Optimal Foreign Reserves, The Dollar Trap and Demand for Global Safe Assets: A DSGE analysis for China

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To My Family

ABSTRACT

The recent surge of foreign reserves in emerging markets has sparked fierce debate about what level of reserves is the optimal amount for a country. Conventional models have achieved important advances in understanding the behaviour of central banks' reserve policy, but fail to find convincing solutions to the puzzle of why emerging economies, and China in particular, would continue to accumulate massive reserves.

With reference to China's massive hoarding of foreign reserves, this thesis develops a representative agent model with elements of dynamic stochastic general equilibrium (DSGE) modelling. The model constructed in this thesis explicitly considers the risky steady state as the equilibrium point when agents take into account future uncertainty but when the shock realizations are zero. In this risky steady state we derive the optimal reserves for emerging markets, with particular reference to the Chinese case. The precautionary savings motivation for holding reserves is then analysed within this framework. This thesis derives the optimality of Chinese reserve accumulation, and provides a plausible explanation for reserve build-up in China and its underlying driving forces.

In order to better understand the foreign reserves accumulation, this thesis further attempts to analyse current external wealth allocation in a portfolio perspective within a DSGE framework. A two-country model is employed, and a Value at Risk (VaR) constraint is introduced to reproduce the risk averse behaviour of investors. After accounting for risk diversification, our findings imply that an investor would shift their portfolio holding to bond related assets.

Finally, China has accumulated a huge amount of foreign reserves. The majority of these assets are denominated in the US dollar. Furthermore, in terms of asset type, the US T-bill is the dominant investment instrument in China's international portfolio choice. This raises questions as to why the central bank of China chooses to make such an investment decision, and what the global repercussions might be. Therefore, China's role in the growing demand for global safe assets deserves exploration. Given the world-wide shortage of global safe assets, to what extent China will continue the current international investment decision, and the driving forces behind such policy inertia, are major concerns. In order to gain a better understanding, this thesis applies a global solving method, as well as a standard local solving method.

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Qingjing, we met in Durham and become lifetime partners; you have made me the luckiest man in the world. I love you. Finally, I owe great thanks to my parents. Without them, I would have had no chance to be in this wonderful world. I love you, Mom and Dad. I wish you health and happiness always.

DECLARATION

The content of this doctoral dissertation is based on the research work completed at Durham University Business School, UK. No material contained in the thesis has previously been submitted for a degree in this or any other university.

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Introduction

In this chapter we start by introducing the background and giving the main motivations behind the research. Then we present the contributions and main findings of this thesis. Finally, we outline the organization and structure of the entire thesis.

Chapter 1

Introduction

1.1Research Background and Motivations

In recent decades, the surge of foreign reserves in major emerging market economies, especially China, has drawn extensive attention from around the world. By the end of 2012 China, the largest holder of foreign reserves worldwide, had accumulated an astonishing amount of 3,387.51 billion US dollars, in equivalent to 41.18% of its own GDP. The colossal holdings of foreign assets help make China become a significant force in the world economy. In the meantime, however, the massive size of Chinese international reserves begs the question as whether China is having too much of a good thing.

Managing such a large hoard of reserve also proves to be a serious challenge. The majority of China's international reserves are denominated in the US dollar. While it is reasonable for China to hold dollar-denominated assets (Prasad, 2013), the dominance of the dollar would mean that China is excessively exposed to the risk of international currency movements. More profoundly, the inadequate portfolio diversification of Chinese reserve management may result from the eventuality that China is stuck in the dollar trap. In this trap, the fate of Chinese external assets would be under grip of the dollar, and hence China would suffer great capital loss due to a decline in the dollar.

Closely related with the dollar trap issue, there emerges the problem of China's demand for global safe assets. The growing risk associated with China's hoard of foreign reserves implies the country would turn to safe assets that are liquid debt claims with almost nil default risk. The great demand for safe assets our of China's large reserve build-up would fuel the securitization boon in the US and increase the financial vulnerabilities in America, and hence the financial stability of the world economy. China's reserve accumulation thus would have significant global repercussions.

There are critical voids in the literature answering the above concerns raised by China's large holdings of international assets. As the leading foreign reserve holder, China has become the focal point of the debate about what level of foreign assets is appropriate for a country and how to manage the reserve stocks. But existing theories and empirical methods have been limited in explain China’s behaviour of reserve policy. To achieve a better understanding of the dynamics of reserve holdings in China and to shed light on its global repercussions, further research is required in the first instance to work out a benchmark for judging the appropriateness of the reserve build-up in China.

The first motivation for this study is the need for a fundamental methodology for analysing large reserve holdings. To date the conventional literature, both theoretical and empirical, has failed to explain the rapid growth of China's foreign exchange reserves. Meanwhile the fact that dollar-denominated assets dominate China's foreign exchange reserves has caused considerable concern among policy makers and academics. Many scholars trace a link between imbalances in global capital markets and China's huge foreign exchange reserves, while even more worrying is the possibility that China has entered a dollar trap. Further, the interactions among China, its investment in the US dollar and its demand for world safe assets, require extensive investigation. In order to study these problems, it is necessary to introduce a new approach. We attempt to do so by using a general equilibrium approach called DSGE.

The second motivation is to provide effective policy recommendations for China's foreign exchange policy makers. Based on the results derived from our research, we attempt to provide such policy advice. In light of the argument put forward by Bussiere et al. (2013) that countries with higher reserves have suffered less from financial crisis than others, particularly if the high level of reserves is associated with a less open capital account, it would be interesting to investigate the underlying theory foundation for this proposition.

Last but not least, this thesis attempts to provide investment advice for practical investors. Given China’s status as one of the largest world economies, its portfolio of foreign reserve holding significantly affects the exchange rate market. Therefore, a good understanding of China’s reserves investment strategy would be beneficial to investors, particularly international investors.

1.2.Research Questions

China’s accumulation of foreign reserves and the dominance of dollar-denominated assets in the reserves allocation raise three clear issues for academic research. The first is why China has accumulated this large amount of foreign reserves. In order to better understand the motivation and potential repercussions of China’s reserve build up, it is necessary to establish a quantity criterion to measure the optimal level of reserves. The optimal amount identified will provide a benchmark for evaluating the desirability of the quantity of reserves China holds, and the extent to which the actual level of those reserves has deviated from the desirable level; hence, economic cost can be estimated.

The second issue concerns the asset allocation of this enormous external wealth, and the question of whether China has fallen into the Dollar Trap. According to Treasury International Capital (TIC), over 50% of China's total foreign reserve is invested in dollar-denominated assets. The returns on the dollar assets are sensitive to fluctuations of the dollar exchange rate. Given the huge size of China's dollar build-up, this would become particularly problematic in a time of wide swings of the dollar exchange rate, since wide fluctuations of the dollar have significant valuation effects on China's holdings of dollar assets. Consequently, this raises a concern over the risk to which China's dollar positions are exposed.

The third issue is also closely related to China's status as the largest holder of dollar assets. Specifically, since dollar assets constitute the main proportion of global safe assets, it is interesting to question why China shows such favour towards safe assets, and what role the country might play in the world safe assets markets. It is interesting to ask which currency safe assets, other than the US dollar, might be most attractive to China’s portfolio manager, and finally, whether China might exhaust the world safe assets.

1.3.Gaps in the Literature

Studies on reserve accumulation have provided diverse perspectives. However, conventional approaches fail to offer a reasonable explanation for large reserve holdings in countries like China. Triffin (1946, 1960) introduces the ratio approach, which provides a benchmark for judging the minimum level of reserves that a country must hold in order for it to avoid plummeting into a payment crisis. This approach proved to be useful in the circumstances prevailing during the immediate post-war period. However, after steady rehabilitation of the world economy, and the gradual advent of the multi reserve currency system, many countries become able to hold more reserves than just the minimum amount necessary to enable them to keep external payments afloat.

Beyond the bare minimum reserves, countries need to know how much more would make economic sense. Consequently, the cost-benefit approach (Heller, 1966; Frenkel and Javanovic, 1981; Ben-Bassat and Gottlieb, 1992) was developed to provide an answer to this question. This approach claims that the optimal level of foreign reserves should be determined by the point at which the marginal benefit of holding reserves is equal to the marginal costs. However, the cost-benefit approach assumes that reserve benefits are directly comparable to the cost, which overstates the argument. More importantly, in this approach costs and benefits are not measured according to social welfare criteria, and thus the optimal amount of foreign reserves derived in this manner may not actually be best for society as a whole.

Since the Asian financial crisis in 1997-1998, many emerging economies have opted to accumulate foreign reserves to provide self-insurance against the risk of adverse shocks (Jeanne, 2007). These reserves have become so large that conventional approaches such as the ratio approach and the cost-benefit approach would struggle to account for them. Therefore, the precautionary saving approach to foreign reserves has emerged in order to try to provide a more plausible explanation. This approach stresses the necessity for emerging countries to insure against the adverse effects of financial globalization, and shows that foreign reserves comprise the "war chest" that these countries hold as a self-insurance policy designed to smooth out fluctuations in income caused by the growing risks associated with such globalization. This approach is appropriate for those countries that are prone to shocks of a global nature.

However, all of these conventional approaches are unable to fully explain the recent reserve accumulation in China. The precautionary saving approach has opened a very promising avenue through which some improvements have been made in our understanding of the large stockpiling of foreign reserves in the emerging world. Nevertheless, so far, the standard precautionary saving model has evaluated the desired level of reserves based on partial equilibrium, which isolates the analysis by ignoring the interaction between micro foundation and the macro economy, and between domestic and international factors that were traditionally seen as the factors leading to dual drainage of foreign reserves. Furthermore, in its analytical framework, the consumer confronts an exogenously predetermined interest rate and the income level, hence the model fails to evaluate the interrelation and inter-dependencies between economic variables. These deficiencies in the existing models render it desirable and necessary to apply Dynamic Stochastic General Equilibrium modelling to the determination of optimal reserves.

Portfolio composition of reserve holdings also matters. The literature on currency composition of official reserves can be traced back at least to Heller and Knight (1978). They argue that a country’s exchange rate regime and its trade patterns are significantly related to the currency composition of its reserves. Hence the authors conclude that export-led growth strategy plays a major role in determining the currency composition of reserves. However, empirical work on the determinants of currency composition of reserves has been hampered by a lack of publicly available data on reserve portfolios of individual countries. This is especially so in the case of China, which does not release the currency shares of its official reserves to the International Monetary Fund. Therefore, the focus has been mainly on the theoretical side.

The existing theoretical literature derives the currency composition of optimal reserves as the solution to an international version of a portfolio problem. The discussion then mainly revolves around the correct method of applying the optimal portfolio theory in an international context, including such questions as the choice of deflator to calculate real returns from nominal returns, and the derivation of exchange rate return expectations. In parallel with the mean-variance approach, a strand of literature has attempted to explain portfolio choice under the macro circumstance. Kouri (1976) and Dooley and Isard (1983) attempted to integrate portfolio structure into theoretical economy models. However, their models lacked clear micro-foundations and generated incorrect empirical results. Lane and Milesi-Ferretti (2008) and Obstfeld (2004) renewed efforts to study portfolio choice in a dynamic open economy model, which was limited by technical difficulties. More recently, there has been a rapid growth in the literature on incorporating portfolio dynamics in DSGE models (Devereus and Sutherland, 2006, 2007, 2009; Evans and Hnaktkovska, 2005; Tille and Van Wincoop, 2007). These studies attempt to understand portfolio choice using dynamic models with approximation methods in higher order to overcome the technical difficulties.

Prior research has made extensive study of the growing demand by emerging economies for reserves and safe assets, and the effects of this demand on the US economy. This study adds to the literature by investigating this growing demand from the perspective of China. By offering rationales for China's seeking of these assets, we rationalize China's investing behaviour in relation to international portfolio choice, and shed light on the possible magnitude and composition of China's demand for US assets and global safe assets.

1.4.Main Findings and Contributions

This research represents one of the first significant attempts to fill the critical void in the literature by employing DSGE modelling to analyse the optimal reserves for China in a risky world. We set up a baseline model to understand the Chinese foreign reserves policy in a general equilibrium setting. The model is straightforward, but does not lack necessary complexity. The behaviour of three major sectors is modelled, as well as the law of motion of foreign reserves. In addition, the consumer's behaviour is explicitly modelled by including habit formation in the model. Furthermore, we allow for future uncertainty and extend the research into foreign reserves from the perspectives of both perfect foresight and risk aversion.

The main model findings are based on local approximation of the second order for a risky world. Differing from the deterministic steady state, we re-calculate the optimal foreign reserve with future variance. Through the analysis, we find that optimal holdings of foreign reserves in risky steady state are influenced by the risk-premium, due to imperfect international risk sharing and the effects of precautionary saving. As a result, defining the steady state value as the optimal level of foreign reserves, the optimal amount of reserves is greater in a risky world than in a world with certainty. The numerical results confirm our conclusion. By accounting for consumption uncertainty, we discover that with a higher level of consumption volatility, optimal foreign reserves increase substantially. We derive an indicator of optimal holdings of foreign reserves for China in a risky world in the form of the ratio of foreign reserves to GDP: specifically, around 56% with 1% standard deviation.

The second contribution of this thesis is the application of DSGE modelling to better understand the situation of China in the Dollar Trap from a portfolio choice perspective. In other words, this thesis employs an investment approach for the analysis, in which the asset composition of China's reserve holdings is considered as a result of asset allocation. The accumulation and management of international reserves are viewed as an integral process. More specifically, the growing foreign reserve holdings are a result of households' precautionary savings behaviour. Meanwhile, the central bank is required to invest this massive external wealth among the global market. These two separate motives combine into one when the central bank portfolio managers make investments on behalf of households. Therefore, in order to analyse these two motives together, we employ a portfolio allocation approach under a DSGE framework.

For clarification, we set a two-country model which consists of Home Country (China) and Foreign Country (US). We extend the standard three-sector DSGE model by introducing a financial intermediation sector which serves as the investment agent for the country. Furthermore, we follow Aoki and Sudo (2012) to place a VaR constraint in the financial intermediation sector to capture the risk of loss to the portfolio choice. This constraint allows the financial intermediary to repay the deposit to the household even if the portfolio investment experiences a massive loss. Therefore, this approach enables us to understand the portfolio allocation between two countries that are leading world economies. The contribution of this chapter is to show that portfolio choice is sensitive to the maximum loss return of each asset. With a higher maximum loss return asset, the portfolio holder shifts the portfolio choice from risky assets to less risky assets, in an attempt to avoid capital loss. Our results indicate that when considering future uncertainty, the dominance of US assets in China's portfolio is an equilibrium. In the long run, China will continue to accumulate dollar-denominated assets as its main portfolio choice.

The third contribution of this thesis is the introduction of a new solving method for portfolio choice within a DSGE framework. Rather than the standard perturbation solving method, this thesis employs the Parameterized Expectations Algorithm (PEA) as the main solving method. This approach is a non-finite state space method for computing equilibria in nonlinear stochastic dynamic models (e.g., Wright and Williams, 1982; Miranda and Helmberger, 1988; Marcet, 1988; den Haan and Marcet, 1990; Christiano and Fisher, 2000). It allows us easily to solve the indeterminacy problem, overcoming the difficulties of using perturbation based methods, which are highly reliant on deterministic steady-state as the starting point for approximation and so prevent investigation of the problem with permanent shocks, which change the deterministic steady-state. Our results show that the US dollar and American government debt, including Treasuries and Agencies, are not a safety trap for China. Rather, by holding a dominant weight of the dollar and US government debt instruments in its international portfolio, the Chinese central bank is carrying out an equilibrium act. In the long run, the sustained capital flows from China to the US and the consequent holdings of American safe assets will be rewarded with only low interest rates, to counter any adverse impact on the safe asset share.

1.5 Organisation of the Thesis

This thesis is divided into six chapters. Following this introductory chapter, the remaining chapters are structured as follows:

Chapter 2: Literature

This chapter presents a comprehensive review of the literature related to the optimal foreign reserves theories. In addition, it provides an extensive review of the DSGE modelling approach, setting out the research gap and main research method for the subsequent chapters.

Chapter 3: Optimal Foreign Reserves in a Risky World: The Case of China

This chapter employs a standard DSGE model, introducing a second order approximation method to evaluate the puzzling growth of China's foreign reserves in a risky world situation.

Chapter 4: China and the Dollar Trap

In Chapter 4, a two-country model is employed to analyse the current dollar dominated state in foreign reserves management. In particular, this approach allows us to gain a deeper understanding of the assets allocation between two leading countries, China and the US.

Chapter 5: China and Global Safe Assets

This chapter attempts to understand the portfolio choice of China's foreign assets from a different perspective, focusing on the dollar's reputation as a global safe asset. In order to analyse this issue under a DSGE framework, this chapter applies a global solution method rather than a local solution method

Chapter 6: Conclusions

The final chapter presents the main findings of the thesis and possible directions for future research.

Chapter 2

Related Literature

This chapter presents a comprehensive review of the literature related to this thesis. It begins by presenting a detailed survey of the studies on foreign reserves theory. Then, it identifies the gap in the literature. Finally, following an overview of the DSGE modelling approach, the chapter reviews the challenge presented by that approach, with particular reference to China.

Chapter 2

Related Literature

2.1 Problem of Foreign Reserves Theory and the New Frontier

2.1.1 Legacy of the International Gold Standard

The principal form of international reserves under the gold standard was gold. As a universal means of exchange, gold represented the world currency which was generally accepted for settling international payments. For a country on the gold standard, gold reserves in the custody of its central bank also constituted the monetary base that provided the backing for the issue of domestic currency. The dual role played by gold and its intimate relationship with monetary conditions determined the research needs and interests under the gold standard.

Studies of international reserves in this period were duly focused on the interaction between a country's gold stock and money supply (Thornton, 1802, Bagehot, 1973). This is the overture that signals a macro approach to international reserves. In this approach, reserves are viewed as a key variable in a country's macroeconomic stabilization. This is one of the most important intellectual inheritances from early theorists, which has had a long lasting effect until the present day. Increasingly, modern thinking on foreign reserve management is undertaken from a macroeconomic perspective (Jeanne, 2006, Carroll and Jeanne, 2009).

A central concern at the time was the possible depletion of gold. If that were to happen, the consequences would be calamitous, since it would mean that the country in question might have to default on its external obligations, and domestically it would depress the economy through the deflationary effects on the money supply. Given the potentially dire consequences of reserve depletion, Thornton (1802) took the view that a central bank should hold large enough gold reserves to avoid unnecessary domestic deflation caused by monetary contractions as a result of gold depletion. Thornton proposed that central banks should hold international reserves for precautionary purposes and use the reserves as a means of self-insurance against negative shocks to the economy. Also, interestingly, the benefits of holding gold reserves were gauged by the saving of macro adjustment costs that the reserves may provide by avoiding such an adjustment. These types of insights would find their modern day incarnations in, for example, Heller (1966), Ben-Basst (1992) and Jeanne (2006). However, Thornton (1802) did not specify how large a country's international reserves should be, and neither was any attempt made in the early studies to quantitatively derive the optimal level of international reserves.

Depletion of gold reserves was found by early researchers to be caused by drains from two sources, internal and external. Internal drains tended to occur when, in a persistent inflationary environment, domestic money-holders sought to convert bank notes into gold. External drains resulted when gold specie started to flow out of the country. Through the working of the price-specie-flow mechanism, these external drains could be caused by the persistent over-issue of paper money, and hence high inflation or negative shocks to the balance of payments occurred (Thornton cited two successive crop failures as an example). These historical discourses are enlightening and shed critical light on what may cause the reserves to become exhausted. The upshot of the double drain thesis is that domestic as well as international factors can drive the depletion of reserves. Despite the world economy having undergone fundamental changes since then and the form and content of the double drains having also changed, researchers still follow the dichotomy of internal and external drains in their analyses of demand for international reserves (Obstfeld, Shambaugh, Taylor 2010).

The extensive discussions in the early literature on possible causes of reserve depletion and the measures to be taken to arrest it reflect an unrelenting fear by central banks at the time. Indeed it was believed that, if not arrested, the reserve depletion would lead to the depression of aggregate production and reduction of output available for domestic consumption and export. Wary of these dire consequences, the fear of losing gold reserves became entrenched in the psyche of central banks. Maintaining, and if possible accumulating, international reserves thus became the persistent quest of central banks. This fear of losing reserves has continued until today (Aziemen, Sun, 2009). Consequently, this historically-rooted trait of a central banker's mind is still key to understanding the attitude of central banks towards risk in assessing the needs of reserves holdings and the actual behaviour of reserve policy.

2.1.2 Reserve Adequacy as an Indicator for Early Warning: The Ratio Approach

While early writers underscored the importance of domestic factors in reserve management, Keynes (1913, 1930) emphasized the external economy, particularly fluctuations in foreign trade, as a critical parameter. In the face of the disintegration of the international gold standard during the inter-war period and the resultant turmoil in international economic relations, Keynes (1930) indicated that central banks should not evaluate the appropriateness of the level of reserves in terms of what is required under normal conditions. Rather, holdings of gold reserves should be sufficient to meet the extraordinary demand of foreign trade.

Academics in the 1940s reached a general agreement that international reserves serve as a means of payment for international trade rather than the backing of the money supply (Badinger, 2000). In the process, external factors, and foreign trade in particular, became prominent as a determinant of reserve demand. This shift of emphasis towards trade, combined with a regime change in the international monetary system, has led to the emergence of a new paradigm, known as the ratio approach, in the aftermath of the Second World War.

The 1950s witnessed a sea-change in the international monetary system. The gold standard was formally replaced by the Bretton Wood system, under which the dollar became the key currency providing international liquidity in parallel to gold. The new gold-foreign-exchange regime got off to a shaky start because of the over-concentration of gold in a few countries and the severe shortage of the dollar. Many countries then resorted to exchange controls to mobilize domestic foreign exchange resources. This practice has historically proved disruptive to international trade and investment relations, and consequently there was a strong desire among the international community to prevent it from re-emerging after the War. Robert Triffin was then sanctioned by the United Nations to conduct an investigation into by which criteria it would be deemed reasonable for a country to adopt this extreme measure. The result was the ratio approach that Triffin (1946, 1960) developed. The basic idea behind this approach is to use some kind of benchmark to judge whether a country's reserve holdings are so low as to have fallen to a level beyond which the country would reach crisis point in relation to external payments, so that the adoption of exchange controls as an extreme measure would be justified. As such, this benchmark is essentially an early warning indicator to prompt countries to implement measures against plummeting into a fully-fledged payment crisis. The original intention of the benchmark was therefore to assess whether a country's proposal to adopt exchange controls was warranted. However, it later evolved to be widely regarded as a reference point for judging whether the level of reserves that a country holds is adequate to cover its external obligations.

Triffin (1946) formulated the benchmark as a ratio of reserves to imports. Specifically, according to this approach, the least amount of reserves a country must hold is equivalent to the country's imports in value over a period of 3 to 4 months, or the ratio of a country's reserves to monthly imports (the R/M ratio), which should be in the range of 3-4 as a minimum. This is based on the assumption that, once a country is faced with a payment difficulty and is forced to introduce measures to adjust the economy, it takes about 3 to 4 months for the adjustment to take effect.

The R/M ratio approach is popular with practitioners for its simplicity and practicality, but is often criticized by academic scholars because it is viewed as lacking a sound theoretical underpinning. For one thing, the approach is exclusively focused on trade, yet in reality the functionality of foreign reserves covers more than just trade payments. Furthermore, even in relation to trade itself, Triffin's ratio fails to take into account other aspects of the issue, such as fluctuations in trade (Nukse, 1944; Heller, 1968). This also implies that this approach ignores the uncertainty involved in international payments. Thirdly, the approach implicitly assumes that reserves are held for transaction purposes only, while more often than not countries hold reserves not for direct payments of trade but to cover financial deficits (Nurkse, 1944). More recently, countries have tended to hold reserves as a prudential self-insurance policy.

Overall, the ratio approach provides a practically attractive benchmark for the minimum level of international reserves a country must hold in order to buy time to avoid plummeting into a payment crisis while waiting for economic adjustment to take effect. However, the approach does not offer clear guidance on what is appropriate beyond this minimum level. Even as a benchmark for the minimum level reserves, this approach may be called into question as it fails to consider the uncertainty inherent in international payments, and transactions involving anything other than physical merchants such as financial assets which drive international capital movements.

2.1.3 Hoarding of Foreign Reserves under the Multi-Reserve Currency System: The Cost-Benefit Analysis

The ratio approach proposes a benchmark for the minimum level of reserves that a country should hold in order to avoid running into an external payment crisis. This serves as an early warning indicator, which is particularly useful at a time when the dollar is scarce and international liquidity is in short supply. With the rehabilitation of the world economy after the War and the restoration of currency convertibility in Europe at the end of 1950s, the international monetary system in the 1960s saw the gradual emergence of more currencies, in addition to the dollar, as reserve currencies, including the Deutsche Mark, French franc, pound sterling, Japanese yen and Swiss franc etc. (Horii, 1986). As the acute shortage of international liquidity gradually eased, many countries were able to hold reserves greater than the bare minimum required. In these circumstances, researchers started to turn their attention to the optimal level of reserves that a country may hold above the minimum.

Heller (1966) pioneered a novel cost-benefit framework to investigate the demand for foreign reserves. He argued that transaction payment is not the chief motive behind central banks' holding of reserves, since in general monetary authorities are not directly involved in international trade. Instead, it is the precautionary purpose that mainly drives their demand for accumulating foreign reserves. Heller proposed that the optimal level of precautionary reserves is achieved when the marginal benefits of holding reserves are equal to their marginal cost. The optimal level of reserves is determined by three factors: (1) the cost of adjusting the external imbalance which, if it is avoided by using reserves, could be regarded as the benefit of holding reserves; (2) the opportunity cost of holding reserves; and (3) the probability of trade imbalance. Heller's seminal work has shifted the focus of research onto the precautionary motive and established a framework for analysing such precautionary reserves with a sound theoretical underpinning. The cost-benefit analysis has been followed by many subsequent studies, particularly those concerned with precautionary reserves (Hamada and Ueda, 1977; Bahmani-Oskooee and Brown, 2002).

However, Heller's model leaves much scope for improvement. For example, it concentrates only on income reducing adjustment, hence ignoring other adjustment avenues related to income transfer and the cost thereof. He sensibly views trade imbalances as a stochastic process but it is too mechanical to treat the process as a fifty-fifty chance event. To improve the original model, on the benefit side, Kreinin and Heller (1973) took into account the different adjustment policies and their consequences. Sellekaerts and Sellerkaerts (1973) examined the cost of adjusting external imbalances through changes in the interest rate. Classen (1976) considered the adjustment cost through a combination of various policy tools. On the cost side, Agarwal (1971) extended Heller's work to developing countries in order to claim that, for such countries, the opportunity cost of holding international reserves should be measured by the forgone output due to resources being used for acquiring reserves.

The research by Hamada and Ueda (1977) and Frenkel and Javanovic (1981) further developed Heller’s approach by underscoring the importance of the stochastic property of external imbalances. In particular, by way of an inventory management approach developed by Miller and Orr (1966) and their earlier work on an individual’s demand for cash in a continuous time framework, Frenkel and Jovanovic (1981) offered a buffer stock model of optimal foreign reserves. They showed that reserves serve as a buffer stock by absorbing fluctuations in external payments. The demand for reserves is therefore driven by the stochastic variability of international payments. They demonstrated that the desired amount of reserves is a positive function of the volatility of international payments and is negatively related to the opportunity cost of reserve holdings. The optimal reserves are derived at the point where the benefits from saving the cost of adjusting macro imbalances intersect with the opportunity cost of holding reserves. This model has received extensive support from empirical research. However, again, the model failed to analyse the social welfare implications of the cost and benefits.

Hamada and Ueda (1977) modified Heller's optimal reserve model by including the effects of drift, serial correlation in the balance of payments, time lags of policies and speculative capital movements. Another important outcome of their modification is the introduction of inventory management theory developed by Miller and Orr (1966). They argued that, when facing external deficit, the government will use the foreign exchange reserves to finance the deficits until the reserves are depleted. By employing their formula, a higher level of optimal reserves may be obtained.

Based on Miller and Orr's research (1966), Frenkel and Jovanovic (1981) developed a buffer stock model which has become very influential in the field. This model suggests that reserves function as a buffer stock which absorbs fluctuations in external payments and so the level of reserve holdings is determined by the variability of international transactions.

Along with Heller's cost-benefit framework, Frenkel and Jovanovic (1981) formulated a stochastic process into the process of payments and receipts, the analytics of which stem from their earlier work (Frenkel and Jovanovic 1980). They applied this stochastic process to mimic the demand for reserves, and the inventory controls of the reserves were analysed in a continuous time framework. The demand for reserves is therefore driven by the stochastic variability of international payments. They showed that the desired amount of reserves is a positive function of the volatility of international payments and is negatively related to the opportunity cost of reserve holdings. The optimal reserves are derived at the point where the benefits from saving the cost of adjusting macro imbalances intersect with the opportunity cost of holding reserves. The theoretical predictions of the model are widely supported by the empirical literature of the 1980s, making the model a prominent one at the time. However the model failed to analyse the social welfare implications of the cost and benefits. A theoretical improvement on the buffer stock model was later attempted by Jung (1995). He claimed that the omission of an upper boundary of reserves in Frenkel and Jovanovic's model may lead to infinity, which is unrealistic for reserve accumulations. Hence, Jung (1995) incorporated a maximum limit of reserves (U) into the model and claimed that the calibration results show that the revised model outperforms the standard buffer stock model in terms of cost minimization.

The Frenkel-Jovanovic model remained popular until around the time of the emergence of Ben-Bassat and Gottlieb's (1992) model. Following Heller's approach to characterizing precaution as the main motivation for holding reserves, the Ben-Bassat and Gottlieb model focused on the sovereign risk and the cost of default. Ben-Bassat and Gottlieb (1992) argued that the optimal level of reserves is reached when the expected total opportunity cost of holding reserves is minimized. An important advantage of this model is the fact that it allows the authorities to assess the optimal reserves taking into account conditions specific to each country (Ozyildirim and Yaman, 2005). In addition, this model extends the cost of holding reserves more widely to include that of economic or financial crises. It also represents one of the early studies on the nexus of foreign reserves and the sudden stopping of international capital inflows in emerging economies.

The research framework developed by Ben-Bassat and Gottlieb (1992) has been employed by later researchers to analyse the optimal reserves required in developing countries such as Thailand (Vimolchalao, 2003) Turkey (Ozyildirim and Yaman, 2005) and India (Gupta, 2008). Zhang et.al (2009) applied Ben-Bassat and Gottlieb's approach to the Chinese case. Their results indicated that the precautionary consideration seems to constitute the dominant motive behind China's reserve policy. Since 2002, the Chinese monetary authorities have become more cautious in estimating the probability of an economic crisis, which is a critical argument for determining the optimal reserves for China.

2.1.4 International Reserve as a Means for Macro-Stability in the Era of Capital Movements

Recent reserve accumulation in the emerging market has generated a fierce debate over what is the optimal level of reserves for emerging economies. Since the Asian financial crisis in 1997-1998, many emerging economies have opted to accumulate foreign reserves to provide self- insurance against the risk of adverse shocks (Jeanne, 2007). Along with the integration of world financial markets and large-scale capital movements, there is an increasing interest among academics in focusing on the reserve depletion that may be caused by sudden interruptions to capital movements.

Unlike previous studies that focus on achieving a balance between the cost and benefit of holding reserves assets, recent literature has turned its attention to the welfare implications of reserve accumulation and offering an analytical framework based on utility maximization. The cost-benefit approach highlights the trade-off between the cost and benefit of holding reserves, implying that there is a directly comparable relation between the two. Nevertheless, such an assumption implicitly assumes that the behaviour of central banks is always identical, and hence it fails to consider the policy preferences of different countries (Badinger 2000).

The utility maximizing approach represents the new frontier of research on foreign reserves. The underlying format of this approach is comprised of two basic procedures. First, it proposes a social welfare function, the form and shape of which may vary in different models, and second, it includes foreign reserves as one of the arguments in utility maximization. The early contributions to this approach came from Clark (1970) and Kelly (1970), which show that a higher level of reserves can reduce consumption fluctuations, because a growing amount of reserves lowers the probability of reserve depletion. Meanwhile, larger reserves mean greater opportunity costs, and hence less consumption. Solving this trade-off in a welfare maximizing framework gives rise to the optimal reserve level. However, the early model only established a loose connection between social welfare and the level of reserves (Badinger, 2000).

Hipple (1974) tried to combine the Clark and Kelly analyses and also took into consideration some elements of the cost-benefit model developed by Heller (1966). He reached similar conclusions to Heller, claiming that optimal reserves in his model is an increasing function of the total wealth of a country, adjustment costs and uncertainty, and a decreasing function of opportunity costs. Building on Kelly's (1970) research, the utility maximizing framework developed by Claassen (1975) features the consideration of precautionary reserves, thereby incorporating Miller and Orr's (1966) model of the precautionary demand for cash balances. The model discusses the optimal management of reserves for central banks with discontinuous and asymmetric properties under which central banks react to reserve movements only when they move beyond certain limits and are more concerned about trade deficits than surpluses.

Since the 1990s, particularly since the Asian financial crisis, there has been a revival of interest in the utility maximizing approach (Durdu, Mendoza and Terrones, 2009; Jeanne, 2007; Barnichon, 2008; Caballero and Panageas, 2008; Jeanne and Ranciere, 2011; and Obstfeld, Shambaugh, and Taylor, 2010). In the meantime, research on foreign reserves has also paid particular attention to sudden interruptions in capital flows to emerging economies and their consequences (Caballero and Panageas, 2004; Garcia and Soto, 2006; Aizenman and Lee 2007). These two developments gave rise to a surge in research that analysed the welfare implications of the reserve holdings of emerging markets within the utility maximizing framework.

These researchers believe that, in the cost-benefit models of optimal foreign reserves, the objective function of central banks is only loosely related to the national welfare. This has at least two consequences. First, in these models, the consequences of external disequilibrium are not well defined. Second, because of the lack of a sound welfare criterion, the opportunity costs of holding foreign reserves are also not clearly defined.

Initially, scholars tended to use the same determinants as those of the cost-benefit approach with regard to the optimal reserves, although the theoretical underpinning may be fundamentally different. Jeanne (2007) and Jeanne and Ranciere (2011) incorporated the cost-benefit trade-off into a formal framework of utility maximization to determine the optimal level of foreign reserves. They argued that emerging nations use foreign reserves to smooth consumption and to manage the fallout caused by sudden stoppages in capital movements. This approach takes the view that the main motive for emerging nations' holding of foreign reserves is precaution. From an inter-temporal equilibrium perspective based on utility maximization, these models were constructed on the assumption of a small, open economy where there is a representative a consumer holding a certain level of foreign assets. To maximize his welfare, the consumer is faced with a decision about allocating his wealth to different assets over three periods in the face of possible sudden interruptions in capital flows. The optimal reserves are reached when the level of reserves maximizes the representative consumer's expected utility of period (t + 1) consumption.

Compared to previous models, the new utility maximizing models have achieved some important advances. In particular, they specify that the trade-off in the inter-temporal choice of optimal reserves is between the opportunity to carry reserves and the avoidance of, or mitigating the consequences of, economic adversity. In the previous class of models, the trade-off is between the opportunity cost and reduction in the fluctuation of income. However, these new utility maximizing models assume that capital movements are the only cause of economic crisis and decline in output. This is too restrictive an assumption. Studies have shown that a country's reserve depletion could be caused by both external and internal factors.

The studies by Jeanne (2007) and Jeanne and Ranciere (2011) have been supplemented by some recent contributions. Obstfeld, Shambaugh and Taylor (2010) analyzed the dual depletion of reserves in modern times, when not only foreign money leaves the country, but also potentially the entire M2, in a period of severe crisis. Barnichon (2008) followed a similar utility maximizing approach to Jeanne and Rancière (2008). However, his work established an analytical framework for a small open economy with limited access to foreign capital and subject to more than one shock, such as natural disasters or terms of trade shocks. This shows that it is both feasible and fruitful to extend the utility analysis to include domestic as well as international shocks to the economy.

2.1.5 The Precautionary Saving Model

While some research on emerging markets is concerned with the interaction between sudden interruptions and reserve accumulation, studies are now emerging that pay attention to a wide range of domestic and international factors that may cause income fluctuations (Dhasmana and Drummond 2008, Barnichon 2008). Although these studies focus on distinct causes of economic adversity, a key theme that they all share is the precautionary motive for holding foreign reserves, behind which lies the notion that reserves can be a means of self-insurance for the macro-stability of emerging economies. Thus, nations tend to hoard a large amount of reserves as a form of precautionary saving to insure against future adverse shocks.

The surge in precautionary reserves has sparked further research into what determines the precautionary motive with regard to central banks' holding of reserves in the emerging world (Aizenmann and Marion 2003; Durdu et al. 2009; Mendoza et al. 2009; and Carroll and Jeanne 2009).One critical advance in this field was made by Carroll and Jeanne (2009) who presented a tractable model of the net foreign assets of a small open economy. Based on many of the insights from the existing specialized literature on the precautionary motive, they derived a convenient formula for the economy's target value of assets, which balances impatience, prudence, risk, inter-temporal substitution, and the rate of return.

The model is an extension of the original precautionary savings model developed by Carroll (2009) and the prudent inter-temporal choice model designed by Kimball (1990) with a standard Cobb Douglas production function. In a small open economy, population and productivity grow at constant rates. A resident of this economy accumulates precautionary wealth in order to insure against the risk of unemployment. The saving decisions made by individuals are aggregated to produce "net foreign assets" for the economy as a whole.

An important feature of the model is that its discussions include the macro economy as well as the microeconomics of consumer inter-temporal choice under conditions of uncertainty. This is exactly the significant feature of the Dynamic Stochastic General Equilibrium (DSGE) modelling approach which has recently emerged. However, the model solution proposed by Carroll and Jeanne (2009) is derived through the reduced form rather than resorting to the DSGE technique. Although the model is capable of evaluating plausible orders of magnitude for the ratio of desired foreign reserves to GDP, there are several possible dimensions that could be improved and completed which would make the model more realistic. One potential extension is the exchange rate implications of a multi-country model.

Valencia (2010)extended Carroll's (2004) work to model a precautionary savings approach to optimal reserves, Again, the basic premise is that countries hold extra precautionary savings as a buffer against future uncertainty (Carroll and Kimball, 2008). Valencia computed the level of optimal reserves for emerging markets with particular reference to Bolivia. The work focuses on factors other than sudden stoppages, such as current account shocks. Following Carroll's (2004) approach, the representative consumer maximizes his utility in terms of foreign reserves and transitory shocks to income. The model is then extended to consider the case of investment, i.e. the case where the consumer not only saves for precautionary purposes, but also to finance the stock of capital.

Valencia's (2010) approach was built upon a standard precautionary savings model in the sense of partial equilibrium, under which the consumer confronts an exogenously predetermined interest rate and income level (Carroll 2004). Specifically, this analysis framework fails to evaluate the interrelation and inter-dependencies between economic variables.

Valencia's work (2010) marks an interesting application of the precautionary approach to foreign reserves made popular by Carroll and Jeanne (2009), among others. This class of models incorporates the consumer's micro decision within the macro economic analysis, which has opened a very promising avenue through which to improve our understanding of the large accumulation of foreign reserves in the emerging world. Nevertheless, so far, the standard precautionary saving model evaluates the desired level of reserves based on partial equilibrium which isolates the analysis by ignoring the interaction between micro foundation and the macro economy, and between domestic and international factors that were traditionally seen as the factors leading to dual drainage of foreign reserves. In order to fill these gaps in the literature, we will now move on to apply the Dynamic Stochastic General Equilibrium modelling to the determination of optimal reserves level.

2.1.6 Concluding Remarks: Gaps in the Reserve Literature and Possible Improvement

To sum up, early research on foreign reserves was mainly shaped by the institutional features of the gold standard. Due to their deep concern with the relation between gold and domestic monetary conditions, thinkers at the time were wary about the possible depletion of gold, which could be caused by both domestic and foreign factors. To insure against the possible effects of gold depletion, early scholars called for the central bank to hold enough gold reserves to avoid having to endure domestic monetary contraction and output decline due to periodic gold outflow. This was the historical legacy of the precautionary view of reserve holdings, which one still can find today in its modern incarnation. Linked to this notion was the phenomenon of "fear of losing reserves", which has continued until the present day among central bankers. The early research also shed light on the necessity of understanding the holding of reserves as a process in which monetary and real sectors within the domestic economy are intertwined, as are domestic and international influences.

The period since the end of the Second World War has witnessed three fundamental changes in the international monetary system, all of which have had a crucial bearing on the course of research into international reserves. The first is the emergence of the dollar as a key international currency in parallel to gold. The rise of the dollar was, however, accompanied by a severe shortage of the currency in the early days and so the ratio approach was developed as a result. The ratio approach provides a benchmark for judging the minimum level of reserves that a country must hold in order for it to avoid plummeting into a payment crisis, which proved to be very useful in the circumstances prevailing during the immediate post-war period. The second development is related to the solid growth of the world economy in the 1960s and the gradual advent of the multi reserve currency system. Countries may now hold more reserves than just the minimum amount necessary to enable them to keep external payments afloat.

However, beyond this bare minimum, countries have a need to know how much more would make economic sense. Consequently, the cost-benefit approach was developed to provide a solution to this problem. This approach powerfully argues that the optimal level of foreign reserves should be determined by the point at which the marginal benefit of holding reserves is equal to their marginal costs. However, the cost-benefit approach assumes that reserve benefits are directly comparable to the cost, which overstates its argument. More importantly, in this approach, costs and benefits are not measured according to social welfare criteria, and thus the optimal amount of foreign reserves derived by this manner may not actually be best for society as a whole.

The cost-benefit approach has been influential until recently, when the rapid development of financial globalisation sharply raised the stake of the role that foreign reserves may play in mitigating the fallout from adverse shocks to the economy. This is the third critical development that prompted research into foreign reserves. In the age of financial globalisation, particularly since the Asian financial crisis, emerging economies have been accumulating a growing amount of reserves, which have become so large that conventional approaches including the ratio approach or the cost-benefit approach would struggle to account for them. At this juncture, the precautionary saving approach to foreign reserves arose in order to try to provide a more plausible explanation. This approach stresses the necessity for emerging countries to insure against the adverse effects of financial globalisation and shows that foreign reserves comprise the “extra savings” that these countries hold as a self-insurance policy designed to smooth out fluctuations in income caused by the growing risks associated with globalisation. This approach is fitting to the case of countries that are prone to shocks of a global nature. The methodology of the approach is also novel in that it uses a macroeconomic framework combined with the micro-focused optimization behaviour of economic agents. However, the reduced form of the expression to optimal foreign reserves is subject to the Lucas Critique.

2.2 Appeals of DSGE Models

2.2.1 DSGE Modelling as a Framework for Policy Analysis

In recent years, there has been tremendous development in the specification and estimation of Dynamic Stochastic General Equilibrium (DSGE) models (Tovar, 2009). As a branch of applied research within the general equilibrium of the macro-economy, DSGE models study how the economy evolves over time in the face of random shocks. Wide applications of the DSGE methodology have been found in policy discussions and analysis (Woodford, 2003). Smets and Wouters (2003a, 2004), for example, show that the DSGE approach is particularly useful for analysing the impact and consequences of alternative monetary policies.

One distinctive feature of this methodology is its application of microeconomic principles to the explanation of aggregate economic phenomena. In this approach, households, firms and the government (or the central bank) make individual choices and inter-temporal decisions based on their own preferences and information about the future. The sum of these decisions is then taken by the DSGE models as a characterization of the macro-economy under examination. As such, under this approach, the economic system is the result of interactions between agents whose objectives and constraints are modelled and interpreted based on microeconomic theory (Avouyi-Dovi, Metheron, and Feve, 2007).

Another distinctive element of the DSGE approach is its assignment of a central role to agents' expectations in the determination of current macroeconomic outcomes (Sbordone, Tambalotti, Rao, and Walsh, 2010). Under this assumption, rational agents solve inter-temporal optimization problems. The dependence of current choices on future uncertain outcomes makes the models dynamic and moves the DSGE models away from academic circles and into the policymaking community (Sbordone, et al., 2010).

As part of a micro-founded macroeconomic approach, DSGE models have been shown to possess several advantages over other classes of macroeconomic models for policy analysis. Through restricting the acceptable behaviour of agents to dynamic utility maximization and rational expectations, the DSGE modelling approach provides a coherent framework of analysis (De Grauwe, 2010). This coherence can help researchers to structure policy discussions and make it possible for researchers to offer an account that is internally consistent and informed by a wide range of empirical indicators (Huntley and Miller, 2009). In addition, off-model structural parameters can be employed to calibrate and estimate the model, which is particularly useful in cases where time series are short (Tovar, 2009).

Another important advantage of DSGE modelling is the fact that the dynamic mechanics in DSGE models are transparent. Through, for example, analyses of impulse response functions with structural shocks, DSGE models can help to identify sources of fluctuations, and hence the type of shocks most likely to be hitting the economy, and thus provide a better understanding of structural changes. Closely related to this, DSGE modelling offers a further advantage, i.e. not only can it identify various shocks but it can also do so in a theoretically consistent way, whereas other macroeconomic models such as VAR modelling are a theoretical (Iiboshi, Nishiyama, and Watanabe, 2006). For reduced form estimations, DSGE models allow a link to be established between structural features of the economy and reduced form parameters, which is not always possible with other macroeconomic models (Tovar, 2009). Furthermore, in a reduced-form representation of a more complex process, DSGE models can offer more precise insights into the nature of the shocks than those that may be possible with alternative modelling strategies (Huntley and Miller, 2009).

The structural nature of DSGE modelling and, in particular, its capacity to explicitly account for the role of expectations, makes the DSGE analysis less subject to the Lucas Critique and policy experiments more reliable (Galí et al., 2011). Lucas (1981) argues that if economic agents follow optimal decision rules and rationally process available information, they will be able to alter their behaviour when anticipating a change in the course of economic policy. This implies that, in order to predict the effects of a policy experiment, one should model the deep parameters that govern individual behaviour and then aggregate possible reactions of individuals to derive the macroeconomic consequences of the policy change. Otherwise, relying only on historical data, policy analysis would be futile. By modelling the expectation formation process explicitly within the general equilibrium context, the micro-founded DSGE models, if properly specified, have the ability to correctly capture economic dynamics despite policy changes. As such, DSGE models are less susceptible in principle to the Lucas Critique (Woodford, 2003, Tovar, 2009). The DSGE modelling methodology has therefore become a powerful toolkit for policy formulation. DSGE models may be used to predict the effects of policy changes and perform counterfactual experiments, and hence they are useful for testing policy experiments. They can also offer a better understanding of which parameters are likely to be policy invariant and which are not.

These advantages suggest that DSGE modelling can be adapted to suit a wide variety of applications. Whereas conventional macro-econometric models such as VAR models can be used to describe past economic shocks, they cannot be easily adapted to policy analysis. DSGE models, by contrast, can be used to test policy experiments. Because all the agents in the DSGE model make decisions, the model is able to capture the endogenous effects of changes in policy in a way that a VAR model cannot.

Like all other economic models, DSGE modelling has attracted increasing criticism, especially over its failure to predict the global financial crisis (Galí et al., 2011). For example, Schorfheide (2013) evidences five main challenges confronted by DSGE modelling. The first is the fragility of the parameter estimates, and hence the quality of inference and forecasting exercises. This may be largely caused by a lack of identification of the parameters (Canova and Sala, 2009). Second, in a DSGE model, the disturbances that drive macroeconomic fluctuations are exogenously given. It is difficult to distinguish whether the estimated shock processes accurately capture the aggregate uncertainty or are simply a reflection of misspecifications. Third, many time series are of low frequency which makes it difficult to estimate the model and implement sound inference. Fourth, DSGE models often appear to be statistically poor in being fitted to data, relative to VAR models, for example. Fifth, their predictions about the effects of rare policy changes often rely exclusively on extrapolation by theory. This makes it difficult to provide measures of uncertainty, and thus may compromise the reliability of policy predictions.

Currently, efforts are being made by academic researchers around the world to meet these challenges. While the improvement process is ongoing, one may be sure that the combination of a sound micro-founded and theoretically consistent model structure and good policy analysis performance suggests that DSGE models will be further developed as a powerful toolkit that can work effectively in a variety of policy environments (Smets and Wouters 2004).

2.2.2 Development of DSGE Modelling Framework

The DSGE modelling strategy grew largely from macroeconomists' responses to the Lucas Critique. Kydland and Prescott (1982) were among the first to address the concerns that Lucas (1981) raised about the traditional macroeconomic models in relation to their lack of invariance to policy changes in the reduced form parameters. Kydland and Prescott (1982) presented a Real Business Circle model (the RBC model), which made a pioneering contribution to the emergence of the DSGE analysis framework. Since Kydland and Prescott's (1982) study, researchers have tried many extensions to improve and perfect the formulation and empirical fit of the model. Among them, the work by Obstfeld and Rogoff (1995) represents a very important development. It is known as the New Open Economy Macroeconomic model (NOEM), and introduced the modelling strategy that carries almost every characteristic of future DSGE models. These include tackling aggregate macroeconomic events with micro-founded analysis, optimization based modelling, incorporation of the macroeconomists criticisms, postulated factors such as habit persistence in consumption and shocks, and evaluation of money policy based on social welfare, etc.

Another well-developed extension of the DSGE model is the incorporation of more sectors to specify the economic system. The original Kydland and Prescott model implies that the whole economy is represented by individual households, firms and government (through monetary and fiscal policies). Researchers added to the classification with the open-economy sector (Mendoza 1991; Backus et al.1992) and the financial sector (Bernanke et al.1998). Methodologically, the analysis framework has been developed so that forecasting can now be attempted, as well as reduced-form models, for policy analysis (Christiano et al. 2005; Smets and Wouters 2003, 2007).

The RBC model relies on the neoclassical economics under which economic agents maximise their welfare with flexible prices. While agents' maximising behaviour takes place at the micro level, the general macro economy is subject to fluctuations caused exclusively by real shocks, such as technological changes or government spending shocks. These assumptions appear to be too restrictive and later studies of DSGE models therefore turned to Keynesian short-run macroeconomics to introduce nominal and real rigidities into the modelling framework. This new DSGE modelling framework proved to be closer to economic reality and so has become very popular in more recent research. By sharing a number of features with RBC model and introducing several new features such as monopolistic competition and nominal rigidities, the New Keynesian DSGE model has become the dominant workhorse model for current analysis. Consequently, DSGE modelling is now known as the new-neoclassical synthesis or new-Keynesian modelling paradigm. The RBC model and New Keynesian framework serve as two fundamental cornerstones for DSGE modelling.

DSGE models not only establish a coherent connection between micro-foundation and macroeconomics but also provide a feasible path through which to access to quantitative policy analysis. These features have made DSGE models increasingly popular with central banks around the world. Many central banks have incorporated the methodology into the service of monetary policymaking by building their own DSGE models. Those central Banks that have developed their own DSGE models include the Bank of Canada (with their model known as ToTEM), the Bank of England (BEQM), the Central Bank of Chile (MAS), the Central Reserve Bank of Peru (MEGA-D), the European Central Bank(NAWM), the Norges Bank (NEMO), the Sveriges Riks bank (RAMSES) and the US Federal Reserve (SIGMA). As an example of modelling specifics, the Federal Reserve Board measured the impact of a wide variety of shocks such as monetary policy shocks, government spending, consumption demand, productivity growth and capital tax rates using the SIGMA model (Erceg, Guerrieri, and Gust, 2006). The Central Bank of Chile employed the MAS model to evaluate copper-price shocks under different fiscal policies and the factors that account for current account fluctuation (Medina and Soto, 2007).

Not everyone agrees that fitting a few variables or establishing some benchmarks is sufficient for the DSGE models to be readily applied to practical policy work at central banks. Sims (2006) criticizes the inadequacy of the DSGE model in the real world for two reasons. First, the aggregate consumption good and aggregate capital do not exist in a real economy. Second, DSGE models have not included the financial market which is the essential component of the real economy. The debate over the adequacy of DSGE models recently rose to the level of a U.S Congressional hearing in 2008. Solow (2010) argued that DSGE models are deeply deficient since they are unable to precisely describe the dynamics of real economic events due to their feature of stylization.

The debate about DSGE models is on-going and represents longstanding divergent views within macroeconomics. Despite its deficiencies, to the academic community and policy institutions, DSGE modelling methodology has been shown to be capable of playing a role in assisting policy discussions and analysis. As argued by Faust and Gupta (2012), first, while DSGE modelling has undoubtedly come a long way, there remains room for improvement in areas that are materially important for policymaking. Second, prior and particularly posterior predictive analysis can be valuable tools in assessing the strengths and weaknesses of the existing DSGE models. Gupta (2010) evaluated the strengths and weaknesses of DSGE models from the standpoint of their usefulness in relation to monetary policy analysis. He found that model misspecification causes certain pairs of structural shocks in the model to be correlated, a problem which may be address with better modelling techniques. In this regard, criticism of DSGE models only serves to improve DSGE modelling methodology, and DSGE models will continue to function as a practical tool for assessing economic and monetary policies.

The DSGE approach may also offer several advantages for assessing foreign reserve policy. First, it allows the functions of international reserves in the whole economy to be identified, as DSGE models view the economy as an interactive system using insights taken from analysing the micro-foundation of the economy. Second it helps to establish a closer relationship between the real empirical data and the reserve theory. The real economic figures and the fundamental theory support and reflect each other. Such features are perfectly suited to analysing the international reserves, serving as a key component of macroeconomic policy.

2.2.3 Solution and Evaluation of DSGE Models

A typical DSGE model incorporates three components: an environment in which agents interact in the process of maximizing their utility subject to their respective constraints; decision rules that represent their behaviours; and a set of shocks affecting their decisions (Dejong and Dave 2007). These components are typically expressed in the form of nonlinear and expectation difference equations. Closed form solutions for the system are not normally directly derivable. Therefore, researchers resort to linear approximation in order to characterize the model, the methods of which have been explored extensively in the literature (Kydland and Prescott, 1982; Taylor and Uhlig, 1990; Judd, 1992; Handsen and Prescott, 1995 and Danthine and Donaldson, 1995).

Blanchard and Kahn (1980) documented a method of solving linear-quadratic dynamic stochastic models or linear stochastic difference equations. This approach marks a key cornerstone for solving DSGE models. Subsequently, a large body of literature has emerged which extensively studied this approach, such as Sargent (1987), Mcgrattan (1994), Hansen, McGrattan and Sargent (1994). Uhlig (1995) presented a toolkit for solving nonlinear dynamic models, which offers a simplified and unified solution method. This general solution is based on the solution method of matrix-quadratic equations proposed by Binder and Pesaran (1997), and it also uses frequency-domain techniques to calculate the second-order moments of the model, as in that of King and Rebelo (1993). The solution developed by Uhlig (1995), together with earlier contributions such as those of McGrattan (1994) and Hansen et al. (1995) which solve the model through maximizing a quadratic objective function, and Blanchard and Kahn (1980) which solve the systems via searching for the stable manifold in the entire system, has laid the foundations for the standard approach to solving DSGE models (Dejong and Dave 2007).

Subsequent to the linear approximation around the steady state, empirical analysis of the DSGE model was applied. Estimation of the DSGE model can be divided into two categories: informal calibration; and a formal econometric method. According to Dejong and Dave (2007), calibration is a technique which involves several steps towards providing quantitative answers to a particular economic question by specifying values for the parameters of the model. The formal econometric method involves the application of traditional statistical techniques. The most commonly used methods in DSGE estimation are Matching Moments, Maximum Likelihood Estimation and Bayesian Methods.

The seminal work by Kydland and Prescott (1982) as a cornerstone of DSGE modelling also resulted in a shift in the execution of empirical work in macroeconomics by employing calibration which enables researchers to cast the DSGE model. Compared to econometrics methods, it is difficult to estimate and test calibration and to use it to make forecasts. However, two important developments have significantly weakened the appeal of econometric methods. Kydland and Prescott (1991) documented that, during the stagflation of the early 1970s, systems-of-equations models suffered from spectacular failures in making predictions. Moreover, the Lucas Critique implies that fixed parameters of behavioural equations are inconsistent with decision makers' optimizing behaviour, and hence simulations based on these models are of little relevance for policy analysis (Dejong and Dave 2007). In response, researchers have proposed two broad solutions. First, Sim (1972) and Hansen and Sargent (1980) attempted to modify the methodology by allowing it to impose theoretical discipline on reduced-form models of macro-economy. However, such an approach represents an intermediate step at best. In any case, the empirical results generated by this approach proved to be disappointing (Hansen and Sargent, 1981; Hansen and Singleton 1982).

The second solution was the development of calibration. According to Dejong et al. (1996), in the standard calibration procedure, the empirical performance of a theoretical model is evaluated by specifying values for the parameters of the model, and comparing simulated data with the actual data. In this approach, estimation and testing are purposefully neglected, but numerical answers are produced as a result of calibration. This facilitates the empirical implementation of DSGE models and makes them a useful empirical tool for generating concrete answers to quantitative questions. As such, many researchers have attempted to use this method and have made efforts to improve the approach by formalizing aspects of the calibration while taking various other issues into consideration, such as model misspecification. Notable examples can be found in Smith (1993), Watson (1993), Canova (1994), DeJong et al. (1996), Diebold et al. (1998), Geweke (1999), Schorfheide (2000), Dridi et al. (2007), and Bierens (2007).

Compared to the calibration method, the econometrics method has several advantages. Ruge-Murcia (2007) documents three benefits of employing a formal statistical method. First, parameter values are obtained by imposing on the data the restrictions suggested by the model, making the estimates of the

DSGE model consistent with the micro studies. Second, it allows researchers to obtain estimates of parameters that might be hard to obtain using disaggregated data alone. Third, parameter uncertainty can be explicitly incorporated into the impulse-response analysis. Finally, standard tools of model selection and evaluation can be readily applied.

Of the commonly used formal methods for DSGE estimation, the matching moments method seeks to determine the parameterizations by matching preselected moments of the underlying structural model. The empirical performance is explored via hypothesis testing. This is a limited-information approach, which, according to Canova (1994), has both strengths and weaknesses. This approach entails a loss of efficiency that can sometimes be problematic, especially when working with a small sample size. The resulting inferences are also sensitive to the particular collection of moments chosen for the analysis. Nevertheless moment-matching procedures do not require the assumption that shocks are realised. Therefore the moment-matching approach is not subject to potential concerns regarding misspecification along this dimension (Dejong and Dave, 2007).

The maximum likelihood and Bayesian approaches are full-information methods as they can provide a complete statistical characterization of the data (Dejong and Dave, 2007). The maximum likelihood and Bayesian approaches can be differentiated in terms of whether the data or parameters are interpreted as random variables (Poirier, 1995). Under the maximum likelihood method, parameters are interpreted as fixed, and the data are interpreted as the realization of a random drawing from the corresponding likelihood function. Parameter estimates are chosen as those that maximize the likelihood function.

Under the Bayesian approach, parameters are interpreted as random and the data as fixed. The Bayesian approach uses information from both the available data and prior knowledge to obtain the posterior estimates. In the estimation stage, the objective is to make conditional probabilistic statements regarding the parameterization of the model (Dejong and Dave, 2007). The principal advantage of using the Bayesian method is that a priori distribution of the parameters is accounted for in parameter estimation. Since it efficiently uses all of the available information about the data, the Bayesian approach is optimal in terms of its information processing rule, as defined by Zellner (1988). Consequently, the Bayesian method has been favoured by those who have conducted empirical research into DSGE models.

As mentioned above, the literature on estimation of the DSGE model by a full-information, likelihood-based approach has been growing rapidly. However, when we attempt to evaluate the fitness of the model, we encounter two problems. First, without an alternative model it is difficult to assess how well the model fits the data. Second, in the absence of a comparable model with different specifications, it is hard to detect problems of misspecification. One straightforward solution is to conduct a robustness exercise using alternative specifications for the model (Rabanal and Rubio-Ramirez, 2005; Taylor and Wieland, 2009). However, this approach has been challenged, because additional theoretical work is required to derive a suitable model for comparison. A more practical and convenient approach is to implement the robustness test for DSGE models. Del Negro and Schorfheide (2004) introduced a Bayesian-vector autoregression (VAR) approach which uses a VAR model as the benchmark model. Because the VAR model does not have a priori structural restrictions, it is convenient to implement it to compare the fit and forecasting performance of a DSGE model.

2.3 Challenge of DSGE Modelling

2.3.1 Challenges of DSGE Modelling in Emerging Markets

DSGE models feature a micro foundation with nominal and real rigidities as in Smets and Wouters (2003) and Christiano et al. (2005). To such models applicable to emerging markets in general, China in particular, there are serious challenges to overcome. To make DSGE models suitable for analysing the Chinese economy that is undergoing fundamental changes in its economic structure and policy environment one has to consider such special features as frictions in financial markets, interaction between openness in trade and financial globalisation, and vulnerability brought about by economic transition. These challenges could be particularly severe when the economy is experiencing regime shifts in the face of global turmoil in the world financial markets (Tovar, 2009). In the circumstances, great efforts are required to adapt benchmark DSGE models to the economic realty in China.

Of the possible modelling challenges when analysing the Chinese case in a DSGE framework, financial market frictions are of particular important as a lack of profound discourse on this issue is widely claimed to be the primary weaknesses of the current DSGE modelling. Without the inclusion of the financial sector in the DSGE framework, current models fail to explain the behaviour of the economy during financial crises, such as the 1997 Asian financial crises and the most recent financial tsunami triggered by the US subprime mortgage crisis. Moreover, the weak modelling of financial markets excludes any possibility of analysing financial vulnerabilities, stability and illiquidity (Tovar 2009).

A typical approach to incorporating financial frictions into a DSGE framework is by way of considering the financial accelerator (Cespedes et al. 2004). The key feature of such an approach is that the external finance premium is determined endogenously. It also allows the effects of firms' balance sheets on investment to be captured. Nevertheless, the financial accelerator approach can only account for one aspect of many possible financial frictions. Therefore, Lacovello (2005) introduced collateral constraints for households and firms, limiting not only consumption but also investment. While this attempt offers a promising avenue to improve DSGE modelling by adding credit frictions, financial frictions are greatly exacerbated due to different stages of development of an emerging market's financial sectors. This poses further challenges to DSGE modelling of China where economic structure is subject to regime shift and financial deepening of the nation develops fast.

Currency risk premium is another important challenge encountered in DSGE modelling. To rule out the behaviour that consumers can finance consumption streams indefinitely, the most common approach which has been employed is a reduced-form risk premium to foreign assets which is modelled as a monotonically increasing function of the level of borrowing (Schmidt-Grohé and Uribe 2003). On top of this, an exogenous shock to the risk premia is often added to capture the volatility in the data. This problem can be severe, particularly in emerging markets like China, due to the highly volatile sovereign risk.

A final challenge comes from currency substitution or dollarization, a term particularly used interchangeably to indicate that dollars have taken over the basic functions of the domestic currency. According to Castillo et al. (2013), it is possible for dollarization to lead to three outcomes. The first, also known as the basic effect, is transaction dollarization. This occurs when the dominant medium of exchange is occupied by the foreign currency. The second effect is price dollarization, whereby all products are priced in the foreign currency. The third outcome is financial dollarization, which may result from either asset dollarization or liability dollarization. Asset dollarization means that most of the country's savings exist in the form of the dollar. Meanwhile, liability dollarization is defined as foreign currency denominated debt and domestic currency denominated revenues, which results in a currency mismatch. The role of these different forms of dollarization in DSGE frameworks has been analysed in the literature produced mainly for Latin America (Castillo et al., 2013 and Tovar, 2006). But for the case of China, the challenge may take a different form, i.e. China's external assets may concentrate in the dollar and hence the management of China's foreign reserves becomes excessively vulnerable to international currency movements.

The exchange rate has a greater influence on emerging market economies than in developed economies. For China, appreciation of its currency, the renminbi, has been a strong trend in recent year, the impacts of which are not very clear to the researchers in the field so far. To build the consistent currency appreciation, largely unprecedented for emerging markets, is certainly a formidable challenge to DSGE modelling of China.

2.3.2 DSGE Modelling of the China Economy

Although DSGE modelling is popular in the international academic community and among major central banks around the world, the DSGE framework has rarely been used in the literature to explore the Chinese economy. One possible reason could be that DSGE modelling may be unsuited to capturing China's economy effectively since it is not yet a mature market economy. Thus it is inappropriate to characterize the Chinese economy by applying a class of models that were developed for advanced economies. Nevertheless, with the introduction of reforms and the opening-up of the Chinese economy, the economic system in China has been transformed from a central-control regime to a market-oriented economy (Scheibe and Vines, 2005, Chow, 2002, Sun and Sen, 2011). As such, DSGE models can now potentially be employed to analyse China's economic policies. Besides, Chow (2002) has long maintained that the theoretical-quantitative approach is crucial to understanding China's economic policy, along with historical-institutional approaches. In light of this, literature on DSGE models for China has recently started to emerge.

Among the few works that have studied the Chinese economy using a DSGE approach, Zhang (2009) has recently developed a closed-economy DSGE model, which compares the effectiveness of money supply and interest rate rules in managing the Chinese macro-economy. The empirical results indicate that the price rule, or the interest rate rule, is likely to be more effective than the quantity rule, or the money supply rule, in the Chinese context. In line with Christiano et al. (2005) and Smets and Wouters (2003), Zhang (2009) deployed a standard closed-economy DSGE model where all agents maximize their utility subject to an inter-temporal budget constraint. He modified the monetary policy setting of the model following the critical insights given by Liu and Zhang (2010) who documented that fluctuations in the Chinese interest rate cannot be captured effectively by the standard Taylor rule. Zhang (2009) applied a rule that can be considered as a combination of the rules created by Clarida et al. (1998) and Smets and Wouters (2003). Such an approach highlights the inflation expectations and changes in inflation and output. Nevertheless, parameterization represents the real challenge in Zhang's (2009) model for two reasons. First, data for China are relatively scarce. Second, as China is shifting from a planned economy to a market one, it is likely that structural changes will have happened during the past decade. The author estimated some of the parameters with available data and assigned values to others according to related literature.

Another recent work by Mehotra et al. (2011) also considered a closed-economy model, in line with Zhang (2009), but with a focus on the impact of technological and monetary policy shocks under different structures of the Chinese economy. Unlike Zhang (2009), this study employs the traditional Taylor rule to simulate monetary policy. Furthermore, their model is based on the assumption of capital controls which invalidates the uncovered interest parity. Under the model setting, the domestic monetary policy is relatively independent from international counterparts since, under capital controls, the Chinese monetary policy is not subject to the influence of international factors. Mehotra et al.'s findings indicate that a rebalancing of current account imbalances would reduce the volatility of China's real economy in the event of a technological shock, which provides support for policies designed to increase the share of consumption in China.

Sun and Sen (2011) modified the Smets-Wouters model using a monetary growth rule to assess the monetary transmission mechanism of China's monetary policy. They argued that the Smets-Wouters model assumes two production sectors: the intermediate firms in a monopolistic market; and the final firm in a perfect competition market, which coincides with the reality of the Chinese economy. To be specific, owing to their power granted by the government over control of raw materials and energy supplies, state-owned enterprises are in a monopolistic position in the intermediate product market. Meanwhile, private and small-sized firms producing final goods are faced with a competitive market. In contrast with previous studies, this paper not only focuses on monetary policy but also fiscal policy. This feature reflects China’s macro policy environment, in which not only monetary policy, but also fiscal policy, play a very important role in stabilizing and promoting growth in the Chinese economy.

Liu (2007) proposed a model based on the modelling framework developed by Christiano, Motto, Roatagon (2002), but with some important extensions. First, the closed economy model was extended to the case of an open economy, taking into account changes in net foreign assets and domestic economic interaction. Second, according to the characteristics of China's banking sector, the financial sector was added with some modifications. Additionally, monetary policy rules were modified based on the approach to targeting money supply. The model parameters were estimated using a combination of calibration and Bayesian estimation. Unlike the closed version, this open economy approach shifts the focus from domestic to international interaction.

In an open-economy multi-country DSGE framework, Straub and Thimann (2010) analysed economic adjustments in China under flexible and fixed exchange rate regimes. This multi-country version of the DSGE model follows that of Jacquinot and Straub (2007, 2008) and consists of four symmetrical blocks representing the United States, the Euro Zone, China (or emerging markets, especially Asia), and the rest of the countries of the world. The model is formulated in such a way that relevant features of the economy are highlighted with regard to internal and external adjustments by China. In essence, this modified model has kept almost all the essential features of the original multi-country model created by Jacquinot and Straub (2007, 2008)

To account for the mechanism under which the exchange-rate is passed through to consumers as well as the fact that the international risk sharing is imperfect and there are price fluctuations, they extended Coenen and Straub's (2005) original set-up by incorporating two distinct types of households. These two types of households possess different abilities to assess asset markets. One type of household only accumulates money and the other owns financial assets as well as physical capital. They argue that the monetary authorities follow an inertial Taylor rule with interest-rate smoothing, specified in terms of annual consumer-price inflation and quarterly output growth for all economies except China. In the case of China, the authors assume that the Chinese monetary authority follows a fixed exchange rate regime. The model-based results indicate that shocks caused by permanent technology or labour supply cannot explain China's persistent current account surplus. Compared to the case of a flexible exchange rate regime, the fixed exchange rate regime with an inefficient financial market could have negative effects on GDP growth in the medium term.

To summarize, as a powerful toolkit for policy analysis, the DSGE approach has now begun to be adopted for analysing emerging markets including China. Researchers have extended the DSGE approach in two distinct directions: one focuses on domestic policy analysis assuming that China is a closed economy; and the other emphasizes the influence of international factors in an open economy framework. In order to derive the optimal reserves for China, it is reasonable to apply an open economy DSGE model focusing on international links between China and the rest of the world. Currently, there is no research that evaluates the optimal level of reserves in a DSGE framework, which leaves a critical void in the literature. This research will contribute to the literature by filling this gap in the context of China's foreign reserve policy.

2.4 Conclusions

The development of the theory of international reserves has effectuated a shift of research focus from transaction purposes onto the precautionary motive as the main force driving the accumulation of reserves. Along with the failure of previous benchmark to evaluate the current reserve level of China that is rapidly ascending, the precautionary saving approach rises to provide some plausible explanation. The methodology of the approach is also novel in that it builds a macroeconomic framework with micro-foundation featuring optimization behaviour of economic agents. This is exactly the key feature of the DSGE modelling approach. Whereas the standard precautionary saving model derives the optimal level of reserves on partial equilibrium base, it isolates the analysis by ignoring the interaction between micro foundation and the macro economy and between domestic and international factors behind possible dual drainage of foreign reserves. Therefore we will fill this gap in the literature by employing the DSGE modelling to analysis the optimal reserves for China.

The DSGE approach has become a dominant toolkit for analysing macro-economy policies in developed countries. The DSGE literature on emerging markets including China is, however, scarce. Challenges from modelling such countries are formidable, for instance modelling frictions in financial markets, interaction between financial openness and international trade, various shocks facing China that is in economic transition. Also, China, as other emerging economies, exhibits the features of idiosyncratic structure and vulnerabilities to external factors. To meet these challenges, modifications are imperative to make the DSGE model applicable to the Chinese case. The current study represents an important effort towards this improvement by way of studying the optimal reserves in a DSGE framework that takes into account of special properties of the Chinese institutional environment and behaviour of economic agents.

Chapter 3

Optimal Foreign Reserves for China in a Risky World

This chapter develops a representative agent's model with elements of DSGE modelling. The model constructed in this paper explicitly considers the risky steady state as the equilibrium point when agents take into account future uncertainty. In this risky steady state we derive the optimal reserves for emerging markets, with particular reference to the Chinese case.

Chapter 3

Optimal Foreign Reserves for China in a Risky World

3.1 Introduction

The recent surge of foreign reserves in emerging markets has sparked fierce debate about how much reserves are the best amount for a country. China as the largest holder of foreign reserves in the world since 2006 is the focal point of this debate. While in 1990 China's holdings of foreign reserves were 29.5 billion dollars or around 8% of its GDP, in 2006 China overtook Japan to become the world's largest reserve holder. By the end of 2012, China's reserves rose to $3.3 trillion, which is 700 percent of the figure in January 2004. During the period, China's average ratio of the reserves to GDP has also surged dramatically, to 45%. The rapid accumulation of reserves in China challenges the conventional thinking of prudent reserve management. While sufficient reserves are helpful in promoting a country's macroeconomic stability, excess reserve holdings can incur detrimental costs.

In order to better understand the motivation and potential repercussions of China's reserve buildup, it is necessary and desirable to establish some normative criterion to measure the optimality of China's reserve level. The optimal amount of reserves thus identified provides a yardstick for evaluating the desirability of the quantity of reserves that China holds and to what extent the actual level of China's reserves has deviated from the desirable level hence economic costs can be estimated. It is also imperative to establish such a normative criterion to evaluate the current state of China's reserve management.

This research intends to contribute to a large body of literature on international reserves. Studies of international reserves in the classical gold standard period were duly focused on interaction between a country's gold stocks and money supply. Adequate gold reserves were viewed as a critical determinant of macroeconomic stability. The early literature realized the necessity of understanding reserve holdings as an intertwined process influenced by both the monetary and real factors as well as domestic and international factors. However, in this period, quantifying the optimal level of foreign reserves was never made to the top of the research agenda of the day.

In the context of severe dollar shortage in the immediate period after the World War II, the reserve literature shifted its attention to establishing some benchmark for judging whether a country's reserves were dangerously insufficient for meeting its external obligations. A ratio of reserves to imports is proposed as the benchmark giving an early warning indicator to prompt countries to take action before plummeting into a payment crisis. The ratio approach is popular with policy makers for its simplicity and practicality. However the approach's usefulness is largely limited to its service as an indicator for minimum reserves that a country must have. Beyond this bare minimum, no indications can be meaningfully drawn to judge what the optimal level of reserves is for a country.

Heller's (1960) cost-benefit approach extends the analysis of optimal reserves to a new dimension. He argues that it is the precautionary purpose, rather than the transaction motive, that is the main driving force behind demands for foreign reserves. In the model, optimal reserves are determined by the intersection where the marginal benefits of holding reserves are equal to their marginal costs. This work has shifted the research attention to the precautionary motive and established an analytical framework thereof. But the approach assumes that reserve benefits are directly comparable to the costs, which over stretches its argument. More important, in this approach, costs and benefits are not measured according to some social welfare criteria.

In the age of financial globalization, particularly since the Asian financial crisis, conventional wisdom in the ratio approach and cost-benefit approach finds difficulty in explaining reserve hoardings in emerging economies, which are huge. The precautionary savings approach to foreign reserves then rises to provide some plausible explanations. The earlier work may be found in Ghosh and Ostry (1997), while recent examples include those by Durdu et al. (2009), Carroll and Jeanne (2009), Valencia (2010), Jeanne and Rancie`re (2011), and Sandri (2011). However, the explanatory power of these models remains debatable when they are applied to the massive accumulation of foreign reserves by emerging countries, particularly China (Bird and Rajan, 2003; Jeanne, 2007; Yan and Kumhof, 2011).

The seemingly excessive reserve holdings by China represent a vexing challenge for the conventional wisdom about prudent reserve management. In an attempt to work out the puzzle of China's massive hoarding of foreign reserves, this paper develops a model based on the precautionary motive approach to optimal reserves (Carrol and Jeanne, 2008), but with a critical extension to consider factors more than precautionary influences and we cast the analysis in a risky world. Despite some success achieved by the canonical precautionary models in explaining countries' holdings of reserves in an era of volatile capital movements, this class of models is based on partial equilibrium. However, as the complex of the model increase there is little chance to achieve an analytical solution especially under non-linear models which pursued by us. A key feature of our approach is its examination of the aggregate macro event on the basis of economic agents' inter-temporal choice in general equilibrium. This is the area where the Dynamic Stochastic General Equilibrium (DSGE) modeling approach can best offer its advantages. Adopting a DSGE model, we examine the dynamic of foreign reserves in an intertwined process with interactions between consumers, the production sector and the central bank. This strategy will enable us to derive at the optimal level of foreign reserves in equilibrium state.

Conventional DSGE modeling nevertheless lacks incorporation of an appropriate evaluation method of risk variation. In dynamic macroeconomics, it is common to consider the limit behaviour of the economy while assuming agents do not anticipate the effect of future shocks. This approach ascribes that agents know the perfect foresight path of the economy and the corresponding equilibrium in the circumstances is known as the deterministic steady state. By contrast, risk-averse agents are aware of the existence of future shocks hitting the economy. Such agents thus would care for the gap between the actual economy and the risky steady state values and would take optimal decisions by choosing a decision rule that maximizes inter-temporal utility of returning to the risky steady state. Consequently, the risky steady state is a more appropriate candidate for us to use when trying to understand reserve holdings in a risky world. Study of the risky steady state incorporates information about the stochastic nature of the economic environment. To overcome this problem, we apply in our research the technique of risky steady state analysis proposed by, among others, Coeurdacier et al. (2011).This leaves a critical methodological gap in the reserve literature and this research aspires to fill towards this void. We will attempt to employ the DSGE modeling with risky stead state analysis to estimate optimal reserves in a risky world for China.

The rest of this paper is organized as follows. After a review of the related literature in Section 2, we introduce in Section 3 the background of the Chinese reserve policy and specify model assumptions. We then move on to build a DSGE model with future uncertainty. The numerical solution of the model is presented in Section 4, along with the benchmark calibration results and sensitivity analysis. Section 5 presents conclusions and discussion.

3.2 Related Literature

3.2.1 Conventional Theories on Foreign Reserves

While early reserve studies were focused almost exclusively on the relation between a country's gold stocks and money supply (Thornton, 1802, Bagehot, 1973), modern days have witnessed the emergence of three major schools that concern with a whole array of different aspects of reserve policy. The first school to emerge after the World War II was the ratio approach made popular by Triffin

(1946, 1960). This was followed by the cost-benefit analysis of Heller (1960), and others including Frenkel and Jovanovic (1981). Advent of the third strand of the literature which highlights the precautionary motive as the main driving force behind reserve hoarding in the emerging world is promoted by the financial globalization and the related financial turmoil. In response to the Asian financial crisis in the 1990s, emerging economies move to stockpile large amount of reserves; this in turn has stimulated the rise of the precautionary school.

The basic idea behind the ratio approach (Triffin, 1946, 1960) is to use some kind of benchmark to judge whether a country's reserve holdings are so low that beyond which the country would reach crisis point in relation to external payments, so that the adoption of exchange controls as an extreme measure would be justified. However, the approach does not offer clear guidance on what is appropriate beyond this minimum level. Even as a guide for the minimum level reserves, this approach may be called into question as it fails to consider the uncertainty inherent in international payments, and transactions involving anything other than physical merchants such as financial assets which drive international capital movements.

The second school of Heller’s (1960) cost-benefit approach extends the analysis of optimal reserves to a new dimension. He argues that it is the precautionary purpose, rather than transaction motive, that mainly drives demand for accumulating foreign reserves. Along with Heller's cost-benefit framework, Frenkel and Jovanovic (1981) formulated a stochastic process into international payments and receipts, the analytics of which stem from their earlier work (Frenkel and Jovanovic 1980). They deploy this stochastic process to mimic the demand for reserves, and the inventory controls of the reserves are analyzed in a continuous time framework. However, the model largely ignores the social welfare implications of the costs and benefits.

The Frenkel-Jovanovic model remained popular until around the early 1990s. Later, Ben-Bassat and Gottlieb (1992) argue that the optimal level of reserves is reached when the expected total opportunity costs of holding reserves are minimized. An important advantage of this model is the fact that it allows the authorities to assess optimal reserves by taking into account conditions specific to each country (Ozyildirim and Yaman, 2005). The Ben-Bassat and Gottlieb' model is also among the first to highlight the importance of foreign reserves in protecting the economy from the fallout of sudden stops of international capital movement.

In the age of financial globalization, particularly since the Asian financial crisis, conventional wisdom in the ratio approach and the cost-benefit approach find difficult in explaining reserve build-up by emerging economies, which is huge. The surge in reserves has sparked further research into what determines the precautionary motive with regard to central banks' holding of reserves in the emerging world (Aizenmann and Marion 2003; Durdu et al. 2009; Mendoza et al. 2009; and Carroll and Jeanne 2009).One critical advance in this field is made by Carroll and Jeanne (2009) who presented a tractable model of the net foreign assets for small open economies.

Under the classical gold standard, early writers underscored the importance of domestic influences in reserve management. Keynes (1913, 1930), however, emphasized the external economy, particularly fluctuations in foreign trade, as a critical parameter. This emphasis in turn catalyzes the rise of the ratio approach (Triffin 1946, 1960). This ratio approach provides a practically attractive benchmark for the minimum level of international reserves a country must hold. This minimum holding will enable a country to buy time to avoid plummeting into a payment crisis while waiting for economic adjustment to take effect. However, the approach does not offer clear guidance on what is appropriate beyond this minimum level. Even as a benchmark for the minimum level reserves, this approach can be called into question as it fails to consider the uncertainty inherent in international payments, and transactions involving financial assets which are the prominent drive of international capital movements cross the world.

Heller (1966) pioneered a novel cost-benefit framework to investigate the demand for foreign reserves. He argued that transaction payments were not the chief motive behind central banks' holding of reserves, since in general monetary authorities are not directly involved in international trade. Instead, it is the precautionary motive that mainly drives the official demand for foreign reserves. Heller proposed that the optimal level of precautionary reserves is achieved when the marginal benefits of holding reserves are equal to their marginal cost. Heller's seminal work has shifted the focus of research onto the precautionary motive and established a framework for analyzing such precautionary reserves with a sound theoretical underpinning. The cost-benefit analysis has been followed by many subsequent studies (Hamada and Ueda, 1977; Bahmani-Oskooee and Brown, 2002).

The research by Hamada and Ueda (1977) and Frenkel and Javanovic (1981) further developed Heller's approach by underscoring the stochastic property of external imbalances. In particular, following an inventory management approach developed by Miller and Orr (1966), Frenkel and Javanovic (1981) extended their earlier work on an individual's demand for cash in a continuous time framework, to offer a buffer stock model of optimal foreign reserves. They showed that foreign reserves serve as a buffer for absorbing fluctuations in external payments and hence the demand for reserves is driven by the stochastic variability of international payments. They demonstrated that the desired amount of reserves is a positive function of the volatility of international payments and is negatively related to the opportunity costs of reserve holdings. The optimal reserves are derived at the point where the benefits from saving the costs of adjusting macro imbalances intersect with the opportunity costs of holding reserves. This model has received extensive support from empirical research. However, again, the model missed analysing the social welfare implications of the costs and benefits.

A theoretical improvement on the buffer stock model was later attempted by Jung (1995). He claimed that the omission of an upper boundary of reserves in Frenkel and Jovanovic's model may lead to infinity, which is unrealistic for reserve holdings. Hence, Jung (1995) incorporated a maximum limit of reserves into the model and claimed that the calibration results show that the revised model outperforms the standard buffer stock model in terms of cost minimization.

Following Heller's approach to characterizing precaution as the main motivation for holding reserves, the Ben-Bassat and Gottlieb model focused on the sovereign risk and the cost of default. Ben-Bassat and Gottlieb (1992) argue that the optimal level of reserves is reached when the expected total opportunity cost of holding reserves is minimized. This model extends the cost of holding reserves more widely to include that of economic or financial crises. It also represents one of the early studies on the nexus of foreign reserves and sudden stops of international capital inflows to emerging economies.

The research framework developed by Ben-Bassat and Gottlieb (1992) has been employed by later researchers to analyze the optimal reserves required in developing countries such as Thailand (Vimolchalao, 2003), Turkey (Ozyildirim and Yaman, 2005) and India (Gupta, 2008). Zhang et.al (2009) applied Ben-Bassat and Gottlieb's approach to the Chinese case. Since 2002, the Chinese monetary authorities have become more cautious in estimating the probability of an economic crisis, which is a critical argument for determining the optimal reserves for China.

Development of recent research is increasingly influenced by the concern that sudden interruptions to capital movements may cause reserve depletion in developing countries, and hence catastrophic disruption to their economies. Meanwhile, recent literature has paid increasing attention to the welfare implications of reserve management in an analytical framework based on utility maximization. This is conducive to a better understanding of the optimal level of reserves. The cost-benefit approach highlights the trade-off between the costs and benefits of holding reserves, implying that there is a directly comparable relation between the two. Nevertheless, such an assumption implicitly assumes that the behaviour of central banks is always identical, and hence it fails to consider the policy preferences of different countries (Badinger, 2000).

The utility maximizing approach represents the new frontier of research in foreign reserves. The underlying format of this approach is comprised of two basic procedures: First, it proposes a social welfare function the form of which may vary in different models for different countries; and second, it includes foreign reserves as one of the arguments in utility maximization. The early contributions to this approach came from Clark (1970) and Kelly (1970), which show that a higher level of reserves can reduce consumption fluctuations because a greater amount of reserves lowers the probability of reserve depletion. Meanwhile, larger reserves mean greater opportunity costs, and hence less consumption. Solving this trade-off in a welfare maximizing framework gives rise to the optimal reserve level. However, the early models only established a loose connection between social welfare and the level of reserve holdings (Badinger, 2000).

Hipple (1974) tried to combine the Clark and Kelly analyses while also taking into consideration some elements of the cost-benefit model developed by Heller (1966). He reached similar conclusions with Heller's, claiming that optimal reserves are an increasing function of the total wealth of a country, adjustment costs and uncertainty, and a decreasing function of opportunity costs. Building on Kelly's (1970) research and incorporating Miller and Orr's (1966) model of precautionary demand for cash balances, the utility maximizing framework developed by Claassen (1975) features consideration of precautionary reserves.

The model discusses the optimal management of reserves for central banks with discontinuous and asymmetric properties under which central banks react to reserve movements only when they move beyond certain limits and are more concerned about trade deficits than surpluses.

Since the 1990s, particularly since the Asian financial crisis, there has been a revival of interest in the utility maximizing approach (Durdu, Mendoza and Terrones, 2009; Jeanne, 2007; Barnichon, 2008; Caballero and Panageas, 2008; Jeanne and Ranciere, 2011; and Obstfeld, Shambaugh, and Taylor, 2010). In the meantime, the reserve literature has also paid particular attention to sudden interruptions in capital flows to emerging economies and the consequences (Caballero and Panageas, 2004; Garcia and Soto, 2006; Aizenman and Lee 2007). These two developments gave rise to a surge in research that focuses on the precautionary motive for holding foreign reserves in a time of volatile capital movements. Behind this strand of literature lies the notion that reserves can be a means of self-insurance for the macro-stability of emerging economies. Thus, nations tend to hoard a large amount of reserves as a form of precautionary saving to insure against future adverse shocks.

Jeanne (2007) and Jeanne and Ranciere (2011) incorporate the cost-benefit trade-off into a formal framework of utility maximization to determine the optimal level of foreign reserves. They argue that emerging nations use foreign reserves to smooth consumption and to manage the fallout caused by sudden stoppages in capital movements. This approach takes the view that, for emerging nations, the main motive for holding foreign reserves is precaution. From an inter-temporal equilibrium perspective, these precautionary models are constructed on the assumption of a small open economy where a representative consumer holds a certain level of foreign assets. The consumer is faced with a decision about allocating his wealth to different assets over three periods before, during and after a sudden stop of international capital flows. The optimal reserves are reached when the level of reserves maximizes the representative consumer's overall expected utility.

The precautionary models specify the trade-off in the inter-temporal choice of optimal reserves as between the opportunity cost of carrying reserves and the benefits stemming from using these reserves to avoid or mitigate consequences of economic adversity. However, whereas sudden stops are an important factor undermining economic stability of emerging markets, assuming such events are the only cause of economic crisis and output disruption is too restrictive an assumption. Studies have shown that a country's reserve depletion could be caused by both external and internal factors.

In response, new research in recent years has paid attention to a wide range of domestic and international factors that may cause income fluctuations and hence the holdings of foreign reserves. Using a two-good endowment economy model, Dhasmana and Drummond (2008) extends the literature to examine the case of low-income countries facing terms of trade and aid shocks. They derive the optimal level of reserves by comparing the cost of holding reserves with their benefits as an insurance against a shock. Barnichon (2009) applies an analytical framework for a small open economy with limited access to foreign capital. In the model, the economy is subject to more than one shock such as natural disasters or terms of trade shocks. In analyzing depletion of reserves in modern times, Obstfeld, Shambaugh and Taylor (2010) find it could be a dual process since, in a period of severe crisis, the depletion can be caused not only by foreign money leaving the country but also by potentially the entire domestic M2 escaping the country.

Other research examines what determines the precautionary motive (Aizenmann and Marion 2003; Durdu et al. 2009; Mendoza et al. 2009; and Carroll and Jeanne 2009). One prominent advance in this field was made by Carroll and Jeanne (2009) who present a tractable model of the net foreign assets of a small open economy. Based on many of the insights from the existing specialized literature on the precautionary motive, they derived a convenient formula for the economy's target value of foreign assets, which balances impatience, prudence, risk, inter-temporal substitution, and the rate of return. The model is an extension of the original precautionary savings model developed by Carroll (2009) and the prudent inter-temporal choice model introduced by Kimball (1990). Following the work of Carroll and Jeanne (2009), there have further extensions or applications such as Valencia (2010), Sandri (2011), Dabla-Norris, Kim and Shirono (2011).

The development of the theory of international reserves has effectuated a shift of research focus from transaction purposes onto the precautionary motive as the main force driving the holding of reserves. Along with the failure of previous benchmark to evaluate the current reserve levels of emerging market including China, the precautionary saving approach rises to provide some plausible explanation. Departing from its too restrictive assumptions on sudden stops as the almost exclusive reason for holding precautionary reserves, new research in recent years has started to widen its focus to consider other factors such as current account shocks. In these models, the representative consumer maximizes utility given transitory shocks to income. Some analysis is then extended to consider the case of investment, where the consumer not only saves for precautionary purposes, but also to finance the stock of capital (Valencia, 2010).

This class of models has opened a very promising avenue through which some improvements in our understanding of the large stockpiling of foreign reserves in the emerging world have been made. Nevertheless, so far, the standard precautionary saving model evaluates the desired level of reserves based on partial equilibrium which isolates the analysis by ignoring the interaction between micro foundation and the macro economy, and between domestic and international factors that were traditionally seen as the factors leading to dual drainage of foreign reserves. Moreover, in its analytical framework, the consumer confronts an exogenously predetermined interest rate and income level, hence fails to evaluate the interrelation and inter-dependencies between economic variables. Deficiencies in these aspects of the existing models render it desirable and necessary to apply the Dynamic Stochastic General Equilibrium modelling to the determination of optimal reserves.

3.2.2 DSGE as a Modelling Strategy for Policy Analysis

In recent years, DSGE approach has become a prominent modelling strategy adopted by central banks and multilateral international organisations to analyse policies or conduct policy experiments. It has found fruitful applications in a growing number of areas with expanding coverage reaches out to developed as well as emerging economies. As a powerful toolkit for policy analysis, the DSGE approach has now begun to be adopted for analyzing emerging markets including China. For DESGE modelling of the Chinese economy, it has also become a budding enterprise. Researchers have made significant efforts to develop and estimate DSGE models for the whole economy, or use it to analyse specific domestic policy.

DSGE models possess several advantages over other macroeconomic models for policy analysis. Through restricting the acceptable behaviour of agents to dynamic utility maximization and rational expectations, DSGE modelling provides a coherent framework of analysis (De Grauwe, 2010). This coherence can help researchers structure policy discussions and make it possible to offer an account that is internally consistent and informed by a wide range of empirical indicators (Huntley and Miller, 2009). In addition, off-model structural parameters can be employed to calibrate and estimate the model, which is particularly useful in cases where time series are short (Tovar, 2009). Another advantage is the transparency of the models' dynamic mechanics. Through, for example, analysis of impulse response functions with structural shocks, DSGE models can help identify sources of fluctuations and the extent that each shock accounts for the total economic volatility, and thus provide a better understanding of structural changes. Closely related to this, analysis of shocks in DSGE models are conducted in a theoretically consistent way, whereas other macroeconomic models such as VAR modelling are theoretical (Iiboshi, Nishiyama, and Watanabe, 2006). For reduced form estimations, DSGE models allow a link to be established between structural features of the economy and reduced form parameters, which is not always possible with other macroeconomic models (Tovar, 2009). Also, in a reduced-form representation of a more complex process, DSGE models can offer more precise insights into the nature of the shocks than alternative modelling strategies (Huntley and Miller, 2009). Because all the agents in the DSGE model make decisions, the model is able to capture the endogenous effects of changes in policy in a way that other macroeconomic models cannot. In all, the combination of a sound micro-founded and theoretically consistent model structure and good policy analysis performance suggests that DSGE models will be further developed as a powerful toolkit that can work effectively in a variety of policy environments (Smets and Wouters, 2004).

A typical DSGE model incorporates three components: an environment in which agents interact in the process of maximizing their utility subject to their respective constraints; decision rules that represent their behaviours; and a set of shocks affecting their decisions (Dejong and Dave, 2007). These components are typically expressed in the form of nonlinear and expectation difference equations. Closed form solutions for the system are not normally directly derivable. Therefore, researchers resort to linear approximation in order to characterize the model, the methods of which has been explored extensively in the literature (Kydland and Prescott, 1982; Taylor and Uhlig, 1990; Judd, 1992; Handsen and Prescott, 1995 and Danthine and Donaldson, 1995).

Blanchard and Kahn (1980) documented a method of solving linear-quadratic dynamic stochastic models or linear stochastic difference equations. This approach marks a key cornerstone for solving DSGE models. Subsequently, a large body of literature has emerged which extensively studied this approach, as in e.g. Sargent (1987), Mcgrattan (1994), Hansen, McGrattan and Sargent (1995). Uhlig (1995) presented a toolkit for solving nonlinear dynamic models, which offers a simplified and unified solution method. This general solution is based on the solution method of matrix-quadratic equations proposed by Binder and Pesaran (1997), and it also uses frequency-domain techniques to calculate the second-order moments of the model (King and Rebelo, 1993). The solution developed by Uhlig (1995), together with earlier contributions such as McGrattan (1994), Hansen et al. (1995) and Blanchard and Kahn (1980), has laid the foundations for the standard approach to solving DSGE models (Dejong and Dave, 2007).

Although there are several methods of solving DSGE models, they invariably approximate around the deterministic steady state. In dynamic macroeconomics, behaviour of the economy is generally evaluated assuming agents have a perfect foresight of future shocks. The corresponding equilibrium is defined as the equilibrium position of the system without uncertainty. However such an approach completely ignores that agents' behaviour is determined by future uncertainty. Especially for risk-averse agents, the existence of future shocks hitting the economy matters as they would anticipate the convergence of economic variables to risky steady state which, according to Coeurdacier et al. (2011), may be defined as the point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is zero at this date.

While there is a general agreement on the definition of risky steady state, it is quite challenging to derive the convergence of economic variables to risky steady state. The method proposed by Coeurdacier et al (2011) uses an iterative algorithm to find the fixed point of a problem in which one function maps steady states in second moments and another maps second moments into steady states. However this process becomes very complex as the dimensions of the model increase. de Groot (2013) presents an alternative method which dispenses with the fixed point problem and makes use of the now common method for solving a second-order approximation of the model around its deterministic steady state.

Liu (2007) proposes a model based on Christiano, Motto, and Roatagon (2002), but with some important extensions. First, the closed economy model was extended to the case of an open economy, taking into account changes in net foreign assets and domestic economic interaction. Second, according to the characteristics of China's banking sector, the financial sector was added with some modifications. Additionally, the Chinese monetary policy is modified to one that targets money supply. The model parameters are estimated using a combination of calibration and Bayesian estimation. Unlike the closed version, this open economy approach shifts the focus from domestic to international interaction.

Zhang (2009) presents a closed-economy model, which compares the effectiveness of money supply and interest rate rules in the Chinese context. In line with Christiano et al. (2005) and Smets and Wouters (2003), Zhang (2009) specifies that all agents maximize their utility subject to inter-temporal budget constraints. Following the insights offered by Liu and Zhang (2010), Zhang (2009) applies a rule that is a combination of the rules proposed by Clarida et al. (1998) and Smets and Wouters (2003). The approach highlights the importance of inflation expectations and changes in inflation and output. The author estimated some of the parameters with available data and assigned values to others according to related works in the literature. The empirical results indicate that the interest rate rule is likely to be more effective than the quantity rule in the Chinese context.

Mehotra et al. (2011) also consider a closed-economy model, but with a focus on the impact of technological and monetary policy shocks under different settings of the Chinese economy. Unlike Zhang (2009), this study employs the traditional Taylor rule to simulate monetary policy. Furthermore, the model is based on the assumption of capital controls which invalidates the uncovered interest parity. Under the model setting, the domestic monetary policy is relatively independent from international counterparts since, under capital controls, the Chinese monetary policy is not subject to the influences of international factors. Mehotra et al.'s findings indicate that a rebalancing of current account imbalances would reduce the volatility of China's real economy in the event of a technological shock, which provides support for policies designed to increase the share of consumption in China.

Sun and Sen (2011) modify the Smets-Wouters model using a monetary growth rule to assess the monetary transmission mechanism of China's monetary policy. They argue that the Smets-Wouters model assumption of two production sectors: the intermediate firms in a monopolistic market; and the final goods firms in a perfect competition market echoes the reality of China. Owing to the power granted by the government over control of raw material and energy supplies, state-owned enterprises are in a monopolistic position in the intermediate product market. Meanwhile, private and small-sized firms producing final goods are faced with a competitive market. In contrast with previous studies, this paper discusses monetary policy and fiscal policy as well.

In an open-economy multi-country DSGE framework, Straub and Thimann (2010) analyse economic adjustments in China under flexible and fixed exchange rate regimes. This multi-country version of the DSGE model follows Jacquinot and Straub (2007, 2008) to specify a world economy composed of four symmetrical blocks representing the United States, the euro zone, China (or emerging markets, especially Asia) and the rest of the world. The model is formulated in such a way that relevant features of the economies are highlighted with regard to internal and external adjustments by China. In essence, this model is a close variant of Jacquinot and Straub (2007, 2008). But to account for the mechanism through which exchange rate changes pass through to consumers and the fact that international risk sharing is imperfect, they extend Coenen and Straub's (2005) work by incorporating two types of households who have distinct abilities of assessing asset markets. One type of household only accumulates money and the other owns financial assets as well as physical capital. The monetary authorities follow a Taylor rule with interest-rate smoothing. The Chinese monetary authority however operates a fixed exchange rate regime. The model-based results indicate that shocks caused by permanent technological progress or labour supply changes cannot explain China's persistent current account surplus. Compared to the case of a flexible exchange rate regime, the fixed exchange rate regime with an inefficient financial market could have negative effects on GDP growth in the medium term.

Zheng and Guo (2013) extend a DSGE model proposed by Lubik and Schorfheide (2007) in a small open economy framework. Their work provides a theoretical foundation for the empirical investigation by deriving the boundary conditions between determinacy and indeterminacy. Their empirical results indicate that the nominal interest rate reacts not only to inflation and output gaps, but also to changes in the RMB exchange rate. The instability in China's monetary policy is driven by two sources, the sunspot shock and the indeterminate propagation of fundamental shocks. In addition, macroeconomic dynamics is significantly affected by monetary policy shocks in the short run.

3.3 The Model

We now present a backbone New Keynesian DSGE model as a foundation for modelling foreign reserve holdings of China. We start by building a simple three-sector closed economy model in the spirit of Gabriel et.al (2011). In what follows, the agent's behaviour and their optimizing problems are first described in detail.

3.3.1 Households

We consider a closed economy in a competitive market. The economy is inhabited by infinitely-lived households. The representative consumer chooses consumption and labour supply to maximize his lifetime utility. The single-period utility is,

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{3.1}$$

where C_t denotes the consumer's consumption level, N_t is the labour supply. The parameters σ and φ represent the inverse intertemporal elasticity of substitution with respect to consumption and labour supply. The household's problem at time t is to choose paths for consumption C_t, labour supply N_t and aggregate financial assets holdings B_t to maximize utility as given by:

$$U_t = E\sum_{t=0}^{\infty}\beta(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}) \tag{3.2}$$

where β represents the discount factor. The household decides his consumption C_t based on the total financial assets level and the wage income level W_tN_t; the remaining assets B_{t-1} are then invested in the financial market to yield R_t. CS represents the consumption shock.

Thus the individual's period budget constraint is：

$$B_t = R_tB_{t-1} + W_tN_t - P_tC_t * CS \tag{3.3}$$

To derive the necessary first order condition for the optimal problem, we form a Lagrangian:

$$L = \max_{C_tX_t} E\sum_{t=0}^{\infty}\beta^t(\beta\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}) - \lambda_t(R_tB_{t-1} + W_th_t - P_tC_t - B_t) \tag{3.4}$$

For convenience the marginal utility of consumption is denoted as $U'_{C,t}$, marginal utility of labour supply is denoted as $U'_{N,t}$. Then the necessary first order conditions are,

$$\frac{\partial \mathrm{L}}{\partial \lambda_{\mathrm{t}}}: 0 = R_t B_{t-1} + W_t h_t - P_t C_t - B_t \tag{3.5}$$

$$\frac{\partial \mathrm{L}}{\partial \mathrm{C_t}}: 0 = U'_{C,t} - \lambda_{\mathrm{t}} \mathrm{P}_t \tag{3.6}$$

$$\frac{\partial \mathrm{L}}{\partial \mathrm{B_t}}: 0 = \lambda_{\mathrm{t}} R_t - \lambda_{\mathrm{t-1}} \beta \tag{3.7}$$

$$\frac{\partial \mathrm{L}}{\partial \mathrm{N_t}}: 0 = U'_{N,t} - \lambda_{\mathrm{t}} \mathrm{W}_t \tag{3.8}$$

From equations (3.6) and (3.7), we arrive at a standard Euler equation of consumption as follows:

$$U'_{C,t} = E_t \frac{R_t}{P_{t+1}/P_t} \beta \mathrm{E_t} U'_{C,t+1} \tag{3.9}$$

And the optimal labour decision is:

$$-\frac{\mathrm{U'_{N,t}}}{\mathrm{U'_{C,t}}} = \frac{\mathrm{W_t}}{\mathrm{P_t}} \tag{3.10}$$

3.3.2 Firm Sector

3.3.2.1 Wholesale Sector

Turing to the production sector, we assume that production follows the standard Cobb-Douglas production function. Output Y_t^W is produced by the representative competitive firm using labour and capital:

$$Y_t^W = F(A_t, N_t, K_{t-1}) = (A_t N_t)^{\alpha} K_{t-1}^{1-\alpha} \tag{3.11}$$

Let investment in period t be I_t. Then capital accumulates according to:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3.12}$$

where A_t represents a technology parameter, N_t represents labour supply and K_{t-1} represent beginning-of-period capital. All these variables are in per-capita units. We assume that capital adjusts instantly without costs. The first order conditions for production function are:

$$F_{h,t} = \alpha \frac{Y_t}{h_t} = W_t \tag{3.13}$$

$$F_{K,t} = (1-\alpha)\frac{Y_t}{K_t} = R_t + \delta \tag{3.14}$$

where equation (13) equates the marginal product of labour $F_{h,t}$ with the real wage, and equation (14) equates the marginal product of capital $F_{K,t}$ with its costs.

Apart from the above assumption of no investment cost, we introduce investment costs to the law of capital accumulation. At the beginning of the period, firms convert I_t amount capital into $(1 - S(X_t))I_t$ which is sold at a real price Q_t. We define $X_t \equiv {}^{I_t}/_{I_{t+1}}$, the investment cost function $S(X_t)$ satisfies $S', S'' \geq 0$. Firms maximize their expected discounted profits:

$$E_t \sum_{k=0}^{\infty} D_{t,t+k}[Q_{t+k}(1 - S(I_{t+k}/I_{t+k-1}))I_{t+k} - I_{t+k}] \qquad (3.15)$$

where $D_{t+k} \equiv \beta \frac{U'_{c,t+k}}{U'_{c,t}}$ is the real stochastic discount rate and the law of motion for capital accumulation is:

$$K_{t+1} = (1 - \delta)K_t + (1 - S(X_t))I_t \qquad (3.16)$$

We can arrive at the necessary first order conditions:

$$Q_t\big(1 - S(X_t) - X_t S'(X_t)\big) + E_t\left[D_{t,t+1}Q_{t+1}S'(X_{t+1})\frac{I_{t+1}^2}{I_t^2}\right] = 1 \qquad (3.17)$$

Firms’ demand for capital must satisfy the following equation:

$$E_t[(1+R_{t+1})RPS_{t+1}] = \frac{E_t\left[(1-\alpha)\frac{P^W_{t+1}Y_{t+1}}{K_{t+1}}+(1-\delta)Q_{t+1}\right]}{Q_t} \tag{3.18}$$

The right-hand-side of equation (18) represents the gross return to holding a unit of capital for one period from time t to t+1. The left-hand-side is the gross return from holding financial assets which is the opportunity cost of capital and includes an exogenous risk-premium shock RPS_t. We complete the set-up with investment costs by defining the functional form:

$$S(X_t) = \phi_X(X_t - (1+g))^2 \tag{3.19}$$

3.3.2.2 Retail Sector

The monopolistic competition retail firms produce differentiated goods and are price-setters. Instead of a Walrasian determination of prices, firms adjust prices under constraints on the frequency. Therefore we assume that the firm uses a homogeneous wholesale good to produce a basket of differentiated goods for consumption：

$$C_t = (\int_0^1 C_t(i)^{(\zeta-1)/\zeta}\, di)^{\zeta/\zeta-1} \quad (3.20)$$

where ζ is the elasticity of substitution. This indicates a pair of demand equations for each intermediate good i with price $P_t(i)$ of the form:

$$C_t(i) = (\frac{P_t(i)}{P_t})^{-\zeta} C_t \quad (3.21)$$

where $P_t = (\int_0^1 P_t(i)^{1-\zeta}\, di)^{1/1-\zeta}$ is the aggregate price index CPI. Converting good i from wholesale to a homogeneous output requires a cost $cY_t^W(i)$ where c is a cost parameter. The wholesale sector follows the standard Cobb-Douglas production function (3.11) as before:

$$Y_t(i) = (1-c)Y_t^W(i) \quad (3.22)$$

$$Y_t^W = (A_t h_t)^{\alpha} K_{t-1}^{1-\alpha} \quad (3.23)$$

To introduce price stickiness into our model, wholesale firms have a probability of $1-\xi$ at each period to set optimal intermediate good price $P_t^0(i)$. Otherwise the price is not re-optimized and holds fixed. Therefore the intermediate goods producer's objective is to maximize discounted profits by choosing price $P_t^0(i)$:

$$E_t \sum_{k=0}^{\infty} \xi_t D_{t,t+k} Y_{t+k}(i)[P_t^0(i) - P_{t+k} MC_{t+k}] \quad (3.24)$$

subject to equation (3.21), where $D_{t,t+k} \equiv \frac{U'_{C,t+k}}{U'_{C,t}}$ is the stochastic discount factor over the interval $[t, t+k]$. MC_{t+k} is the mark-up shock MS_{t+k} to the steady state mark-up $\frac{1}{(1-1/\zeta)}$. Therefore the optimal solution is,

$$E_t \sum_{k=0}^{\infty} \xi_t D_{t,t+k} Y_{t+k}(m) \left[P_t^0(m) - \frac{1}{(1-1/\zeta)} P_{t+k} MC_{t+k} MS_{t+k} \right] = 0 \qquad (3.25)$$

Evolution of the price index and the real marginal costs are defined as the following:

$$P_{t+1}^{1-\zeta} = \xi P_t^{1-\zeta} + (1-\xi)(P_{t+1}^0)^{1-\zeta} \qquad (3.26)$$

$$MC_t = \frac{P_t^W}{P_t} \qquad (3.27)$$

3.3.3 Central Bank

It is customary to assume that the nominal interest rate is the main instrument of monetary policy. Nominal and real interest rates are related by the Fischer equation as follows:

$$E_t[1 + R_{t+1}] = E_t \left[\frac{1 + R_{n,t}}{\pi_{t+1}} \right] \qquad (3.27)$$

where $R_{n,t}$ is the nominal interest rate, $R_{t+1}$is the real interest rate and $\pi_{t+1} \equiv$

$\frac{P_{t+1}}{P_t}$ is the inflation rate. $R_{n,t}$ serves as a policy variable set at the beginning of period t and the monetary policy follows a Taylor-type rule:

$$\log\left(\frac{1+R_{n,t}}{1+R_n}\right) = \rho\log\left(\frac{1+R_{n,t-1}}{1+R_n}\right) + \theta\log(\frac{\pi_t}{\pi}) \qquad (3.28)$$

Finally the model is completed applying a market clear condition with lump-sum taxes T_t,

$$Y_t = C_t + G_t + I_t \qquad (3.29)$$

$$G_t = T_t \qquad (3.30)$$

where G_t represent government spending, and we assume all taxes are used for government spending.

3.3.4 Dynamics of Foreign Assets

In the above model there is no specific description of the law of motion for foreign assets. Foreign reserves are part of the aggregate financial wealth and the aggregate financial wealth B_t can be decomposed into foreign bonds and claims on physical capital. Hence,

$$B_t = F_t + V_t \qquad (3.31)$$

where F_t is the value of the end-of-period holdings of foreign assets, and V_t the value of the end-of-period holdings of claims on physical capital. Such capital earns a real rate of return if one assumes that assets are perfect substitutes among those in the household's portfolio. Following the similar setting for wholesale firms, investment in physical capital is determined by maximizing cash flows each period subject to the law of motion of capital accumulation. Therefore the profit from investing in physical capital is:

$$E_t \sum_{k=0}^{\infty} D_{t,t+k}[Q_{t+k}(1 - S(I_{t+k}/I_{t+k-1}))I_{t+k} - I_{t+k}] \tag{3.32}$$

where $D_{t+k} \equiv \beta \frac{U'_{c,t+k}}{U'_{c,t}}$ is the real stochastic discount rate, Q_t is the capital price.

Capital accumulates according to:

$$K_{t+1} = (1 - \delta)K_t + (1 - S(X_t))I_t \tag{3.33}$$

The first-order conditions can be derived as:

$$Q_t\big(1 - S(X_t) - X_t S'(X_t)\big) + E_t\left[D_{t,t+1}Q_{t+1}S'(X_{t+1})\frac{I_{t+1}^2}{I_t^2}\right] = 1 \tag{3.34}$$

Firm's demand for capital must satisfy the following equation:

$$E_t[(1 + R_{t+1})] = \frac{E_t\left[(1-\alpha)\frac{P^W_{t+1}Y_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1}\right]}{Q_t} \quad (3.35)$$

The above equation corresponds to the standard no-arbitrage condition, where the future gross returns on physical capital are equal to the future returns of holding one unit of financial capital:

$$V_t = Q_t[(1 - S(I_t/I_{t-1}))I_t - I_t] \quad (3.36)$$

The right-hand side of equation (3.36) represents the discounted future cash flows from claims on physical capital.

Combining equations (3), (31), (32) and (36) we get the following law of motion for net foreign assets:

$$F_{t+1} = R_tF_t + W_tN_t - P_tC_t - Q_tI_{t-1} + Q_t(1 - S(I_t/I_{t-1}))I_t \quad (3.37)$$

3.3.5 Shocks Processes

To complete the benchmark model, we introduce the structural shock processes in log-linearized form.

$$LogA_t - Log\overline{A_t} = \rho_A(LogA_{t-1} - Log\overline{A_{t-1}}) + \varepsilon_{A,t} \tag{3.38}$$

$$LogG_t - Log\overline{G_t} = \rho_G(LogG_{t-1} - Log\overline{G_{t-1}}) + \varepsilon_{G,t} \tag{3.39}$$

$$Log\left(\frac{1+R_{n,t}}{1+R_n}\right) = \rho\, Log\left(\frac{1+R_{n,t-1}}{1+R_n}\right) + \theta Log(\frac{\pi_t}{\pi}) \ \ + \varepsilon_{MS,t} \tag{3.40}$$

where all innovation $\varepsilon \sim N(0, \sigma^2)$, $\sigma = 1\%$. The above are standard setting represent technology shock $\varepsilon_{A,t}$, government spending shock $\varepsilon_{G,t}$, and monetary policy shock $\varepsilon_{MS,t}$. Apart from general setting, we introduce a consumption shock to represent future variation on represented agents. This shock is also followed AR(1) processes.

$$LogCS_t - Log\overline{CS_t} = \rho_{CS}(LogCS_{t-1} - Log\overline{CS_{t-1}}) + \varepsilon_{CS,t} \tag{3.41}$$

3.3.6 Steady State Analysis

In dynamic macroeconomics, behaviour of an economy is usually evaluated assuming agents have perfect foresight of future shocks. The corresponding steady state is defined as the equilibrium position of the system in absence of shocks. However such an approach ignores the fact that agents' behaviour is determined by future uncertainty. Especially for risk-averse agents, the existence of future shocks hitting the economy matters. Thus they anticipate the convergence of economic variables to some risky steady state. Coeurdacier et al. (2011) propose to define risky steady state as the point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is zero at this date.

Deriving at a solution to risky steady state proves a difficult challenge. The method proposed by Coeurdacier et al. (2011) uses an iterative algorithm to find the fixed point a problem in which one function maps steady states in second moments and another maps second moments into steady states. However this process becomes more complex as the dimensions of the model increase. de Groot (2013) suggests an alternative method which dispenses with the fixed point problem and makes use of the now common method for solving a second-order approximation of the model around its deterministic steady state.

To illustrate, we assume that the representative agent maximizes his lifetime utility:

$$U_t = E\sum_{t=0}^{\infty}\beta(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}) \qquad (3.42)$$

subject to the budget constraint:

$$B_t = R_t B_{t-1} + W_t N_t - P_t C_t \tag{3.43}$$

Then we are able to derive the Euler type equation by assuming

$$\pi_{t+1} \equiv \frac{P_{t+1}}{P_t} \equiv 1 \tag{3.44}$$

in the deterministic steady state, so that we have:

$$U'_{C,t} = E_t R_t \beta E_t U'_{C,t+1} \tag{3.45}$$

After some manipulation the above equation becomes:

$$\frac{1}{\beta}\left(\frac{E_t[c_{t+1}]}{c_t}\right)^{\sigma} = E_t[R_{t+1}] \tag{3.46}$$

For illustration purpose, we assume the price level $P_t = 1$, wage W_t and the rate of return R_t exogenous variables following two auto-correlated log-normally distributed processes with mean $\bar{W}$ and $\bar{R}$, autocorrelation ρ_W and ρ_R, and standard deviations σ_W and σ_R. Hence the deterministic steady state for the maximize problem becomes:

$$\beta\bar{R} = 1 \tag{3.47}$$

$$B^*(1-\bar{R}) = \bar{W}N^* - C^* \tag{3.48}$$

$$\bar{W} = C^{*,\sigma}N^{*,\varphi} \tag{3.49}$$

Ascribe to limitations of the usual perturbation approach (Coeurdacier et al., 2011), the above system of three equations are unable to uniquely define the equilibrium values of C^*, B^*, N^*. By including the law of motion for foreign assets and assuming $S(X_t) = 1$ to rule out domestic investment hence all investments are in foreign assets ($B^* = F^*$), we have:

$$F^*(1-\bar{R}) = \bar{W}N^* - C^* \tag{3.50}$$

Turning to risky steady state, the Euler equation can be approximated as follows:

$$\frac{1}{\beta}\left(\frac{E_t[c_{t+1}]}{c_t}\right)^{\sigma} = E_t[r_{t+1}]\left(1 + \frac{\sigma(\sigma+1)}{2}\frac{Var_t(c_{t+1})}{E_t[c_{t+1}]^2}\right) - \sigma\frac{Cov_t(c_{t+1}, r_{t+1})}{E_t[c_{t+1}]} \tag{3.51}$$

Taking one step further, when evaluated at the risky steady state, this Euler equation becomes:

$$\frac{1}{\beta} = \bar{R}\left(1 + \frac{\sigma(\sigma+1)}{2}\frac{\mathrm{Var}_t(C_{t+1})}{\hat{C}^2}\right) - \sigma\frac{\mathrm{Cov}_t(C_{t+1}, r_{t+1})}{\hat{C}} \qquad (3.52)$$

where variables with a hat indicate that they are evaluated at the risky steady state values. With above equations, we are now in a position to conclude that the rate of investment return multiplied by time preference is equal to 1 at deterministic steady state, which takes no consideration of the effect of risk. But in risky steady state, we have a risk-premium term represented by $\frac{\sigma(\sigma+1)}{2}\frac{\mathrm{Var}_t(c_{t+1})}{\hat{c}^2}$, and also a negative term $\sigma\frac{\mathrm{Cov}_t(c_{t+1}, r_{t+1})}{\hat{c}}$ indicating the effect of precautionary saving. It implies with higher variance of consumption growth, households intend to keep extra saving.

By plugging above equations into the budget constraint, and recall $B^* = F^*$, we have:

$$\hat{F} = \frac{\bar{W}\hat{N} - \hat{C}}{\left[1 - \left[\bar{R}\left(1 + \frac{\sigma(\sigma+1)}{2}\frac{\mathrm{Var}_t(C_{t+1})}{\hat{C}^2}\right) - \sigma\frac{\mathrm{Cov}_t(C_{t+1}, R_{t+1})}{\hat{C}}\right]\right]} \qquad (53)$$

Optimal holdings of foreign assets under risky steady state are related to the risk-premium due to imperfect international risk sharing and the effects of precautionary saving. With the increase in the model complexity, however, it would become increasingly difficult to obtain a closed form analytical solution to optimal amount of foreign reserves in risky steady stage. This necessitates our efforts being devoted to estimating optimal foreign reserves in risky steady state with numerical methods.

3.4 Method of Calculating Risky Steady State

To illustrate the method employed to calculate the risky steady state value of optimal foreign reserves, we assume a medium-scale macroeconomic model and the solution to its deterministic steady state takes the following form:

$$E_t\left[f\left(y_{t+1}, y_t, x_{t+1}, x_{t,} z_{t+1,} z_t\right)\right] = 0 \tag{3.54}$$

$$z_{t+1} = \Lambda z_t + \eta\sigma\varepsilon_{t+1} \tag{3.55}$$

All the variables are vectors. Where y_t represents endogenous non-predetermined variables, $x_{t,}$ indicates endogenous predetermined variables and z_t being exogenous predetermined variables. Λ and η are respective parameters in an AR(1) process, and σ is a parameter measuring the degree of uncertainty in the economy.

The deterministic steady state and risky steady state are defined as, where variables with bar denote DSS and variables with hat represent two states, respectively:

$$f(\bar{y}, \bar{y}, \bar{x}, \bar{x}, 0,0) = 0 \quad (3.56)$$

$$E_t f(\hat{y}, \hat{y}, \hat{x}, \hat{x}, , z_{t+1}, 0) = 0 \quad (3.57)$$

It is clear that the risky steady state solution takes into account the future uncertainty of the economy while the deterministic steady state solution assumes a perfect foresight path. To calculate the risky steady state, we follow the two step method introduced by de Groot (2013). First of all, define two policy functions for solving the model as:

$$y_t = g(x_t, z_t, \sigma) \quad (3.58)$$

$$x_{t+1} = h(x_t, z_t, \sigma) \quad (3.59)$$

Following the method introduced by Schmitt-Grohe and Uribe (2004), a second-order approximation is being taken to the policy function around its deterministic steady state:

$$x_{t+1} = \bar{x} + h_x(x_t - \bar{x}) + \frac{1}{2}(x_t - \bar{x})' h_{xx}(x_t - \bar{x}) + \frac{1}{2} z_t' h_{zz} z_t + (x_t - \bar{x})' h_{zz} z_t + \frac{1}{2} h_{\sigma\sigma} \sigma^2 \quad (3.60)$$

where $\bar{x} = h(\bar{x}, 0,0)$ is the value of the vector in deterministic steady state. The value of risky steady state is $\hat{x} = h(\hat{x}, z_{t+1}, 0)$. By definition, in risky steady state we have $E_t x_{t+2} = E_t x_{t+1} = \hat{x}$, and $E_t z_{t+1} = 0$.

The value of variables in risky steady state can be derived by taking second-order approximation around the deterministic steady state and then scrolling time forward to have:

$$\hat{x} = \bar{x} + h_x(\hat{x} - \bar{x}) + \frac{1}{2}(\hat{x} - \bar{x})' h_{xx}(\hat{x} - \bar{x}) + \frac{1}{2}E_t(z_t' h_{zz} z_t) + \frac{1}{2}h_{\sigma\sigma}\sigma^2 \tag{3.61}$$

3.5 Extensions with Habit Formation

Habit formation has been widely employed as an assumption in explaining behaviour of consumption dynamics in financial economics, e.g. for resolving the equity premium puzzle such as in Constantinides (1990) and Abel (1990) where they show that the equity premium puzzle can be partially explained by habit formation. A bunch of studies has been inspired by such an approach and has taken into account consumption habit in their examination of optimal consumption and optimal savings (Campbell and Cochrane 1999, Lettau and Uhlig 2000, Guvenen 2009). The most recent study applying habit formation in consumption can be found in Kim el al. (2013), which argues that habit formation preference provides an incentive for higher savings, resulting in higher demand for bonds over equity demand. Given the eventuality that consumption habit is also a critical feature of the Chinese economy, we extend our model to include exogenous habit formation in line with Christiano et al. (2005).

The representative household maximizes his utility subject to budget constraints. Into the standard model, external habit formation (H_t) is introduced. The consumer's optimal problem now becomes:

$$U_t = E\sum_{t=0}^{\infty}\beta(\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}) \quad (3.62)$$

$$B_t = R_t B_{t-1} + W_t N_t - P_t C_t \quad (3.63)$$

$$H_t \equiv \omega C_{t-1} \quad (3.64)$$

where $H_t \equiv \omega C_{t-1}$ implies that the level of habit is positively proportionate to the level of past consumption. Parameter ω is the constant habit persistence parameter, with $0 < \omega < 1$. Following the setting in the previous section where it is assumed that $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t} \equiv 1$, we derive the standard Euler equation of consumption as follows:

$$U'_{C,t} = E_t R_t \beta E_t U'_{C,t+1} \quad (3.65)$$

After some manipulation, we obtain:

$$\frac{1}{\beta}(\frac{E_t[c_{t+1} - \omega C_t]}{c_t - \omega C_{t-1}})^{\sigma} = E_t[R_{t+1}] \quad (3.66)$$

For illustration purposes, we follow the previous setting to assume that the price level $\mathrm{P_t} = 1$, while wage $\mathrm{W_t}$ and rate of return $\mathrm{R_t}$ are exogenous variables following two auto-correlated log-normally distributed processes with mean $\bar{W}$ and $\bar{R}$, autocorrelation ρ_W and ρ_R, and standard deviations $\sigma_W$and σ_R. Hence the deterministic steady state for the maximization problem becomes:

$$\beta\bar{R} = 1 \tag{3.67}$$

$$B^*(1-\bar{R}) = \bar{W}N^* - C^* \tag{3.68}$$

$$\bar{\mathrm{W}} = \mathrm{C}^*(1-\omega)^{\sigma}\mathrm{N}^{*,\varphi} \tag{3.69}$$

We specify the law of motion of foreign assets and rule out investment by assuming $S(X_t) = 1$, to obtain:

$$F^* = \frac{\bar{W}N^* - C^*}{(1-\bar{R})} \tag{3.70}$$

By combining equations (69) and (71) we have:

$$F^* = \frac{\mathrm{C}^*(1-\omega)^{\sigma}\mathrm{N}^{*,\varphi} * N^* - C^*}{(1-\bar{R})} \tag{3.71}$$

For comparison purposes, we rule out the habit formation effect by assuming $\omega = 0$, and hence we have:

$$F^* = \frac{C^{*\sigma} N^{*,\varphi} * N^* - C^*}{(1-\bar{R})} \tag{3.72}$$

It is clear that the optimal level of foreign reserves is very sensitive to the value of habit parameter. As argued by Deaton (1992), a positive ω indicates that consumers generate less pleasure from the same level of consumption. Hence households need to purchase more amounts to generate the same benefit, resulting in less wealth being available for investment. On the other hand, a negative ω implies that households make their decisions on the basis of not only the current level of consumption but also past consumption. Guariglia and Rossi (2002) identify such a behaviour as consumption durability, which results in a lower level of consumption and a higher level of investment in foreign assets.

Finally if we take risky steady state into account to derive the optimal foreign assets,

$$\hat{F} = \frac{C^*(1-\omega)^{\sigma} N^{*,\varphi} * N^* - C^*}{\left[1-\left[\bar{R}\left(1+\frac{\sigma(\sigma+1)}{2}\frac{Var_t(C_{t+1})}{\hat{C}^2}\right)-\sigma\frac{Cov_t(C_{t+1},R_{t+1})}{\hat{C}}\right]\right]} \tag{3.73}$$

3.6 Numerical Results

Table 3.1 Parameters in the Model

Parameter	Description	Parameter	Description
β	Discount factor	c	Conversion cost
σ	Risk Aversion	$\frac{1}{(1-1/\zeta)}$	Mark-up
φ	Inverse of the Frisch elasticity	ω	Habit parameter
α	Labour share	ρ	Interest rate smoothing
δ	Depreciation rate	θ	Feedback from expected inflation
ϕ_X	Investment adjustment cost parameter	ρ_A	Technology persistence
g	Balanced growth rate	ρ_G	Government spending persistence
ξ	Calvo prices	ρ_{CS}	Consumption persistence
ζ	Substitution elasticity		

Choosing appropriate parameters to characterize the Chinese economy requires good care. At this stage, we set the parameter value to those calibrated and reported in the literature. Staring with the household sector, we set the discount factor $\beta =$, which corresponds to an equilibrium annual interest rate of %. Risk aversion is set to $\sigma = 2$ by following Walsh (2003) and the inverse of the Frisch elasticity is set to be $\varphi = 6.16$, in line with Zhang (2009). The value of marginal disutility of working is considerably greater than many other countries to reflect China's low marginal disutility and the higher rate of household savings. Regarding consumption habit, the parameter value is set to 0.76, which results from the GMM estimation by Mehrotra et al. (2011).

For the firm sector, we set the output elasticity of labour to $\alpha = 0.4$, which represents a higher investment share in equilibrium (He et al. 2007). To capture the shorter depreciation period featuring the Chinese industrial policy, we set the depreciation rate of capital $\delta = 0.05$ (He et al. 2007). For the investment adjustment costs, the parameter value is set to be $\phi_X = 2$ with a balanced growth rate $g = 0.007$. The Calvo price parameter is set $\xi = 0.84$ indicating a higher average price-change duration (Zhang 2009). Moving to the parameter of mark-up, the value of substitution elasticity to intermediate goods is $\zeta = 1/7$, implying a 17% mark-up.

For the government sector, the possible value of inflation parameter has been under extensive debate. Mehrotra et al. (2011) report a value of $\theta = 1.34$, however Fan et al. (2011) argue that this parameter should be less than 1. We follow Chen et al (2012) to set the value at 1.01 implying a passive monetary policy.

Table 3.2 Steady State Values of Different State

	Output	Consumption	Interest Rate	Foreign Reserves Holding
RSS without Habit	2.692	1.247	1.02	1.509
DSS without Habit	2.687	1.246	1.006	1.249
RSS with habit	4.71	2.182	1.02	2.218
DSS with habit	4.702	2.181	1.006	2.207

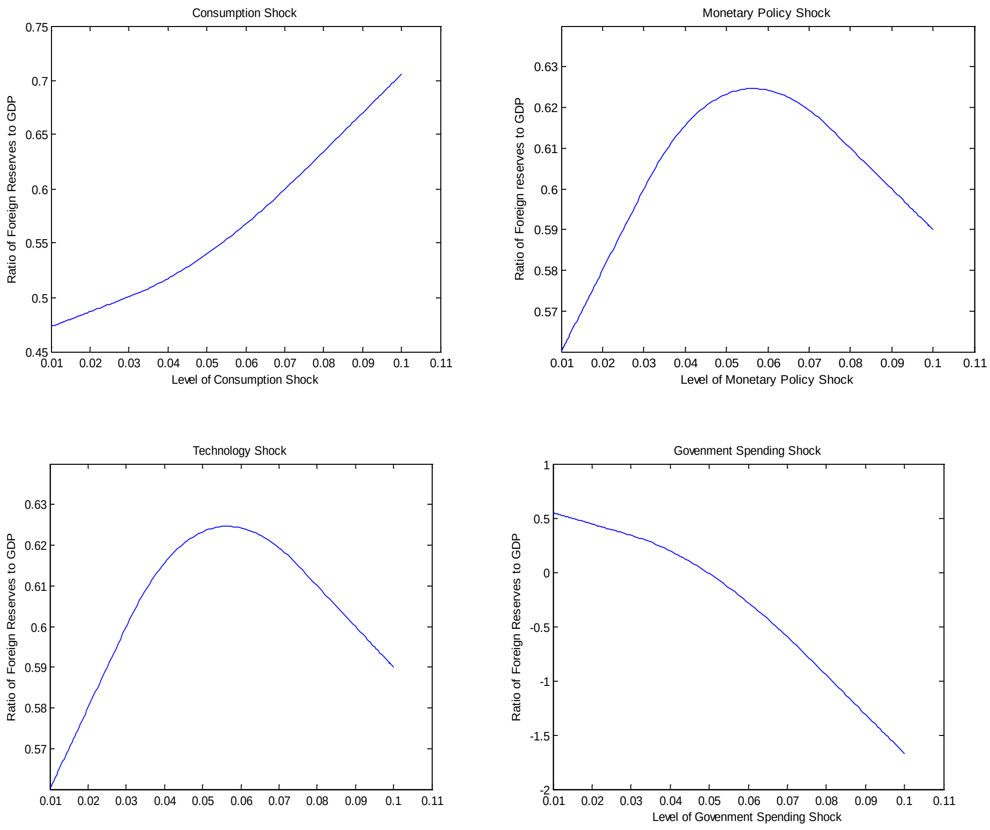
Notes: RSS denotes risky steady state, and DSS stands for deterministic steady state

Table 3.2 reports selected variables' risky steady state and deterministic steady state value respectively with and without habit formation. The economy represented by risky steady state is richer indicated the economy is perform in a higher level. The result is consistent with other publications (de Groot 2013, Michel 2011), by considering the future uncertainty the economy would stay at a higher level steady state, which is a result from precautionary capital accumulation. Another distinctive difference is the effect from consumer's habit. No matter which steady state, the value is triggered by almost 50% increase in each state. However, the interest rate is performed in a complete different way, the value of interest rate is lower by 1.37% at deterministic steady state resulting by the combination effect from risk premium and precautionary effect. The lower interest rate in risky steady state implying precautionary effect is outweigh by risky premium.

Turning now to the level of foreign reserves, the optimal ratio foreign reserves to GDP=Y/F= 56% at risky steady state without habit formation, which is 10% higher than the value at deterministic steady state. Interestingly, consumer's habit drags down the optimal ratio by 10% at risky steady state (46%). This result consist with the finding from previous literature (Deaton ,1992) , a positive ω imply that consumers generate less pleasure from the same level of consumption resulting in less wealth being available for capital accumulation.

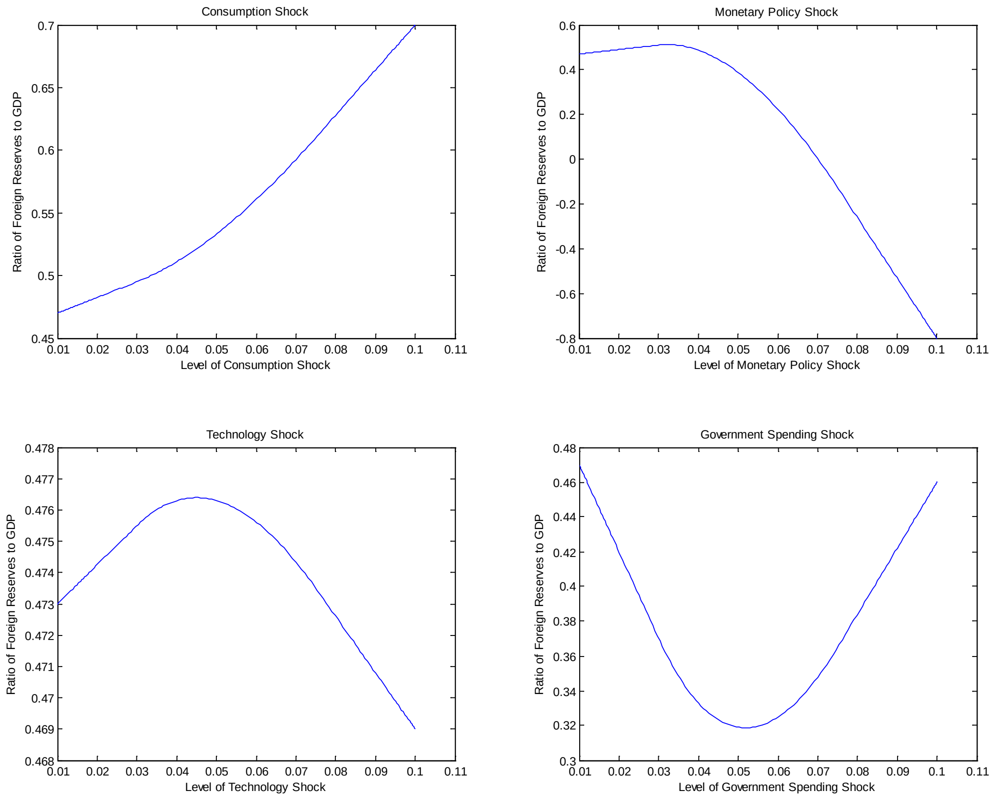
To illustrate the effect of future uncertainty, we recalculate optimal ratio of foreign reserves to GDP with different level of standard deviation from 1% to 10% for each shock. The results are presented below.

Figure 3.1 Ratio of Foreign Reserves to GDP



On the one hand, with the increase in variance of future shocks, there is a significant positive relationship between consumption shock and level of foreign reserves. On the other hand the optimal ratio of foreign reserves drops sharply when standard deviation exceeds 5%. Coming to monetary policy shock and technology shock, the relationship between level of uncertainty and optimal ratio becomes ambiguous, where response behaviour is complete different in different range.

Figure 3.2 Ratio of Foreign Reserves to GDP (Habit Formation)



The above charts report the optimal ratio of foreign reserves to GDP by introducing habit formation. The consumption shock represent a similar pattern compared with the baseline model without habit formation. Optimal ratio increases with the higher standard deviation. Also monetary policy shock and technology shock show ambiguous relationship as baseline model. It is noticeable that with habit formation, the effect of monetary policy shock is magnified where over 5% standard deviation from the optimal ratio decrease significantly. Last government spending shock becomes neutral without noticeable indication.

3.7 Conclusions

The development of the theory of international reserves has effectuated a shift of research focus from transaction purposes onto the precautionary motive as the main force driving central banks' holdings of reserves. Along with the failure of previous benchmark to evaluate the reserve level of China that is rapidly ascending, the precautionary saving approach rises to provide some plausible explanations. The methodology of the approach is also novel in that it builds a macroeconomic framework with micro-foundation featuring optimizing behaviour of economic agents. This is where the DSGE approach can provide its best service as a modelling strategy. For the standard precautionary savings models, their analysis is essentially based on partial equilibrium and ignores interaction between the micro foundation and the macro economy, and between domestic and international factors causing possible dual drainage of foreign reserves.

This chapter represents one of the first significant efforts of trying to fill the critical void in the literature by employing the DSGE modelling to analyze the optimal reserves for China in a risky world. We set up a baseline model to understand the Chinese foreign reserves policy in a general equilibrium setting. The model is straightforward but not lacks of necessary complexity. The behaviour of three major sectors is modelled, as well as the law of motion of foreign reserves. Besides, the consumer's behaviour is further explicitly modelled by including habit formation in the model. Furthermore, we allow for future uncertainty and extend the research into studying the perspectives of both the perfect foresight and risk aversion.

The main model findings are based on local approximation of the second order for a risky world. Differing from deterministic steady state, we re-calculate the optimal foreign reserve with future variance. Through the analysis, we find that optimal holdings of foreign reserves in risky steady state are influenced by the risk-premium due to imperfect international risk sharing and the effects of precautionary savings. As a result, defining the steady state value as the optimal level of foreign reserves, the amount of optimal reserves is greater in a risky world than in a certainty world. The numerical results confirm our conclusion. By way of introducing consumption uncertainty, we discover that with a higher level of consumption volatility, optimal foreign reserves increase substantially. We derive at an indicator of optimal holdings of foreign reserves for China in a risky work, which is, in the form of the ratio of foreign reserves to GDP, around 56% with 1% standard deviation.

Overall, our model attempts to better our understanding of the optimal reserves for China under uncertainty. The DSGE approach has proved to be very useful toolkit for analyzing macroeconomics including the reserve policy. However, despite the recent progress of DSGE modelling, challenges for modelling emerging economies in general, China in particular, are vexing. The difficult tasks ahead for our future research include, for instance, modelling frictions in financial markets, interaction between financial openness and international trade, various shocks facing China, and bearings of all these factors on China's holdings of foreign reserves. In addition, such future research should not only consider features of China's idiosyncratic structure of the domestic economy, but the nation's vulnerabilities to external factors as well. In the coming chapters we will gradually deepen our analysis by dealing with these critical research tasks.

Chapter 4

China and the Dollar Trap

In this chapter, a two-country model is introduced to analyse the dollar dominated state of China's foreign reserves management. In particular, this approach contributes to a deeper understanding of the assets allocation between two US assets. Furthermore, this chapter investigates the co-dependence relationship between two leading countries in the world economy, namely.

Chapter 4

China and the Dollar Trap

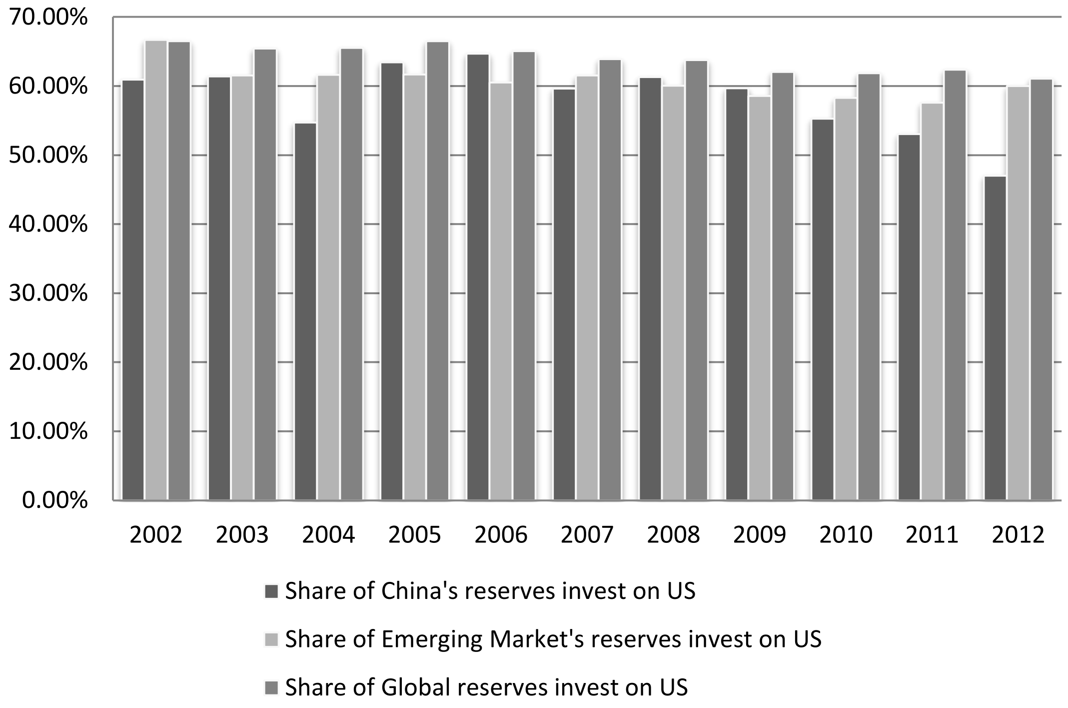
4.1 Introduction

4.1.1 Foreign Reserve Management in China

Since the Asian financial crisis of 1997-1998, China's foreign reserves have experienced a significant surge. According to data from the IMF COFER and the World Bank, China's foreign reserves rose from $161.41 billion in 1999 to $3387.51 billion in 2012, with an annual growth rate of 26%. China now accounts for nearly 31% of total world foreign reserves; consequently, the country has become a major influence in the international foreign exchange market. Furthermore, with reference to another macroeconomic benchmark, in 2012 China's reserves are in equivalent of 41.18% of its GDP (See Table 1).

Most of China's international reserves are held in US dollars. Given the dollar's status as a key currency in the international monetary system, holding dollar-denominated assets by international investors is reasonable and China is no exception in this regard (Prasad, 2013). It is generally estimated that holdings of US securities by China could account for around 50% of the country's total international reserves. Some even argue that this tends to be an underestimation since the data from the US government, or the Treasury International Capital (TIC) reporting are likely to underestimate China's investment (Setser and Pandey, 2009). By using the TIC report and COFER data together, Shi and Nie (2012) deduce that the dollar assets held by China represent around 60%-65% of the country's total foreign reserves. While the Chinese government does not make public the dollar composition of its foreign reserve holdings, by the TIC data it is evident that China has become the largest holder of US assets.

Figure 4.1 Holdings of US Securities, 2002-2012



Source: IMF COFER and US Treasury (TIC)

Table 4.1 China's Foreign Exchange Reserves, 1999-2012

Year	Billions of US Dollars	As % of World FX Reserves	As % of Chinese GDP
1999	161.41	9.06%	14.90%
2000	171.76	8.87%	14.33%
2001	220.06	10.74%	16.61%
2002	297.74	12.37%	20.48%
2003	416.20	13.76%	25.36%
2004	622.95	16.62%	32.25%
2005	831.41	19.25%	36.84%
2006	1080.76	20.57%	39.84%
2007	1546.36	23.06%	44.26%
2008	1966.04	26.76%	43.48%
2009	2452.90	30.04%	49.14%
2010	2913.71	31.45%	49.13%
2011	3254.67	31.89%	44.45%
2012	3387.51	30.93%	41.18%

Source: IMF COFER and the World Bank

The dollar assets are favored by China for a number of reasons. First, in order to maintain the exchange value, it makes great economic sense for China to invest the acquired US dollars into dollar-denominated securities. Second, the United States is the world's largest economy and has the biggest capital market. In 2011, the combined value of US private and public debt securities was $33.7 trillion (compared with $15.4 trillion for Japan and $5.3 trillion for Germany), and accounted for 34.2% of global debt securities. AS such, the US debt securities market is the only global market that has the capacity to absorb a large part of China's growing foreign reserve holdings. Third, US securities are safe and liquid relative to other types of investments. They are generally regarded as global safe assets. The global economic slowdown and the European sovereign debt crisis in recent years have also boosted the attractiveness of US securities for China. According to China's State Administration of Foreign Exchange (SAFE), the main objectives for its administration of China's foreign reserves are "security, liquidity, and increases in value, among which security is the primary principle". To fulfil these objectives, investing in US dollars is a good choice since dollar assets are safe, liquid and, on average, can reap sound returns.

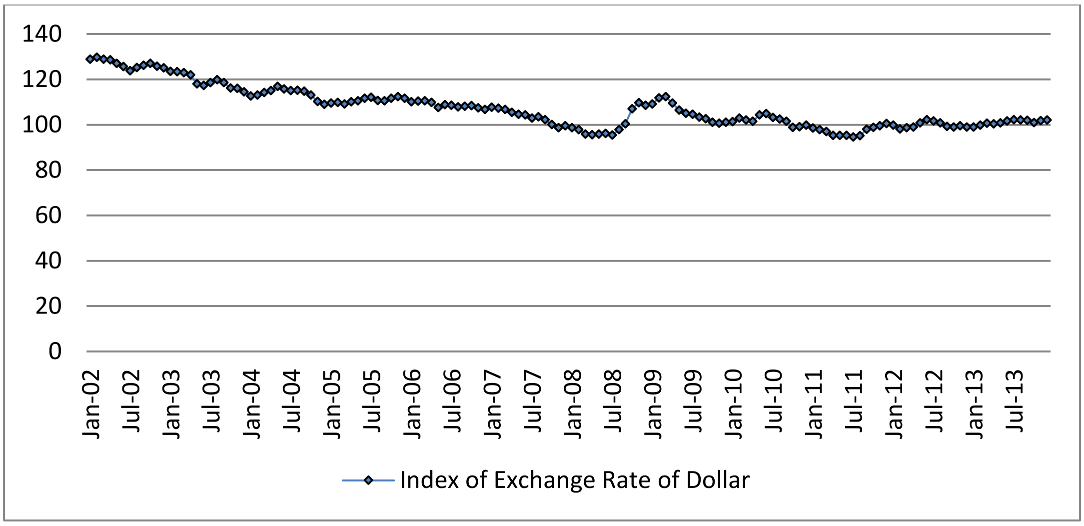
Figure 4.2 China's Dollar Assets Holdings as Percentage of Total Foreign Holdings of US Securities

Source: US Treasury (TIC)

4.1.2 The Dollar Trap

The returns on the dollar assets are sensitive to fluctuations of the dollar exchange rates. Given the huge size of the China's dollar build-up, this would become particularly problematic in a time of wide swings of the dollar exchange rate. From a peak in early 2002 through the first half of 2008, the trade-weighted dollar exchange rate, for the most part, steadily depreciated, falling by a total of about 25% (Figure 3). The dollar's fall over this six-year period was moderately paced at about 3% to 4% annually. For the next nine months, as the wider economy was reeling from the effects of the global financial crisis and recession, the dollar appreciated sharply, increasing more than 11% on a trade-weighted basis. This appreciation was a market response to the great uncertainty associated with the economic troubles. As economic conditions began to stabilize in mid-2009, the dollar began to depreciate again; by mid-2011 it had fallen by about 16%, and more or less returned to its pre-recession trend of depreciation. However, a second bout of market uncertainty caused by the European sovereign debt crisis caused the dollar to appreciate by more than 5% by the end of 2011. In early 2012, the dollar resumed its depreciation, down about 2% in February 2012 (see Figure 4.3). The wide fluctuations of the dollar have significant valuation effects on China's holdings of the dollar assets. Consequently, this raises the concern over risk to which China's dollar positions are exposed.

Figure 4.3 Index of Trade-Weighted Exchange Rate of the Dollar



Source: Board of Governors of the Federal Reserve System

Krugman (2009) claims that China has driven itself into a dollar trap from which it seems impossible to extricate itself. China's fall into this dollar trap is firstly related to China's build-up of massive foreign reserves. Existing literature has proposed several drives behind this build-up, such as the precautionary savings motive (Mendoza, Quadrini and Rios-Rull, 2009; Carroll and Jeanne, 2009; Sandri, 2011) and the mercantilist view (Aizenman and Lee, 2007). Krugman (2009) argues that there are two major reasons for China's hoarding of foreign reserves. One is the fixed and undervalued exchange rate policy that China has deployed to stimulate exports, which has resulted in current account surpluses. The other is the impressive growth of the economy, which attracts substantial inflows of foreign capital including foreign direct investment. Therefore, both current account and capital account surplus are driving China's continued accumulation of reserves.

Given the fact that these massive holdings of Chinese international reserves are mostly invested in the US dollar assets, China is highly likely to suffer significant capital loss as a result of fluctuations of the dollar. To make the matter worse, China is placed in a position without many options: even though the Chinese government makes efforts to diversify its foreign asset holdings by shifting to a different currency composition or into the assets denominated in currencies other than the dollar, the move would lead inevitably to the devaluation of the dollar. Through the valuation effect, this will depress the value of dollar-denominated assets that China holds. Krugman then concludes that China can neither get itself out nor change the policies that put it in that trap in the first place.

4.2 Literature Review

4.2.1 The Debate

The Chinese dollar trap is controversial. In a wider picture, the first question to ask is why China would accumulate such a vast amount of foreign assets. The literature has already proposed several alternative theories about China's motivation for reserve hoarding: the deficient US saving view, the new economy view, the global savings glut view (Bernanke, 2005), and the Sino-American co-dependency view (Eichengreen, 2009). Although the deficient US saving view is the most prominent argument to explain global imbalances, the Sino-American co-dependency view is perhaps the most suitable explanation for the imbalance between China and the US.

These theories are not mutually exclusive and each or even all of them may contain some measure of truth about the reason behind China's reserve build-up. But to the current research interests, a deeper question is why China's reserve structure would be so dominantly lean towards the dollar assets, particularly the US government securities.

There is long debate on the prospect of the dollar's role in international reserve portfolios. Over the past decade, many commentators have discussed whether the advent of the euro in 1999 would reduce that role. The theme recurred in the context of the sustainability of the US current account deficits, high and rising government debt, risks associated with the Fed's aggressive use of unconventional monetary policies, and political gridlock in Washington. These developments have shaken confidence in the US economy, eroding the dollar's global importance. In the circumstances, the looming if not imminent displacement of the dollar as the world's leading currency challenges the wisdom of China's dollar investment policy. Prasad (2014) on the other hand believes that a case can be made that the financial crisis has actually strengthened the dollar's position as the predominant store of value in the world. Financial assets denominated in dollars, especially US government securities, are still the preferred destination for investors interested in safeguarding their investments.

Discussion of the symbiotic relationship between China and the US was highlighted by Dooley, Folkerts-Landau and Garber (2004a, 2004b, 2005, and 2009). Their Bretton Woods II hypothesis argues that trade imbalances between China and the US persist because both countries benefit from this relationship.

On the one hand, China prefers to rely on exports is a major driving force of economic growth. To maintain the growth of exports, the Chinese authorities in turn promote the competitive advantage through adoption of an undervalued currency, resulting in accumulation of large amounts of foreign reserves. This is consistent with the findings of Aizenman and Lee (2008), who indicate that from a long-run perspective, manufacturing exporters in East Asia adopt financial mercantilism to subsidize the cost of capital during decades of high growth.

From the US perspective, China's undervalued currency have triggered overvaluation of the dollar, so enabling the US to live beyond its means, maintaining the smooth running of the economy with external and internal deficits. Meanwhile, since the US financial market is one of the most developed, and much more efficient than China's market, the US imports of Chinese savings provides an important source of funding for foreign direct investment and for domestic deficits (Caballero et al., 2008 and Mendoza et al., 2009). Therefore the US government is equally satisfied with this state of affairs.

4.2.2 Export Promotion Policy

Dooley et al. (2004a, 2004b, 2005, and 2009) propose that under the new Bretton Woods system, emerging market economies maintain a development strategy featuring export-led growth. This strategy is supported by undervalued exchange rates, capital controls and accumulation of foreign reserves implying official capital outflows and accumulation of claims on the center country, the US. The success of this strategy in fostering economic growth is well documented.

Under the traditional Bretton Woods system of the 1950s, the US was the center of the system with essentially uncontrolled capital and goods markets. Europe and Japan, whose capital markets had been destroyed by the war, constituted the emerging new economies at the time. They chose a development strategy of currency undervaluation, controls on capital flows and trade, reserve accumulation, and use of the central region as a financial intermediary that lent credibility to their own financial systems. In turn, the US provide long-term support to these emerging markets, generally through foreign direct investment (FDI). Once the capital of these zones had been rebuilt and their institutions restored, the emerging economies become part of the central region.

Dooley et al. emphasize the success of a development strategy that has subordinated the objective of maximizing the value of reserve assets in order to subsidize and build a domestic capital stock capable of competing in international markets. As a result, other economies, mainly in Asia, chose to follow suit the same strategy as adopted by the immediate post-war Europe and Japan.

In spite of the growing US deficits, this revived Bretton Woods system has been stable and sustainable. In the context of China, however, some commentators do not consider the growth and employment benefits of an undervalued RMB as exceeding its costs (Goldstein, 2006; Goldstein and Lardy, 2006). They argue that there are several problems requiring attention. First, the fixed dollar exchange rate limits the independence of China's monetary policy and has contributed thereby to the pronounced macroeconomic fluctuations of recent years. Second, China's undervalued currency has contributed to growing trade surpluses and, at least in some years, also to very large portfolio capital inflows to China, which appear motivated by an expectation of appreciation. Third, a highly undervalued RMB encourages excess investment in tradable goods. Fourth, China is accumulating a large exposure to potential capital losses. Finally, China's heavy intervention in the foreign exchange market fuels protectionist pressure in the United States and elsewhere.

4.2.3 Co-dependence

This school views the financial deals between China and the US as reflection of a co-dependency relationship. While it was difficult to identify whether such cooperation between the two countries was beneficial or problematic, the global financial crisis of 2008 has demonstrated that the imbalances cannot continue. Hence, there is growing recognition of the need for policy changes in both countries. Prominent in the recent studies that promote policy changes in China is the one that highlights the imperatives of rebalancing China's growth by shifting away from relying on exports and current account surpluses towards boosting domestic demand and seeking a balanced current account (Blanchard and Giavazzi, 2006; Kuijs and Wang, 2006; Lardy, 2007; Prasad and Rajan, 2006; Prasad, 2009; Zheng, Bigsten and Hu, 2009).

While it is evident that large trade imbalances between China and the US are not sustainable, Bonatti and Fracasso (2010) attempt to understand the Sino-American financial relation from a different angle. In their recent research, they discuss the economic and political feasibility of policy shifts when focusing on the short-term and long-term trade-offs faced by the policy-makers. In particular, they argue that improper rebalancing of the Sino-US relation might be costly for both countries. A rapid closure of the imbalances would destroy China's export-led growth strategy as well as the expansion of US consumption. Other things being equal, this would lead to a lower growth rate in both countries. In addition, a dramatic shift from the tradable goods sector to the non-tradable goods sector might reduce the economy's longer-term growth rate and complicate the mobilization of the Chinese labour force, as discussed in McKinnon (2006), Hua (2007), Bonatti and Fracasso (2009), McKinnon and Schnabl (2009) and Rodrik (2009, 2010). On the other hand, downsizing the imbalances by making the US economy less dependent on domestic consumption and more oriented toward exports might bring about a relative shrinking of the labour-intensive non-tradable sector that provides most jobs, and thus generating structural unemployment (Bonatti and Fracasso, 2010).

Therefore, both the US and China face serious challenges in choosing and implementing new growth policies that are both economically feasible and politically viable. Although it is impossible to maintain the imbalances between the two countries, the rebalancing will of necessity be gradual and partial, because of the costs associated with a major shift in the growth models adopted by both China and the US (Bonatti and Fracasso, 2010). In theoretical attempts to probe this question, Obstfeld and Rogoff (OR) (2000, 2005 and 2007) quantify the consequence of closing the US current account deficits by using a two-country model. They argue that the US dollar would have to depreciate by 30% to balance the US current account. Méjean Rabanal and Sandri (2011) extend OR's model to a three-country setting comprising the US, China and the rest of Asia. Their study highlights the shift in demand of tradable goods from China to other Asian countries with a reduction of the US current account deficits.

Despite the global attention attracted by this debate, there are few analyses taking a theoretical approach from the Chinese perspective. Bonatti and Fracasso (2013a) extend their study by employing a two-country two-stage growth model to capture the characteristic of the special US-China relation. The Chinese export-led growth policy is modelled in a setting where China accumulates foreign reserves with a pegged exchange rate at an undervalued level, which stimulates production of the tradable sector. They identify the possible consequences of various policy changes from the Chinese side in three scenarios. First, the Chinese authorities shift domestic demand towards tradable goods by fiscal policy adjustment. Second, a flexible exchange rate is adopted, though this approach may turn out to be too risky to slow down the growth of the economy. Third, the export-led growth strategy is continued as the policy deems to be economically beneficial for the Chinese economy. In a further study, Bonatti and Fracasso (2013b) argue that a mercantilist motive for holding massive foreign reserves is consistent with the Chinese goal of fast GDP growth. That is, the Chinese export-led growth policy serves the Chinese economic policies well, assuming that the US maintains a sufficiently loose monetary policy.

4.2.4 Co-dependence from a Portfolio Perspective

The co-dependence between China and the US can be further understood from a portfolio perspective, since portfolio choice plays an important role in macro fluctuations. Beck and Weber (2011) argues that there is a significant positive relation between reserves allocation and the level of reserves holdings. That is, countries with a dollar anchor tend to increase dollar holdings as the level of reserves increases. However, this pattern may not always hold since the valuation effect brought about by exchange rate changes may change the level of foreign reserve holdings, such as in the case of the euro which had appreciated by about 64% relative to the US dollar by the end of 2007 (Sheng, 2013). Similarly, a moderate portfolio shift from the US dollar to the euro, without further depreciating the exchange rate of the dollar, would probably achieve a significant return from the appreciation of the euro. An additional influence to consider is the "concentration risk" proposed by Greenspan (2004), which suggests that the value of Chinese foreign reserves is closely related to the value of the US dollar. All these influences have generated growing pressure on the Chinese central bank to diversify its allocation of foreign assets.

However, current literature has not been particularly successful in explaining the portfolio choice considerations on co-dependency between China and US, either empirically or theoretically. Furthermore, there are two plausible explanations why diversification of Chinese foreign reserves seems not to have occurred. First, as the largest investor of US dollar-denominated assets, China cannot afford the declining value of the dollar due to its own action. Second, because China pursues an export-led growth strategy it has to target the US market and to maintain a stable undervalued exchange rate policy (Dooley et al., 2004a, 2004b). Therefore, if the exchange rate policy and the export-led development strategy are the main considerations of the Chinese central bank, it makes little sense to adjust the assets allocation away from the dollar. Rather, the Chinese central bank may consider buying even more dollars to maintain the value of the US currency (Lim, 2007; Truman and Wong, 2006).

4.2.4.1 The Currency Composition Issue

The literature on currency composition of official foreign exchange reserves can be traced back at least to Heller and Knight (1978). The literature can broadly be classified into two general categories, one is empirical models attempting to understand the currency allocation of reserve portfolios, this literature employs observable data from countries that publicly report relevant information, and the other is a series of theoretical studies that derive the optimal currency composition of reserves by using the portfolio management theory.

Empirical work on the determinants of currency composition of reserves has been hampered by a lack of publicly available data on reserve portfolios of individual countries. This is especially so in the case of China, which does not release its currency shares of official reserves to International Monetary Fund. The well-known data source for analysts to gauge foreign holdings of US dollar securities is the US Treasury International Capital (TIC) System database. However, as argued in Prasad and Wei (2007), the TIC reports only the dollar denominated assets, not other currencies. Even for the dollar securities, it does not cover the US dollar-denominated instruments purchased by the Chinese central bank outside the US, while it includes the US dollar-denominated instruments purchased by other financial institutions from China. The Currency Composition of Official Foreign Exchange Reserves (COFER) provided by the International Monetary Fund (IMF) is another popular data source for Currency Composition analysis. However, China does not report its currency portfolio to the IMF. Thus there is no publicly available resource to obtain a direct measure of the currency composition of China's official foreign reserves.

In a study using for the first time a confidential data set on the currency composition of official reserves of 76 countries, Heller and Knight (1978) find that a country's exchange rate regime and its trade patterns are significantly related with the currency composition of its reserves. Hence the authors conclude that export-led growth strategy plays a major role in determining the currency composition of reserves. Similar results are found in Dooley et al. (1989), and Eichengreen and Mathieson (2000), who also account for the currency composition of external debt or financial flows. Using the aggregate COFER data, Chinn and Frankel (2007, 2008) find that the size of the home country, size of the home financial market, exchange rate volatility and inflation rate of the reserve currency are significant determinants of the currency shares of central bank reserves. This is consistent with findings in the previous literature. Meanwhile, Lim (2007) and Wong (2007) and examine the impact of exchange rate changes on aggregate currency shares of foreign exchange reserves and document that currency diversification in response to exchange rate changes has thus far tended to be stabilizing for foreign exchange markets; that is, central banks have tended to pursue portfolio rebalancing (Perold and Sharpe, 1995) in which they buy falling currencies rather than following the market trend strategies in which one would buy rising currencies. Lim (2007) concludes that these findings are consistent with the relatively stable currency shares in the COFER database. He also suggests that the findings may support the view that optimal reserve portfolios have hardly changed over time. Furthermore, Lim (2007) suggests that his findings may also support the view that optimal reserve portfolios may have changed over time, but reserve managers have, in general, implemented the change very gradually.

The empirical literature finds evidence for not only a relatively stable share of currency composition, but also a strong relationship between reserves currency composition and undervalued exchange rate policy. However, the existing theoretical literature has for the most part ignored these findings and derived the currency composition of optimal reserves as the solution to an international version of a Markowitz type portfolio problem. The discussion then mainly revolves around the correct method of applying the optimal portfolio theory in an international context, including, the choice of deflator to calculate real returns from nominal returns, derivation of exchange rate return expectations, etc. The literature does not explicitly take into account the fact that central banks pursue objectives different from those of a conventional investor. The resulting optimal portfolio is then compared to actual portfolios and the small difference between the two is interpreted as support for the hypothesis that central banks pursue portfolio objectives. Thus, Ben-Bassat (1980) suggests applying mean-variance optimization in terms of a basket of import currencies. Using data from 1976 to 1980, he compares optimal to actual reserve portfolios and suggests portfolio objectives are an important determinant of the currency composition of reserves of the emerging market economies, but not of industrialized countries. With data on South Korean reserve composition and the actual currency denomination of the country's imports, Dellas and Yoo (1991) test both an import-based version of the consumption capital asset pricing model (CCAPM) and a mean-variance optimization model. Their results indicate that the actual central bank portfolio is quite close to the efficient frontier computed and that the restrictions implied by the CCAPM could not be rejected, but admit that the power of these tests is low.

4.2.4.2 Asset Portfolio Choice under the Macro Circumstance

In parallel with the research on currency composition, a series of studies has attempted to explain portfolio choice under the macro circumstance, using portfolio balance models. For example, rather than assuming a closed economy in which a representative agent holds an indifferent market portfolio, Lucas (1982), established a fully optimizing model of portfolio choice by assuming households trade bonds, equities and government monetary transfers. However, these were partial equilibrium models and the portfolio choice elements were never fully integrated in the general equilibrium models. From the early 1980s these models have been gradually replaced by general equilibrium models with optimizing agents (Branson and Henderson, 1985). But these general equilibrium international macro models at the time ignore portfolios, that is, the existence of and choice between several available asset types, altogether. Typically, models adopt either a setup of complete international financial markets, or the case where only a single asset is traded, and focus their attention only on net external flows.

While a large and growing body of literature has considered portfolio choice, this literature is concerned with portfolio choice in the steady state rather than time-variation in portfolio choice that gives rise to capital flows. In addition, most studies assume completeness of international financial markets, which is not realistic. Contributions to this literature include Engel and Matsumoto (2009), Heathcote and Perri (2013), Kollmann (2006), Coeurdacier and Gourinchas (2011), and Coeurdacier et al. (2007). Until recently, portfolio choice issues are only considered as a special case of complete financial markets, in which the behaviour of macro variables and portfolios are independent, and the latter can be backed out after having obtained a solution to macro dynamics.

With regard of the solution method under the general equilibrium assumptions, several papers have attempted to understand portfolio choice using dynamic models with approximation methods. In particular, Devereux and Sutherland (2010) derive an approximation method for an economy with incomplete markets, but constant portfolio shares, and apply it to a two-country general equilibrium model with production and trade in equities, and to a two-country endowment model with trade in real bonds. Devereux and Sutherland (2011) apply this model to a sticky-price monetary model that allows for portfolios of bonds and equity trade. Then, they extend the approximation method to allow for time-varying portfolios, and apply the method to a two- country endowment model with trade in real bonds (Devereux and Sutherland, 2007). In a subsequent study (Devereux and Sutherland, 2009), the authors examine a similar model, with a focus on the role of changes in valuation for the international distribution of wealth. Tille and van Wincoop (2010a) use a similar approximation to solve a two-country general equilibrium model with capital and production and trade in equities. Then, they use these methods to examine the response of the current account and net foreign assets to changes in savings (Tille and van Wincoop, 2010b). Also, Evans and Hnatkovska (2012) examine a similar model with a related solution methodology.

4.3 The Baseline Model

Our model is developed in a dynamic stochastic general equilibrium framework, which consists of two countries: the home country, denoted by H; and the foreign country, denoted by F. Given research purposes of this chapter, the home country implies China and the foreign country is represented by the United States. There are five types of agent in our model: households, intermediate goods producers, final goods producers, financial intermediaries and government.

In such a model setting, we focus on examining the puzzling phenomenon that China would hold its official reserves mostly in the US dollar assets and at such a large scale. Ultimately, this would answer the question of what are the rationale behind China's choice of falling and staying in the dollar trap. To sovlve the puzzle of China's stuck in the dollar trap, we take an investment approach for the investigation, in which the asset composition of Chinese foreign reserve holdings is viewed as a result of asset portfolio choice. With this approach, we consider accumulation of foreign reserves and reserve management as an integral process. On the one hand, China's stockpiling of foreign reserves in final analysis is a result of households' prudential savings behaviour; on the other hand, the management of foreign assets is the duty of fund managers appointed by the central bank for handling the investment of Chinese foreign reserves. These two processes combine into one when the reserve managers make investment of foreign reserves held on behalf of households.

The population of each country has a continuum of households, measured in units, who live infinite horizons. We assume that the representative household chooses labour supply, consumption, and the wealth level in each period. Intermediated goods are produced by intermediate producers using labour and capital, and sold to final goods producers. Differentiated products are supplied to the consumption market by wholesale producers as imperfectly substitutable goods. Government maintains a balanced budget by collecting a lump-sum tax and issuing government bonds. The central bank maintains a conventional monetary policy according to a Taylor rule.

Our model is distinct from conventional two-country DSGE models in three aspects. First, while representative households in both countries make their savings in both home and foreign assets, financial market in the home country is so seriously underdeveloped that its international investment is intermediated through the domestic agent, i.e. the reserve management fund. The representative only owns assets as deposits. Second, the reserve management fund are the key financial intermediation that collects all deposits from the household to make investment in foreign capital markets. Also, following Devereux and Sutherland (2009), we assume that only foreign bonds are traded in international capital markets. This assumption captures the widely accepted view that US bonds dominate the international bond market and emerging market bonds are almost untradeable in that market. Third, financial intermediaries allocate their portfolio under a Value-at-Risk (VaR） constraint as suggested in Aoki and Sudo (2012).

4.3.1 Households

Consumption of each country is a composite of goods demanded for home country and foreign country, which is aggregated via a CES function:

$$C_t = \left\{ (\omega)^{\frac{1}{\rho}} (C_{H,t})^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} (C_{F,t})^{\frac{\rho-1}{\rho}} \right\}^{\frac{\rho}{\rho-1}} \tag{4.1}$$

where $\omega = 0.5$ represents the degree of home bias in consumption and $\rho > 0$ is the elasticity of substitution between goods from different countries. The consumer price index for the home country can be derived from the households' expenditure minimization problem and is given by:

$$P_t = \left\{ \omega P_x^{\frac{\rho}{\rho-1}} + (1-\omega)(e_t P_x^*)^{\frac{\rho}{\rho-1}} \right\}^{\frac{\rho-1}{\rho}} \tag{4.2}$$

where P_x and P_x^* denote price of home product goods and foreign product goods, respectively in terms of home currency. Also $\omega = 0.5$ represents the degree of home bias in consumption and $\rho > 0$ is the elasticity of substitution between goods from different countries. We assume that the law of one price holds, therefore both countries faces the same prices after adjustment by the nominal exchange rate (e_t).

Taking the home goods as the numeraire, the above consumer price index is given by:

$$P_t = \left\{\omega + (1-\omega)e_t^{\frac{\rho}{\rho-1}}\right\}^{\frac{\rho-1}{\rho}} \quad (4.3)$$

where e_t represents the exchange rate, defined as the relative price of the goods produced in the two countries. Therefore an increase in the exchange rate represents an appreciation of the home currency, or the RMB. The demands for domestic goods and imported goods by the home consumer are respectively given by:

$$C_{H,t} = \omega(P_t)^{\rho} C_t \quad (4.4)$$

$$C_{F,t} = (1-\omega)(e_t P_t)^{\rho} C_t \quad (4.5)$$

From the above relationships, an appreciation of the RMB shifts consumer demand from home country to foreign country. Therefore, the allocation of goods across the countries is determined by the exchange rate.

The representative consumer chooses consumption and labour supply to maximize his lifetime utility. c_t represents the consumer's consumption level at the current period, while h_t represents labour supply. The parameters σ and υ represent the inverse inter-temporal elasticity of substitution with respect to consumption and labour supply. The household's problem at time t is to choose paths for consumption c_t, labour supply h_t and aggregate saving holdings d_t to maximize utility, given by:

$$maxE_t\left[\sum_{j=0}^{\infty}\beta^j\,(\zeta_{c,t+j}^{\frac{1}{\sigma}}\frac{c_{t+j}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}-\frac{\psi}{1-\upsilon}\zeta_{h,t+j}^{-\upsilon}h_{t+j}^{1+\upsilon})\right] \tag{4.6}$$

where β represents the discount factor. The representative household decides his consumption c_t based on the level of total saving holding, wage income $w_t h_t$ and lump sum tax τ_t ; saving holding is deposited through financial intermediaries, yielding R_t. We assume that deposits are risk-free assets and the real deposit rate is the real risk-free rate. Therefore, the representative household's period budget constraint is:

$$P_t d_t + P_t c_t + P_t i_t = P_{t-1}d_{t-1} + P_t w_t h_t - P_t \tau_t \tag{4.7}$$

The first order necessary conditions yield:

$$\zeta_{c,t+j}^{\frac{1}{\sigma}} c_{t+j}^{1-\frac{1}{\sigma}} = \Lambda_t P_t \tag{4.8}$$

$$\psi \zeta_{h,t}^{-\upsilon} h_t^{1+\upsilon} = \Lambda_t P_t \omega_t \tag{4.9}$$

$$\Lambda_t = \beta R_t E_t \Lambda_{t+1} \tag{4.10}$$

where Λ_t denote the Lagrange multiplier of the constrain. The behaviour of the foreign country is denote by a $*$. We also have the following risk sharing condition:

$$\Lambda_t = \frac{\Lambda_t^*}{e_t} \tag{4.11}$$

4.3.2 Financial Intermediation

A risk-neutral financial intermediary collects deposits from the household and purchases domestic equity claims k_t and two international tradable assets: equity claims on the foreign country k_t^* and foreign currency denominated bonds b_t. Therefore this financial intermediary serves as the investment agent for the country, aiming to maximize the portfolio holding position of the home country. The financial intermediary's balance sheet in each period is given by:

$$k_t + k_t^* + b_t^* = r_{k,t} k_{t-1} + r_{k,t}^* k_{t-1}^* + r_{b,t}^* b_{t-1}^* - r_t d_{t-1} + d_t \tag{4.12}$$

where $r_{k,t}$ $r^*_{k,t}$ $r^*_{b,t}$ denote the return from three different types of assets, and in each case they are the ex-post real returns. Defining the net worth of financial intermediary as nw_t, the intermediary receives returns on the three types of assets invested in the previous period and repays the household deposit with a risk free rate of return r_t. Consequently, the net worth's law of motion evolves according to the following form:

$$nw_t, = r_{k,t}k_{t-1} + r^*_{k,t}k^*_{t-1} + r^*_{b,t}b^*_{t-1} - r_t d_{t-1} \qquad (4.13)$$

The position of net foreign assets can therefore be written as $nfa_t = k^*_t + b^*_t$, and the law of motion of net foreign assets is given by:

$$nfa_t = nw_t + d_t - b^*_t \qquad (4.14)$$

The financial intermediary accumulates net worth up until when it exits from the economy. We assume that, in each period, the financial intermediary has an exit probability of γ. Therefore, the financial intermediary's maximization problem is to maximize the discount value of net worth:

$$V_t = \beta E_t M_{t,t+1}[\gamma V_{t+1}(nw_{t+1}) + (1-\gamma)nw_{t+1}] \qquad (4.15)$$

where $M_{t,t+1}$ represents the household's stochastic discount factor from period t to period t+1.

4.3.3 VaR Constraint

In financial risk management, VaR is a widely used measure of the risk of loss on a specific portfolio of financial assets. To capture the risk of loss to the portfolio choice, we follow Aoki and Sudo (2012) to place a VaR constraint on the financial intermediary's maximization problem. This constraint enables the financial intermediary to repay the deposit to the household even if the portfolio investment experiences the maximum loss. Denoting the maximum loss return of the assets as $\underline{r}_{k,t}$, $\underline{r}^*_{k,t}$, $\underline{r}^*_{b,t}$, the VaR constraint is given by:

$$E_t \underline{r}_{k,t} k_{t-1} + E_t \underline{r}^*_{k,t} k^*_{t-1} + E_t \underline{r}^*_{b,t} b^*_{t-1} - r_t d_{t-1} \geq 0 \qquad (4.16)$$

Here, we assume that foreign equity claims carry a larger risk than foreign government bond holdings, and domestic equity claims are risky compared with foreign bond holding. Therefore, we have the following relationship between different asset returns: $E_t \underline{r}^*_{k,t} \geq E_t \underline{r}_{k,t} \geq E_t \underline{r}^*_{b,t}$. However, the maximum loss could vary over time which is associated with shocks to the economic circumstance, such as increasing uncertainty in terms of current global economy outlook and downside risks regarding to the retunes of each assets. Besides this loss is associated with institutional changes, such as a strengthening of capital requirements, which changes directly financial intermediaries' risk tolerance.

In contrast to the original application of VaR constraint by Aoki and Sudo (2012), the VaR constraint in our model not only influences the financial intermediary's leverage level but also, and more importantly, it affects the choice of asset portfolio allocation. With VaR constraint, the financial intermediary's maximization problem becomes as follows: The financial intermediary maximizes its value function (4.17) subject to the balance sheet constraint (4.12), the VaR constraint and the net worth's law of motion (4.13). To solve the maximization problem, we apply the standard procedure to derive the solution with Bellman function. First we guess the value function of the intermediation as:

$$V_t = \phi_t nw_t \tag{4.17}$$

By plugging equation (4.17) into equation (4.12), we have:

$$maxV_t$$
$$= \beta E_t \mathrm{M}_t \begin{bmatrix} \gamma\phi_t\left(\left(r_{k,t+1} - r_t\right)k_t + \left(r^*_{k,t+1} - r_t\right)k^*_t + \left(r^*_{b,t+1} - r_t\right)b^*_t + r_{t+1}nw_t\right) \\ +(1-\gamma)\left(\left(r_{k,t+1} - r_t\right)k_t + \left(r^*_{k,t+1} - r_t\right)k^*_t + \left(r^*_{b,t+1} - r_t\right)b^*_t + r_t nw_t\right) \end{bmatrix} \tag{4.18}$$

The corresponding first order condition is derives as:

$$E_t\left[\frac{(\gamma\phi_{t+1}+1-\gamma)M_t(r_{k,t}^*-r_t)}{\underline{r}_{k,t}^*-r_t}\right]=E_t\left[\frac{(\gamma\phi_{t+1}+1-\gamma)M_t(r_{k,t}-r_t)}{\underline{r}_{k,t}-r_t}\right]=$$

$$E_t\left[\frac{(\gamma\phi_{t+1}+1-\gamma)M_t(r_{b,t}^*-r_t)}{\underline{r}_{b,t}^*-r_t}\right] \qquad (4.19)$$

The above equation provides the condition for asset allocation condition of three different assets: domestic equity, foreign equity and foreign bonds. Accounting for the VaR constraint, the expected excess returns of these assets are not necessarily equal in equilibrium. It worth note that all three equation have a sharing part $(\gamma\phi_{t+1}+1-\gamma)$, the expect return of each assets is subject to the variation of the maximum loss return of each asset. Recalling the above assumption of asset returns, the maximized loss returns of these assets exhibit such a relationship $\underline{r}_{k,t}^* \geq r_{k,t} \geq \underline{r}_{b,t}^*$. Accordingly, to maintain the equivalence of equation (4.18), the expected returns of these three assets should follow the relationship $E_t r_{b,t}^* \geq E_t r_{k,t} \geq E_t r_{k,t}^*$.Also, from equations (4-16) and (4-19) we can obtain the expression for ϕ_t,

$$\phi_t=\beta E_t\left[M_t(\gamma\phi_{t+1}+1-\gamma)r_t\frac{(1-(r_{k,t}-r_t)}{r_{k,t}-r_t}\right] \qquad (4.20)$$

4.3.4 Retail Firms

Turing to the production sector, the retail firms produce a non-tradable final good by using foreign and domestic goods. It determines its optimal production to maximize its profit,

$$\max_{\{x_t^d, x_t^f\}} P_t Y_t - P_{x,t} x_t^d - e_t P_{x,t}^* x_t^f \tag{4.21}$$

where $P_{x,t}$ and $P_{x,t}^*$ represent the price of the home country and foreign good respectively, which denominated in terms of the currency of the export country. And the final good production function is as follows,

$$y_t = (\omega^{\frac{1}{1-\rho}} x_t^{d\rho} + (1-\omega)^{\frac{1}{1-\rho}} x_t^{f\rho})^{\frac{1}{\rho}} \tag{4.22}$$

where ω denotes the degree of home bias in consumption and ρ as the elasticity of substitution between goods from different countries. Optimal behaviour of the retailer determined the demand for the home and foreign goods.

$$x_t^d = (\frac{P_{x,t}}{P_t})^{\frac{1}{1-\rho}} \omega y_t \tag{4.23}$$

$$x_t^f = \left(\frac{e_t P_{x,t}}{P_t}\right)^{\frac{1}{1-\rho}} (1-\omega) y_t \tag{4.24}$$

4.3.5 Intermediate Firms

The intermediate firms has a local monopoly power, and each intermediate firm j, $j \in (0,1)$, apply a constant returns to scale technology,

$$x_t(j) = a_t k_t(j)^{\alpha} h_t(j)^{1-\alpha} \tag{4.25}$$

where k_t and h_t represent capital and labour used for production. The intermediate firms minimizes its real cost $s_t x_t(j)$ subject to (4.25). The real marginal cost is given by,

$$s_t = \frac{z_t^{\alpha} \omega_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha} a_t} \tag{4.26}$$

Integrating across all the intermediate firms, we have,

$$\int_0^1 \left(\frac{P_{x.t}(j)}{P_{x.t}}\right) dj y_t = a_t k_t^{\alpha} h_t^{1-\alpha} \tag{4.27}$$

where $P_{x.t}$ denote the price level of all intermediate goods.

4.3.6 Government or the Central Bank

Given the general practice of central banks, it is customary to assume the nominal interest rate to be the main instrument of monetary policy. Nominal and real interest rates are related by the Fischer equation as follows:

$$E_t(1 + r_{t+1}) = E_t(\frac{1+R_t}{\pi_{t+1}}) \quad (4.28)$$

where R_t is the nominal interest rate, r_{t+1} is the real interest rate and $\pi_{t+1} = P_{t+1} / P_t$ is the inflation rate. R_t serves as a policy variable set at the beginning of period t. Hence conventional monetary policy follows a Taylor-type rule:

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1 - \rho_r)(\log(\overline{\mathrm{R}}) + \gamma_\pi(\log(\pi_t) - \log(\overline{\pi})\big) +$$

$$\gamma_y\big(\log(y_t) - \log(\overline{y})\big)\big) + \varepsilon_M \quad (4.29)$$

where all the variables with hat represent the steady state value, ρ_r is the autoregressive coefficient of the policy rate, γ_π is the policy weight attached to the inflation rate and γ_y denotes the weight attached to the output gap . Last ε_M represents shocks to the monetary policy rule.

The budget constraint of the government is given by:

$$g_t = \tau_t \tag{4.30}$$

The government finances its expenditure g_t by collecting a lump-sum tax τ_t.

4.4 Steady State Analysis

To emphasize the asset allocations of financial intermediaries, we explore the determination of the portfolio choice problem before investigating the dynamics of the whole model. Given our interests in the level and structure of international portfolio of China's foreign reserves, here we focusing on investigating into how the expected returns from holding the two foreign risky assets in the steady state, denoted as r_k^* , r_b^* , are affected by the VaR constraint of financial intermediaries. Hence, the domestic asset is not included in our steady state analysis. For the sake of straightforward illustration, in this section we also assume that the labour supply by households is inelastic, i.e. $h = 1$.

By evaluating the variables included in the portfolio choice equation, the VaR constraint and the law of motion of financial intermediary's net worth at the steady state values, we have:

$$\frac{\bar{r}_k^* - \bar{r}}{\bar{r} - \underline{r}_k^*} = \frac{\bar{r}_b^* - \bar{r}}{\bar{r} - \underline{r}_b^*} = \frac{\bar{r}_k - \bar{r}}{\bar{r} - \underline{r}_k} \tag{4.31}$$

$$\left(\underline{r}_k^* - \bar{r}\right)\bar{k}^* + \left(\underline{r}_b^* - \bar{r}\right)\bar{b}^* + \left(\underline{r}_k - \bar{r}\right)\bar{k} = -\bar{r} * \overline{nw} \tag{4.32}$$

$$\overline{nw} = \frac{\gamma}{1-\gamma\bar{r}}\left[\left(\left(\underline{r}_k^* - \bar{r}\right)\bar{k}^* + \left(\underline{r}_b^* - \bar{r}\right)\bar{b}^* + \left(\underline{r}_k - \bar{r}\right)\bar{k}\right)\right] \tag{4.33}$$

The household's Euler equation in the steady state implies that:

$$\bar{r} = \frac{1}{\beta} \tag{4.34}$$

The three equations (4.31) - (4.33) yield the excess returns from holding two foreign risky foreign assets, and the spreads between the returns on the assets are given by:

$$\bar{r}_k^* - \bar{r} = \frac{1-\gamma\bar{r}}{\gamma\bar{r}}\bar{r} - \underline{r}_k^* \tag{4.35}$$

$$\bar{r}_b^* - \bar{r} = \frac{1-\gamma\bar{r}}{\gamma\bar{r}}\bar{r} - \underline{r}_b^* \tag{4.36}$$

In equations (4.35) and (4.36), the excess returns of those on foreign risky assets over the steady state returns, are expressed in the expected maximum loss from holding the foreign assets $\underline{r}_k^*$ and $\underline{r}_b^*$, together with financial intermediary's survival probability γ.

In this setting, when the maximum loss of holding equity investment $\underline{r}_k^*$ rises, the financial intermediary's VaR constraint becomes tighter as the value of

$\underline{r}_k^*$ declines, which implies a reduction in the financial intermediary's risk tolerance. If the financial intermediary maintains the same amount of saving holdings form consumer, then compared with government bonds it would require a higher spread to hold equity investment compared with bond investment. However, the government bond yield is unaffected by the change in $\underline{r}_k^*$. There is a similar mechanism in operation if the maximum loss of holding the government bonds $\underline{r}_b^*$ increases.

In contrast to the case of maximum loss of holding returns, a reduction in survival probability γ leads to a rise in the excess returns associated with the two types of assets. As indicated by equation (4.33), the smaller survival probability prevents financial intermediaries from increasing their net worth. Because by increasing the risk of default a lower level of net worth tightens the VaR constraint, and hence, financial intermediaries would accordingly downsize their capital sheets. Since financial intermediaries' demand for both assets falls, the excess returns on them needs to rise to meet asset market supply.

Next we discuss how financial intermediaries allocate their assets between investments in equity $\bar{k}^*$ and in government bonds $\bar{b}^*$. However it is impossible to calculate the real holding of each type of assets. Therefore we calculate the ratio of bond holding to equity holding to illustrate the allocation between this two types of assets. Because the return from holding equity investment $\bar{r}_k^*$ equals the return to capital stocks in the foreign economy, we have:

$$\bar{r}_k^* = \alpha\bar{a}{\bar{k}_{Total}^*}^{\alpha-1} - (1-\delta) \qquad (4.37)$$

Rearrange the above equation we have the total capital stocks for foreign country:

$$\bar{k}_{Total}^* = \left[\frac{\bar{r}_k^* - (1-\delta)}{\alpha\bar{a}}\right]^{\frac{1}{\alpha-1}} \qquad (4.38)$$

Taking into consideration that $\alpha - 1 < 0$, a higher return on equity investment implies a smaller amount of total capital stocks and thus a lower amount of investment in the economy. Based on the considerations above, an increase in the maximum loss on equity investment or a decline in financial intermediaries' survival probability reduces their investment to foreign equity through a rise in the return on the capital stocks $\bar{r}_k^*$.

The financial intermediary's decision regarding the holding of government bonds is determined by the foreign government's policy on taxes and the budget balance. The budget constraint of the government is given by:

$$r_{b,t-1}^* b_{Total,t-1}^* + g_t^* = \tau_t^* + b_{Total,t}^* \qquad (4.39)$$

The government finances its expenditure g_t^* and the payment to previous period government bonds by collecting a lump-sum tax τ_t^* and issuing government bonds $b_{Total,t}^*$. The government lump-sum tax is specified as the following:

$$\tau_t^* = b_{Total,t-1}^* (\frac{b_{Total,t-1}^*}{y_t})^{\psi} T \tag{4.40}$$

where χ is the elasticity of the lump-sum tax with respect to the amount of outstanding government debt. Therefore we have,

$$\bar{r}_b^* \bar{b}_{Total}^* = T \bar{b}_{Total}^{*}{}^{\chi} + \bar{b}_{Total}^* \tag{4.41}$$

$$\bar{b}_{Total}^* = \left[\frac{\bar{r}_b^* - 1}{T}\right]^{1/\chi} \bar{y}^* = \left[\frac{\bar{r}_b^* - 1}{T}\right]^{1/\chi} \bar{a} \bar{k}_{Total}^{*}{}^{\alpha} \tag{4.42}$$

Here, we assume that the inflation rate is unity in the steady state. These equations suggest that, for $\chi > 1$, financial intermediaries shift their investment holding towards government bonds as the returns on them increase. Along with an increase in the bond return, an equivalent increase in government bond issuance is followed, which eventually leading to a higher level of government bond holdings by financial intermediaries under the tax policy described by the equations above.

Finally, we discuss how financial intermediaries allocate their assets between government bond holdings and equity investment. From equations (4.38) and (4.42), the ratio of government bond holdings relative to equity holding is given by:

$$\frac{\bar{b}^*_{Total}}{\bar{k}^*_{Total}} = \left[\frac{\bar{r}^*_b - 1}{T}\right]^{1/\chi} \left[\frac{\bar{r}^*_k - (1-\delta)}{\alpha}\right] \tag{4.43}$$

According to equation (4.43), any changes in the economic environment that increases the returns on the two assets, $\bar{r}^*_k$ and $\bar{r}^*_b$, e.g. an increase in the maximum loss on equity investment or of bonds holding, or a deterioration of financial intermediaries' net worth, leads financial intermediaries shift their portfolio allocation to bond investment.

4.5 Empirical Analysis

In this section, we first attempt to explore the effeteness of the VaR constraint to our two country model. We calibrate the model using predetermined parameter values in the literatures. Later based on data from both China and US, we estimate the model's parameters and a couple of structural shocks.

Table 4.2 Parameters in the Model

Parameter	Description	Parameter	Description
β	Discount factor	α	Elasticity of labour to capital
σ	Risk Aversion	ρ_r	Coefficient of the policy rate
υ	Inverse elasticity of labour supply	γ_π	Coefficient of the policy rate
$1-\alpha$	Labour share	γ_y	Coefficient of the policy rate
δ	Depreciation rate		
γ	Financial intermediation survive rate	ρ_a	Technology persistence
ω	Import share	ρ_c	Consumption persistence
ρ	Substitutability between domestic and foreign goods	ρ_h	Labour persistence
χ	the elasticity of the lump-sum tax		

To characterize the Chinese economy we choose appropriate parameter value along with previous literature. Staring with the household sector, we set the discount factor $\beta = 0.99$, which corresponds to an equilibrium annual interest rate of 4%. Risk aversion is set to $\sigma = 2$ by following Walsh (2003) and the inverse elasticity of labour supply is set to be $\upsilon = 0.1$.For the firm sector, we set the output elasticity of labour to capital $\alpha = 0.4$, which represents a higher investment share in equilibrium (He et al. 2007). To capture the shorter depreciation period featuring the Chinese industrial policy, we set the depreciation rate of capital $\delta = 0.05$ (He et al. 2007). Apart from above parameters, other parameters are set with standard value.

Table 4.3 Steady State Value of Selected Variables

	Without VaR Constraint	With VaR Constraint
Output H	0.59	0.73
Output F	0.59	0.73
Consumption H	0.45	0.44
Consumption F	0.45	0.44
Equity Investment	3.97	4.8
Bond Investment	0.6	2.06
Ratio of Bond to Whole Portfolio	13.00%	41.60%
Return of Equity	1.05	0.99
Return of Bond	1.01	1.07

Table 3 reports the steady state value of selected variables with and without VaR constraint. The economy with VaR constraint is slightly richer compared with the economy without VaR constraint. However, the consumer's behaviour is shifted by the risk consideration to a lower level. Another distinctive difference is the investment allocation for the financial intermediary. With VaR constraint, the bond component of the portfolio increases from 13% to nearly 42% of the whole portfolio. This result is backed by the rate of return from these two types of assets, where the risk adjusted return of bond is clearly higher than the return from equity investment. Therefore, it can be concluded that the VaR constraint shapes the consumer's behaviour significantly, resulting in a remarkable increase in investment. Furthermore, the financial intermediary rebuilds their portfolio into a more profitable asset in terms of future uncertainty. The quantitative result is consistent with the previous qualitative analysis, indicating that VaR constraint has an effective influence. Moreover, the two countries experience a greater economic performance with VaR constraint, which might be evidence of a co-dependence relationship between China and the US.

4.5.1 Simulation Results with Predetermined Parameters

From Figure 4 to Figure 9 we report the impulse response functions to a one standard deviation from six different shocks with and without VaR constraint for 50 periods. The dashed line shows the responses in the specification without VaR constraint, while the solid line depicts the responses in the specification with VaR constraint. The red line represents the steady state of the economy. It is clear that the VaR constraint shifts the trend of the economy dramatically in terms of responses to all exogenous shocks.

Figure 4.4 Impulse Response Functions to Technology Shocks (H)

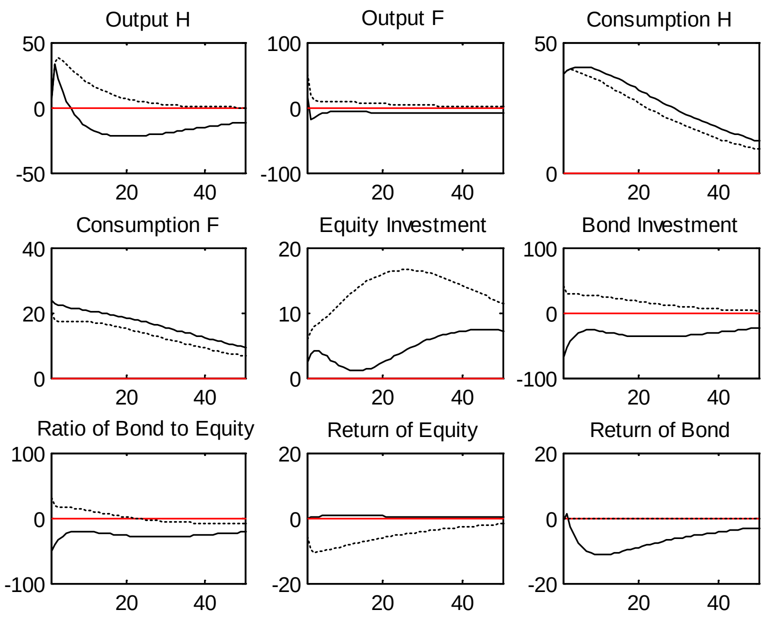


Figure 4.5 Impulse Response Functions to Technology Shocks (F)

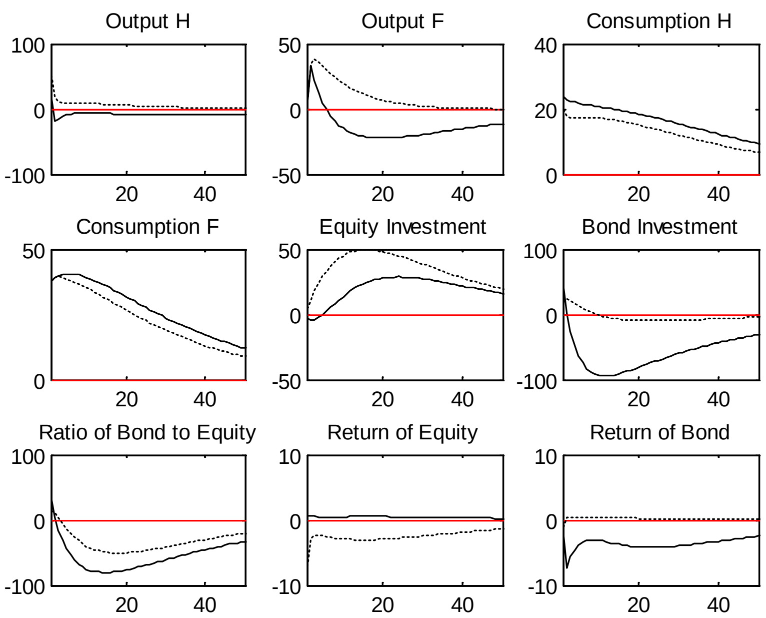


Figure 4.6 Impulse Response Functions to Consumption Shocks (H)

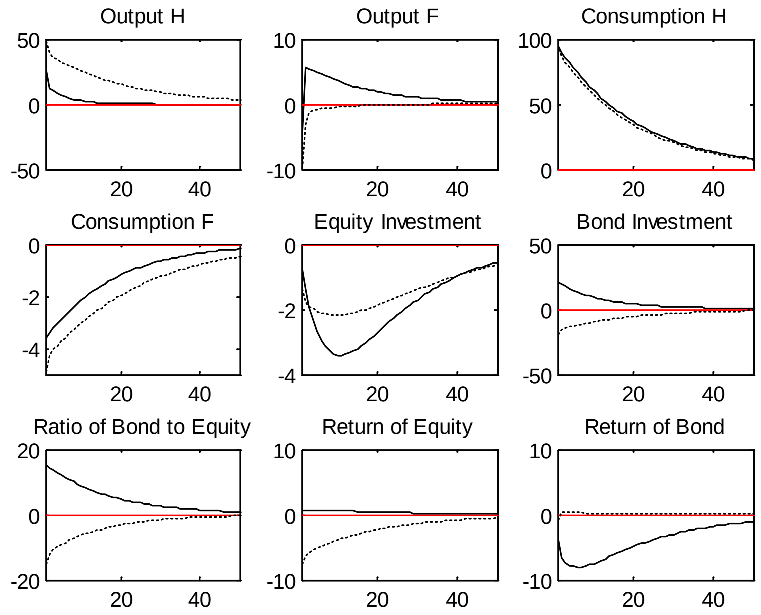


Figure 4.7 Impulse Response Functions to Consumption Shocks (F)

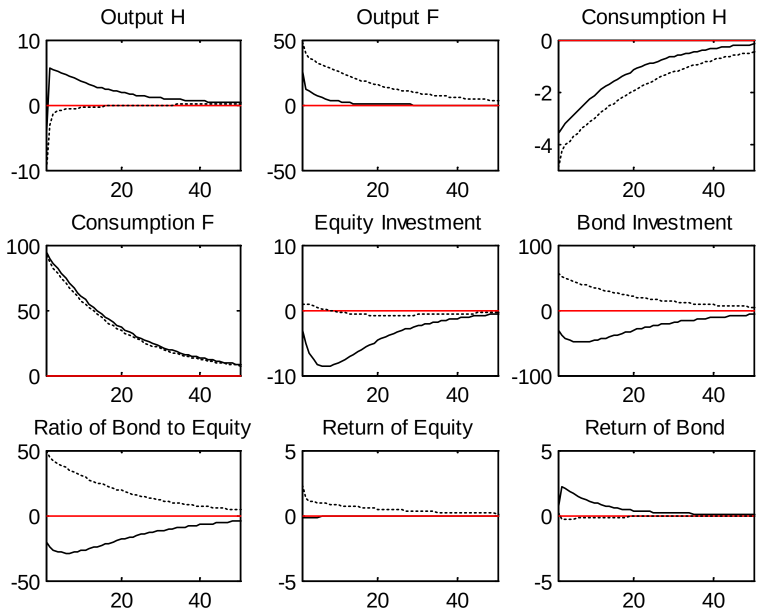


Figure 4.8 Impulse Response Functions to Labour Shocks (H)

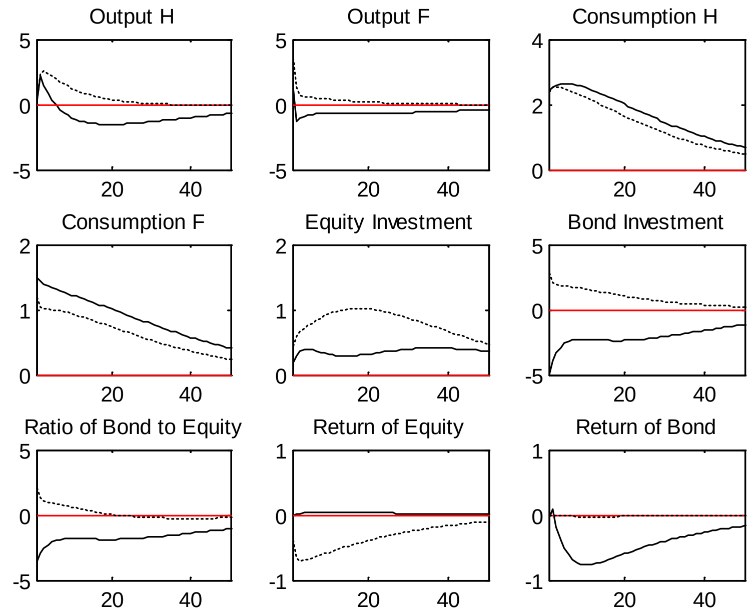


Figure 4.9 Impulse Response Functions to Labour Shocks (F)

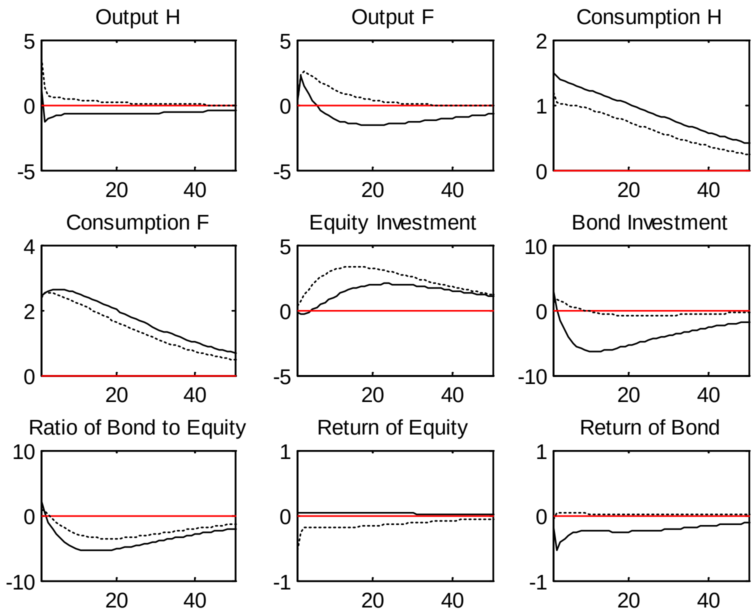


Figure 4.10 Impulse Response Functions to Monetary Shocks (H)

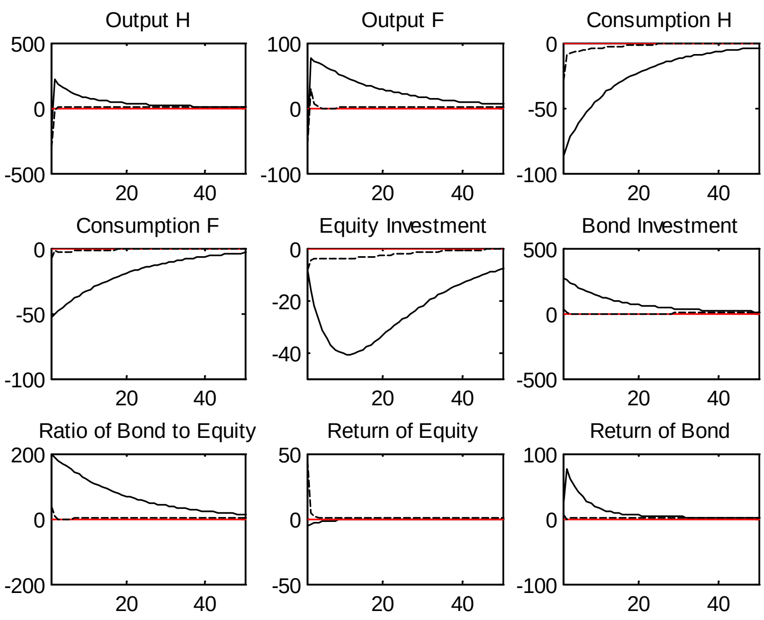
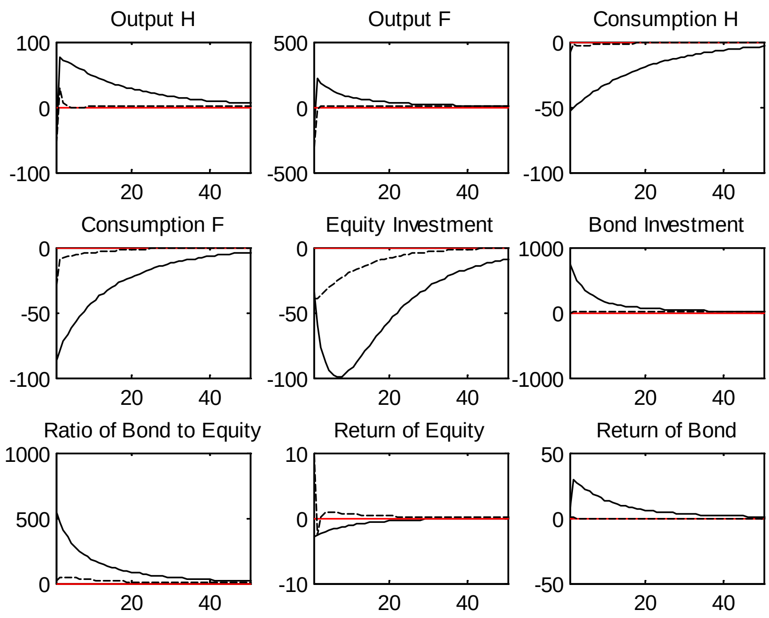


Figure 4.11 Impulse Response Functions to Monetary Shocks (F)



It can be concluded from the above figures that VaR constraint plays a significant role in the economic performance. From a portfolio perspective, we focus on the response to the return from equity and bond. Combined with the variance decomposition of variables by all exogenous shocks, monetary policy shock accounts for the majority of variance of return of equity and return of bond. With VaR constraint, foreign country monetary policy shock accounts for 54.50% of variation of the return of equity investment. Domestic monetary shock accounts for 63.39% variation of the return of bond. This result is consistent with the finding of Devereux and Sutherland (2006) that monetary policy shock plays a significant role in the optimal portfolio choice. More specifically, with one standard deviation of positive home monetary policy or foreign monetary policy shock, there is an immediate positive increase in the return of bond, while the return of equity drops to negative territory. In line with the impulse response function to return on equity and bond, the actual holding of each type of investment is consistent with the return of each asset: with a positive monetary policy shock from the home country, the portfolio weight shifts immediately to bond, while a similar effect is observed with a positive monetary policy shock from the foreign country.

Table 4.4 Variance Decomposition of Variables with VaR Constraint

Variables	ea	eas	exc	excs	exh	exhs	em	ems
Output (H)	6.91	0.93	0.44	0.12	0.03	0.01	73.36	18.19
Output (F)	2.16	3.16	0.12	0.46	0.01	0.03	18.69	75.37
Consumption (H)	20.94	3.34	44.88	0.06	0.08	0.02	20.14	10.54
Consumption (F)	8.12	10.06	0.06	48.49	0.02	0.09	11.39	21.76
Equity Holding	1.49	7.21	0.14	0.67	0.01	0.08	17.1	73.3
Bond Holding	3.25	4.83	0.13	2.21	0.01	0.05	19.55	69.97
Ratio of bond to equity	2.94	5.65	0.14	1.06	0.01	0.06	19.57	70.58
Return of equity	7.81	3.35	5.94	0.16	0.03	0.03	54.5	28.18
Return of Bond	12.2	1.52	5	0.12	0.05	0.01	63.39	17.7

Finally, another interesting point that ought to be addressed is that the existence of VaR constraint enhances the effect of monetary shock on portfolio choice. From the table, we can see that variations in the ratio of bond to equity are mainly driven by technology shock and consumption preference shock from the foreign country. On the other hand, foreign country monetary policy shock accounts for the deviation of the ratio of bond to equity. This result implies that the portfolio manager is more prudent, or more sensitive to the portfolio return. Therefore, the manager is more vulnerable to monetary policy shock.

Table 4.5 Variane Decomposition of Variables without VaR Constraint

Variables	ea	eas	exc	excs	exh	exhs	em	ems
Output (H)	8.89	1.74	15.7	0.1	0.04	0.02	70.53	2.98
Output (F)	4.03	4.06	0.11	16.15	0.02	0.04	3.07	72.54
Consumption (H)	26.89	3.51	68.32	0.2	0.1	0.02	0.86	0.1
Consumption (F)	8.83	13.36	0.22	76.37	0.02	0.11	0.11	0.96
Equity Holding	21.48	60.15	0.24	0.04	0.07	0.58	0.63	16.81
Bond Holding	27.81	3.71	4.65	56.89	0.12	0.05	2.28	4.49
Ratio of bond to equity	7.61	41.5	1.55	29.86	0.02	0.45	1.68	17.33
Return of equity	45.29	4.02	10.15	0.5	0.19	0.03	37.8	2.02
Return of Bond	8.84	4.15	5.1	2.3	0.05	0.05	76.74	2.78

4.5.2 Bayesian Estimation

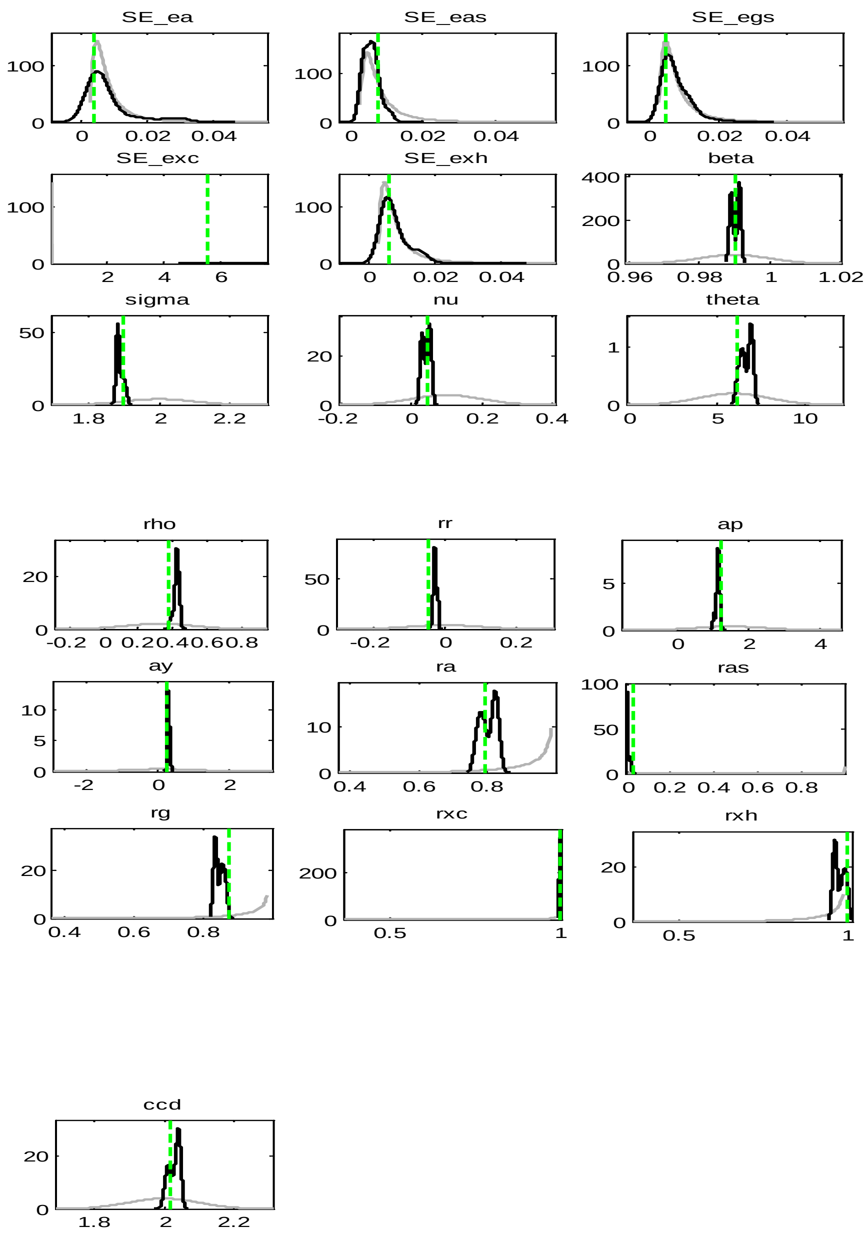
Table 4.6 Prior and Posterior Distributions

Parameters	Prior Mean	Post. Mean	90% HPD	Interval	Prior	Pstdev
			Parameters			
β	0.99	0.9903	0.9887	0.9916	norm	0.01
σ	2	1.8881	1.8758	1.9027	norm	0.1
υ	0.1	0.0412	0.0257	0.0551	norm	0.1
θ	6	6.6687	6.1655	6.9959	norm	2
ρ	0.33	0.4243	0.4052	0.4509	norm	0.2
ρ_r	0	-0.0271	-0.0342	-0.0191	norm	0.1
ρ_π	1.5	1.1247	1.0128	1.1904	norm	1
ρ_y	0.125	0.3015	0.2609	0.3524	norm	1
ρ_a	0.95	0.8032	0.7702	0.8316	beta	0.09
$\rho_{a,f}$	0.01	0.0045	0	0.0176	beta	0.09
ρ_m	0.95	0.8473	0.8318	0.8652	beta	0.09
ρ_c	0.95	0.9986	0.9966	1	beta	0.09
ρ_h	0.95	0.9777	0.9626	1	beta	0.09
χ	2	2.0288	2.0073	2.0475	norm	0.1
			Standard Deviations of Shocks			
$\boldsymbol{\varepsilon_a}$	0.01	0.008	0.0019	0.0187	invg	Inf
$\boldsymbol{\varepsilon_{a,f}}$	0.01	0.0059	0.0028	0.0091	invg	Inf
$\boldsymbol{\varepsilon_m}$	0.01	0.0075	0.0028	0.0129	invg	Inf
$\boldsymbol{\varepsilon_c}$	0.01	6.0182	5.4081	6.8681	invg	Inf
$\boldsymbol{\varepsilon_h}$	0.01	0.0077	0.0033	0.0155	invg	Inf

In order to test our model with the real economy, we attempt to estimate the model against macroeconomic data from China and the US. Bayesian estimation enables us to evaluate the predetermined parameters with real data. The sample period of our data is from 2000 Q1 to 2012 Q4, for China and the US. Data for China are obtained from International Financial Statistics (IFS), while data for the US come from Federal Reserve Economic Data (FRD). The variables used are Gross Domestic Product (GDP), interest rate, and foreign reserves. In order to make the data stationary, we employ the widely-used Hodrick-Prescott filter.

In the table 4.6, we present the estimation result, including prior and posterior distributions. It is clear that by employing real macro-economic data, the parameter values for our model are updated. This enables us to perform a more accurate impulse response function based on real-life data.

Figure 4.12 Prior and Posterior Distributions



The figures below present the Bayesian IRFs using the posterior mean, where the grey shaded areas indicate the highest posterior density intervals (Highest Posterior Density Interval (HPDI)). Compare this result with the previous IRF, which is computed with the calibrated parameter. As can be seen, the Bayesian IRFs provide us with a much more sensible result based on the real data.

Figure 4.13 Bayesian IRFs to Technology Shocks (H)

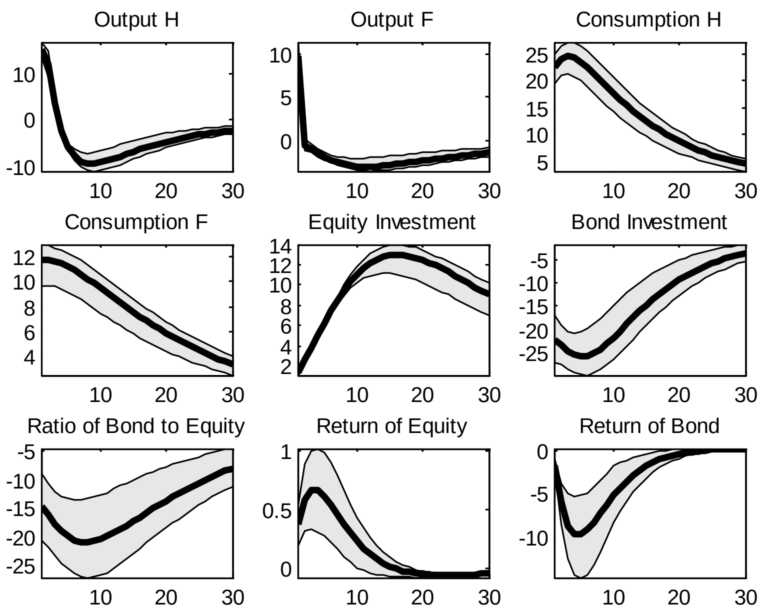


Figure 4.14 Bayesian IRFs to Technology Shocks (F)

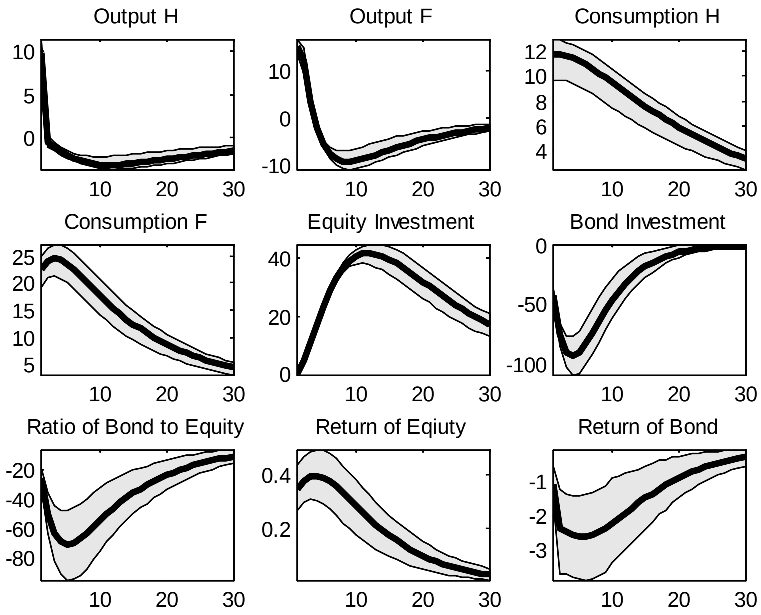


Figure 4.15 Bayesian IRFs to Consumption Shocks (H)

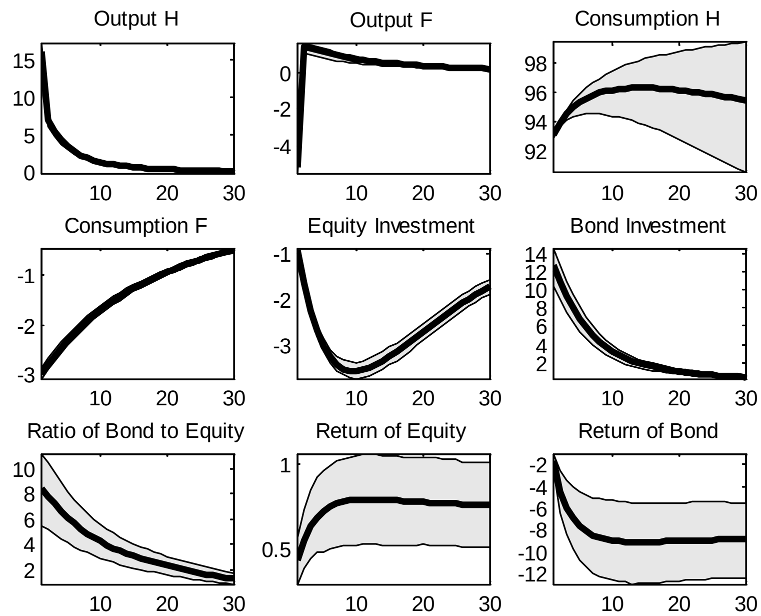


Figure 4.16 Bayesian IRFs to Consumption Shocks (F)

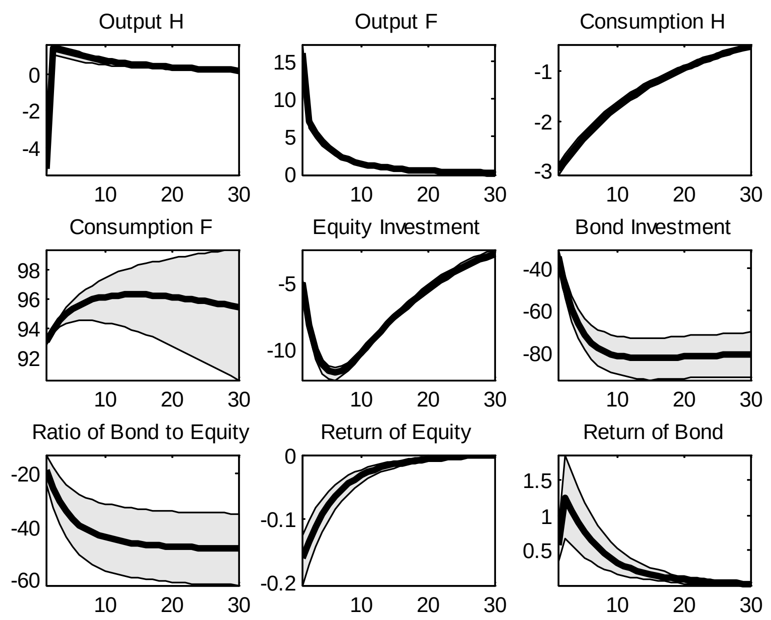


Figure 4.17 Bayesian IRFs to Labour Shocks (H)

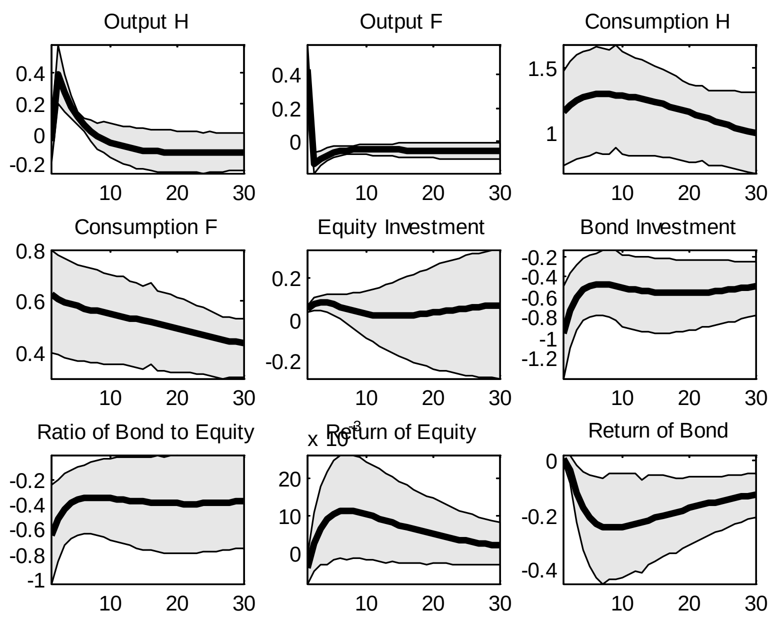


Figure 4.18 Bayesian IRFs to Labour Shocks (F)

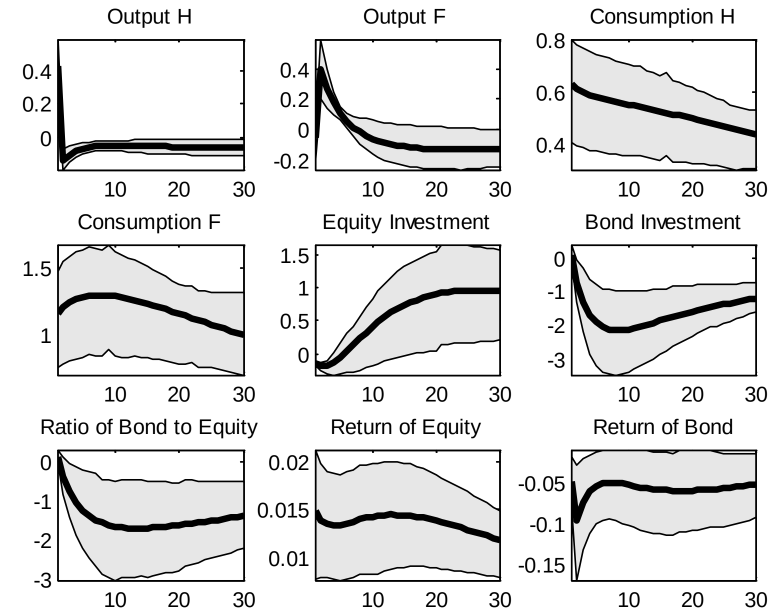


Figure 4.19 Bayesian IRFs to Monetary Policy Shocks (H)

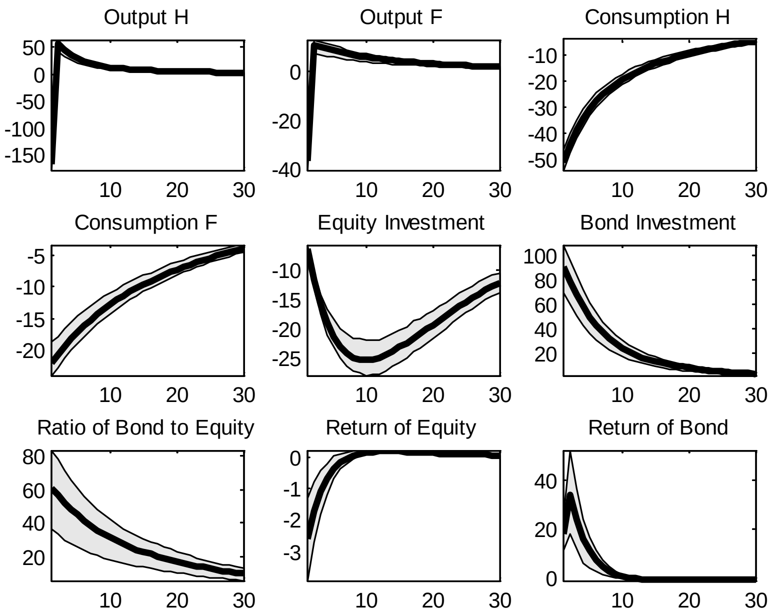
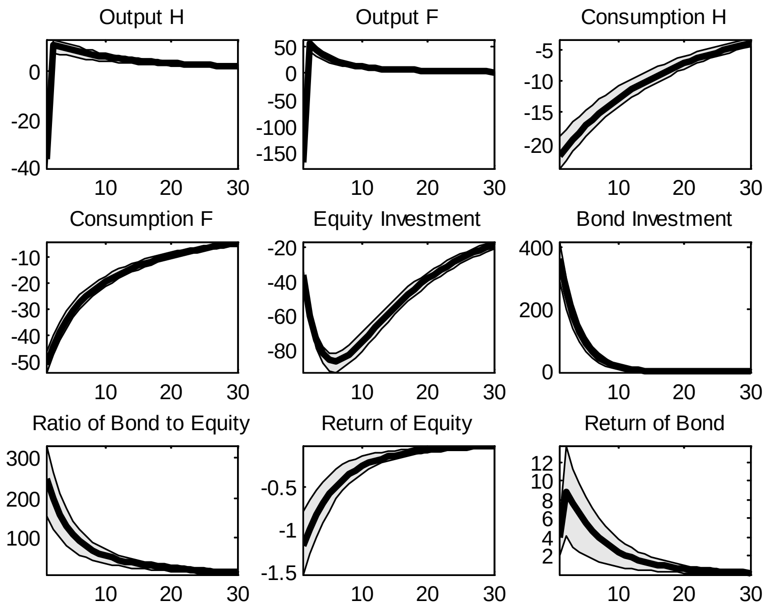


Figure 4.20 Bayesian IRFs to Monetary Policy Shocks (F)



4.6 Conclusions

Among China's massive foreign reserves, dollar denominated assets dominate, compared with other currency assets. Furthermore, China favours bond related assets, i.e. the Treasury Bills. Recent research offers some explanation for this situation. However, the literature to date has not been particularly successful in explaining the portfolio choice perspective on co-dependency between China and the US, either empirically or theoretically.

This chapter investigates the underlying reason for this “unrealistic” investment behaviour. We develop a two-country DSGE model to tackle the portfolio choice of foreign assets. A VaR constraint is also included implying a risk averse investor. The results show that VaR constraint serves as a significant explanation for the current build-up of US bond holdings. This result is an equilibrium act for the financial intermediary. The consumer’s behaviour is shifted by the risk consideration to a lower level. Further, both China and the US experience higher production compared with an economy without VaR constraint. Therefore, our result indicates that in the short run there will be no sudden decline of China’s accumulation of American bond related assets.

Chapter 5

China and Global Safe Assets

This Chapter investigates China's demand for global safe assets in an international portfolio approach. In the context of China, we examine the driving forces behind China's growing demand of global safe assets. A global method is developed to solve international portfolio choices problem for the central bank in DSGE models using the parameterized expectations algorithm.

Chapter 5

China and Global Safe Assets

5.1 Introduction

China has accumulated a massive amount of foreign reserves. The majority of these assets are held in US dollars and the dominant investment vehicles are US Government bonds and Federal Agency Instruments. Internationally, the dollar and the US Treasuries are generally regarded as global safe assets. China's this international portfolio choice raises the question of the soundness of the country's reserve investment as its overwhelming concentration on the dollars seems deviating from the basic diversification principle of investment. But of more concern is China's colossal and growing demands for US Treasuries which would disturb the supply-demand relationship of global safe assets in the world economy. In this light, this chapter seeks to dissect China's international portfolio choice in a DSGE framework with a focus on assessing the prospect of China's demand for global safe assets and the repercussions it may have.

A safe asset is a liquid debt claim with negligible default risk (Gourinchas and Jeanne, 2012). Safe assets play a critical role in financial transactions. The IMF summarizes their four main functions as a reliable store of value, a safe form of collaterals, a pricing benchmark and a key ingredient to the prudency regulatory framework for banks (IMF, 2012).

Safe assets are socially valuable. The essential trait of safe assets is their information insensitiveness, i.e. they are immune to adverse selection in trading (Dang, Gorton, and Holmström, 2012, 2013). With this, they can be used efficiently as collateral in finance, just like the role that money plays in commerce (Gorton, Lewellen, and Metrick, 2012). Because safe assets are deemed to be relatively low risk while at the same time offering high liquidity, safe assets would not suffer from financial frictions that affect many other financial assets (Gorton, 2012). As safe assets carry virtually no default risk, and promise a fixed return in the future, they are desired by risk-averse investors. These properties are also demanded by reserve managers of central banks.

Despite their fundamental importance in financial systems, very little is known about the demand for and supply of safe assets (Gorton, Lewellen and Metrick, 2012). Generally, the supply of safe debt comes from three layers: central bank liabilities, government debt and private-label safe assets (Bernanke et al., 2011). On the demand side, the major demander groups are both domestic and international. In the context of US, and other mature economies alike such as the UK, Germany and France, the private real sector's demand for safe assets has been stable since 1952, whereas demands the US financial system and the rest of the world are increasing (Gorton, Lewellen and Metrick, 2012).

Recently, the IMF has expresses a great concern for the prospect of the shrinking set of safe assets coupled with growing demand can negatively affect the global financial stability (IMF, 2012). While not all economists agree with the suggestion that in the world economy there exists a severe shortage of safe assets, Caballero and Krishnamurthy (2009) point out that capital flows into the US in recent years, many from foreign central banks and governments, have been predominantly seeking safe assets as a store of value. As a result, the US sells riskless assets to foreigners, which raises the effective leverage of US financial institutions. This means that a securitization boom in the US fuelled by elevated demand for high grade US assets by emerging market economies is a major contributing factor to financial vulnerabilities in the US This highlights the role of emerging market central banks in affecting global financial stability through elevated demand for U.S safe assets.

Demand for safe assets reflects frictions and inefficiencies (Gourinchas and Jeanne, 2012; Gorton and Ordoñez, 2013). Driven by precautionary and mercantilist motives, emerging market economies in the recent decades have accumulated a large amount of foreign reserves. How to invest these massive funds soundly becomes a serious challenge to these countries. Given financial frictions in these countries, safe assets denominated in major reserve currencies, or the global safe assets, are the sensible vehicles for their management of external wealth. Further, of the global safe assets, the US Treasuries are the primary investment instrument to be chosen for international portfolios of emerging countries. Widely considered free of default risk with a very high degree of liquidity, US Treasuries affords an assurance of convertibility, and thus fast and easy access to capital. Along with private investors, foreign central banks have become the major investor in Treasuries for a riskless, performing asset with liquidity second only to currency (Gorton and Ordoñez, 2012).

Indeed, foreign exchange reserve holdings of central banks in the developing world are overwhelmingly invested in liquid financial assets denominated in the major reserve currencies, namely the US dollar, the euro, and to a lesser extent the pound sterling and the Japanese yen. About 95% of all global foreign exchange reserve holdings for which the composition is disclosed are invested in these four currencies. The fact that these four major reserve currencies are the only units (along with the Swiss franc) reported in IMF statistics on the currency composition of official foreign exchange reserves (COFER) further illustrates their dominant reserve currency status.

As the largest holder of international reserves in the world, China is a prominent demander for global safe assets, particularly US Treasuries and other government debt such Agencies. Over 60% of China's reserve holdings have been in the US market. Some even argue that this is an underestimation. First, the Treasury International Capital (TIC) data are likely to underestimate China's investment (Setser and Pandey, 2009). Secondly, TIC data include some investments by domestic institutions, which should not be counted in foreign reserves. In any case China now is also the largest foreign holder of US official debt. Given the dollar's status as the key currency in the international monetary system, holding of dollar-denominated assets by international investors is reasonable and China is not an exception in this regard (Prasad, 2014). By using the TIC report and COFER data together, Shi and Nie (2012) estimate that the dollar assets held by China represent around 60%-65% of the country's total foreign reserves.

Prior research has extensively studied the growing global demand by emerging economies for safe assets and the effects on the US economy. This study adds to the literature by investigating this growing demand from the perspective of China as a major demander for global safe assets. By offering rationales for China's seeking of global safe assets, we rationalise China's investing behaviour in relation to international portfolio choice and shed lights on possible magnitude and composition of China's demand for global safe assets.

5.2 Literature Review

The demand for safe assets can be decomposed into two parts: the demand for safe heaven currencies as the denomination of the safe assets and the demand for the type of assets. These two components can be examined from a portfolio prospective, so we adopt a portfolio approach to examining demand for global safe assets in a DSGE framework.

Engel and Matsumoto (2009) show that when there is a high degree of price stickiness, international diversification may turn out to be about the foreign exchange denomination of nominal assets in international portfolios rather than the composition of the asset portfolio. So we first consider the currency composition in international portfolio choice.

5.2.1 General Solving Methods for DSGE Models

To explore international portfolio choice in a general equilibrium setting, we apply a modelling strategy based on the dynamic stochastic general equilibrium (DSGE) framework. While studying currency composition in a portfolio approach allows for an avoidance of having to model time variation of individual currency's properties, the DSGE model can capture the dynamics of the portfolio choice and non-linearity in the system. However, this comes at a cost, that is, solving portfolio choice in DSGE models is a very challenging task.

Solving DSGE models is rarely straightforward. This is because the stochastic problems associated with nonlinear functions would aggravate the difficulty of finding the model solution. In addition, expectations of future endogenous variables complicate the problem even further. Currently, the main solution approaches to overcoming this difficulty in the DSGE literature are presented in what follows.

5.2.1.1 Value Function Iteration

The most direct approach in the solution method family is the value function iteration (VFI), which offers accuracy, safety, reliability and, most importantly, good convergence properties. This approach is therefore often employed as a reference benchmark for the comparison with other numerical methods. Extensive applications of this VFI method can be found in, for example, Taylor and Uhlig (1990), Aruoba et al. (2006) and Heer and Maussner (2009). However, despite its possessing of desirable properties in many ways, VFI is criticized as having slow computing speed relative to other methods. Also, VFI suffers from the curse of dimensionality. These disadvantages prevent the application of this method to medium and large size DSGE models. Therefore, in order to solve more complicated systems, researchers have developed other solution procedures, such as perturbation methods and projection algorithms, which not only maintain the good convergence properties but also can compute with much faster speed than VFI.

5.2.1.2 Perturbation

Perturbation is a widely-used solving method for DSGE models, due to its computation speed. The idea behind this method is that all the nonlinear systems of optimality conditions and resource constraints are converted into a linear system through a linear approximation of all the equations based on the non-stochastic steady state. It is much easier to derive a solution to the linear system, and this serves as the approximation solution to nonlinear systems. A common approximation method is to build a Taylor series expansion of the agents' policy functions around the steady state and a perturbation parameter. Hall (1971) and Magill (1977) present the procedure to complement the perturbation method. Since the first order approximation is linear and many models exhibit the behaviour that is close to a linear law of motion, this method is also known as linearization. However, first order approximation may fail to account for the future uncertainty of the system, since the approximation assumes a perfect foresight path of the economy. To improve, Schmitt-Grohé and Uribe (2004) propose a second order approximation approach to account for future uncertainty. This improvement is crucial to address issues related to attitudes toward risk, which cannot be evaluated under first order approximation. Higher order approximation is also used by researchers including Jin and Judd (2002) and Juillard and Kamenik (2003).

First order perturbation corresponds to the solution obtained by standard linearization of first order conditions. One important advantage of higher order approximation methods is the deviation from the certainty equivalence principle, i.e., higher order moments of the distribution of the shocks (volatility) of the models are taken into account. Second, through the quadratic term of the expansion the curvature of the true decision rule is handled better. This is important when the curvature of utility is high. In addition to these benefits, in the case of welfare evaluation, quadratic approximation techniques may be better in approximating the system when one is a little farther away from the steady state. Over the last few years, many algorithms have been developed to take a second order expansion of the equilibrium conditions of DSGE models (see, for instance, Judd, 1998; Collard and Juillard, 2001; Jin and Judd, 2002; Schmitt-Grohé and Uribe, 2004; Kim et al., 2008; Swanson et al., 2006). All these methods approximate the policy functions locally.

Although perturbation methods had already been applied to continuous time, Judd's (1998) pioneering work presents a general method for deterministic discrete-time models showing the adjustments that must be made. Jin and Judd (2002) describe in detail how to compute approximations of arbitrary order in discrete-time rational expectations models. They analyse regularity conditions which justify the local approximations and provide methods for checking the validity of the approximations. Jin and Judd (2002) argue that deriving perturbation solutions is no more difficult than deriving the linear approximation solutions, but they also discuss some cases where linear approximation is better than higher order approximation.

Accuracy criteria of higher order perturbation methods are provided by Collard and Juillard (2001). They compare the solutions of perturbation to closed form solutions of an asset pricing model, and find that in comparison to linearization, second order and fourth order approximation perform better. Swanson et al. (2006) focus on Taylor series expansions of nth-order to approximate solutions to dynamic, discrete-time rational expectations models. These higher order approximations have the advantage (compared to first or second order approximated solutions) that they are valid everywhere within the domain of convergence of the Taylor series; i.e., they are valid globally (in a very rigorous sense). An algorithm for computing second order approximation, forecasts and impulse response functions in dynamic models can be found in Kim et al. (2008).

5.2.1.3 Projection

The underlying idea of the projection method is the use of basic functions to build an approximated policy function that minimizes a residual function; therefore, this method is also known as the minimum weighted residual method. There are two alternative approaches to implementing projection methods: finite element methods (Aruoba et al., 2006; McGrattan, 1996) and orthogonal polynomials (Judd, 1992).

While perturbation is labelled a local method, projection is known as a global one. The main difference between the two is computation speed versus accuracy. Since local methods compute solutions at just one point, a deterministic steady state, using Taylor expansions of optimality conditions, the computation speed is much faster than that of global methods (Judd and Guu, 1993; Kollmann et al., 2011). However, global methods such as projection, which compute solutions in larger domains, derive more accurate solutions (Judd, 1992; Kollmann et al., 2011). Therefore, the choice between these methods represents a trade-off between computation speed and accuracy.

5.2.1.4 A Comparison

The relative performance of the above solution methods has been studied by e.g. Aruoba et al. (2006). They attempt to implement all three methods to the canonical stochastic neoclassical growth model with leisure choice, and evaluate the performance of each method in terms of computing time, implementation complexity and accuracy.

Here we highlight five main aspects. First, perturbation delivers an interesting compromise among speed, accuracy, and programming burden. For instance, for the benchmark calibration, compared with other solution methods, fifth order perturbation is shown to be more accurate. Aruba et al. (2006) quantitatively assess the speed and the extent perturbations deteriorate when they move away from the steady state. Also, they illustrate how the simulations tend to explode and the reasons for such behaviour.

Second, for the reason that higher order perturbations preserve a better performance over linear methods for a trivial marginal cost, It is desirable to perform some computation with linear methods by at least a second order approximation.

Third, even if the application of linear methods is disappointing along a number of dimensions, linearization in levels is preferred to log linearization for both the highly nonlinear cases and the benchmark calibration. Based on the fact that the exact solution to the model with log utility, inelastic labour, and full depreciation is log linear, the result contradicts common practice.

Fourth, for all parameterizations, finite elements perform well. The result is extremely accurate and stable over the range of the state space even for large values of the risk aversion and the variance of the shock. This property is essential to produce unbiased estimation with accurate result. (Fernández-Villaverde and Francisco Rubio-Ramírez, 2004). Also, they employ simple linear basis functions. Given the smoothness of the solution, finite elements with higher order basis functions would perform even better. However, finite elements are probably the most complicated to implement in practice.

Finally, Chebyshev polynomials share all the good properties with the finite elements method and are easier to perform. For the reason that the neoclassical growth model has smooth policy functions, it is predictable that Chebyshev polynomials do well in this implement. However, Chebyshev polynomials might underperform finite elements in a model where policy functions have complicated local behaviour. Therefore, although our results depend on the particular model we have used, perturbation should be more widely used, to suggest the reliance on finite elements for problems that demand high stability and accuracy, and support the progressive phasing out of pure linearization.

5.2.2 Methods for Solving Portfolio Choice in DSGE Models

According to Pavlova and Rigobon (2010), international macro-finance of open economy macroeconomics is an important area calling for growing research attention. Research in this area takes portfolio choice and asset pricing to models of international macroeconomics. Conventional models have seriously overlooked international portfolio choice in both equity and bond. Nevertheless, the recent financial crisis and global imbalances promote the research interest in analysing international portfolio choice within the macroeconomics' framework leading to a shift of research efforts from, for example, the international variant of the CAPM to the more refined DSGE models. However, this process is only to be hampered by the indeterminacy problem. Devereux and Sutherland (2010) document that the standard log-linear approximation method struggles to deal directly with the portfolio choice due to portfolio indeterminacy. This indeterminacy problem can be decomposed into two distinct parts. First, for model with stochastic shocks, the portfolio choice in the first-order approximation of the model is indeterminate. Secondly, for the model without stochastic shocks, the equilibrium portfolio is indeterminate at steady state. Although there is distinct difference between these models, the main problem is that all the assets in the model yield the same return both in first-order approximation and non-stochastic steady state. Therefore, in order to achieve international portfolio choice, one has to overcome this problem first.

5.2.2.1 Local Methods

The literature on portfolio choice in DSGE models can at least be grouped into three different approaches. The first approach relies on higher-order approximations around a deterministic steady state which is represented by Devereux and Sutherland (2011). Their paper proposes a simple approach to solve the country portfolio choice based upon the cornerstone paper by Samuelson (1970). Specifically, Devereux and Sutherland (2011) derive an approximation method for an economy with incomplete markets but constant portfolio shares. This is applied to a two-country general equilibrium model with production and trade in equities, and to a two-country endowment model with trade in real bonds. Devereux and Sutherland (2008) also apply this approach to a sticky-price monetary model that allows for portfolios of bonds and equity trade. In addition, Devereux and Sutherland (2007) extend the approximation method to allow for time-varying portfolios, which is applied to a two-country endowment model with trade in real bonds. Devereux and Sutherland (2010) examine a similar model, with a focus on the role of changes in valuation in the international distribution of wealth. Tille and Van Wincoop (2010a) use a similar approximation to solve a two-country general equilibrium model with capital and production and trade in equities. In another paper, i.e. Tille and Van Wincoop (2010b), they apply the method to examine the response of the current account and net foreign assets to changes in saving. More recently, Evans and Hnatkovska (2014) examine a similar model with a similar solution methodology.

The second approach makes use of bifurcation theory in conjunction with the implicit function theorem to characterise the portfolio choice problem. Judd and Guu (2001) employ this approach to analysis optimal portfolio when shocks are small enough by resorting to a bifurcation theorem. Compared with the method developed by Devereux and Sutherland (2011), both methods generate a similar result. In particular, the steady-state portfolio holdings derived by this second approach correspond to a bifurcation point in the set of non-stochastic equilibria. However, both the first and second approaches share a common disadvantage. When the economy experiences large underlying volatilities away from steady state, the corresponding behaviour of the economy is unable to clarify.

The third approach aims to analysis the closed-form solutions from simplified models. This approach is preferred for the reason that it allows researchers to analyze the economy away from the steady state. Nevertheless the analytical solution is difficult to achieve as it often only exists in few special cases (Helpman and Razin, 1978; Cole and Obstfeld, 1991; and Zapatero, 1995).

The method introduced by Devereux and Sutherland (2011) contributes to the research on portfolio in many aspects. This method is applied explicitly to models with incomplete markets. Also, the development of the method has enabled researchers to analyse a number of important questions in open economy macroeconomics such as the existence of substantial gross external asset and liability positions and their rapid growth in recent decades, the increasing empirical importance of asset trade, the role of portfolio allocation in determining capital flows, and the potential influences of size and composition of gross portfolios on macroeconomic outcomes.

However, this method can be criticized for being inaccurate when evaluating the effects of large shocks, especially for emerging markets. Also, in asymmetric setups local approximation methods have an additional difficulty that when the true policy functions are very nonlinear, results rely heavily on the choice of the approximation point of net foreign assets, the ‘right’ value of which is, however, itself determined endogenously in a true solution. Furthermore, the (closed economy) finance literature has shown that nonlinearities may be of particular importance in providing joint explanations for macro and asset pricing stylized facts. Such nonlinearities may derive from the presence of explicit borrowing or short-selling constraints or from consideration of richer preference specifications, for example the Epstein-Zin preferences.

5.2.2.2 Global Methods (Parameterized Expectations Algorithm)

Conventional solution methods for portfolio choice highly rely on deterministic steady-state as the start point for approximation. Therefore it hampers investigation of the problem with permanent shocks which change its deterministic steady-state. Furthermore, compared with perturbation based methods, global methods is more accurate though with a cost of computation time. Last but not the least, the method developed by Devereux and Sutherland (2011) is mainly interested in finding a simple solution in the direction of the model's state variables whereas it neglects the effect of shocks. Thus in order to overcome the shortcomings, we turn to apply the Parameterized Expectations Algorithm which is a global solution method.

The Parameterized Expectations Algorithm (PEA) is a non-finite state space method for computing equilibria in nonlinear stochastic dynamic models (e.g., Wright and Williams 1982; Miranda and Helmberger 1988; Marcet 1988; den Haan and Marcet 1990; Christiano and Fisher 2000). The main idea of the method is to approximate the conditional expectations in Euler's equations by a parametric function of state variables and find the parameters. This can minimize the distance between the expectation and the approximating function.

Several properties make the PEA an attractive tool for analysing economic dynamics. First, if a low-degree polynomial approximation delivers a sufficiently accurate solution, the cost of the algorithm does not practically depend on the dimensionality of the state space. Second, the PEA can be applied to analyzing not only the optimal parameters vital to the economy but also the economy with externalities, distortions, liquidity constraints, and so on. Finally, the algorithm is fast and simple to program. For a detailed discussion of the method and its applications, see Marcet and Lorenzoni (1999) and Christiano and Fisher (2000).

5.3 The Model

5.3.1 Model Setups

We first set a standard utility function with CRRA preference as in Equations 1 and 2. The representative household, which receives an endowment (Y_t), chooses consumption (C_t) to maximize its lifetime expected utility. In the equations, $\beta \in (0,1)$ represents the subjective discount rate and σ represents the degree of risk aversion. The net foreign assets are assumed to be invested in assets denominated in three currencies: the US dollar denominated assets ($\theta_{1,t}$), euro denominated assets ($\theta_{2,t}$) and Japanese yen denominated assets ($\theta_{3,t}$). So we have:

$$U = E_t \sum_{t=1}^{\infty} \beta (\frac{C_t^{1-\sigma}}{1-\sigma}) \tag{5.1}$$

$$U' = \frac{C_{a,t}^{1-\sigma}}{(1-\sigma)} \tag{5.2}$$

The household is subject to a sequence of budget constraints:

$$\theta_{1,t} q_{1,t} + \theta_{2,t} q_{2,t} + \theta_{3,t} q_{3,t} = \theta_{1,t-1} \left(q_{1,t} + Y_{1,t}^K \right) + \theta_{2,t-1} \left(q_{2,t} + Y_{2,t}^K \right) + \theta_{3,t-1} \left(q_{3,t} + Y_{3,t}^K \right) + Y_t - C_t \tag{5.3}$$

where $q_{1,t}$, $q_{2,t}$ and $q_{3,t}$ stand for the different currency assets prices, but they are expressed in the same currency unit (the US dollar could be a natural choice). Uncertainty in the model is introduced by three exogenous stochastic processes: $Y_{1,t}^k, Y_{2,t}^k$ and $Y_{3,t}^k$, which represent income uncertainty of assets denominated in each currency. We postulate each uncertainty with a first order autoregressive process:

$$\log(Y_{1,t}^k) = \rho_1^k \log(Y_{1,t-1}^k) + \varepsilon_{1,t}^k \tag{5.4}$$

$$\log(Y_{2,t}^k) = \rho_2^k \log(Y_{2,t-1}^k) + \varepsilon_{2,t}^k \tag{5.5}$$

$$\log(Y_{3,t}^k) = \rho_3^k \log(Y_{3,t-1}^k) + \varepsilon_{3,t}^k \tag{5.6}$$

where ρ_1^k , ρ_2^k and ρ_3^k denote the level of persistence; $\varepsilon_{1,t}^k$, $\varepsilon_{2,t}^k$ and $\varepsilon_{3,t}^k$ represent i.i.d. white noises. Thus, the covariance matrix of the vector $\varepsilon_t = \left[\varepsilon_{1,t} \varepsilon_{2,t} \varepsilon_{3,t} \varepsilon_t \right]$ is given by:

$$\sum = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix} \tag{5.7}$$

Real return on each asset is defined as:

$$R_{1,t} = \frac{q_{1,t} + Y_{1,t}^k}{q_{1,t-1}} \tag{5.8}$$

$$R_{2,t} = \frac{q_{2,t} + Y_{2,t}^k}{q_{2,t-1}} \tag{5.9}$$

$$R_{3,t} = \frac{q_{3,t} + Y_{3,t}^k}{q_{3,t-1}} \tag{5.10}$$

The first order conditions for consumption derived for each currency are:

$$C_t^{-\sigma} = \beta E_t \left[C_{t+1}^{-\sigma} R_{1,t+1} \right] \tag{5.11}$$

$$C_t^{-\sigma} = \beta E_t \left[C_{t+1}^{-\sigma} R_{2,t+1} \right] \tag{5.12}$$

$$C_t^{-\sigma} = \beta E_t \left[C_{t+1}^{-\sigma} R_{3,t+1} \right] \tag{5.13}$$

To determine allocation of each currency asset in the portfolio, we also need to define the total net foreign assets. We assume that the Chinese central bank holds foreign reserves W_t. Although evaluated in the US dollar, these external wealth consists of three currency assets:

$$W_t = \theta_{1,t} q_{1,t} + \theta_{2,t} q_{2,t} + \theta_{3,t} q_{3,t} \tag{5.14}$$

Correspondingly, the budget constraint is written as follows:

$$W_t = \theta_{1,t-1}\left(q_{1,t} + Y_{1,t}^K\right) + \theta_{2,t-1}\left(q_{2,t} + Y_{2,t}^K\right) + \theta_{3,t-1}\left(q_{3,t} + Y_{3,t}^K\right) + Y_t - C_t \tag{5.15}$$

In order to identify the composition of foreign reserves' growth rate, we subtract the above equation by W_{t-1} and divide by W_{t-1}:

$$\frac{W_t - W_{t-1}}{W_{t-1}} = \left\{ (\frac{q_{1,t} + Y_{1,t}^K}{q_{1,t-1}})(\frac{\theta_{1,t-1} q_{1,t}}{W_{t-1}}) + (\frac{q_{2,t} + Y_{2,t}^K}{q_{2,t-1}})(\frac{\theta_{2,t-1} q_{2,t}}{W_{t-1}}) + (\frac{q_{3,t} + Y_{3,t}^K}{q_{3,t-1}})(\frac{\theta_{3,t-1} q_{3,t}}{W_{t-1}}) \right\} + \frac{Y_t - C_t}{W_{t-1}} \tag{5.16}$$

Thus the growth rate of foreign reserves can be decomposed into two parts: the weighted average rate of return and the purchase rate of national savings. The law of motion of foreign reserves is obtained as follows:

$$W_t = W_{t-1}R_{1,t} + Y_t - C_t + \frac{\theta_{2,t-1}q_t}{W_{t-1}}(R_{2,t} - R_{1,t}) + \frac{\theta_{3,t-1}q_t}{W_{t-1}}(R_{3,t} - R_{1,t}) \tag{5.17}$$

Finally, the portfolio allocation is characterized by imperfect substitutability between currency assets. We follow a similar specification as Blanchard et al. (2005):

$$\frac{\theta_{1,t-1}q_{1,t}}{\theta_{1,t-1}q_{1,t} + \theta_{2,t-1}q_{2,t}} = b_{12}\left\{E_t \frac{R_{1,t+1}}{R_{2,t+1}} - \frac{\overline{R_1}}{\overline{R_2}}\right\} \tag{5.18}$$

$$\frac{\theta_{1,t-1}q_{1,t}}{\theta_{1,t-1}q_{1,t} + \theta_{3,t-1}q_{3,t}} = b_{13}\left\{E_t \frac{R_{1,t+1}}{R_{3,t+1}} - \frac{\overline{R_1}}{\overline{R_3}}\right\} \tag{5.19}$$

where b_{12} and b_{13} are the parameters that capture the degree of substitutability between different currency assets. A higher level of substitutability implies a more sensitive response to changes in the expected relative returns on different currency assets. Equations (18) and (19) comprise two components, expected relative returns and steady state relative returns. It is noticeable that the reaction of substitutability comes mainly from expected relative returns, because at steady state, the return on each asset is predetermined. it becomes clear allocation of the portfolio shares of each currency assets requires calculation of expected returns on each currency asset.

5.3.2 Portfolio Construction

Now we explore portfolio allocation in a two-country setting. We assume that the representative agent in country $a \in \{h, f\}$ maximizes his utility according to the standard CRRA function (5.20), where h represents the home country (China) and f denotes foreign (US). Households from both countries maximize their consumption utility under their respective budget constraints.

$$U = E_t \sum_{t=1}^{\infty} \beta (\frac{C_{a,t}^{1-\sigma}}{1-\sigma}) \tag{5.20}$$

where $C_{a,t}$ represents the consumption level; β is the endogenous discount factor; σ is a country-specific coefficient of risk-aversion. Both countries receive a labour endowment (Y_{at}^{l}) at the beginning of each period. We also assume that consumers from each country are able to invest in both equity and bond markets globally, implying that investors are unable to hedge their country-specific shocks directly.

The budget constraint for the home country is given by:

$$\frac{\theta_{h,t}^{e} q_{h,t}^{e}}{P_{h,t}} + \frac{b_{h,t}^{e} q_{h,t}^{b}}{P_{h,t}} + \frac{\theta_{f,t}^{e} q_{f,t}^{e}}{P_{f,t}} + \frac{b_{f,t}^{b} q_{f,t}^{b}}{P_{f,t}} = \frac{\theta_{h,t-1}^{e}\left(q_{h,t}^{e} + Y_{h,t}^{k}\right)}{P_{h,t-1}} + \frac{b_{h,t-1}^{b}}{P_{h,t-1}} + \frac{\theta_{f,t-1}^{e}\left(q_{f,t}^{e} + Y_{f,t}^{k}\right)}{P_{f,t}} + \frac{Y_{h,t}^{l}}{P_{h,t}} - C_{h,t} \tag{5.21}$$

where $\theta^a_{h,t}$ represents the share of investment in the home equity market from 'a' investor, while $b^a_{h,t}$ is his bond investment holding in domestic bonds. The price of each type of home asset is denoted as $q^e_{h,t}$ for equity and $q^b_{h,t}$ for bonds. The price level of each economy is denoted as $P_{h,t}$ and $P_{f,t}$. Also, we assume that the endowments from capital and labour follow the below processes:

$$\log(Y^k_{h,t}) = \rho^k_h \log(Y^k_{h,t-1}) + \varepsilon^k_{h,t} \tag{5.22}$$

$$\log(Y^k_{f,t}) = \rho^k_f \log(Y^k_{f,t-1}) + \varepsilon^k_{f,t} \tag{5.23}$$

$$\log(Y^l_{h,t}) = \rho^l_h \log(Y^l_{h,t-1}) + \varepsilon^l_{h,t} \tag{5.24}$$

$$\log(Y^l_{f,t}) = \rho^k_f \log(Y^l_{f,t-1}) + \varepsilon^l_{f,t} \tag{5.25}$$

where ρ^k_h , ρ^k_f , ρ^l_h and ρ^k_f denote the level of persistence and $\varepsilon^k_{\mathrm{h},t}$ $\varepsilon^k_{\mathrm{f},t}$ $\varepsilon^l_{\mathrm{h},t}$ and $\varepsilon^l_{\mathrm{f},t}$ represent white noises with symmetrically distributed intervals. The goods market clearing condition is:

$$C_{ht} + C_{ft} = Y^k_{h,t} + Y^k_{f,t} + Y^l_{h,t} + Y^l_{f,t} \tag{5.26}$$

$$Y_{h,t} = Y^k_{h,t} + Y^l_{h,t} \tag{5.27}$$

$$Y_{\mathrm{f},t} = Y^k_{\mathrm{f},t} + Y^l_{\mathrm{f},t} \tag{5.28}$$

Consumer prices are assumed to be determined by simple quantity theory of money in the following form:

$$M_{h,t} = P_{h,t} Y_{h,t} \tag{5.29}$$

$$M_{\mathrm{f},t} = P_{\mathrm{f},t} Y_{\mathrm{f},t} \tag{5.30}$$

where home and foreign money supplies, $M_{h,t}$ and $M_{\mathrm{f},t}$ are assumed to be exogenous and autoregressive processes of the form:

$$\log(M_{h,t}) = \rho_h^M \log(M_{h,t-1}) + \varepsilon_{h,t}^M \tag{5.31}$$

$$\log(M_{f,t}) = \rho_f^M \log(M_{\mathrm{f},t-1}) + \varepsilon_{\mathrm{f},t}^M \tag{5.32}$$

The consumer’s intra-temporal consumption decision is given by the following first order conditions:

$$q_{h,t}^e = \beta\, E(q_{h,t+1}^e + Y_{h,t+1}^k) \frac{P_{h,t+1}}{P_{h,t}} \tag{5.33}$$

$$q_{f,t}^e = \beta\, E(q_{f,t+1}^e + Y_{f,t+1}^k) \frac{P_{f,t+1}}{P_{f,t}} \tag{5.34}$$

$$q_{h,t}^b = \beta \frac{P_{h,t+1}}{P_{h,t}} \tag{5.35}$$

$$q_{f,t}^b = \beta \frac{P_{f,t+1}}{P_{f,t}} \tag{5.36}$$

Therefore, we are able to obtain returns on each asset:

$$R_{h,t}^{e} = \frac{(q_{h,t+1}^{e} + Y_{h,t+1}^{k})}{q_{h,t}^{e}} \bullet \frac{P_{h,t+1}}{P_{h,t}} \tag{5.37}$$

$$R_{f,t}^{e} = \frac{(q_{f,t+1}^{e} + Y_{f,t+1}^{k})}{q_{f,t}^{e}} \bullet \frac{P_{f,t+1}}{P_{f,t}} \tag{5.38}$$

$$R_{h,t}^{b} = \frac{1}{\beta} \bullet \frac{P_{h,t+1}}{P_{h,t}} \tag{5.39}$$

$$R_{f,t}^{b} = \frac{1}{\beta} \bullet \frac{P_{f,t+1}}{P_{f,t}} \tag{5.40}$$

The portfolio choice is determined by Equations (5.41) and (5.42), where Eq. (5.41) represents the home country portfolio allocation between home equity and home bonds, and Eq. (5.42) determines the asset allocation between foreign equity and foreign bonds by the home country consumer:

$$\frac{\theta_{h,t}^{b} q_{h,t}^{b}}{\theta_{h,t}^{e} q_{h,t}^{e}} = b_{h}^{be} \left\{ E_{t} \frac{R_{h,t+1}^{b}}{R_{h,t+1}^{e}} - \frac{\overline{R_{h}^{b}}}{\overline{R_{h}^{e}}} \right\} \tag{5.41}$$

$$\frac{\theta_{f,t}^{b} q_{f,t}^{b}}{\theta_{f,t}^{e} q_{f,t}^{e}} = b_{h}^{be} \left\{ E_{t} \frac{R_{f,t+1}^{b}}{R_{f,t+1}^{e}} - \frac{\overline{R_{f}^{b}}}{\overline{R_{f}^{e}}} \right\} \tag{5.42}$$

From the above two equations, we are able to calculate the total portfolio allocation between home (China) and foreign (US) countries as follows:

$$\alpha = b\left\{E_t\left(\frac{\frac{\theta^b_{h,t}q^b_{h,t}}{\theta^e_{h,t}q^e_{h,t}}R^b_{h,t+1}+(1-\frac{\theta^b_{h,t}q^b_{h,t}}{\theta^e_{h,t}q^e_{h,t}})R^e_{h,t+1}}{\frac{\theta^b_{f,t}q^b_{f,t}}{\theta^e_{f,t}q^e_{f,t}}R^b_{f,t+1}+(1-\frac{\theta^b_{f,t}q^b_{f,t}}{\theta^e_{f,t}q^e_{f,t}})R^e_{f,t+1}}\bullet\frac{e_{t+1}}{e_t}\right)-\frac{\frac{\theta^b_{h,t}q^b_{h,t}}{\theta^e_{h,t}q^e_{h,t}}\overline{R^b_h}+(1-\frac{\theta^b_{h,t}q^b_{h,t}}{\theta^e_{h,t}q^e_{h,t}})\overline{R^e_h}}{\frac{\theta^b_{f,t}q^b_{f,t}}{\theta^e_{f,t}q^e_{f,t}}\overline{R^b_f}+(1-\frac{\theta^b_{f,t}q^b_{f,t}}{\theta^e_{f,t}q^e_{f,t}})\overline{R^e_f}}\right\} \tag{5.43}$$

5.4 Solving Algorithms

5.4.1 Algorithm of Parametrized Expectation Algorithm

Step 1: Discretizing the Exogenous Shocks

In order to discretize the exogenous shocks in the model, we choose to approximate by finite-state Markov Chains as in Gospodinov and Lkhagvasuren (2013) (henceforth GL). What makes GL stand out is that it enhances the approximation accuracy compared with previous methods, especially when the value of the persistence parameter is close to one, or when the target is a VAR process rather than simply a univariate case. By using the GL approach, we can obtain two sets of matrices, one containing the discrete values of each target variable and another with their transitional probabilities.

We approximate six exogenous variables separately, i.e. $Y_{K1,t}$, $Y_{L1,t}$, $Y_{K2,t}$, $Y_{L2,t}$, $M_{1,t}$ and $M_{2,t}$. Given the corresponding transitional probability matrix, we can know the value of the variable in the next period, and the associated probability for it to transfer from current state to the next-period state.

Step 2: Approximating Policy Functions

There are two questions to be answered here. First, we need to find out which function is to be approximated, and then we have to determine what to do the job. For the first question, we choose 6 policy functions: C_1 , C_2 , Z_1 , Z_2 , Z_3 and Z_4 . For the second, we adopt the idea proposed in Rabitsch et al. (2014), to introduce a new state variable in order to avoid the 'curse of dimensionality'. More specifically, the variable is defined as the home country's share in the world financial wealth, which is shown in the following equations. In this way, we recast the equilibrium conditions in a form that is consistent with a wealth-recursive equilibrium:

$$w_{t+1} = \frac{\sum_{i=1}^{4} \theta_{i,t} \cdot r_{i,t+1} + Y_{K1,t+1} + Y_{L1,t+1}}{Y_{K1,t+1} + Y_{L1,t+1} + Y_{K2,t+1} + +Y_{L2,t+1}} \tag{5.44}$$

$$w_{t+1}^{*}=\frac{\sum_{i=1}^{4}\theta_{i,t}^{*}\cdot r_{i,t+1}+Y_{K2,t+1}+Y_{L2,t+1}}{Y_{K1,t+1}+Y_{L1,t+1}+Y_{K2,t+1}++Y_{L2,t+1}} \tag{5.45}$$

Now all we need to do is to find the relationship between each of the policy functions and the state variables, consisting of $Y_{K1,t}$, $Y_{L1,t}$, $Y_{K2,t}$, $Y_{L2,t}$, $M_{1,t}$, $M_{2,t}$ and w_t . This relationship is presented in the following coefficient matrix:

$$\begin{pmatrix} Z_{1,t} \\ Z_{2,t} \\ Z_{3,t} \\ Z_{4,t} \\ C_{1,t} \\ C_{2,t} \end{pmatrix}_{6\times 1} = \begin{pmatrix} coefficient \\ matrix \end{pmatrix}_{6*7} \cdot \begin{pmatrix} Y_{K1,t} \\ Y_{L1,t} \\ Y_{K2,t} \\ Y_{L2,t} \\ M_{1,t} \\ M_{2,t} \\ w_t \end{pmatrix}_{7\times 1} \tag{5.46}$$

Step 3: Updating the Coefficient Matrix

We start our iterative algorithm by making an initial guess of the coefficient matrix. It is worth noting that given the state value of w_t and each possible realization of next period $Y_{K1,t+1}$, $Y_{L1,t+1}$, $Y_{K2,t+1}$, $Y_{L2,t+1}$, $M_{1,t+1}$ and $M_{2,t+1}$, we can work out the current period equilibrium values. After taking into account all possible next period values of $Y_{K1,t+1}$, $Y_{L1,t+1}$, $Y_{K2,t+1}$, $Y_{L2,t+1}$, $M_{1,t+1}$ and $M_{2,t+1}$, we solve the current values of C_1, C_2, Z_1, Z_2, Z_3 and Z_4.

Our main interest is in the price variables (Z_1, Z_2, Z_3 and Z_4). To evaluate the approximation accuracy of each policy function in the iteration, we set a predetermined threshold. It seems that 0.0001 has been a popular choice in similar research. By comparing this threshold with the differential between the estimated price levels and the calculated levels, we can then decide whether to continue this iteration or not. Specifically, the stopping criterion set here is that the differential of all four price variables in every grid point is less than the threshold. If not satisfied, we need to update the coefficient matrix after this iteration, as shown in the equation. In equation 48, the value of the update ratio lies between 0 and one, and the 'regressed coefficient matrix' is obtained from the regression as indicated in equation (5.47).

$$\begin{pmatrix} Z_{1,t} \\ Z_{2,t} \\ Z_{3,t} \\ Z_{4,t} \\ C_{1,t} \\ C_{2,t} \end{pmatrix}_{model\_calculated} = \begin{pmatrix} coefficient \\ matrix \end{pmatrix}_{regressed} \cdot \begin{pmatrix} Y_{K1,t} \\ Y_{L1,t} \\ Y_{K2,t} \\ Y_{L2,t} \\ M_{1,t} \\ M_{2,t} \\ w_t \end{pmatrix} \qquad (5.47)$$

$$\begin{pmatrix} coefficient \\ matrix \end{pmatrix}_{\substack{new \\ guess}} = (1\text{-update\_ratio}) \cdot \begin{pmatrix} coefficient \\ matrix \end{pmatrix}_{\substack{old \\ guess}} + \text{update\_ratio} \cdot \begin{pmatrix} coefficient \\ matrix \end{pmatrix}_{regressed} \tag{5.48}$$

Whenever the stopping criterion is fulfilled, we assume that the algorithm has achieved its convergence and the coefficient matrix used in that iteration is considered as the best guess of the policy functions.

5.4.2 Algorithm of the Projection Method

Step 1: Discretization of the Exogenous Shocks

The first step of the projection method is exactly the same as in the PEA, so we skip this and will return to this step in the later process if necessary.

Step 2: Approximating the Policy Functions

The approximation methodology in the projection methods is less straightforward than in the PEA case. Projection methods use basis-functions to optimally approximate the policy functions according to the equilibrium conditions of the model. There are two popular methods for choosing basis-functions: finite elements and the spectral methods. Unlike finite elements method, which divides the domain into sub-intervals, the spectral method makes use of higher order polynomials, which cover the whole state variable domain. Hence, the spectral method is a better choice when the underlying policy function is assumed to be continuous and differentiable. Another reason for its popularity is the ease of implementation.

Next, we need to decide the weighting function from three conventional options: the Least Square method, the Galerkin method and the Collation method. As suggested in Caldara et al. (2012), the Collation method delivers the best trade-off between speed and accuracy. When orthogonal polynomials are selected, the collation points are exactly the roots of the polynomial of the highest order. Normally the Chebyshev polynomials are preferred, owing to their flexibility and convenience.

We approximate C_1 and C_2 using NFA as the only state variable. [1] The approximating function is:

$$C_{a,t}(NFA_t, \sigma_Y; \theta) \equiv \sum_{i=0}^{n} \theta_i^{\sigma_Y} T_i(\varphi(NFA_t)) \tag{5.49}$$

[1] It is sufficient to use the projection results of C_1 and C_2 to solve other endogenous variables, including Z_1, Z_2, Z_3 and Z_4.

where $a \in (1,2)$ denoting the home and foreign country respectively; $T_i(\cdot)$ is the Chebyshev polynomial of order $i = 0,...,n$; and $\varphi(NFA)$ is a linear function mapping the domain $[NFA_{\min}, NFA_{\max}]$ into $[-1,1]$. It is important to note that the stochastic shock processes ($Y_{K1,t}$, $Y_{L1,t}$, $Y_{K2,t}$, $Y_{L2,t}$, $M_{1,t}$ and $M_{2,t}$) have been discretized in the first step, so σ_Y is introduced here simply to represent the fact that the algorithm has taken into consideration all possible next-period shock realizations.

The problem now is to find the vector θ such that the residual function, written in the form of Eq. (50), is equal to zero at the $\{NFA_i\}_{i=1}^{n_{NFA}}$ roots of the $n_{NFA} - th$ order Chebyshev polynomial.

$$\Re(NFA_t, \sigma_Y; \theta) = [\begin{matrix} C_{1,t}^{-\rho} - \beta E_t(C_{1,t+1}^{-\rho} \cdot r_{i,t+1}) \\ C_{2,t}^{-\rho} - \beta E_t(C_{2,t+1}^{-\rho} \cdot r_{i,t+1}) \end{matrix}] \quad (5.50)$$

Step 3: Updating the Coefficient Matrix

Similar to what has been discussed above, we start our iterative algorithm by having an initial guess of the vector θ . We set the highest Chebyshev polynomials to be 4 and the threshold as 10^{-4} . By comparing the projected values of both C_1 and C_2 with their respective calculated values, we can determine whether this iteration is successful. If not, we will regress the calculated values of C_1 and C_2 on the Chebyshev polynomials to update the guess of θ. Whenever the differential is less than the threshold in every Chebyshev node, we assume the algorithm has achieved its convergence and the vector θ used in that iteration is considered as the best guess of the policy functions.

5.5 Results

5.5.1 Results for Currency Assets

5.5.1.1 Calibration

In order to solve our currency model, we need to choose parameter values. With prior knowledge about the Chinese economy and the information offered by the previous literature, we choose a set of parameter values for our calculation. Detailed parameter values for the calibration are reported in Table 5.1.

Table 5.1 Parameter Values

Parameter		
Discount Factor	β	0.98
Risk Aversion	σ	2
Persistence	$\rho_1^k, \rho_2^k, \rho_3^k$	0.95
Volatility Dollar	σ_1	0.1
Volatility Euro	σ_2	0.05
Volatility Yen	σ_3	0.02
Asset Substitution	b_{12}, b_{13}	100

As shown in the table, we set the discount factor β=0.98, which corresponds to an equilibrium annual interest rate of 2%. Risk aversion is set to σ=2, following Walsh (2010). The persistence of each asset's exogenous stochastic process is set to be 0.95. To accommodate income volatilities from different currency assets, we set the standard error of yen currency assets as 0.02, euro currency assets as 0.05 and the US currency assets as 0.1. Finally, we assume that the Chinese central bank has a high degree of substitutability between different currency assets; thus the parameter b is set to be 100.

5.5.1.2 Results from global and projection methods

Table 5.2 Summary Statistics of Currency Asset Returns

	r1_expect	*r2_expect*	*r3_expect*
Mean	1.02	1.02	1.02
Standard Error	0.00	0.00	0.00
Median	1.02	1.02	1.02
Standard Deviation	0.00	0.00	0.00
Sample Variance	0.00	0.00	0.00
Kurtosis	-0.62	-0.88	-0.91
Skewness	-0.31	0.15	-0.02
Range	0.00	0.00	0.00
Minimum	1.02	1.02	1.02
Maximum	1.02	1.02	1.02
Sum	15946.15	15960.80	15953.38
Count	15625	15625	15625

Our approach estimates 15625 possible economic states. Table 5.2 reports expected returns on each currency asset calculated by global methods. It is interesting to note that all three assets generate similar levels of real return with different levels of uncertainty. The highest level of return comes along with the moderate level of uncertainty, which in our setting is represented by the euro currency asset. This can be explained with reference to Konno and Wijayanayake (2001). They study the case of an asymmetric mean income-variance-skewness model, which has value in the case of asymmetric distribution yield, and because having the same mean and variance of the portfolio is likely to have different skewness, the possibility of a large portfolio skewness yields also increased. Table 3 reports the assets allocation, where US dollar assets dominate at 82.5%, and the euro comes second with 35%. Surprisingly, the investor holds a conservative attitude towards yen currency assets, with a -17.5% short position.

It is interesting to note the euro's allocation in the portfolio choice. While its mean return is slightly higher than the other two, one may not necessarily choose to give it a dominant weight in our portfolio. We observe that, the euro currency return has a positive skewness, implying that there more occasions where the actual return is under the mean. In contrast, the dollar currency return has a negative skewness, suggesting it has more occasions where actual return is above the mean. In practice, this means investors are more likely to invest in the dollar as a currency asset.

Figure 5.1 Distribution of Currency Assets Returns

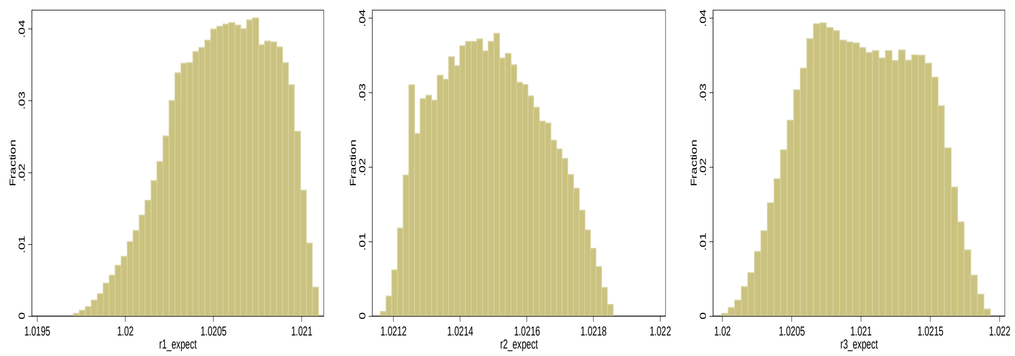


Table 5.3 Summary Statistics of Portfolio Allocation

	Yen share	*Euro share*	*Dollar share*
Mean	-0.18	0.35	0.82
Standard Error	0.05	0.14	0.09
Median	-0.27	0.55	0.72
Standard Deviation	6.65	17.61	10.96
Sample Variance	44.23	310.12	120.13
Kurtosis	2408.54	2409.87	2410.29
Skewness	-6.79	6.83	-6.86
Range	822.81	2179.01	1356.21
Minimum	-461.78	-955.98	-760.25
Maximum	361.03	1223.03	595.96
Sum	-2738.13	5475.44	12887.69
Count	15625	15625	15625

5.5.2 Results for portfolio determination

5.5.2.1 Calibration

Parameter values for the second model with a symmetric setting for the countries are reported in Table 5.4. The discount rate is set as β=0.98, corresponding to an equilibrium annual interest rate of 2%. Risk aversion is given as σ=2, following Walsh (2003). The persistence of capital and labour endowments is set to 0.95. Money supply is believed to have less persistence, so rho=0.7. Finally, the degree of asset substitutability is set to 1 to accommodate Chinese central bank's behaviour.

Table 5.4 Parameter Values

Parameter		***Value***
Discount Factor	β	0.98
Risk Aversion	σ	2
Persistence	$\rho_1^k, \rho_2^k, \rho_1^l, \rho_2^l$	0.95
Volatility	$\sigma_{k1}, \sigma_{k2}, \sigma_{l1}, \sigma_{l2}$	0.02
Money Persistence	ρ_1^M, ρ_2^M	0.7
Asset Substitution	b_h^{be}, b_f^{be}	1

5.5.2.2 Results from Global and Projection Methods

Table 5.5 reports the expected returns on all four types of assets where r1_expect represents home equity return, r2_expect is foreign equity return, r3_expect is home bond return and r4_expect is foreign bond return. It is noticeable that compared with bonds, equities in both countries have higher returns; however, under our symmetric setting of parameter values, the difference between home and foreign asset returns is not great.

Table 5.6 reports allocations of home country's investments in both domestic and foreign capital markets. It can be seen that home investors are more attracted by bond investment. Of total investments, the home investor invests 80% of his fund in both domestic and foreign markets. As far as external investment is concerned, this is consistent with China's investing behaviour. The Chinese central bank has built a massive position in the US bond market. According to TIL data, around 85% of its external assets are held in American Treasuries and Agencies.

Table 5.5 Summary Statistics of Assets Returns

	r1_expect	*r2_expect*	*r3_expect*	*r4_expect*
Mean	1.20	1.20	1.00	1.00
Standard Error	0.01	0.01	0.00	0.00
Median	1.00	1.00	1.00	1.00
Standard Deviation	0.70	0.70	0.00	0.00
Sample Variance	0.48	0.48	0.00	0.00
Kurtosis	-1.00	-1.00	-0.45	-0.26
Skewness	0.60	0.60	0.34	0.08
Range	1.94	1.94	0.00	0.00
Minimum	0.42	0.42	1.00	1.00
Maximum	2.36	2.36	1.00	1.00
Sum	18673.55	18673.52	15625.13	15625.24
Count	15625	15625	15625	15625

Table 5.6 Summary Statistics of Portfolio Shares

	Domestic equity share	*Foreign equity share*	*Domestic bond share*	*Foreign bond share*
Mean	0.20	0.20	0.80	0.80
Standard Error	0.01	0.01	0.01	0.01
Median	0.00	0.00	1.00	1.00
Standard Deviation	0.70	0.70	0.70	0.70
Sample Variance	0.48	0.48	0.48	0.48
Kurtosis	-1.00	-1.00	-1.00	-1.00
Skewness	0.60	0.60	-0.60	-0.60
Range	1.94	1.94	1.94	1.94
Minimum	-0.58	-0.58	-0.36	-0.36
Maximum	1.36	1.36	1.58	1.58
Sum	3048.40	3048.29	12576.60	12576.71
Count	15625	15625	15625	15625

5.6 Conclusions

Safe assets play a central role in the financial system. Fundamental changes to their supply and demand relationship may change the structure of the safe assets share, and hence affecting financial stability. Recent research shows that, in matured economies like the US., the real sector's demand for safe assets has been remarkably stable, while demands from the financial sector and from foreign governments are drastically growing. This has changed financial landscape of the US. and its adverse consequences have been behind many macro imbalances.

It is debatable that whether the global economy has run short of safe assets and in any case their supply may expand in scope and scale with the growing demand. Drastic growth of demand however represents a vital force that may influence financial stability. This is especially so in the light of recent large capital flows into the US from foreign governments.

This research tells a demand side story from China. As the largest reserve holder of the world, China has also been the largest foreign holder of American Treasuries and Agencies. In a DSGE framework, we explore China’s international portfolio decision. Prior literature in this area has been limited by the portfolio indeterminacy in policy-relevant dynamic general equilibrium modelling. This chapter applies global methods to analyse international portfolio allocation for the Chinese central bank. Project methods and the Parametrized Expectation Algorithm are employed to solve for the international portfolio choice including the currency portfolio allocation and weight selection of domestic and foreign equity and bonds.

Our results show that the US dollar and American government debt including Treasuries and Agencies are not a safety trap for China. Rather, it is an equilibrium act for the Chinese central bank to hold a dominant weight of the dollar and US government debt instruments in its international portfolio choice. In the long run, the sustained China' capital flows into US. and the consequent holdings of American safe assets would be rewarded with only low interest rates to discourage its adverse impact on the safe asset share.

Chapter 6

Conclusions

The final chapter presents the main findings of the thesis and possible directions for future research.

Chapter 6

Conclusions

6.1 Main Findings and Policy implications

This thesis evaluates the rapid growth of China's foreign exchange reserves through theoretical modelling and empirical test. The conventional theory of foreign reserves has witnessed a shift of research focus from the transaction purpose to prudential considerations as the main motivation for holding foreign assets. Specifically, this thesis aims to explore the representative agent's decision making, by allowing the agent to account for future uncertainty at the equilibrium point. To characterize the behaviour of each sector of the whole economy, we implement our research within a DSGE framework. Therefore, weare able to achieve a better understanding of the effect of foreign reserve accumulation on other economic sectors.

After providing a comprehensive review of the related literature, this thesis applies DSGE modelling to study the determination of optimal reserves. The main findings are based on local approximation of the second order for a risky world. Rather than the deterministic steady state, we re-calculate the optimal foreign reserve with future variance. Through the analysis, we find that optimal holdings of foreign reserves in the risky steady state are influenced by the risk-premium, due to imperfect international risk sharing and the effects of precautionary savings. As a result, defining the steady state value as the optimal level of foreign reserves, the amount of optimal reserves is greater in a risky world than in a world with certainty. The numerical results confirm our conclusion.

If China's build-up of massive foreign reserves is for prudential purposes, another mystery is why those foreign assets are allocated mainly to dollar denominated assets. Furthermore, bond related assets represent a large proportion of China's portfolio pool. Recent research has offered some potential explanations for this situation. However, the literature to date has not been particularly successful in explaining the portfolio choice considerations on co-dependency between China and the US, either empirically or theoretically. We investigate this circumstance by setting a two-country DSGE model, including a VaR constraint to reflect the investor's risk averse behaviour. Our main finding is that the VaR constraint plays a significant role in shaping the behaviour of all economic actors, from consumer to monetary authority. Both countries experience a higher output compared with an economy without VaR constraint. Also, the portfolio choice is influenced by the constraint; taking account of the risk associated with each type of asset, the portfolio manager is willing to shift the portfolio to bond investment.

Finally, in order to better understand the portfolio choice of China's foreign reserves and its influence on the global safety asset market, this research presents a demand side story from China. As the largest reserve holder in the world, China has also been the largest foreign holder of American Treasuries and Agencies. In a DSGE framework, we explore China's international portfolio decision. Prior literature in this area has been limited by the portfolio indeterminacy in policy-relevant dynamic general equilibrium modelling. This chapter applies global methods to analyse international portfolio allocation for the Chinese central bank. Project methods and the Parametrized Expectation Algorithm are employed to solve the international portfolio choice, including the currency portfolio allocation and weight selection of domestic and foreign equity and bonds. Our main findings imply that the accumulation of US denominated assets is reasonable investment behaviour, which accounts for future uncertainty. In the long run, the sustained capital flows from China into the US, and the consequent holdings of American safe assets, will be rewarded with only low interest rates, to discourage any adverse impact on the safe asset share.

6.2 Policy Implications

The results of our paper provide a good reference point for China's foreign exchange policy. Recommendations for policy makers can be summarized as follows. First, it can be seen from the historical data that China's accumulation of foreign exchange reserves is driven both by an aim to maximize GDP, and by the country's export oriented policy; however, at the same time, an export-oriented economy is more vulnerable to external shocks. Consequently, China's central bank will continue to hold a relatively high proportion of foreign exchange reserves for prudential reasons, and this situation will not change over the short term. Therefore, for foreign exchange policy makers, the main issue of concern should be increasing the value of foreign assets.

The dollar's status as the major component of foreign exchange assets has sparked fears among policy makers that China might have entered a dollar trap. From the results reported in our paper, choosing the US dollar as the main investment asset is rational investment behaviour, especially after weighing the risks. In order to diversify risk, policy-makers may try to extend the range of investments to include some real assets, such as crude oil and gold. Meanwhile, since the dollar is the main component of safe assets, China's position in the safe assets market will remain important. On the demand side, China will continue to hold a stable number of safe assets as the country's reserves. Therefore, it can be concluded that China's demand for safe assets will not disturb the equilibrium of the global safe assets market.

6.2 Limitations of Study and Future Research

The DSGE approach has proved to be a very useful toolkit for analysing macroeconomics, including the reserve policy. However, this thesis has some limitations. First, the current DSGE model we apply is limited to a two-country framework, which might not be sufficient to analyse foreign reserve management globally. Second, despite the recent progress of DSGE modelling, challenges exist when modelling emerging economies in general, and China in particular. Third, the current range of investable assets is extensive, and not all are included in our research.

In response to these limitations, the following plausible directions for future research are suggested: First, given the development of the DSGE framework, a multiple countries model can be introduced. A possible extension for our research would be to extend our two-country model into a three country model, including three different economies (China, US and the rest of the world). Under this setting, it would be possible to better understand the movement of foreign reserves globally.

Second, despite the popularity of DSGE analysis, few studies focus specifically on China's economy. This is partly because of the difficulty of modelling the behaviour of emerging markets under the DSGE framework. Various directions are available: for instance, modelling frictions in financial markets, interaction between financial openness and international trade, various shocks facing China, and the bearing of all these factors on China's holdings of foreign reserves. In addition, future research should consider not only the idiosyncratic features of the structure of China's domestic economy, but also the nation's vulnerabilities to external factors. In future research we will gradually deepen our analysis by dealing with these critical research tasks.

Third, with the development of the international investment market, a possible modification of our research would be to broaden the investment universe of foreign reserves. Gold and oil are particularly interesting assets to consider as an investment choice. In the current situation of volatile financial markets, where safe assets are in great demand, gold and oil become perfect strategic reserves investment assets.

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