exiting net worth is returned to households, purchasing home final goods. Therefore, net transfer to households (Ω_t^E) in the equation (3.3.2) is illustrated as:

$$\Omega_t^E = NW_{X,t} - NW_N - \frac{\mu_\gamma}{2}\left(\gamma_{t+1} - \bar{\gamma}\right)^2. \tag{3.3.45}$$

The home banks' funding rate (R_D, deposit rate) also represents the opportunity cost of entrepreneurs' funds, since entrepreneurs can alternatively put their funds in home banks. As illustrated in BGG (1999), the expected gross return of capital $\left(E_t\left(R_{t+1}^K\right)\right)$ is equated to the marginal cost of external finance in equilibrium. Defining R^{EP} as the external finance premium which is represented by the ratio of 'external finance cost/opportunity cost',

$$R_t^{EP} = \frac{E_t\left(R_{t+1}^K\right)}{R_{D,t}}. \tag{3.3.46}$$

Since ϕ^I is increasing in $E_t\left(R_{t+1}^K\right)/R_{D,t}$, the equation (3.3.39) suggests that the external finance premium (R_t^{EP}) is inversely related to entrepreneurs' net worth.

3.3.4 Capital Producing Firms

During intermediate goods production in both economies, capital stock depreciates by δ per one unit. In the home economy, the capital producing firm purchases used

capital stock $(1 - \delta)K_t$ from the entrepreneurs given the relative price Q_t. Through investment (I_t), the capital producer makes new capital stock (K_{t+1}) and sell it to the entrepreneurs. The capital accumulation is then illustrated by:

$$K_{t+1} = (1 - \delta)K_t + I_t. \tag{3.3.47}$$

Producing one unit of I_t, the capital producer purchases $[1 + g(I_t/I_{t-1})]$ of final goods, where $g(\cdot)$ represents the investment adjustment cost. The investment adjustment cost function is given by:

$$g\left(\frac{I_t}{I_{t-1}}\right) = \frac{\mu_{iv}}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{3.3.48}$$

where μ_{iv} denotes the investment adjustment parameter. Eventually, the capital producing firm solves

$$\max_{I_t} \sum_{t=\tau}^{\infty} \beta^{t-\tau} E_\tau \left\{ \Lambda_{\tau,t} \left[Q_t I_t - I_t - g\left(\frac{I_t}{I_{t-1}}\right) I_t \right] \right\},$$

which yields the first-order condition as follows:

$$Q_t = 1 + g\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} g'\left(\frac{I_t}{I_{t-1}}\right) - \beta E_t \left\{ \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 g'\left(\frac{I_{t+1}}{I_t}\right) \right\}. \tag{3.3.49}$$

For simplicity, the foreign capital producing market is assumed to be competitive and there is no investment adjustment cost; the relative capital price (Q^*) is fixed at unity[17], and the profit of the foreign capital producing sector is zero. In the foreign economy capital stock accumulates as follows:

$$K_{t+1}^* = (1 - \delta)K_t^* + I_t^*. \tag{3.3.50}$$

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[17]In nominal terms, same to the price of final goods

3.3.5 Foreign QE and Lending Market

Foreign Central Bank QE

Quantitative easing (QE) is defined as an expansion of the central bank assets, which is aimed at injecting liquidity into the financial market. In this model, the foreign central bank gets funds from foreign households and purchases assets from foreign banks. One period later, debts are repaid with the interest rate and the funds are transferred to foreign households. The amount of net transfer from households to the central bank is represented by Ω_t^{CB*} in the equation (3.3.3), and it is determined by the foreign central bank's QE decision. Thus, Ω_t^{CB*} plays a similar role of banks' interest-bearing reserves in the central bank[18]. When the foreign central bank purchases the assets from foreign banks, a fraction of financial intermediation is funded by QE. Figure 3.4 illustrates the flows of funds with respect to QE implementation.

The liquidity change caused by the quantitative policy at time t is determined as a fraction $(\psi_t^* - 1)$ of the amount of foreign banks' total funding (liabilities), which is represented by:

$$\tau\left(\psi_t^* - 1\right) R_{D,t}^{*-1} D_{B,t}^* \tag{3.3.51}$$

where τ is the mass of foreign banks, and $R_{D,t}^{*-1} D_{B,t}^*$ denotes the average amount of an individual foreign bank's total liabilities at time t. ψ_t^* is an exogenous variable with a unitary steady state value ($\bar{\psi}^* = 1$); at the steady state, there is no quantitative policy. QE is represented by a positive shock on the variable ψ_t^*.

Foreign Banks and Entrepreneurs

The mass of foreign banks and that of foreign entrepreneurs are both τ - equivalent to the mass of foreign households. Foreign banks identically make loans to foreign and home entrepreneurs, using the funds provided by the foreign households and

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[18]Bank of England (2015) explains that 'the quantity of reserves is determined by accounting identities on the central bank's balance sheet' (p.10).
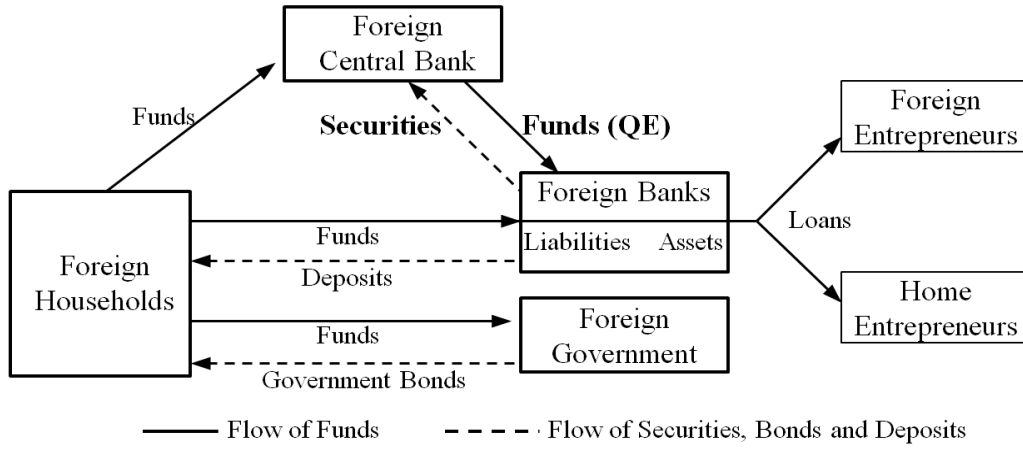
Figure 3.4: Foreign Central Bank QE and Flows of Funds

the foreign central bank (QE). Defining L^* as an individual foreign bank's average amount of total loans, the average balance sheet of an individual foreign bank can be expressed as follows:

$$R_{D,t}^{*-1} D_{B,t}^* = L_t^*. \qquad (3.3.52)$$

The foreign banking sector is competitive, and thus the profit is zero. In the short-run, a fraction (γ_F) of foreign banks' total loans are made to home entrepreneurs - overseas loans. Another fraction ($1-\gamma_F$) of loans are made to foreign entrepreneurs who purchase foreign capital stock. There is no agency problem between lenders and borrowers in the foreign economy; foreign entrepreneurs do not accumulate net worth. Defining K_{t+1}^* as the average amount of capital stock purchased by an individual foreign entrepreneur at time t, with the unitary foreign capital price

$$(1 - \gamma_F)L_t^* = K_{t+1}^*. \qquad (3.3.53)$$

Since there is no auditing cost in the foreign loan market, contrary to the home economy's lending market,

$$E_t \left(R_{t+1}^{K*} \right) = R_{D,t}^* \qquad (3.3.54)$$

where R^{K*} denotes the foreign gross real capital return, which is illustrated as

follows:

$$R_t^{K*} = r_{K,t}^* + 1 - \delta. \tag{3.3.55}$$

$r_{K,t}^*$ is the foreign capital rental rate in the intermediate goods production sector. Consequently, the profit of the foreign entrepreneur sector is also zero.

3.3.6 Intermediate Goods Producers

There is a continuum of identical intermediate goods producers indicated by $h \in (0,1)$ in the home economy. Each home intermediate good firm h produces a differentiated intermediate good $Y_t(h)$ with a Cobb-Douglas technology as below:

$$Y_t(h) = A_t K_t^d(h)^\nu N_t(h)^{1-\nu} \tag{3.3.56}$$

where $K_t^d(h)$ and $N_t(h)$ are the amounts of capital and labour inputs that are used for production. The variable A_t denotes the level of productivity which is common to all firms. The parameter $\nu \in (0,1)$ denotes the share of the capital income, which is common across countries. Similarly in the foreign economy the intermediate goods producers are indexed by $f \in (0,\tau)$, and the production function is given by

$$Y_t^*(f) = A_t^* K_t^{d*}(f)^\nu N_t^*(f)^{1-\nu}. \tag{3.3.57}$$

The producers h and f borrow capital stock ($K_t^d(h)$ and $K_t^{*d}(f)$) from the home and the foreign entrepreneur sectors, respectively. They return capital stock after production, paying real rental rates ($r_{K,t}$ and $r_{K,t}^*$). The real marginal costs (MC_t and MC_t^*) are identical across firms within each country. Given the constant-returns-to-scale technologies, optimality conditions imply

$$\frac{N_t(h)}{K_t^d(h)} = \frac{1-\nu}{\nu}\frac{r_{K,t}}{W_t} \qquad\qquad MC_t = \frac{1}{A_t\nu^\nu(1-\nu)^{1-\nu}}r_{K,t}^\nu W_t^{1-\nu} \tag{3.3.58}$$

$$\frac{N_t^*(f)}{K_t^{d*}(f)} = \frac{1-\nu}{\nu}\frac{r_{K,t}^*}{W_t^*} \qquad\qquad MC_t^* = \frac{1}{A_t^*\nu^\nu(1-\nu)^{1-\nu}}r_{K,t}^{*\nu}W_t^{*1-\nu}. \qquad (3.3.59)$$

The intermediate good of the home producer h is either purchased in the home economy or exported abroad: $Y_t(h) = Y_{H,t}(h) + Y_{H,t}^*(h)$, where $Y_{H,t}(h)$ is the amount of home good h sold in the home market and $Y_{H,t}^*(h)$ denotes the amount purchased in the foreign market (home exports)[19]. Similarly, for the foreign producer $Y_t^*(f) = Y_{F,t}^*(f) + Y_{F,t}(f)$, where $Y_{F,t}^*(f)$ is purchased in the foreign market and $Y_{F,t}(f)$ is exported to the home economy.

The demand function for an individual intermediate good is determined by cost minimization of the final good producer in each economy. Define $P_{H,t}(h)$ and $P_{H,t}^*(h)$ as the nominal prices of home produced goods purchased in the home and the foreign economies, respectively. $P_{H,t}^*(h)$ is denominated in foreign currency. The demand function for the home intermediate good h in each economy is given by

$$Y_{H,t}(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t} \qquad\qquad Y_{H,t}^*(h) = \left(\frac{P_{H,t}^*(h)}{P_{H,t}^*}\right)^{-\varepsilon} Y_{H,t}^* \qquad (3.3.60)$$

where ε is the elasticity of substitution among individual intermediate goods in the home and the foreign economies. $Y_{H,t}$ and $Y_{H,t}^*$ denote the aggregate demands for home intermediate goods in both economies. $P_{H,t}$ and $P_{H,t}^*$ are the aggregate prices. The aggregate demands and prices follow the aggregator form of Dixit and Stiglitz (1977) as below:

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(h)^{\frac{\varepsilon-1}{\varepsilon}}dh\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad\qquad Y_{H,t}^* = \left[\int_0^1 Y_{H,t}^*(h)^{\frac{\varepsilon-1}{\varepsilon}}dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$P_{H,t} = \left[\int_0^1 P_{H,t}(h)^{1-\varepsilon}dh\right]^{\frac{1}{1-\varepsilon}} \qquad\qquad P_{H,t}^* = \left[\int_0^1 P_{H,t}^*(h)^{1-\varepsilon}dh\right]^{\frac{1}{1-\varepsilon}}$$

In a symmetric way, the demand functions for the foreign intermediate good (f) in

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[19]The subscript H or F implies the place of production and the superscript (* or nothing) denotes where it is purchased.

the home and the foreign economies are represented by

$$Y_{F,t}^*(f) = \left(\frac{P_{F,t}^*(f)}{P_{F,t}^*}\right)^{-\varepsilon} Y_{F,t}^* \qquad Y_{F,t}(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\varepsilon} Y_{F,t} \qquad (3.3.61)$$

with aggregators:

$$Y_{F,t}^* = \left[\left(\frac{1}{\tau}\right)^{\frac{1}{\varepsilon}} \int_0^\tau Y_{F,t}^*(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad Y_{F,t} = \left[\left(\frac{1}{\tau}\right)^{\frac{1}{\varepsilon}} \int_0^\tau Y_{F,t}(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$P_{F,t}^* = \left[\frac{1}{\tau} \int_0^\tau P_{F,t}^*(f)^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}} \qquad P_{F,t} = \left[\frac{1}{\tau} \int_0^\tau P_{F,t}(f)^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}}.$$

$Y_{F,t}^*$ and $Y_{F,t}$ are per capita terms, and τ is the mass of the foreign households.

Home and foreign intermediate goods producers set export prices in domestic currency - producer currency pricing (PCP); firms choose identical prices across borders maximizing the total revenue. Assuming that there are no trade costs or trade barriers, the law of one price (LOOP) holds; this means $P_{H,t}(h) = S_t P_{H,t}^*(h)$ and $P_{F,t}(f) = S_t P_{F,t}^*(f)$. Given the identical elasticity of substitution between intermediate goods in both economies (ε), the following equations hold:

$$P_{H,t} = S_t P_{H,t}^* \qquad P_{F,t} = S_t P_{F,t}^*. \qquad (3.3.62)$$

Given the definition of the real exchange rate ($S_{R,t} = S_t P_t^*/P_t$), the equation (3.3.62) can be rewritten by

$$\frac{P_{H,t}}{P_t} = S_{R,t} \frac{P_{H,t}^*}{P_t^*} \qquad \frac{P_{F,t}}{P_t} = S_{R,t} \frac{P_{F,t}^*}{P_t^*}. \qquad (3.3.63)$$

The revenue of the home intermediate goods producer h is the sum of the cash flows in the home and the foreign markets. At time t the real profit of the firm h is

as follows:

$$\frac{P_{H,t}(h)}{P_t}Y_{H,t}(h) + \frac{S_t P_{H,t}^*(h)}{P_t}Y_{H,t}^*(h) - \left(r_{K,t}K_t^d(h) + W_t L_t(h)\right). \tag{3.3.64}$$

Defining $\Theta\left(Y_{H,t}(h) + Y_{H,t}^*(h)\right)$ as the nominal cost of producing $Y_{H,t}(h) + Y_{H,t}^*(h)$, the nominal cash flow at time t can be expressed as

$$P_{H,t}(h)Y_{H,t}(h) + S_t P_{H,t}^*(h)Y_{H,t}^*(h) - \Theta\left(Y_{H,t}(h) + Y_{H,t}^*(h)\right). \tag{3.3.65}$$

Following Calvo (1983), an individual home intermediate good producer can adjust its price with a probability $1 - \xi$ at each period. As Yun (1996), when it cannot optimally change the price, its home price is increasing at the steady state home inflation rate ($\bar{\Pi}$). The steady state inflation rates of the home and the foreign economies are assumed to be same to each other ($\bar{\Pi} = \bar{\Pi}^*$). $\tilde{P}_{H,t}$ is defined as the home price of home produced goods optimized at time t, and the foreign price $\tilde{P}_{H,t}^*$ is determined by the LOOP ($\tilde{P}_{H,t} = S_t \tilde{P}_{H,t}^*$). Also, $P_{H,t+\tau|t}$ and $P_{H,t+\tau|t}^*$ denote the home and the foreign prices of those goods at time $t + \tau$, respectively. When the home producer cannot change its price, the price in the foreign economy is indexed by not only $\bar{\Pi}^*$ but also the inverse of the nominal exchange rate change such that the LOOP holds every period[20]. Therefore, for the home intermediate goods, the following equations hold:

$$P_{H,t+\tau|t} = \bar{\Pi}^\tau \tilde{P}_{H,t} \qquad P_{H,t+\tau|t}^* = \left(\frac{S_t}{S_{t+\tau}}\right)\bar{\Pi}^{*\tau}\tilde{P}_{H,t}^*. \tag{3.3.66}$$

Given the LOOP, the equation (3.3.66) implies $S_{t+\tau}P_{H,t+\tau|t}^* = \bar{\Pi}^{*\tau}S_t\tilde{P}_{H,t}^*$, which is the foreign market price at time $t + \tau$ in home currency. The home producer who has a chance to optimize its price then maximizes

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[20]Assuming the LOOP and $\bar{\Pi} = \bar{\Pi}^*$, $P_{H,t+\tau|t} = \bar{\Pi}^\tau \tilde{P}_{H,t} = \bar{\Pi}^{*\tau}S_t\tilde{P}_{H,t}^*$. From $P_{H,t+\tau|t} = S_{t+\tau}P_{H,t+\tau|t}^*$, $\bar{\Pi}^{*\tau}(S_t/S_{t+\tau})\tilde{P}_{H,t}^* = P_{H,t+\tau|t}^*$.

$$\sum_{\tau=0}^{\infty} \beta^\tau \xi^\tau E_t \left\{ \Lambda_{t,t+\tau} \frac{P_t}{P_{t+\tau}} \left[\bar{\Pi}^\tau \tilde{P}_{H,t} \left(\frac{P_{H,t+\tau|t}}{P_{H,t+\tau}} \right)^{-\varepsilon} Y_{H,t+\tau} + \bar{\Pi}^{*\tau} S_t \tilde{P}_{H,t}^* \left(\frac{P_{H,t+\tau|t}^*}{P_{H,t+\tau}^*} \right)^{-\varepsilon} Y_{H,t+\tau}^* - \Theta(Y_{t+\tau|t}) \right] \right\}$$

where $\Lambda_{t,t+\tau} = (C_{t+1}/C_t)^{-\sigma}$ and $\beta^\tau \Lambda_{t,t+\tau}$ is the stochastic discount factor for real payoffs. $Y_{t+\tau|t}$ denotes output at $t+\tau$ of a firm whose last price reset was at time t. It is the sum of the domestically sold goods ($Y_{H,t+\tau|t}$) and the exported home goods ($Y_{H,t+\tau|t}^*$). Given the LOOP and $\bar{\Pi} = \bar{\Pi}^*$, the optimization problem becomes

$$\max_{\tilde{P}_{H,t}} \sum_{\tau=0}^{\infty} \beta^\tau \xi^\tau E_t \left\{ \Lambda_{t,t+\tau} \frac{P_t}{P_{t+\tau}} \left[\bar{\Pi}^\tau \tilde{P}_{H,t} \left(\frac{P_{H,t+\tau|t}}{P_{H,t+\tau}} \right)^{-\varepsilon} \left(Y_{H,t+\tau} + Y_{H,t+\tau}^* \right) - \Theta(Y_{t+\tau|t}) \right] \right\}$$
(3.3.67)

Profit maximization of the home producer yields the optimal relative price of the home goods in the home economy ($\tilde{P}_{H,t}/P_t$) as follows:

$$\frac{\tilde{P}_{H,t}}{P_t} = \frac{\frac{\varepsilon}{\varepsilon-1} \sum_{\tau=0}^{\infty} \tilde{\beta}^\tau \bar{\Pi}^{-\varepsilon\tau} E_t \left(\Lambda_{t,t+\tau} \Pi_{H,t,t+\tau}^\varepsilon Y_{t+\tau} V_{t+\tau} MC_{t+\tau} \right)}{\sum_{\tau=0}^{\infty} \tilde{\beta}^\tau \bar{\Pi}^{(1-\varepsilon)\tau} E_t \left(\Lambda_{t,t+\tau} \Pi_{t,t+\tau}^{-1} \Pi_{H,t,t+\tau}^\varepsilon Y_{t+\tau} V_{t+\tau} \right)}$$
(3.3.68)

where $MC_{t+\tau}$ is the real marginal cost at $t+\tau$. $\tilde{\beta} = \beta\xi$ and $V_{t+\tau} = \left[\int_0^1 \left(\frac{P_{H,t+\tau}(i)}{P_{H,t+\tau}} \right)^{-\varepsilon} di \right]^{-1}$. $\Pi_{t,t+\tau}$ denotes the cumulative gross inflation rate of home final goods from t to $t+\tau$, and $\Pi_{H,t,t+\tau}$ is the cumulative inflation of home produced goods. The aggregate home goods price $P_{H,t}$ evolves over time as below:

$$P_{H,t} = \left[\xi \left(P_{H,t-1} \bar{\Pi} \right)^{1-\varepsilon} + (1-\xi) \tilde{P}_{H,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$
(3.3.69)

The equation (3.3.69) can be rewritten as follows:

$$\frac{P_{H,t}}{P_t} = \left[\xi \left(\frac{\bar{\Pi}}{\Pi_{t-1,t}} \right)^{1-\varepsilon} \left(\frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\varepsilon} + (1-\xi) \left(\frac{\tilde{P}_{H,t}}{P_t} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$
(3.3.70)

For simplicity, foreign intermediate goods producers adjust foreign goods prices in

a flexible manner, which implies

$$\frac{P^*_{F,t}}{P^*_t} = \frac{\varepsilon}{\varepsilon - 1} MC^*_t \tag{3.3.71}$$

where $\varepsilon/(\varepsilon - 1)$ represents the markup.

3.3.7 Final Goods Producers

Final goods producing firms in the home and the foreign economies produce final goods Z_t and Z^*_t by combining home and foreign intermediate goods. Z_t and Z^*_t are per capita terms. Define $\tilde{Y}_{F,t}$ as the per capita foreign goods consumption in the home economy (home imports) and $\tilde{Y}^*_{H,t}$ as the per capita home goods consumption in the foreign economy (foreign imports)[21]. The representative final good producer in each economy then faces a CES technology as below:

$$Z_t = \left[(1 - \alpha)^{\frac{1}{\theta}} Y_{H,t}^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} \tilde{Y}_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{3.3.72}$$

$$Z^*_t = \left[(1 - \alpha^*)^{\frac{1}{\theta}} Y_{F,t}^{*\frac{\theta-1}{\theta}} + \alpha^{*\frac{1}{\theta}} \tilde{Y}_{H,t}^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{3.3.73}$$

where the parameter θ denotes the intratemporal elasticity of substitution between home and foreign intermediate goods, which is identical in the home and the foreign economies. $\alpha \in (0, 1)$ and $\alpha^* \in (0, 1)$ represent the degrees of openness in the home and the foreign economies, respectively.

Cost minimization by the home final good producer yields the following demand

---

[21]Since $Y_{F,t}$ is per capital value in the foreign economy and the size of the home economy is unity, $\tilde{Y}_{F,t} = \tau Y_{F,t}$. In a similar way, $\tilde{Y}^*_{H,t} = \frac{1}{\tau} Y^*_{H,t}$ where $Y^*_{H,t}$ is per capita exports of the home economy.

equations for the home and the foreign intermediate goods:

$$Y_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} Z_t \tag{3.3.74}$$

$$\tilde{Y}_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} Z_t, \tag{3.3.75}$$

with a price index

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\theta} + \alpha P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.3.76}$$

which implies

$$1 = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{1-\theta} + \alpha\left(\frac{P_{F,t}}{P_t}\right)^{1-\theta}. \tag{3.3.77}$$

The demand functions for the foreign and the home intermediate goods in the foreign country can be written as:

$$Y_{F,t}^* = (1-\alpha^*)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\theta} Z_t^* \tag{3.3.78}$$

$$\tilde{Y}_{H,t}^* = \alpha^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\theta} Z_t^* \tag{3.3.79}$$

with a price index

$$P_t^* = \left[(1-\alpha^*)P_{F,t}^{*1-\theta} + \alpha^* P_{H,t}^{*1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.3.80}$$

which also means

$$1 = (1-\alpha^*)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{1-\theta} + \alpha^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{1-\theta}. \tag{3.3.81}$$

3.3.8 Central Bank Policy Rate

The nominal gross policy rates in the home and the foreign economies (R_N and R_N^*) are determined based on Taylor-type rules (Taylor, 1993). With a monetary policy smoothing parameter $\rho_R \in (0,1)$, the nominal policy rates are chosen by the rules

as below:

$$R_{N,t} = \kappa R_{N,t-1}^{\rho_R} \left(\Pi_t^{\gamma_P} Y_t^{\gamma_Y}\right)^{1-\rho_R} \eta_{m,t} \tag{3.3.82}$$

$$R_{N,t}^* = \kappa^* R_{N,t-1}^{*\rho_R} \left(\Pi_t^{*\gamma_P} Y_t^{*\gamma_Y}\right)^{1-\rho_R} \eta_{m,t}^* \tag{3.3.83}$$

where Y_t and Y_t^* denote the amounts of home and foreign aggregate output per capita; this means $Y_t = \int_0^1 Y_t(h)dh$ and $Y_t^* = \frac{1}{\tau}\int_0^\tau Y_t^*(f)df$. κ and κ^* are scale parameters, and γ_P and γ_Y represent the policy weights on inflation and output, respectively. $\eta_{m,t}$ and $\eta_{m,t}^*$ indicate policy shocks with unitary expected values. The nominal interest rates have the following relationships with the real interest rates:

$$R_t = \frac{R_{N,t}}{E_t(\Pi_{t+1})} \qquad\qquad R_t^* = \frac{R_{N,t}^*}{E_t(\Pi_{t+1}^*)}. \tag{3.3.84}$$

3.3.9   Government

The home government (i) intervenes in the home foreign exchange (FX) market, (ii) holds foreign government bonds (foreign reserves), and (iii) issues government bonds in the home market. Introducing foreign reserves accumulation (public capital flows) enables the sum of the change in foreign loans (private capital flows) and net exports to be non-zero[22]. Thus, net exports and foreign loans are not bound to each other in equilibrium. Alfaro, Kalemli-Ozcan and Volosovych (2011) argue that considering public (government) capital flows helps to understand *upstream capital flows*[23] and global imbalances, although the government's behaviour is still puzzling. In this chapter, the government's behaviour is explained by FX intervention and resulting foreign reserves accumulation. Figure 3.5 illustrates the flows of funds around the

---

[22]In the equilibrium, Net Exports $=\Delta$Private Net Foreign Assets $+\ \Delta$Public Net Foreign Assets.

[23]Contrary to the prediction of the neoclassical model, international capital flows from emerging economies to developed economies - the Lucas paradox (Lucas, 1990). Alfaro, Kalemli-Ozcan and Volosovych (2011) find that the private capital flows do not contradict the prediction, while the public flows do.
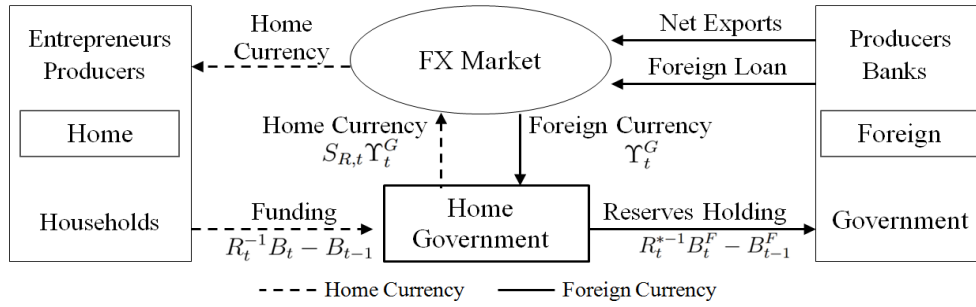
Figure 3.5: Sterilized Government FX Intervention and Foreign Reserves

home government.

The home government purchases or sells the foreign currency based on an FX intervention rule. Following the intervention rules in Montoro and Ortiz (2012) and Chutasripanich and Yetman (2015)[24], the government's demand for the foreign currency (Υ_t^G) depends on the currency value changes ($\Delta S_t = S_t/S_{t-1}$) as below:

$$\Upsilon_t^G = \bar{\Upsilon}_t^G + \bar{B}^F ln\left(\Delta S_t^{\frac{1}{\rho_{FX}}}\right) \tag{3.3.85}$$

where $\bar{B}^F$ denotes the steady state amount of the foreign government bonds held by the home government. $\bar{\Upsilon}_t^G$ is the steady state value of Υ_t^G, and Υ_t^G is in real terms - in units of foreign final goods.

Purchasing (selling) the foreign currency in the FX market, the home government increases (reduces) its foreign reserves[25]. Therefore, the amount of the home government's foreign currency purchase (Υ_t^G) is equivalent to the net funds outflow through the changes in the foreign government bond holding (B^F):

$$\Upsilon_t^G = R_t^{*-1}B_t^F - B_{t-1}^F, \tag{3.3.86}$$

---

[24]In Montoro and Ortiz (2012), the pre-announced FX intervention rule is given by $\omega_t = \phi\Delta S_t + \varepsilon_t$ where ω_t denotes the government's foreign currency demand and ε_t is an unanticipated shock. The intervention rule in Chutasripanich and Yetman (2015) is illustrated as $\Delta FR_t = \phi(S_{t-1} - S_t)$ where FR_t denotes foreign reserves and $\phi > 0$.

[25]The relationship between government intervention and foreign reserves accumulation is well supported by many studies, such as Aizenman and Lee (2006) and Reinhart and Reinhart (2008).

where R_t^* denotes the gross return of the foreign government bonds.

In order to offset the monetary effects of FX intervention on the home economy, the home government issues bonds (B). The nominal cash flow in home currency obtained from bonds issuance, $P_t(R_t^{-1}B_t - B_{t-1})$, is equivalent to the government's home currency supply in the FX market, which is $S_t P_t^* \Upsilon_t^G$ - in return for the foreign currency $(P_t^* \Upsilon_t^G)$. Thus, in real terms, the following equation holds:

$$S_{R,t}\Upsilon_t^G = R_t^{-1}B_t - B_{t-1}. \tag{3.3.87}$$

Foreign government spending is funded by bonds issuance and the tax (T^*). The foreign government bonds are purchased by foreign households (B^*) and the home government (B^F). Define G^* as per capita foreign government spending. Given the mass of foreign households (τ), then the following equation holds:

$$\tau\left(R_t^{*-1}B_t^* - B_{t-1}^*\right) + R_t^{*-1}B_t^F - B_{t-1}^F + \tau T^* = \tau G_t^*. \tag{3.3.88}$$

3.3.10 Market Clearing

In the home FX market, the private net supply of the foreign currency[26] is balanced by the government's foreign currency demand. From the equation (3.3.86), the FX market clearing condition is given by:

$$P_t^*\left(R_t^{*-1}B_t^F - B_{t-1}^F\right) = P_t^*\left(\gamma_{t+1}L_t^F - \gamma_t R_{D,t-1}^* L_{t-1}^F\right) + P_{H,t}^* Y_{H,t}^* - P_{F,t}S_t^{-1}Y_{F,t} \tag{3.3.89}$$

and the real net exports in units of home final goods (NX_t) is

$$NX_t = S_{R,t}\frac{P_{H,t}^*}{P_t^*}Y_{H,t}^* - \frac{P_{F,t}}{P_t}Y_{F,t}. \tag{3.3.90}$$

---

[26]This consists of (i) net exports and (ii) the change in foreign loans.

Defining NX_t^* as the foreign economy's net exports (per capita) in units of the foreign final goods, $NX_t^* = -\frac{1}{\tau}\frac{NX_t}{S_{R,t}}$.

In each country, the amount of aggregate intermediate goods production is equal to the sum of domestically purchased goods and exported goods. From the equations (3.3.60) and (3.3.61), this can be illustrated by:

$$Y_t \;=\; Y_{H,t}\int_0^1 \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\varepsilon} dh + Y_{H,t}^*\int_0^1 \left(\frac{P_{H,t}^*(h)}{P_{H,t}^*}\right)^{-\varepsilon} dh \qquad (3.3.91)$$

$$Y_t^* \;=\; \frac{1}{\tau}Y_{F,t}^*\int_0^\tau \left(\frac{P_{F,t}^*(f)}{P_{F,t}^*}\right)^{-\varepsilon} df + \frac{1}{\tau}Y_{F,t}\int_0^\tau \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\varepsilon} df \qquad (3.3.92)$$

where Y_t and Y_t^* denote the amounts of aggregate intermediate goods production per capita in the home and the foreign economies, which implies:

$$Y_t = \int_0^1 Y_t(h)dh \qquad \text{and} \qquad Y_t^* = \tfrac{1}{\tau}\int_0^\tau Y_t^*(f)df.$$

In the home economy, the amount of final goods production (Z) is equal to the sum of households' consumption, investment, home and foreign banks' monitoring costs (m), the investment adjustment cost, and the entrepreneurs reallocation cost. This is illustrated by:

$$C_t + I_t + m_t + g\left(\frac{I_t}{I_{t-1}}\right)I_t + \frac{\mu_\gamma}{2}\left(\gamma_{t+1} - \bar\gamma\right)^2 = Z_t \qquad (3.3.93)$$

where

$$m_t = R_t^K Q_{t-1}\left[(1-\gamma_t)\mu_t^I M\left(\omega_t^I\right)K_t^I + \gamma_t\mu_t^J M\left(\omega_t^J\right)K_t^J\right]. \qquad (3.3.94)$$

Also, the amount of foreign final goods is equivalent to the sum of consumption, investment, the investment adjustment cost, and government spending as below:

$$C_t^* + I_t^* + g\left(\frac{I_t^*}{I_{t-1}^*}\right)I_t^* + G_t^* = Z_t^*. \qquad (3.3.95)$$

Since the mass of home entrepreneurs and that of home intermediate goods producers are both unity, K_{t+1} in the equation (3.3.16) is equivalent to K_{t+1}^d in the production function. Equivalently, in the foreign economy $K_{t+1}^* = K_{t+1}^{d*}$. In the production sector, the sum of the individual producer's labour demand is same to the sum of the individual labour supply of each household as follow:

$$N_t = \int_0^1 N_t(h)dh = \int_0^1 N_t^s(h)dh \tag{3.3.96}$$

$$N_t^* = \frac{1}{\tau}\int_0^\tau N_t^*(f)df = \frac{1}{\tau}\int_0^\tau N_t^{s*}(f)df \tag{3.3.97}$$

where N_t and N_t^* denote per capita employment in each country.

In the home banking sector, the total amount of home loans is equivalent to the amount of funding (household deposits) as follows:

$$(1-\gamma_{t+1})L_t = R_{D,t}^{-1}D_t. \tag{3.3.98}$$

Also, the home group J entrepreneurs' total loan demands are matched by the foreign banks' overseas loans supply, which is illustrated by:

$$\gamma_{t+1}L_t^F = \tau\gamma_F L_t^* \tag{3.3.99}$$

where τ and γ_{t+1} denote the mass of foreign banks and that of the group J entrepreneurs, respectively. γ_F denotes the share of the foreign banks' overseas loans. Defining $D_t^* = \frac{1}{\tau}\int_0^\tau D_t^*(f)df$ as the average foreign household deposit, from $(3.3.51)$[27]

$$(2-\psi_t^*)\,D_{B,t}^* = D_t^*. \tag{3.3.100}$$

---

[27]The amount of an individual foreign bank's funding is the sum of the household deposit and the central bank's liquidity injection. From the equation (3.3.51) then, $D_{B,t}^* = D_t^* + (\psi_t^* - 1)\,D_{B,t}^*$.

3.3.11 Exogenous Variables

Home and foreign technologies (A_t and A_t^*), the home and the foreign policy rate shocks ($\eta_{m,t}$ and $\eta_{m,t}^*$), and the foreign QE shock (ψ_t^*) follow

$$A_t = \bar{A}^{1-\rho_A} A_{t-1}^{\rho_A} \varepsilon_{A,t} \tag{3.3.101}$$

$$A_t^* = \bar{A}^{*1-\rho_A} A_{t-1}^{*\rho_A} \varepsilon_{A,t}^* \tag{3.3.102}$$

$$\eta_{m,t} = \bar{\eta}_m^{1-\rho_m} \eta_{t-1}^{\rho_m} \varepsilon_{m,t} \tag{3.3.103}$$

$$\eta_{m,t}^* = \bar{\eta}_m^{*1-\rho_m} \eta_{t-1}^{*\rho_m} \varepsilon_{m,t}^* \tag{3.3.104}$$

$$\psi_t^* = \bar{\psi}^{*1-\rho_{qe}} \psi_{t-1}^{*\rho_{qe}} \varepsilon_{qe,t}^*. \tag{3.3.105}$$

where the variables with bars represent the steady state values. The expected values of the variables $\varepsilon_{A,t}$, $\varepsilon_{A,t}^*$, $\varepsilon_{m,t}$, $\varepsilon_{m,t}^*$, and $\varepsilon_{qe,t}^*$ are all unity, and the coefficients $\rho_A, \rho_m, \rho_{qe} \in (0,1)$.

3.3.12 National Income Identity

Home Economy

Aggregating home households' budget constraints (3.3.2) and subtracting the portfolio adjustment cost (κ_t^A) from both sides yields

$$C_t + R_t^{-1} B_t + R_{D,t}^{-1} D_t = W_t N_t + \Omega_t + D_{t-1} + B_{t-1} + \Omega_t^E. \tag{3.3.106}$$

From the equations (3.3.86), (3.3.87), (3.3.89) and (3.3.90), (3.3.106) is rewritten as

$$C_t + S_{R,t} \left(\gamma_{t+1} L_t^F - \gamma_t R_{D,t-1}^* L_{t-1}^F \right) + R_{D,t}^{-1} D_t + NX_t = W_t N_t + \Omega_t + D_{t-1} + \Omega_t^E.$$
(3.3.107)

From the equations (3.3.29) and (3.3.98)

$$D_{t-1} = (1 - \gamma_t) R_t^K Q_{t-1} K_t^I \left[\Gamma \left(\omega_t^I \right) - \mu^I M \left(\omega_t^I \right) \right],$$
(3.3.108)

and using the equation (3.3.94),

$$\begin{aligned}
C_t + \gamma_{t+1} S_{R,t} L_t^F \quad & - \quad \gamma_t R_t^K Q_{t-1} K_t^J \Gamma \left(\omega_t^J \right) + m_t + (1 - \gamma_{t+1}) L_t + NX_t \\
& = \quad W_t N_t + \Omega_t + (1 - \gamma_t) R_t^K Q_{t-1} K_t^I \Gamma \left(\omega_t^I \right) + \Omega_t^E \quad (3.3.109)
\end{aligned}$$

Aggregating the entrepreneurs' balance sheets (3.3.12) and (3.3.13), and using the equation (3.3.16),

$$C_t + Q_t K_{t+1} - NW_t - \gamma_t R_t^K Q_{t-1} K_t^J \Gamma \left(\omega_t^J \right) + m_t + NX_t = W_t N_t + \Omega_t + (1 - \gamma_t) R_t^K Q_{t-1} K_t^I \Gamma \left(\omega_t^I \right) + \Omega_t^E.$$
(3.3.110)

From the equations (3.3.42), (3.3.43), and (3.3.45),

$$\Omega_t^E = R_t^K Q_{t-1} \left\{ (1 - \gamma_t) \left[1 - \Gamma \left(\omega_t^I \right) \right] K_t^I + \gamma_t \left[1 - \Gamma \left(\omega_t^I \right) \right] K_t^J \right\} - NW_t - \frac{\mu_\gamma}{2} \left(\gamma_{t+1} - \bar{\gamma} \right)^2,$$
(3.3.111)

and plugging the equation (3.3.111) into (3.3.110) and using the equation (3.3.17),

$$C_t + Q_t K_{t+1} - R_t^K Q_{t-1} K_t + m_t + NX_t = W_t N_t + \Omega_t - \frac{\mu_\gamma}{2} \left(\gamma_{t+1} - \bar{\gamma} \right)^2.$$
(3.3.112)

The cash flows from the producers (Ω_t) is the sum of the dividends from intermediate goods producers ($\Omega_{IG,t}$) and the capital producer ($\Omega_{K,t}$), where

$$\Omega_{IG,t} = \frac{P_{H,t}}{P_t} \left(Y_{H,t} + Y_{H,t}^* \right) - r_t^K K_t - W_t N_t \qquad \Omega_{K,t} = Q_t I_t - I_t - g(\cdot) I_t. \quad (3.3.113)$$

Combining the equations (3.3.112) and (3.3.113) yields

$$
\begin{aligned}
C_t + Q_t K_{t+1} \quad &- \quad R_t^K Q_{t-1} K_t + m_t + \frac{\mu_\gamma}{2} (\gamma_{t+1} - \bar{\gamma})^2 + N X_t \\
&= \quad \frac{P_{H,t}}{P_t} \left(Y_{H,t} + Y_{H,t}^* \right) - r_t^K K_t + Q_t I_t - I_t - g(\cdot) I_t. \quad (3.3.114)
\end{aligned}
$$

Also, from the definition of the home gross return of capital (3.3.17), the equation (3.3.114) is rewritten by

$$
C_t + Q_t K_{t+1} - (1-\delta) Q_t K_t + m_t + \frac{\mu_\gamma}{2} (\gamma_{t+1} - \bar{\gamma})^2 + N X_t = \frac{P_{H,t}}{P_t} \left(Y_{H,t} + Y_{H,t}^* \right) + Q_t I_t - I_t - g(\cdot) I_t,
$$
$$(3.3.115)$$

and from $Q_t K_{t+1} = Q_t I_t + Q_t (1-\delta) K_t$, the home economy's national income identity is derived as follows:

$$
C_t + I_t + m_t + g \left(\frac{I_t}{I_{t-1}} \right) I_t + \frac{\mu_\gamma}{2} (\gamma_{t+1} - \bar{\gamma})^2 + N X_t = \frac{P_{H,t}}{P_t} \left(Y_{H,t} + Y_{H,t}^* \right). \quad (3.3.116)
$$

Foreign Economy

Aggregating and normalizing foreign households' budget constraints (3.3.3), the following equation holds:

$$
C_t^* + R_t^{*-1} B_t^* + R_{D,t}^{*-1} D_t^* + T^* + \Omega_t^{CB*} = W_t^* N_t^{s*} + \Omega_t^* + D_{t-1}^* + B_{t-1}^*. \quad (3.3.117)
$$

From the equation (3.3.88), the equation (3.3.117) can be rewritten by

$$
C_t^* + R_{D,t}^{*-1} D_t^* + G_t^* - \frac{1}{\tau} \left(R_t^{*-1} B_t^F - B_{t-1}^F \right) + \Omega_t^{CB*} = W_t^* N_t^{s*} + \Omega_t^* + D_{t-1}^*. \quad (3.3.118)
$$

Define $M_{qe,t}^*$ as the amount of QE at time t, which implies $M_{qe,t}^* = \tau \left(\psi_t^* - 1 \right) R_{D,t}^{*-1} D_{B,t}^*$.

Using the equations (3.3.52), (3.3.53) and (3.3.99),

$$
\begin{aligned}
C_t^* \;+\; & \left(\frac{1}{\tau} \gamma_{t+1} L_t^F + K_{t+1}^* - \frac{1}{\tau} M_{qe,t}^* \right) + G_t^* - \frac{1}{\tau} \left(R_t^{*-1} B_t^F - B_{t-1}^F \right) + \Omega_t^{CB*} \\
= \;& W_t^* N_t^{s*} + \Omega_t^* + R_{D,t-1}^* \left(\frac{1}{\tau} \gamma_t L_{t-1}^F + K_t^* - \frac{1}{\tau} M_{qe,t-1}^* \right)
\end{aligned}
\tag{3.3.119}
$$

From the FX market clearing condition (3.3.89) and the definition of NX_t^*, the equation (3.3.119) can be simplified as:

$$
C_t^* + NX_t^* + K_{t+1}^* + G_t^* + \Omega_t^{CB*} = W_t^* N_t^{s*} + \Omega_t^* + R_{D,t-1}^* K_t^* - \frac{1}{\tau} \left(R_{D,t-1}^* M_{qe,t-1}^* - M_{qe,t}^* \right),
\tag{3.3.120}
$$

where $\Omega_t^{CB*} = \frac{1}{\tau} \left(M_{qe,t}^* - R_{D,t-1}^* M_{qe,t-1}^* \right)$. Using $R_{D,t-1}^* = R_t^{K*} = r_{K,t}^* + 1 - \delta$ and $K_{t+1}^* = (1 - \delta) K_t^* + I_t^*$,

$$
C_t^* + I_t^* + G_t^* + NX_t^* = W_t^* N_t^{s*} + r_{K,t}^* K_t^* + \Omega_t^*.
\tag{3.3.121}
$$

Since the profits of foreign banks and entrepreneurs are both zero, Ω_t^* represents only the profit in the production sector, which can be illustrated by

$$
\Omega_t^* = \frac{P_{F,t}^*}{P_t^*} \left(Y_{F,t}^* + Y_{F,t} \right) - \left(W_t^* N_t^{s*} + r_{K,t}^* K_t^* \right).
\tag{3.3.122}
$$

Finally the national income identity is derived as

$$
C_t^* + I_t^* + G_t^* + NX_t^* = \frac{P_{F,t}^*}{P_t^*} \left(Y_{F,t}^* + Y_{F,t} \right).
\tag{3.3.123}
$$

3.3.13 Equilibrium

The equilibrium consists of 68 equations and 68 variables. Home and foreign households' maximization is represented by the equations (3.3.6)-(3.3.11). In the home entrepreneurs sector, the equilibrium is illustrated by the equations (3.3.12), (3.3.13), (3.3.16), (3.3.17), (3.3.33), (3.3.34), (3.3.37)-(3.3.42), (3.3.44) and (3.3.46). Capi-

tal producers' profit maximization and capital accumulation are represented by the equations (3.3.47), (3.3.49) and (3.3.50). The foreign lending market equilibrium consists of the equations (3.3.52)-(3.3.55). Optimization of home and foreign intermediate goods producers is illustrated by the equations (3.3.56)-(3.3.59), (3.3.63), (3.3.68), (3.3.70) and (3.3.71). In the home and the foreign final goods production sectors, the equilibrium consists of the equations (3.3.74), (3.3.75), (3.3.77)-(3.3.79), (3.3.81). The equations (3.3.82)-(3.3.84) denote central bank policy rates and the nominal rates. The home and the foreign government balance sheets are given by the equations (3.3.85), (3.3.86) and (3.3.88). The market clearing conditions are illustrated by the equations (3.3.89), (3.3.91)-(3.3.95), (3.3.98)-(3.3.100). Finally, the exogenous variables are indicated by the equations (3.3.101)-(3.3.105)[28].

Assuming $K = K^d$, $K^* = K^{d*}$, $N = N^s$ and $N^* = N^{s*}$, corresponding variables are as follow: C, C^*, N, N^*, Y, Y^*, I, I^*, K, K^I, K^J, K^*, S_R, ΔS, B, B^F, B^*, D, D^*, D_B^*, L, L^F, L^*, MC, MC^*, Y_H, Y_F, Y_H^*, Y_F^*, G^*, Z, Z^*, W, W^*, r_K, r_K^*, R, R^*, R_D, R_D^*, R_{ND} R_{ND}^*, Π, Π^*, R_N, R_N^*, R^{EP}, γ, NW, ω^I, ω^J, m, R^K, R^{K*}, $\frac{P_H}{P}$, $\frac{P_F}{P}$, $\frac{P_H^*}{P^*}$, $\frac{P_F^*}{P^*}$, $\frac{\tilde{P}_H}{P}$, ϕ, ϕ^I, ϕ^J, Q, A, A^*, η_m, η_m^*, ψ.

3.3.14 Solution Method

The solution method of the model starts with computing the well-defined steady state. The steady state values of variables are found analytically and illustrated in the appendix 3.A.1. Having computed the steady state, equations are approximated around the steady state, using log-linearization. The linearized equations are reported in the appendix 3.A.2. These equations are also represented by the following state space form:

$$AE(x_{t+1}) = Bx_t + Cz_t$$

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[28]Appendix 3.A.1 and 3.A.2 illustrate the steady state and the log-linearized equation system.

where x is the vector of endogenous variables and z denotes the vector of exogenous variables. Following the method of Klein (2000), firstly a generalized Shur decomposition to matrices A and B is conducted. This yields $QAZ = S$ and $QBZ = T$, where S and T are upper triangular matrices. Using block matrices of Q, Z, S and T, eventually reduced form solutions are derived. Given a shock on the vector z, then the impulse response simulation results are computed.

In the Baeyesian estimation, a Markov Chain Monte Carlo (MCMC) simulation is used, with the Metropolis-Hastings (MH) algorithm. It is a process of finding the desired posterior distributions of parameters using a large number of iterations. In the MH algorithm, given a current state variable, a new draw is accepted only when it increases the posterior density (Fernández-Villaverde, 2010).

3.4 Parameters and Steady States

As in Adolfson *et al.* (2007), both calibration and a Bayesian approach are used in this section to set up the values of the parameters and the steady state variables.

3.4.1 Calibration

The quarterly discount factor (β) is set to 0.995 such that the steady state real gross interest rate ($\bar{R}$) is 1.005. The annual capital depreciation rate is 10% ($\delta = 0.025$). Following Coenen *et al.* (2010), the elasticity of substitution between home intermediate goods (ε) is 6. The home central bank's policy rule follows Taylor (1993) which means $\gamma_P = 1.5$ and $\gamma_Y = 0.5$, and as in Gertler and Karadi (2011) the monetary policy smoothing parameter (ρ_R) is 0.8. The price stickiness parameter ξ is 0.75, and the long-run level of net exports of each country ($\bar{NX}$ and $\bar{NX}^*$) is assumed to be zero.

In the home entrepreneur sector, the steady state external finance premium ($\bar{R}^K/\bar{R}_D$) is 1.005 as in Bernanke, Gertler and Gilchrist (1999). The home banks'

Table 3.2: Parameters and Steady State Calibration

Parameter	Value	Description	Parameter	Value	Description
β	0.995	Discount Rate	μ	0.20	Monitoring Cost
ε	6.0	Elasticity of Substitution	ρ_{FX}	-0.29	Exchange Rate
ξ	0.75	Price Stickiness	$\bar{\gamma}$	0.24	Financial Openness
α	0.42	Home Openness	$\bar{\Pi}$	1.007	Inflation Rate
ζ	0.97	Survival Ratio	$\bar{R}^K/\bar{R}_D$	1.005	Finance Premium
γ_P	1.5	Taylor Rule	α^*	0.053	Foreign Openness
γ_Y	1.0		τ	6.336	Foreign Mass
ρ_R	0.8		γ_F	0.023	Overseas Loans

funding rate premium on the risk-free rate ($\bar{R}_D/\bar{R}$) is 1.0025 at the steady state, and that of the foreign banks is set to unity. Following Unsal (2013), the lenders' monitoring cost parameter (μ) is 0.20. The idiosyncratic risk (ω) is log-normally distributed with the standard deviation 0.17. As in Faia (2007) the survival probability of an individual entrepreneur (ζ) is 0.97.

Calibrating some parameters and the steady state values in the home economy, South Korean data are used. The steady state inflation rate ($\bar{\Pi}$) is set to 1.007 based on the CPI during 2000-2015, and thus the steady state nominal risk-free interest rate ($\bar{R}_N$) is roughly 1.012. From the regression results using the exchange rate and the foreign reserves data from 2000:Q1 to 2015:Q3, $\rho_{FX} = -0.29$ in the FX intervention rule. The degree of openness of the home economy (α) is 0.42 based on the average level during 2000-2014. From the foreign loans ratio during 2000:Q1 to 2015:Q3, home financial openness ($\bar{\gamma}$) is set to 0.24.

For some of the foreign economy parameters, the US data are used. The share of the overseas loans in the total loans (γ_F) is 0.023, and the steady state ratio of $\bar{D}_P^*/\bar{B}_P^*$ is assumed to be unity[29]. The mass of the foreign economy ($\tau = 6.336$) and foreign openness ($\alpha^* = 0.053$) are determined such that at the steady state

[29]During 2000:Q1 to 2015:Q3, the average amount of the total loans of the US private depository institutions is $7196.2bn, and the loans to the rest of the world is $161.9bn (Board of Governors of the Federal Reserve System). Also, the average amount of the US Treasury Securities held by domestic agents is $7725.6bn (US Department of Treasury).

Table 3.3: Parameter Estimates using Bayesian Approaches

	Prior Distribution			Posterior Distribution				
	Type	Mean	St.error	Mode	Median	Mean	5%	95%
ν	beta	0.33	0.03	0.309	0.317	0.318	0.268	0.368
σ	gamma	2.00	0.30	1.784	1.809	1.814	1.513	2.115
χ	gamma	2.00	0.30	2.298	2.366	2.367	1.891	2.846
θ	gamma	1.50	0.30	0.675	0.688	0.690	0.649	0.733
μ_A	gamma	0.20	0.05	0.187	0.193	0.197	0.120	0.280
μ_A^*	gamma	0.20	0.05	0.044	0.047	0.049	0.032	0.065
μ_{iv}	gamma	4.00	0.80	1.792	1.882	1.901	1.471	2.338
μ_γ	gamma	4.00	0.80	3.689	3.702	3.762	2.477	5.062
ε_A	inv.gamma	0.10	-	0.061	0.064	0.065	0.049	0.078
ε_m	inv.gamma	0.10	-	0.019	0.019	0.019	0.016	0.022
ε_m^*	inv.gamma	0.10	-	0.037	0.041	0.042	0.029	0.054
ε_A^*	inv.gamma	0.10	-	0.077	0.082	0.083	0.065	0.100
ε_{qeA}^*	inv.gamma	0.10	-	0.177	0.182	0.183	0.150	0.217

$\bar{G}^*/\bar{Y}^* = 0.2$ - following Gertler and Karadi (2011). Table 3.2 summarizes the calibrated values of the parameters and the steady state values.

3.4.2 Bayesian Estimation

A Bayesian estimation is performed for the parameters that are not calibrated and for the second moments of exogenous shocks. These parameters are: (i) the share of the capital income in production (ν), (ii) consumer's preference parameters (σ and χ), (iii) the elasticity of substitution between home and foreign goods (θ), (iv) the home and the foreign assets adjustment costs (μ_A and μ_A^*), (v) the home investment adjustment cost (μ_{iv}), and (vi) the home entrepreneurs' cross-border reallocation cost (μ_γ). The Metropolis-Hastings (MH) algorithm is used with 50,000 draws. Five series of data are used[30]: output, imports, investment and the real exchange rate of South Korea, and total loans in the US. The sample period is between 2000:1Q and 2015:Q3.

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[30]Data sources are presented in the appendix 3.A.5.

Regarding the prior distributions, the share of capital in the production (ν) is assumed to have a beta distribution with a mean 0.33. The coefficient of relative risk aversion of households (σ) and the inverse of the elasticity of the labour supply (χ) are both 2.0, as in Kolasa and Lombardo (2014). The home (μ_A) and the foreign assets adjustment cost parameters (μ_A^*) are both 0.2 in the prior assumption. The prior mean of the entrepreneurs reallocation cost (μ_γ) is set to 4.0. As in Christoffel, Coenen and Warne (2008), the prior means of θ and μ_{iv} are set to 1.5 and 4.0, respectively. Table 3.3 illustrates the prior and the posterior distributions.

According to the results of the Bayesian estimation, the posterior mean of ν is 0.32. Regarding consumer's preference, σ is 1.81, and χ is 2.37. The elasticity of substitution between home and foreign goods (θ) is 0.69. The home assets adjustment cost (μ_A) is 0.20, and that of the foreign economy (μ_A^*) is 0.05. Finally, the posterior mean of μ_{iv} is 1.90 and that of μ_γ is 3.76. The appendix 3.A.6 illustrates parameters identification, prior and posterior distributions, and the convergence statistics.

3.5 Impulse Responses: Foreign QE Shock

3.5.1 Bayesian Impulse Response Analysis

There is a positive shock on ψ^* in the equation (3.3.51), as the foreign central bank injects liquidity into the financial market (QE). Figure 3.6 illustrates the Bayesian impulse responses to the foreign QE shock. The figure describes the mean response (solid line), and 5 and 95 per cent significance bands (dotted lines).

The foreign quantitative policy directly boosts foreign banks' liabilities (funding), and thus their assets (loans) are also enlarged. This raises the amount of the loans to the home entrepreneurs (L^F). In the foreign economy, as more funds are available for the entrepreneurs, the amount of capital stock rises - accompanied by an increase in foreign output. As a result, the foreign capital return declines, and consequently
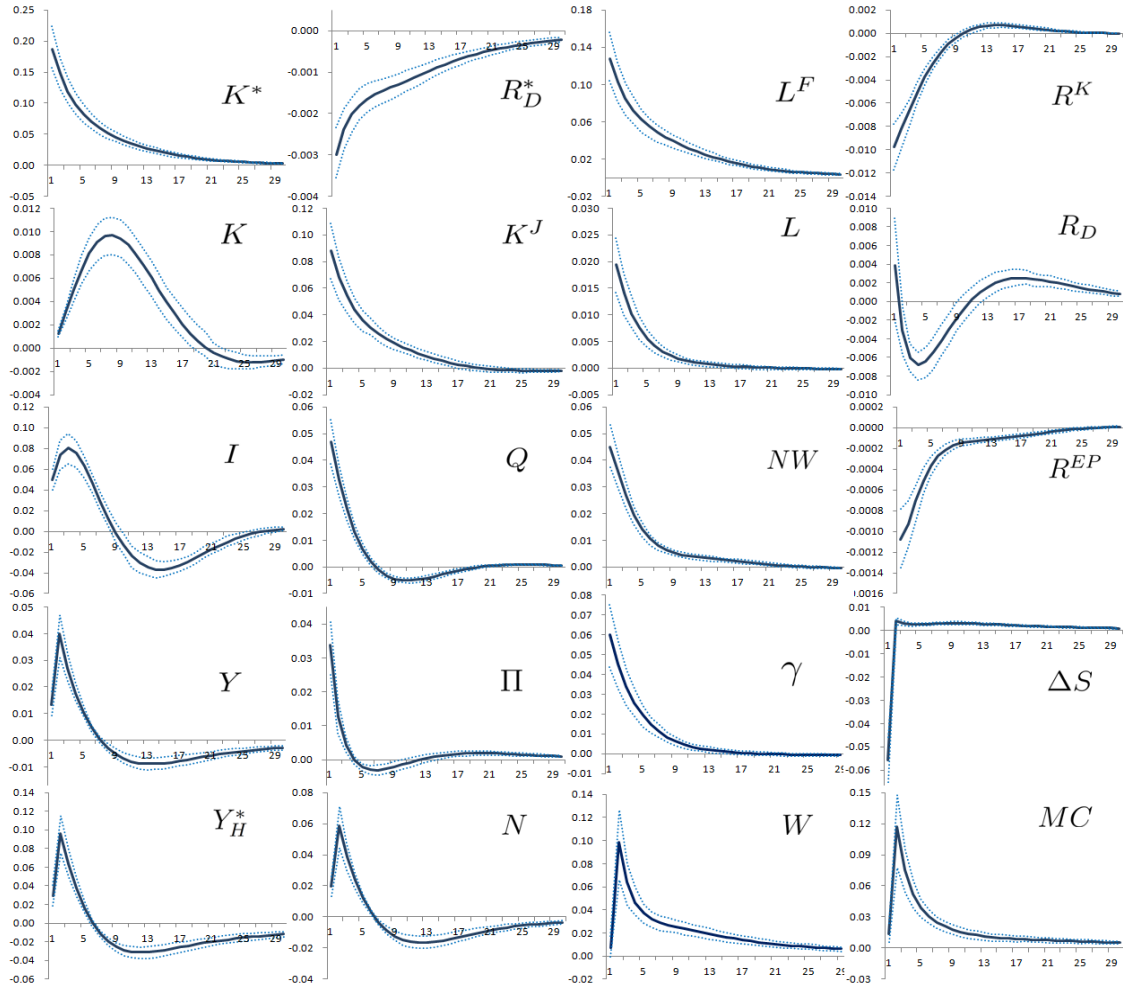
Figure 3.6: Effects of Foreign Country's QE: Bayesian Estimation

the foreign banks' return on loans falls; under the zero profit condition of the foreign banking sector, the foreign banks' funding rate (R_D^*) also declines.

In the financial sector, foreign QE affects the home economy through the changes in cross-border capital flows. As the amount of home entrepreneurs' foreign borrowing increases and the foreign interest rate (R_D^*) falls[31], capital stock of the group J home entrepreneurs (K^J) rises. This also leads to an increase in aggregate capital stock. The rise in capital stock is accompanied by an increase in investment. The

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[31]The optimality condition (3.3.35) suggests that a decline in $R_{D,t}^*$ raises the cut-off value ω_{t+1}^J. This has an upward pressure on K_{t+1}^J through the equation (3.3.39).

unanticipated increase in investment raises the asset price (Q). As this enlarges the balance sheet of the entrepreneur sector, entrepreneurs' net worth (NW) increases, and thus the external finance premium (R^{EP}) declines[32]. As the foreign nominal deposit rate (R^*_{ND}) falls sharply, the home currency appreciates following the modified uncovered interest rate parity - a fall in S.

Foreign QE affects the home economy more broadly as it lowers the home interest rate. Since the foreign funding rate declines, the foreign banks' required return on the loans to home entrepreneurs falls - following the zero profit condition. As the foreign borrowing becomes cheaper, the number of entrepreneurs using foreign loans (γ) rises. This strengthens the effect of the increase in the average foreign borrowing of the group J entrepreneurs (L^F). As the home entrepreneurs' debt repayment depends on the capital return, the gross return of capital in the home economy (R^K) also falls[33]. As a result, the foreign QE effects are not confined to the agents borrowing from the foreign economy. Since the capital return is common to all entrepreneurs, the fall in R^K lowers the home banks' returns on domestic loans. Consequently, under the zero profit condition the home banks' funding rate (R_D) also declines. The drop in R_D then leads to an increase in the amount of home loans (L). These cross-border effects of QE through capital flows can be explained by the portfolio balance channel. As the demand for assets increases and the yields on domestic assets decline in the foreign economy, foreign funds flow into the home economy - increasing foreign agents' assets (loans). As a result, the home interest rate also falls.

Foreign QE affects the home economy also through the trade flow channel. As the foreign aggregate demand rises, the amount of home exports (Y^*_H) increases.

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[32]As in BGG (1999), net worth inversely depends on the external finance premium following the equations (3.3.35) and (3.3.39).

[33]In the equation (3.3.30), a decline in the foreign bank funding rate ($R^*_{D,t}$) lowers the required capital return next period (R^K_{t+1}). The fall in the capital return then leads to a decrease in the home banks' funding rate ($R_{D,t}$) through (3.3.29). Figure 3.6 depicts R^K_{t+1} instead of R^K_t.

The resulting expansion of output is initially driven by a rise in employment (N), and as a result the real wage increases - putting upward pressure on the marginal cost and home inflation. The exchange rate channel of foreign QE[34] appears to be dominated by the trade flow effect.

3.5.2   Sensitivity Analysis

The effects of a parameter value change or model modification on the international spillovers of QE are investigated in this section. All other parameters are fixed at the calibrated or estimated levels. There is a positive 1%p shock on ψ^*.

Financial Accelerator

The financial friction leads to larger fluctuations of economic variables in response to shocks - the financial accelerator (BGG, 1999). The relationship between foreign QE effects and the financial accelerator is illustrated in Figure 3.7, comparing two cases: *with* and *without* the financial friction. In order to focus on the effects of the home friction, in this experiment the level of the domestic friction is relatively high and that of the cross-border friction is low [35]. The model without the friction is described in the appendix 3.A.4. Without the friction, the amount of entrepreneurs' net worth and the external finance premium (R^{EP}) become zero.

In response to foreign QE, the unanticipated rise in home investment is accompanied by an increase in the asset price; with the financial friction, this boosts home entrepreneurs' balance sheets and net worth. Since net worth is inversely related to the external finance premium, as in BGG (1999), R^{EP} declines. The fall in the premium then stimulates investment - magnifying the foreign QE effects. Compared to the case without the friction, home investment and capital stock rise more with

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[34]Through the exchange rate channel of foreign QE spillovers, the home currency appreciation negatively affects home exports (Lavigne, Sarker and Vasishtha, 2014).

[35]In this experiment, $\mu^I = 0.45$ and $\mu^J = 0.05$ in the model with the friction.
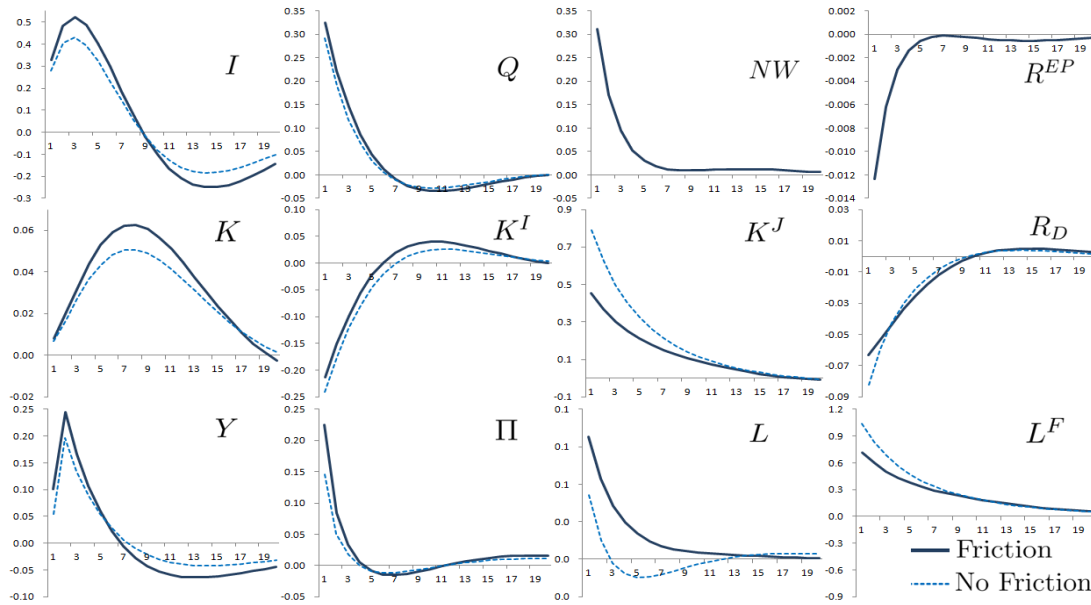
Figure 3.7: Foreign Country's QE with/without Home Financial Friction

the financial friction in response to the foreign QE shock. Eventually, home banks'
domestic loans (L), home output, and inflation are subject to more considerable
fluctuations. The financial accelerator is through the home credit changes, rather
than the foreign loans (L^F) variations.

Different Foreign Assets Adjustment Cost

As the foreign central bank injects liquidity into the banking sector and the foreign
deposit rate declines, foreign households reduce their deposits holding. When the
assets adjustment cost (μ_A^*) is low, the foreign households can reduce deposits easily.
However, when the assets adjustment cost is high, the foreign households would not
change their portfolios much; the decline in deposits would be small. This is well
represented by Figure 3.8.

When the foreign assets adjustment cost is higher $(\mu_A^* = 0.20)$, the amount of
foreign household deposits (D^*) drops less. Combined with QE, this leads to a
greater increase in foreign banks' total funding (D_B^*); the home economy's foreign
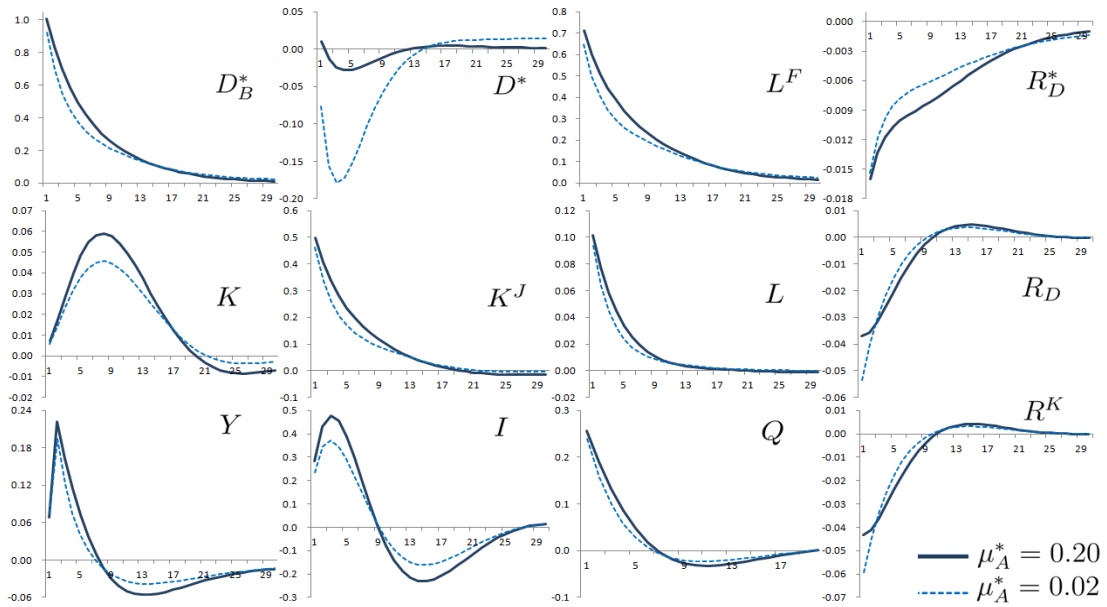
Figure 3.8: Foreign QE with Different Foreign Assets Adjustment Costs

borrowing (L^F), home investment and capital stock then increase more significantly. As a consequence, the cross-border QE effects on the home economy become stronger with the higher foreign assets adjustment cost.

The assets adjustment cost is greater when market liquidity is not enough or substitutability between assets is low; agents have considerable difficulty in finding trade counterparts in the market, and thus pay more to reallocate their assets. The result in this section implies that, as the foreign financial market suffers from low liquidity, the international effects of foreign QE become more significant.

Different Home Investment Adjustment Cost

Figure 3.9 (a) indicates the role of the home economy's investment adjustment cost (μ_{iv}). Since the international spill-over effects of foreign QE are through the changes in home investment and capital stock, as home investment adjustment is easier, the effects of foreign QE become stronger. Home investment and capital stock rise more with the lower cost ($\mu_{iv} = 2$) than the other case ($\mu_{iv} = 8$). The increase in capital
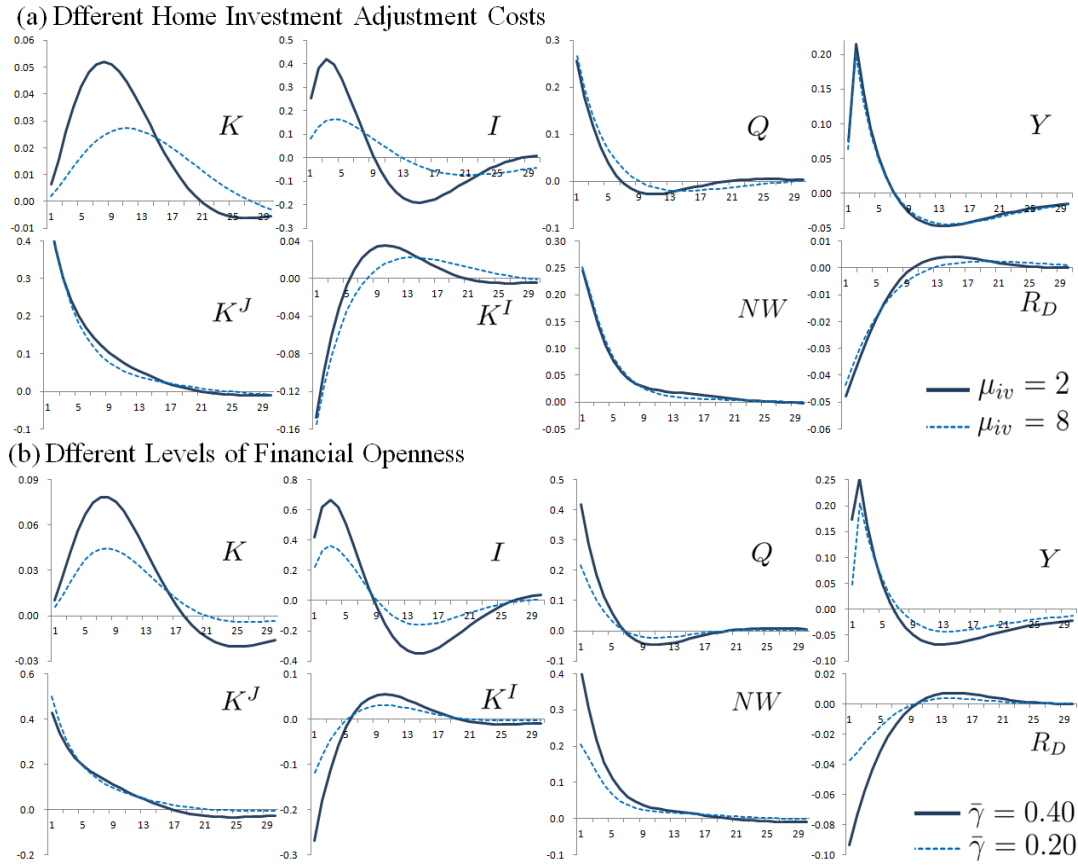
Figure 3.9: Effects of Foreign QE with different Parameters

stock is mainly supported by the group I entrepreneurs' capital stock change. The effects on home output and net worth are not substantial.

Different Financial Openness of Home Economy

Figure 3.9 (b) compares the foreign QE effects with different levels of home financial openness. Home financial openness is represented by the steady state mass of the home entrepreneurs funded by foreign banks ($\bar{\gamma} \in (0,1)$). With a higher value of $\bar{\gamma}$ (=0.40), the propagation channel of foreign QE in the home economy is stronger. Investment, the asset price, and capital stock in the home economy increase more than the other case ($\bar{\gamma} = 0.20$). Moreover, home output rises more rapidly in response to the foreign QE shock. The amount of entrepreneurs' net worth keeps

a higher level and the home interest rate (R_D) falls more with greater financial openness. This result is in line with Dedola and Lombardo (2012) and Goldberg (2009) which emphasize the role of financial integration enhancing the international transmission of shocks.

3.6   Conclusion

The cross-border effects of QE on other economies have been widely investigated in empirical papers. However, there are not enough studies that incorporate the international QE effects into general equilibrium frameworks. This chapter investigates the propagation channels where foreign QE affects the home economy, with a focus on the cross-border capital flows.

The impulse response analyses suggest that capital stock, investment and the asset price of the home country rise in response to a foreign QE shock. As the foreign nominal interest rate falls sharply, the exchange rate declines (home currency appreciation). This explains the findings of the empirical studies investigating the international effects of QE. Foreign QE has far-reaching effects on the home economy as it lowers the home capital return; this reduces the home interest rate, and the amount of home loans increases. As the increase in the foreign aggregate demand boosts home exports, home output expands. The rise in employment raise the wage and inflation. The home financial friction strengthens the spill-over effects of foreign QE through the financial accelerator. The rise in the asset price boosts entrepreneurs' net worth, which lowers the external finance premium. Home investment, capital stock and output then increase more than the case without the friction.

Foreign QE has stronger international effects when the foreign financial market faces a higher assets adjustment cost. The cost causes the foreign private sector to reduce deposits sluggishly in response to QE. As a result, the amount of foreign

banks' loans to home entrepreneurs increases more. This boosts home investment and capital stock more significantly. Moreover, the international effects of foreign QE are stronger with a lower home investment adjustment cost and a higher degree of home financial openness.

3.A Appendix

3.A.1 Steady State

$$\bar{R} = \bar{R}^* = \beta^{-1}$$

$$\bar{\Pi} = 1.007$$

$$\bar{A} = 1$$

$$\bar{Q} = 1$$

$$\bar{\gamma} = 0.24$$

$$\bar{R}_D/\bar{R} = 1.0025$$

$$\bar{R}^k/\bar{R} = 1.005$$

$$\bar{R}^{k*}/\bar{R}^* = \bar{R}_D^*/\bar{R}^* = 1$$

$$\bar{r}_K = \bar{R}^k - (1-\delta)$$

$$\bar{r}_K^* = \bar{R}^{k*} - (1-\delta)$$

$$\bar{M}C = \bar{M}C^* = (\varepsilon - 1)/\varepsilon$$

$$\bar{S}_R = \Delta\bar{S} = 1$$

$$\bar{W} = \left[\frac{1}{\nu^\nu(1-\nu)^{1-\nu}}\bar{r}_K^\nu \bar{M}C^{-1}\right]^{\frac{1}{\nu-1}}$$

$$\bar{W}^* = \left[\frac{1}{\nu^\nu(1-\nu)^{1-\nu}}\bar{r}_K^{*\nu} \bar{M}C^{*-1}\right]^{\frac{1}{\nu-1}}$$

$$\bar{N}/\bar{K} = \frac{1-\nu}{\nu}\bar{r}_K\bar{W}^{-1}$$

$$\bar{N}^*/\bar{K}^* = \frac{1-\nu}{\nu}\bar{r}_K^*\bar{W}^{*-1}$$

$$\left(\overline{\frac{P_{H,t}}{P_t}}\right) = \left(\overline{\frac{P_{F,t}}{P_t}}\right) = \left(\overline{\frac{\tilde{P}_{H,t}}{P_t}}\right) = 1$$

$$\left(\overline{\frac{P_{F,t}^*}{P_t^*}}\right) = \left(\overline{\frac{P_{H,t}^*}{P_t^*}}\right) = 1$$

$$\bar{\phi} = (1-\bar{\gamma})\bar{\phi}^I + \bar{\gamma}\bar{\phi}^J$$

$$\bar{K} = (1-\bar{\gamma})\bar{K}^I + \bar{\gamma}\bar{K}^J$$

$$\bar{K}^I = \bar{\phi}^I N\bar{W}$$

$$\bar{K}^J = \bar{\phi}^J N\bar{W}$$

$$\bar{K} = \bar{\phi}N\bar{W}$$

$$\bar{I} = \delta\bar{K}$$

$$\bar{N} = \frac{\bar{N}}{\bar{K}}\bar{K}$$

$$\bar{Y} = \left(\frac{\bar{N}}{\bar{K}}\right)^{1-\nu}\bar{K}$$

$$\bar{C} = \left(\bar{N}^{-\chi}\bar{W}\right)^{\frac{1}{\sigma}}$$

$$\bar{C}^* = \left(\bar{N}^{*-\chi}\bar{W}^*\right)^{\frac{1}{\sigma}}$$

$$\bar{Z} = \bar{C} + \bar{I} + \bar{m}$$

$$\bar{Y}_H = (1-\alpha)\bar{Z}$$

$$\bar{Y} = \bar{Y}_H + \bar{Y}_H^*$$

$$\bar{Y}_F = \alpha\bar{Z}$$

$$\bar{\tilde{Y}}_H^* = \bar{Y}_H/\tau$$

$$\bar{\tilde{Y}}_H^* = \alpha^*\bar{Z}^*$$

$$(1-\alpha^*)\bar{Z}^* = \bar{Y}_F^*$$

$$\bar{Z}^* = \bar{Y}^*$$

$$\bar{Y}^* = \left(\frac{\bar{N}^*}{\bar{K}^*}\right)^{1-\nu}\bar{K}^*$$

$$\bar{N}^* = \frac{\bar{N}^*}{\bar{K}^*}\bar{K}^*$$

$$\bar{m} = \bar{R}^k\left[(1-\bar{\gamma})\mu^I\bar{M}^I\bar{K}^I + \bar{\gamma}\mu^J\bar{M}^J\bar{K}^J\right]$$

$$\bar{I}^* = \delta\bar{K}^*$$

$$\bar{R}^N = \bar{R}^{N*} = \Pi\beta^{-1} \qquad\qquad \bar{Z}^* = \bar{C}^* + \bar{I}^* + \bar{G}^*$$

$$\bar{L} = \left(\bar{\phi}^I - 1\right) N\bar{W} \qquad\qquad \bar{L}^F = \left(\bar{\phi}^J - 1\right) N\bar{W}$$

$$\bar{B}^F = \left[\bar{\gamma}\left(\bar{R}^* - 1\right)\bar{L}^F\right]/(1-\beta) \qquad\qquad \bar{B} = \bar{B}^F$$

$$\bar{D}^* = \bar{R}_D^* \bar{K}^*/\left(1-\gamma_F\right) \qquad\qquad \bar{D}^* = \bar{D}_B^* = \bar{B}^*$$

$$\bar{\gamma}\bar{L}^F = \tau\gamma_F \bar{R}_D^{*-1}\bar{D}^* \qquad\qquad \bar{G}^* = 0.2\bar{Y}^*$$

$$M\left(\bar{\omega}^I\right) = \int_0^{\bar{\omega}^I} \omega f(\omega)d\omega \qquad\qquad M\left(\bar{\omega}^J\right) = \int_0^{\bar{\omega}^J} \omega f(\omega)d\omega$$

$$\Gamma\left(\bar{\omega}^I\right) = \int_0^{\bar{\omega}^I} \omega f(\omega)d\omega + \bar{\omega}^I \int_{\bar{\omega}^I}^{\infty} \omega f(\omega)d\omega \qquad \Gamma\left(\bar{\omega}^J\right) = \int_0^{\bar{\omega}^J} \omega f(\omega)d\omega + \bar{\omega}^J \int_{\bar{\omega}^J}^{\infty} \omega f(\omega)d\omega$$

$$F\left(\bar{\omega}^I\right) = \int_0^{\bar{\omega}^I} f(\omega)d\omega \qquad\qquad F\left(\bar{\omega}^J\right) = \int_0^{\bar{\omega}^J} f(\omega)d\omega$$

$$\Gamma'\left(\bar{\omega}^I\right) = 1 - F\left(\bar{\omega}^I\right) \qquad\qquad \Gamma'\left(\bar{\omega}^J\right) = 1 - F\left(\bar{\omega}^J\right)$$

$$M'\left(\bar{\omega}^I\right) = \bar{\omega}^I f\left(\bar{\omega}^I\right) \qquad\qquad M'\left(\bar{\omega}^J\right) = \bar{\omega}^I f\left(\bar{\omega}^J\right)$$

$$\left(\frac{\bar{R}^k}{\bar{R}_D}\right)^{-1} = \frac{\left[1 - \Gamma\left(\bar{\omega}^I\right)\right]\left[\Gamma'\left(\bar{\omega}^I\right) - \mu^I M'\left(\bar{\omega}^I\right)\right]}{\Gamma'\left(\bar{\omega}^I\right)} + \Gamma\left(\bar{\omega}^I\right) - \mu^I M\left(\bar{\omega}^I\right)$$

$$\left(\frac{\bar{R}^k}{\bar{R}_D^*}\right)^{-1} = \frac{\left[1 - \Gamma\left(\bar{\omega}^J\right)\right]\left[\Gamma'\left(\bar{\omega}^J\right) - \mu^J M'\left(\bar{\omega}^J\right)\right]}{\Gamma'\left(\bar{\omega}^J\right)} + \Gamma\left(\bar{\omega}^J\right) - \mu^J M\left(\bar{\omega}^J\right)$$

$$f\left(\omega\right) = \frac{1}{\omega}\frac{1}{\sigma_\omega\sqrt{2\pi}}e^{-\frac{1}{2\sigma_\omega^2}(ln\omega)^2}$$

3.A.2   Log-linearized Equations

$$\hat{Y}_t = \hat{A}_t + \nu\hat{K}_t + (1-\nu)\hat{N}_t \tag{3.A.1}$$

$$\hat{Y}_t^* = \hat{A}_t^* + \nu\hat{K}_t^* + (1-\nu)\hat{N}_t^* \tag{3.A.2}$$

$$\widehat{MC}_t = -\hat{A}_t + \nu\hat{r}_{K,t} + (1-\nu)\hat{W}_t \tag{3.A.3}$$

$$\widehat{MC}_t^* = -\hat{A}_t^* + \nu\hat{r}_{K,t}^* + (1-\nu)\hat{W}_t^* \tag{3.A.4}$$

$$\hat{N}_t - \hat{K}_t = \hat{r}_{K,t} - \hat{W}_t \tag{3.A.5}$$

$$\hat{N}_t^* - \hat{K}_t^* = \hat{r}_{K,t}^* - \hat{W}_t^* \tag{3.A.6}$$

$$\left(\widehat{\frac{P_{H,t}}{P_t}}\right) = \xi\left(\widehat{\frac{P_{H,t-1}}{P_{t-1}}}\right) + (1-\xi)\left(\widehat{\frac{\tilde{P}_{H,t}}{P_t}}\right) - \xi\hat{\Pi}_t \tag{3.A.7}$$

$$\left(\widehat{\frac{\tilde{P}_{H,t}}{P_t}}\right) = (1-\beta\xi)\,\widehat{MC}_t + \beta\xi E_t\left(\widehat{\frac{\tilde{P}_{H,t+1}}{P_{t+1}}} + \hat{\Pi}_{t+1}\right) \tag{3.A.8}$$

$$\left(\widehat{\frac{P_{F,t}^*}{P_t^*}}\right) = \widehat{MC}_t^* \tag{3.A.9}$$

$$\left(\widehat{\frac{P_{F,t}}{P_t}}\right) = \left(\widehat{\frac{P^*_{F,t}}{P^*_t}}\right) + \hat{S}_{R,t} \tag{3.A.10}$$

$$\left(\widehat{\frac{P_{H,t}}{P_t}}\right) = \left(\widehat{\frac{P^*_{H,t}}{P^*}}\right) + \hat{S}_{R,t} \tag{3.A.11}$$

$$\hat{Y}_{H,t} = -\theta\left(\widehat{\frac{P_{H,t}}{P_t}}\right) + \hat{Z}_t \tag{3.A.12}$$

$$\hat{Y}_{F,t} = -\theta\left(\widehat{\frac{P_{F,t}}{P_t}}\right) + \hat{Z}_t \left(= \hat{\hat{Y}}_{F,t}\right) \tag{3.A.13}$$

$$\hat{Y}^*_{F,t} = -\theta\left(\widehat{\frac{P^*_{F,t}}{P^*_t}}\right) + \hat{Z}^*_t \tag{3.A.14}$$

$$\hat{Y}^*_{H,t} = -\theta\left(\widehat{\frac{P^*_{H,t}}{P^*_t}}\right) + \hat{Z}^*_t \left(= \hat{\hat{Y}}^*_{H,t}\right) \tag{3.A.15}$$

$$0 = \alpha\left(\widehat{\frac{P_{H,t}}{P_t}}\right) + (1-\alpha)\left(\widehat{\frac{P_{F,t}}{P_t}}\right) \tag{3.A.16}$$

$$0 = \alpha^*\left(\widehat{\frac{P^*_{F,t}}{P^*_t}}\right) + (1-\alpha^*)\left(\widehat{\frac{P^*_{H,t}}{P^*_t}}\right) \tag{3.A.17}$$

$$\hat{W}_t = \sigma\hat{C}_t + \chi\hat{N}_t \tag{3.A.18}$$

$$\hat{W}^*_t = \sigma\hat{C}^*_t + \chi\hat{N}^*_t \tag{3.A.19}$$

$$\hat{C}_t = E_t\left(\hat{C}_{t+1}\right) - \sigma^{-1}\hat{R}_t \tag{3.A.20}$$

$$\hat{C}^*_t = E_t\left(\hat{C}^*_{t+1}\right) - \sigma^{-1}\hat{R}^*_t \tag{3.A.21}$$

$$\hat{R}_{D,t}/\bar{R}_D = \hat{R}_t/\bar{R} + \mu_A\bar{D}\hat{D}_t \tag{3.A.22}$$

$$\hat{R}^*_{D,t}/\bar{R}^*_D = \hat{R}^*_t/\bar{R}^* + \mu^*_A\bar{D}^*\hat{D}^*_t \tag{3.A.23}$$

$$\hat{Q}_t = \mu_{iv}\left(\hat{I}_t - \hat{I}_{t-1}\right) - \beta\mu_{iv}E_t\left(\hat{I}_{t+1} - \hat{I}_t\right) \tag{3.A.24}$$

$$\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \frac{\bar{I}}{\bar{K}}\hat{I}_t \tag{3.A.25}$$

$$\hat{K}^*_{t+1} = (1-\delta)\hat{K}^*_t + \frac{\bar{I}^*}{\bar{K}^*}\hat{I}^*_t \tag{3.A.26}$$

$$E_t\left(\hat{S}_{R,t+1}\right) - \hat{S}_{R,t} = E_t\left(\hat{\Pi}^*_{t+1}\right) - E_t\left(\hat{\Pi}_{t+1}\right) + E_t\left(\Delta\hat{S}_{t+1}\right) \tag{3.A.27}$$

$$\Delta\hat{S}_t = \rho_{FX}\left[\left(\hat{B}^F_t - \hat{R}^*_t\right)\bar{R}^{*-1} - \hat{B}^F_{t-1}\right] \tag{3.A.28}$$

$$\hat{R}_{N,t} = \rho_R\hat{R}_{N,t-1} + (1-\rho_R)\left(\gamma_P\hat{\Pi}_t + \gamma_Y\hat{Y}_t\right) + \hat{\eta}_t \tag{3.A.29}$$

$$\hat{R}_{N,t} = \hat{R}_t + E_t\left(\hat{\Pi}_{t+1}\right) \tag{3.A.30}$$

$$\hat{R}^*_{N,t} = \rho_R\hat{R}^*_{N,t-1} + (1-\rho_R)\left(\gamma_P\hat{\Pi}^*_t + \gamma_Y\hat{Y}^*_t\right) + \hat{\eta}^*_t \tag{3.A.31}$$

$$\hat{R}^*_{N,t} = \hat{R}^*_t + E_t\left(\hat{\Pi}^*_{t+1}\right) \tag{3.A.32}$$

$$\bar{R}^{-1}\left(\hat{B}_t - \hat{R}_t\right) - \hat{B}_{t-1} = (\bar{R}^{*-1} - 1)\hat{S}_{R,t} + \bar{R}^{*-1}\left(\hat{B}^F_t - \hat{R}^*_t\right) - \hat{B}^F_{t-1} \tag{3.A.33}$$

$$\tau\bar{G}^*\hat{G}^*_t = \tau\bar{B}^*\left[\left(\hat{B}^*_t - \hat{R}^*_t\right)/\bar{R}^* - \hat{B}^*_{t-1}\right]$$
$$+\bar{B}^F\left[\left(\hat{B}^F_t - \hat{R}^*_t\right)/\bar{R}^* - \hat{B}^F_{t-1}\right] \tag{3.A.34}$$

$$\bar{R}^k \left(\hat{Q}_{t-1} + \hat{R}_t^k \right) = \bar{r}_K \hat{r}_{K,t} + (1 - \delta) \hat{Q}_t \tag{3.A.35}$$

$$\hat{\phi}_t = \hat{Q}_t + \hat{K}_{t+1} - \widehat{NW}_t \tag{3.A.36}$$

$$\bar{K} \hat{K}_{t+1} = (1 - \bar{\gamma}) \bar{K}^I \hat{K}_{t+1}^I + \bar{\gamma} \bar{K}^J \hat{K}_{t+1}^J + \bar{\gamma} \left(\bar{K}^J - \bar{K}^I \right) \hat{\gamma}_{t+1} \tag{3.A.37}$$

$$\bar{L} \hat{L}_t = \bar{K}^I \left(\hat{Q}_t + \hat{K}_{t+1}^I \right) - \bar{NW} \widehat{NW}_t \tag{3.A.38}$$

$$\bar{L}^F \left(\hat{L}_t^F + \hat{S}_{R,t} \right) = \bar{K}^J \left(\hat{Q}_t + \hat{K}_{t+1}^J \right) - \bar{NW} \widehat{NW}_t \tag{3.A.39}$$

$$\frac{1}{\bar{\phi}^I} \hat{\phi}_t^I = \frac{\bar{R}^k}{\bar{R}_D} \left[\Gamma \left(\bar{\omega}^I \right) - \mu^I M \left(\bar{\omega}^I \right) \right] \left[E_t \left(\hat{R}_{t+1}^k \right) - \hat{R}_{D,t} \right]$$
$$+ \frac{\bar{R}^k}{\bar{R}_D} \bar{\omega}^I \left[\Gamma' \left(\bar{\omega}^I \right) - \mu^I M' \left(\bar{\omega}^I \right) \right] \hat{\omega}_{t+1}^I \tag{3.A.40}$$

$$\frac{1}{\bar{\phi}^J} \hat{\phi}_t^J = \frac{\bar{R}^k}{\bar{R}_D^*} \left[\Gamma \left(\bar{\omega}^J \right) - \mu^J M \left(\bar{\omega}^J \right) \right] \left[E_t \left(\hat{R}_{t+1}^k \right) - \hat{R}_{D,t}^* + \hat{S}_{R,t} - E_t \left(\hat{S}_{R,t+1} \right) \right]$$
$$+ \frac{\bar{R}^k}{\bar{R}_D^*} \bar{\omega}^J \left[\Gamma' \left(\bar{\omega}^J \right) - \mu^J M' \left(\bar{\omega}^J \right) \right] \hat{\omega}_{t+1}^J \tag{3.A.41}$$

$$E_t \left(\hat{R}_{t+1}^k \right) - \hat{R}_{D,t} = \left(\frac{\bar{R}^k}{\bar{R}_D} \right) \mu^I \bar{\omega}^I \left[\eta^I + \bar{\omega}^I f \left(\bar{\omega}^I \right) \right] \hat{\omega}_{t+1}^I \tag{3.A.42}$$

$$E_t \left(\hat{R}_{t+1}^k \right) - \hat{R}_{D,t}^* - E_t \left(\hat{S}_{R,t+1} \right) + \hat{S}_{R,t} = \left(\frac{\bar{R}^k}{\bar{R}_D^*} \right) \mu^J \bar{\omega}^J \left[\eta^J + \bar{\omega}^J f \left(\bar{\omega}^J \right) \right] \hat{\omega}_{t+1}^J \tag{3.A.43}$$

$$\hat{Q}_t + \hat{K}_{t+1}^I = \hat{\phi}_t^I + N\hat{W}_t \tag{3.A.44}$$

$$\hat{Q}_t + \hat{K}_{t+1}^J = \hat{\phi}_t^J + N\hat{W}_t \tag{3.A.45}$$

$$\left(\bar{R}^k \zeta \right)^{-1} N\hat{W}_t = \left\{ (1 - \bar{\gamma}) \left[1 - \Gamma \left(\bar{\omega}^I \right) \right] \bar{\phi}^I + \bar{\gamma} \left[1 - \Gamma \left(\bar{\omega}^J \right) \right] \bar{\phi}^J \right\} \left(\hat{R}^k + N\hat{W}_{t-1} \right)$$
$$+ \left\{ (1 - \bar{\gamma}) \left[1 - \Gamma \left(\bar{\omega}^I \right) \right] \bar{\phi}^I \hat{\phi}_{t-1}^I + \bar{\gamma} \left[1 - \Gamma \left(\bar{\omega}^J \right) \right] \bar{\phi}^J \hat{\phi}_{t-1}^J \right\}$$
$$- \left[(1 - \bar{\gamma}) \Gamma' \left(\bar{\omega}^I \right) \bar{\omega}^I \bar{\phi}^I \hat{\omega}_t^I + \bar{\gamma} \Gamma' \left(\bar{\omega}^J \right) \bar{\omega}^J \bar{\phi}^J \hat{\omega}_t^J \right]$$
$$+ \bar{\gamma} \left\{ \left[1 - \Gamma \left(\bar{\omega}^J \right) \right] \bar{\Phi}^J - \left[1 - \Gamma \left(\bar{\omega}^I \right) \right] \bar{\Phi}^I \right\} \hat{\gamma}_t \tag{3.A.46}$$

$$\hat{R}_t^{EP} = E_t \left(\hat{R}_{t+1}^k \right) - \hat{R}_{D,t} \tag{3.A.47}$$

$$\hat{R}_{ND,t} = \hat{R}_{D,t} + E_t \left(\Pi_{t+1} \right) \tag{3.A.48}$$

$$\hat{R}_{ND,t}^* = \hat{R}_{D,t}^* + E_t \left(\Pi_{t+1}^* \right) \tag{3.A.49}$$

$$\frac{\mu_\gamma}{\bar{\gamma}} \left(\hat{\gamma}_{t+1} - \hat{\gamma}_t \right) = \left\{ \left[1 - \Gamma \left(\bar{\omega}^J \right) \right] \bar{K}^J - \left[1 - \Gamma \left(\bar{\omega}^I \right) \right] \bar{K}^I \right\} \bar{R}^K \left(E_t(\hat{R}_{t+1}^K) + \hat{Q}_t \right)$$
$$- \left[\hat{K}_{t+1}^I - \Gamma \left(\bar{\omega}^I \right) \hat{K}_{t+1}^I - \Gamma' \left(\bar{\omega}^I \right) \bar{\omega}^I \hat{\omega}^I \right] \bar{K}^I \bar{R}^K$$
$$+ \left[\hat{K}_{t+1}^J - \Gamma \left(\bar{\omega}^J \right) \hat{K}_{t+1}^J - \Gamma' \left(\bar{\omega}^J \right) \bar{\omega}^J \hat{\omega}^J \right] \bar{K}^J \bar{R}^K \tag{3.A.50}$$

$$E_t \left(\hat{R}_{K,t+1}^* \right) = \hat{R}_{D,t}^* \tag{3.A.51}$$

$$\bar{R}_K^* \hat{R}_{K,t}^* = \bar{r}_K^* \hat{r}_{K,t}^* \tag{3.A.52}$$

$$\hat{K}_{t+1}^* = \hat{D}_{B,t}^* - \hat{R}_{D,t} \tag{3.A.53}$$

$$\hat{L}_t^* = \hat{K}_{t+1}^* \tag{3.A.54}$$

$$\hat{\gamma}_{t+1} + \hat{L}_t^F = \hat{D}_{B,t}^* - \hat{R}_{D,t} \tag{3.A.55}$$

$$\bar{Y} \hat{Y}_t = \bar{Y}_H \hat{Y}_{H,t} + \bar{Y}_H^* \hat{Y}_{H,t}^* \tag{3.A.56}$$

$$\bar{Z} \hat{Z}_t = \bar{C} \hat{C}_t + \bar{I} \hat{I}_t + \bar{m} \hat{m}_t \tag{3.A.57}$$

$$\bar{Y}^* \hat{Y}_t^* = \bar{Y}_F^* \hat{Y}_{F,t}^* + \bar{Y}_F \hat{Y}_F \tag{3.A.58}$$

$$\bar{Z}^* \hat{Z}_t^* = \bar{C}^* \hat{C}_t^* + \bar{I}^* \hat{I}_t^* + \bar{G}^* \hat{G}_t^* \tag{3.A.59}$$

$$\bar{B}^F \left[\bar{R}^{*-1} \left(\hat{B}_t - \hat{R}_t^* \right) - \hat{B}_{t-1}^F \right] = \bar{\gamma} \bar{L}^F \left[\hat{L}_t^F - \bar{R}_D^* \left(\hat{R}_{D,t-1}^* + \hat{L}_{t-1}^F \right) \right] - \bar{\gamma} \bar{L}^F \left(\hat{\gamma}_{t+1} - \bar{R}_D^* \hat{\gamma}_t \right)$$
$$+ \bar{Y}_H^* \left(\widehat{\frac{P_{H,t}^*}{P_t^*}} + \hat{Y}_{H,t}^* \right) - \bar{Y}_F \left(\widehat{\frac{P_{F,t}}{P_t}} - \hat{S}_{R,t} + \hat{Y}_{F,t} \right) \tag{3.A.60}$$

$$\bar{L}(1 - \bar{\gamma})\hat{L}_t - \bar{\gamma}\bar{L}\hat{\gamma}_{t+1} = \hat{D}_t - \hat{R}_{D,t} \tag{3.A.61}$$

$$\hat{D}_t^* = \hat{D}_{B,t}^* - \psi_t^* \tag{3.A.62}$$

$$\bar{m}\hat{m}_t = (1 - \bar{\gamma})\, \mu^I \bar{R}^k \bar{K}^I \left[\bar{\omega}^{I2} f\left(\bar{\omega}^I \right) \hat{\omega}_t^I + M\left(\bar{\omega}^I \right) \left(\hat{R}_t^k + \hat{Q}_{t-1} + \hat{K}_t^I \right) \right]$$
$$+ \bar{\gamma}\mu^J \bar{R}^k \bar{K}^J \left[\bar{\omega}^{J2} f\left(\bar{\omega}^J \right) \hat{\omega}_t^J + M\left(\bar{\omega}^J \right) \left(\hat{R}_t^k + \hat{Q}_{t-1} + \hat{K}_t^J \right) \right]$$
$$+ \bar{\gamma} \left(\mu^J \bar{R}^K \bar{M}^J \bar{K}^J - \mu^I \bar{R}^K \bar{M}^I \bar{K}^I \right) \hat{\gamma}_t \tag{3.A.63}$$

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \hat{\varepsilon}_{A,t} \tag{3.A.64}$$

$$\hat{A}_t^* = \rho_A \hat{A}_{t-1}^* + \hat{\varepsilon}_{A,t}^* \tag{3.A.65}$$

$$\hat{\eta}_{m,t} = \rho_m \hat{\eta}_{m,t-1} + \hat{\varepsilon}_{m,t} \tag{3.A.66}$$

$$\hat{\eta}_{m,t}^* = \rho_m \hat{\eta}_{m,t-1}^* + \hat{\varepsilon}_{m,t}^* \tag{3.A.67}$$

$$\hat{\psi}_t^* = \rho_{qe} \hat{\psi}_{t-1}^* + \hat{\varepsilon}_{qe,t}^* \tag{3.A.68}$$

In the equations (3.A.42) and (3.A.43),

$$\eta^I = \frac{\partial \left[\omega \left(1 - \Gamma(\omega) \right) \left(1 - F(\omega) \right)^{-1} f(\omega) \right]}{\partial \omega} \mid \omega = \bar{\omega}^I$$

$$\eta^J = \frac{\partial \left[\omega \left(1 - \Gamma(\omega) \right) \left(1 - F(\omega) \right)^{-1} f(\omega) \right]}{\partial \omega} \mid \omega = \bar{\omega}^J.$$

3.A.3 Entrepreneurs' Optimization and Leverage Ratio

The optimization problem of the equation (3.3.31) under the constraint (3.3.29) yields the following Lagrangian function:

$$\mathcal{L} = \left[1 - \Gamma\left(\omega_{t+1}^I(i)\right)\right] E_t\left(R_{t+1}^K\right) Q_t K_{t+1}(i)$$

$$+\lambda \left\{E_t\left(R_{t+1}^K\right) Q_t K_{t+1}(i) \left[\Gamma\left(\omega_{t+1}^I(i)\right) - \mu^I M\left(\omega_{t+1}^I(i)\right)\right] - R_{D,t}\left[Q_t K_{t+1}(i) - NW_t(i)\right]\right\}$$

$$(3.A.69)$$

where $L_t(i) = Q_t K_{t+1}(i) - NW_t(i)$. The first-order conditions with respect to $\omega_{t+1}^I(i)$ and $K_{t+1}(i)$ are as below:

$$\Gamma'\left(\omega_{t+1}^I(i)\right) - \lambda\left[\Gamma'\left(\omega_{t+1}^I(i)\right) - \mu^I M'\left(\omega_{t+1}^I(i)\right)\right] = 0 \quad (3.A.70)$$

$$\frac{E_t\left(R_{t+1}^K\right)}{R_{D,t}} \left\{\left[1 - \Gamma\left(\omega_{t+1}^I(i)\right)\right] + \lambda\left[\Gamma\left(\omega_{t+1}^I(i)\right) - \mu^I M\left(\omega_{t+1}^I(i)\right)\right]\right\} = \lambda. (3.A.71)$$

Combining the equations (3.A.70) and (3.A.71) yields the equation (3.3.33). Similarly, optimization of (3.3.32) under the constraint of (3.3.30) leads to the first-order conditions that yield the equation (3.3.34).

Combining the equations (3.3.12), (3.3.13), (3.3.29) and (3.3.30) leads to:

$$E_t\left(R_{t+1}^K\right) Q_t K_{t+1}(i) \left[\Gamma\left(\omega_{t+1}^I(i)\right) - \mu^I M\left(\omega_{t+1}^I(i)\right)\right] = R_{D,t}\left[Q_t K_{t+1}(i) - NW_t(i)\right]$$

$$(3.A.72)$$

$$E_t\left(R_{t+1}^K\right) Q_t K_{t+1}(j) \left[\Gamma\left(\omega_{t+1}^J(j)\right) - \mu^J M\left(\omega_{t+1}^J(j)\right)\right] = R_{D,t}^* \frac{E_t\left(S_{R,t+1}\right)}{S_{R,t}}\left[Q_t K_{t+1}(j) - NW_t(j)\right].$$

$$(3.A.73)$$

Rearranging the equations (3.A.72) and (3.A.73) then yields:

$$\frac{E_t\left(R_{t+1}^K\right)}{R_{D,t}} \left[\Gamma\left(\omega_{t+1}^I(i)\right) - \mu^I M\left(\omega_{t+1}^I(i)\right)\right] = 1 - \frac{NW_t(i)}{Q_t K_{t+1}(i)} \quad (3.A.74)$$

$$\frac{E_t\left(R_{t+1}^K\right)}{R_{D,t}^*} \left[\Gamma\left(\omega_{t+1}^J(j)\right) - \mu^J M\left(\omega_{t+1}^J(j)\right)\right] = \frac{E_t\left(S_{R,t+1}\right)}{S_{R,t}}\left[1 - \frac{NW_t(j)}{Q_t K_{t+1}(j)}\right](3.A.75)$$

The equations (3.A.74) and (3.A.75) can be rewritten by the equations (3.3.39).

3.A.4   Model Description without Home Financial Friction

Without the financial friction in the home economy, there is no agency problem between lenders and borrowers; home and foreign banks can take all the remaining income of home entrepreneurs in case of default without any auditing cost. Therefore, home entrepreneurs do not set up the cut-off values of idiosyncratic shocks. From the equations (3.3.33) and (3.3.34), when $\mu^I = \mu^J = 0$,

$$E_t\left(R_{t+1}^K\right) = R_{D,t} \qquad \text{and} \qquad E_t\left(R_{t+1}^K\right) = R_{D,t}^* \frac{E_t\left(S_{R,t+1}\right)}{S_{R,t}}. \tag{3.A.76}$$

Without default, there is no borrowing limit which depends on the amount of home entrepreneurs' net worth; home entrepreneurs can borrow optimal amounts of funds without using net worth. From the equations (3.3.12) and (3.3.13) then the following equations hold:

$$L_t = Q_t K_{t+1}^I \qquad \text{and} \qquad S_{R,t} L_t^F = Q_t K_{t+1}^J. \tag{3.A.77}$$

Also, the non-default loan rates of home and foreign loans would satisfy $R_{L,t+1} = R_{D,t}$ and $R_{L,t+1}^F = R_{D,t}^*$ due to the zero profit condition of home and foreign banks.

Combining the equations (3.A.76) and (3.A.77) then suggests that the expected profit is zero in the home entrepreneur sector, since the expected value of the individual shock is unity[36]. Thus, there is no incentive to change the share of the group J entrepreneurs (γ_{t+1}); γ_{t+1} is assumed to be fixed at its steady state level $(\bar{\gamma})$. Therefore the lending market clearing conditions (3.3.98) and (3.3.99) are transformed into

$$(1 - \bar{\gamma}) L_t = R_{D,t}^{-1} D_t \qquad \text{and} \qquad \bar{\gamma} L_t^F = \tau \gamma_F L_t^*. \tag{3.A.78}$$

Also, since there is no auditing cost (m_t), the equation (3.3.93) becomes

$$C_t + I_t + g\left(\frac{I_t}{I_{t-1}}\right) I_t = Z_t. \tag{3.A.79}$$

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[36]The expected income of capital is $E_t\left(R_{t+1}^K\right) Q_t K_{t+1}$, which can be rewritten as $E_t\left(R_{t+1}^K\right) S_{R,t} L_t^F$ for the group J by (3.A.77). Since the expected debt repayment becomes $R_{D,t}^* E_t\left(S_{R,t+1}\right) L_t^F$, the expected income is equivalent to the expected repayment by (3.A.76).
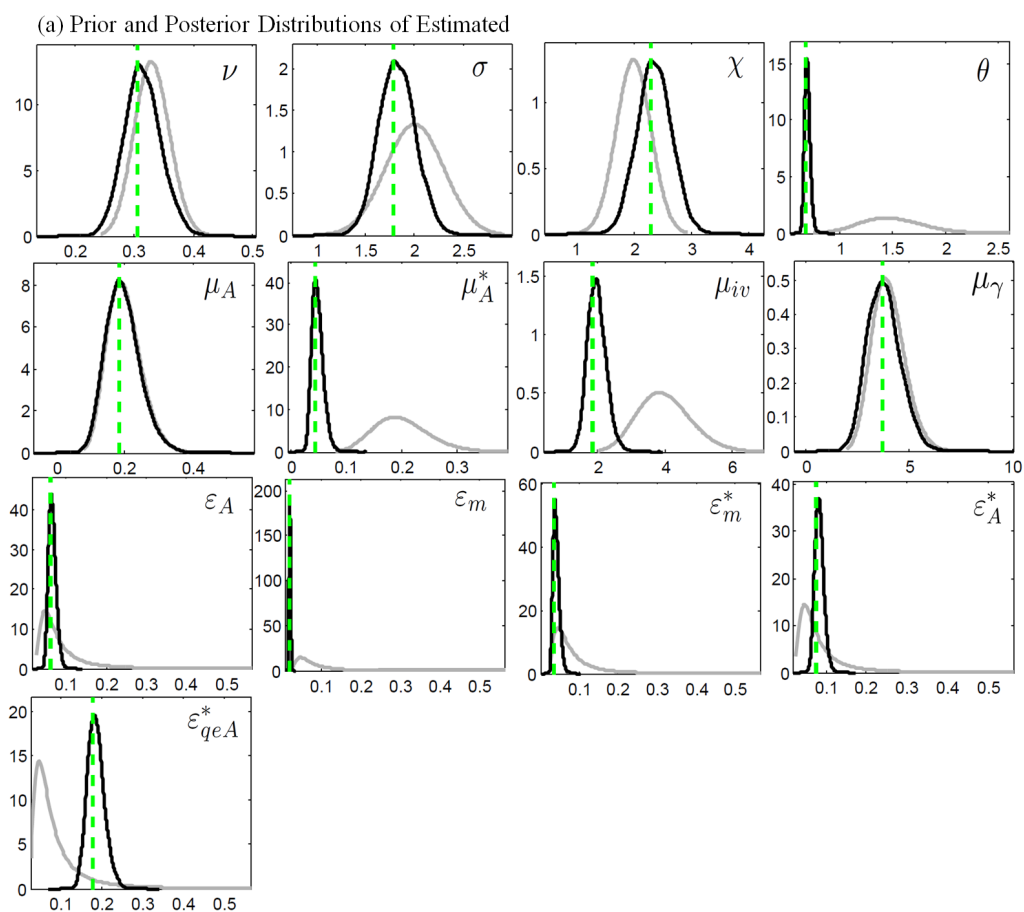
3.A.5   Data Source

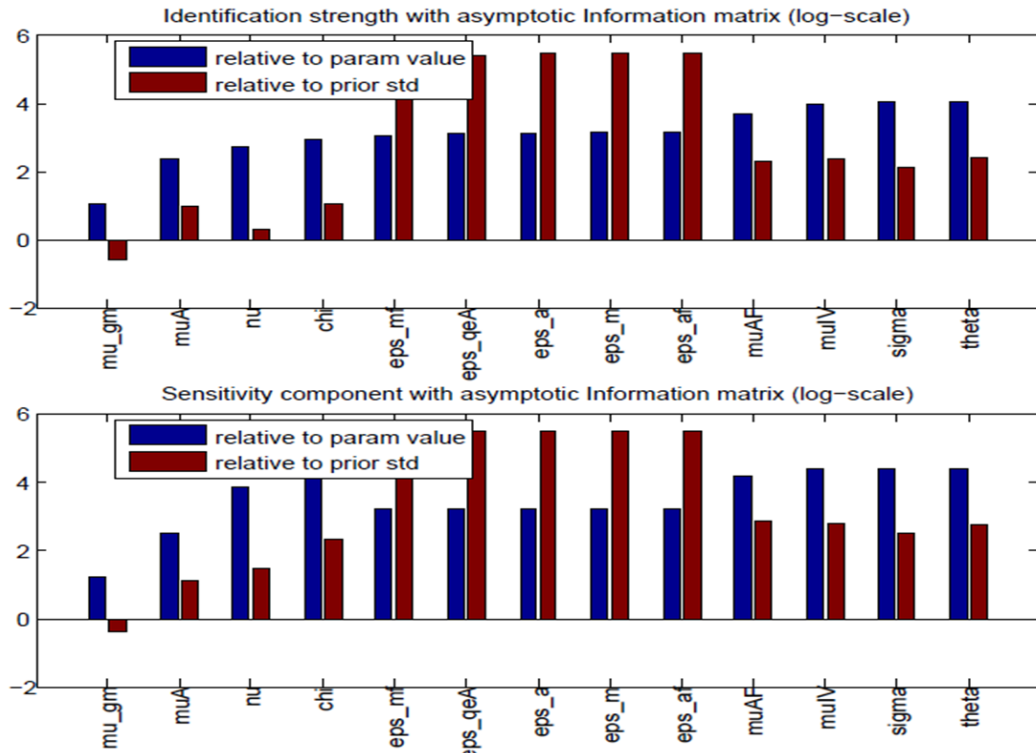| Country	Data	Source (code)
South Korea	Output (GDP)	Datastream (KOGDP...D)
	Inflation (CPI)	Bank of Korea: ECOS
	Real Exchange Rate	Datastream (KOQCC011H)
	Nominal Exchange Rate	Datastream (KOXRUSD.)
	Investment	Datastream (KOGFCF..D)
	Imports	Datastream (KOIMNGS.D)
	Total and Foreign Loans	Bank of Korea: ECOS
	Foreign Reserve	Datastream (KOEXAUHDA)
US	Total Loans	Federal Reserve: Financial Accounts
	Overseas Loans	Federal Reserve: Financial Accounts
	Treasuries Outstanding	Datastream (USSECMNSA)

Country	Portfolio Assets	Other Assets	Portfolio Liabilities	Other Liabilities
Argentina	AGI3B9AAA	AGI3D9AAA	AGI3B9LAA	AGI3D9LAA
Brazile	BRI3B9AAA	BRI3D9AAA	BRI3B9LAA	BRI3D9LAA
Chile	CLI3B9AAA	CLI3D9AAA	CLI3B9LAA	CLI3D9LAA
China	CHI3B9AAA	CHI3D9AAA	CHI3B9LAA	CHI3D9LAA
Czech	CZI3B9AAA	CZI3D9AAA	CZI3B9LAA	CZI3D9LAA
India	INI3B9AAA	INI3D9AAA	INI3B9LAA	INI3D9LAA
Indonesia	IDI3B9AAA	IDI3D9AAA	IDI3B9LAA	IDI3D9LAA
Israel	ISI3B9AAA	ISI3D9AAA	ISI3B9LAA	ISI3D9LAA
Mexico	MXI3B9AAA	MXI3D9AAA	MXI3B9LAA	MXI3D9LAA
New Zealand	NZI3B9AAA	NZI3D9AAA	NZI3B9LAA	NZI3D9LAA
Philipines	PHI3B9AAA	PHI3D9AAA	PHI3B9LAA	PHI3D9LAA
Poland	POI3B9AAA	POI3D9AAA	POI3B9LAA	POI3D9LAA
South Africa	SAI3B9AAA	SAI3D9AAA	SAI3B9LAA	SAI3D9LAA
South Korea	KOI3B9AAA	KOI3D9AAA	KOI3B9LAA	KOI3D9LAA
Thailand	THI3B9AAA	THI3D9AAA	THI3B9LAA	THI3D9LAA
Turkey	TKI3B9AAA	TKI3D9AAA	TKI3B9LAA	TKI3D9LAA

Note: Datastream codes for International Investment Position (IMF-IFS)

3.A.6 Bayesian Estimation Results

(a) Prior and Posterior Distributions of Estimated

(b) Identification

Identification strength with asymptotic Information matrix (log−scale)

Sensitivity component with asymptotic Information matrix (log−scale)

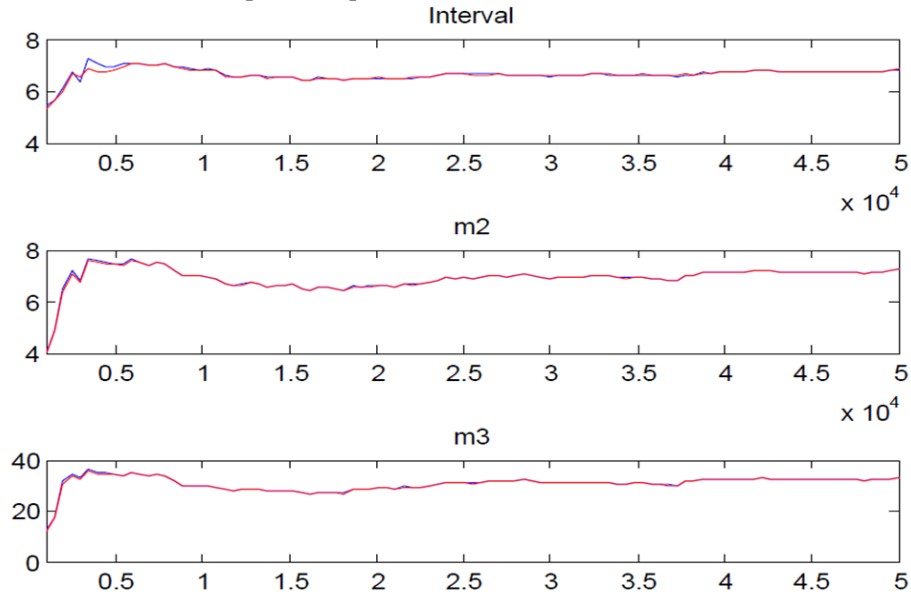(c) Multivariate MH Convergence Diagnosis

Interval

m2

m3

Figure 3.10: Identification and Convergence Diagnosis

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