M.O. 420t

AIR MINISTRY

METEOROLOGICAL OFFICE

PROFESSIONAL NOTES NO. 100

(Twentieth Number of Volume VI)

PREDICTION DIAGRAMS FOR RADIATION FOG

By W. C. SWINBANK, B.Sc.

LONDON
HIS MAJESTY'S STATIONERY OFFICE

1949

FOURPENCE NET

Decimal Index
551.509.39 : 551.575.1

Prof. Notes Met. Off.
London, 6, No. 100, 1949

PREDICTION DIAGRAMS FOR RADIATION FOG

By W. C. Swinbank, B.Sc.

Previous inquiries into the incidence of radiation fog have been concerned solely with observations of surface elements such as dry-bulb temperature, dew point and to some extent wind. But all these quantities, and their variations, depend largely on their vertical distribution and more especially on their distribution in the lowest layers of the atmosphere. Hence an attempt was made to correlate the incidence of fog on clear radiation nights with the vertical distribution of water vapour.

The vertical decrease of vapour pressure with height is termed the "hydrolapse"; it is defined as the decrease in the vapour content expressed as milligrams of water vapour per kilogram of air per millibar in the layer from the surface to the base of the region of markedly increased stability or even inversion, which is usually associated with the anticyclonic conditions favourable for radiation nights. This level may occur as low as 950 mb. or as high as 850 mb. In the absence of such a stable region, the hydrolapse was found to be rather independent of height throughout about 200 mb., and can therefore be reasonably estimated from any two values of water vapour below about 800 mb. Values of the hydrolapse were obtained from aircraft ascents, first at Mildenhall and later at Bircham Newton. The observations were taken at the surface, at a height of 1,000 ft. and then at intervals of 50 mb.; this is not really sufficiently detailed for the purpose.

A radiation night is defined as any night during which the cover of low cloud did not exceed 5 tenths at any of the synoptic hours of observation. High cloud was regarded as having little effect on radiation; medium cloud is uncommon.

The process of condensation of the water vapour in the air near the earth's surface is aided in two ways :—

(1) Cooling of the surface layer. This is most effective in conditions of calm, since there will be no tendency for eddy transfer of heat in the vertical. Therefore the air near the surface will, under such conditions, undergo nearly the same cooling as the surface itself, which on clear nights is considerable.

(2) Retention of the water vapour already present in the surface layer, or, better still, increase of this water-vapour content by diffusion downwards. This is determined by the hydrolapse; a large lapse will tend to remove water vapour from the surface layer; a small lapse or an inversion will operate so as to retain water vapour in this layer.

The preliminary investigation took no account of absolute vapour content as it was expected that there would be a high correlation between this quantity and the hydrolapse. In the final form of the prediction diagram, however, dew point in the screen at 1800 G.M.T. is introduced as one of the variables.

The processes unfavourable to the formation or persistence of fog are turbulence and the removal of moisture by deposition.

The tendency towards turbulence in a layer of air is governed by wind shear in a twofold manner. In turbulent motion in stable air, the eddy stresses must provide the necessary work against the gravity forces. Now the eddy stresses are determined by the wind shear, and the sliding motion in unit time of one layer over another is also proportional to the wind shear. Hence the work done by the eddy stresses is proportional to the square of the wind shear, and so the tendency to turbulence is governed by this factor. Wind shear was measured as the average of the squares of the scalar differences in miles per hour between wind speed at the surface and the gradient wind over the area concerned for the three hours 1800, 0100 and 0700. No account was taken of the difference in direction between surface and gradient wind. Note that the scalar differences are squared first, and the averages of the squares taken.

In practice it was found that the effect of wind shear was proportional to a power of less than two.

Houghton and Radford in their investigations of fog at Round Hill, Mass., showed that country fogs have a spectrum of drop sizes such that the maximum frequency of occurrence is for drops of about 10μ radius, whilst the drop size which contributes most to the water content of a fog is about 40μ radius.

An average thickness for a radiation fog is about 300 ft. The terminal velocity of a drop of 10μ radius is 1 cm./sec., so that a drop this size would fall 300 ft. in about three hours in the absence of turbulence, whilst a drop of 40μ radius would travel in the same distance in a little over ten minutes. The flux of liquid water in the absence of turbulence is given by

$$F \text{ (downwards)} = \Sigma\, nvm$$

where n is the number of droplets of given size per cubic centimetre, m the mass of a drop and v the terminal velocity. The summation is taken over all drop sizes.

With turbulence active the tendency is for drop concentration, like other properties in the atmosphere such as water vapour, momentum, smoke content, etc., to become equally distributed in the vertical in a saturated atmosphere and the flux equation is now

$$F = mnv + mK\frac{\partial n}{\partial z}$$

where K is the coefficient of eddy diffusion and v is the terminal velocity.

The rate of accumulation of drops at a point will be given by

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial z}\left(nv + K\frac{\partial n}{\partial z}\right)$$

$$= n\frac{\partial v}{\partial z} + v\,\frac{\partial n}{\partial z} + K\frac{\partial^2 n}{\partial z^2}$$

assuming K independent of height.

Now v is dependent only on the drop size, and for the distances considered here it is easily shown that the rate of condensation of water on a droplet will cause no appreciable change in size, and so v can be assumed independent of z.

Thus

$$\frac{\partial n}{\partial t} = v\,\frac{\partial n}{\partial z} + K\,\frac{\partial^2 n}{\partial z^2}$$

The solution of this equation presents some awkward aspects, partly because of the difficulty in giving a correct representation of the boundary conditions. It should be possible to apply the solution to the discussion of the lowering of a cloud base under certain conditions.

Thus the vertical distribution for no flux of particles of a given size is

$$n = n_0 e^{-vz/K}$$

where n_0 is the number of drops per cubic centimetre at some arbitrary level.

No information is available to show the distribution of drop concentration in fogs, but it is probable that, in very stable conditions, *i.e.* with small K, then the flux term due to the terminal velocity will outweigh that due to turbulence at any rate for the bigger drops. It seems that in such conditions there will be a tendency for the fog to be removed by deposition in the absence of any source of replenishment of the drops. There are two ways in which the drops can be replenished.

(1) By the eddy flux of moisture downward from the layer above the fog. It will be shown later that the rate of transfer of water vapour by this means is very small and insufficient to replenish the water removed by deposition of drops. Furthermore there is no reason why removal of droplets from the top of the fog should result in flux of moisture downwards since the air in the fog layer will remain saturated.

(2) By continued cooling within the fog layer or, more probably, at its upper boundary.

It should be possible, by examination of the humidity of the air above the fog layer, to determine whether there is a sufficiency of water vapour there to replace the loss through deposition of droplets from within the fog itself. At Dunstable, on the evening of December 2, 1941, anticyclonic radiation fog, which was in its second day, had become very wet, and there appeared to be a rapid rate of deposition of water. The afternoon tephigram from Bircham Newton, which could be taken as representative, showed a very dry layer above the fog layer. The next morning the fog had cleared from Dunstable, and was much less widespread elsewhere, though there had been no change in the synoptic situation.

The flux of water vapour in the vertical due to eddy diffusion is given by $F = K\partial q/\partial z$ where K is the coefficient of eddy diffusion and q the specific humidity. Giving K the value 10^4 in C.G.S. units and an increase of q of 2 gm./Kg./50 mb. then water vapour will diffuse downwards at a rate of about 3×10^{-7} gm./sq. cm./sec. This value of K is probably much greater than will actually occur in such stable conditions as exist in a fog layer ; 10^3 is probably a more representative value.

Next we consider a fog composed, for the sake of computation, of drops of radius 10μ, with 200 drops per cubic centimetre of air. Then the rate of deposition of water due to the downward velocity of the drops is about 8×10^{-7} gm./sq. cm./sec. in the absence of turbulence. Thus it appears that the rate of deposition is probably much greater than that of flux of water downwards. This result has some bearing on Petterssen's discussion on the effect of snow as a fog-dissipating agent.

Construction of fog-prediction diagrams.—For the first diagram (Fig. 1) the months of October and November were considered together, and the weather at 0700 G.M.T. on the morning after a radiation night was divided into three types :—

(i) widespread fog,
(ii) local fog, and
(iii) no fog.

The area considered was England south-east of an approximate line, The Wash–Cranwell–Birmingham–Upper Heyford–Southampton. The investigation covered 56 nights over the six years 1936–41, including 15 occasions of widespread fog, 24 of local fog and 17 of no fog. These were plotted in Fig. 1 as crosses, points or circles against the square of the wind shear as abscissa and the hydrolapse as ordinate. It is seen that the diagram can be divided into three distinct areas :—

A Fog is certain and may be widespread.
B Local fog only may be expected.
C No fog will form.

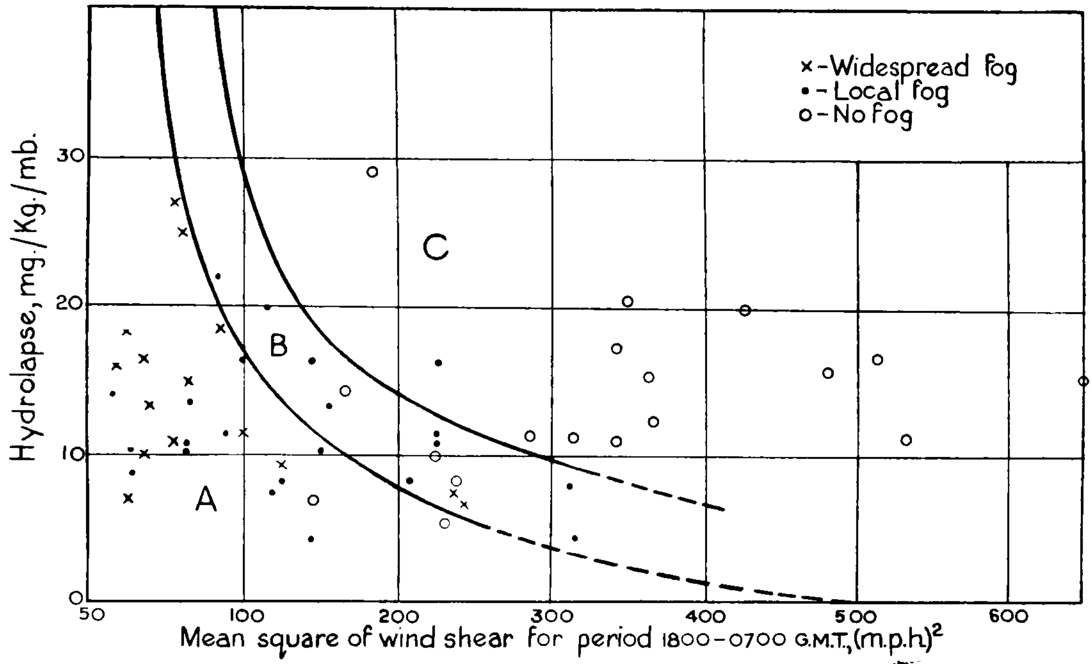

FIG. 1.—OCCURRENCE OF FOG IN RELATION TO SQUARE OF WIND SHEAR AND HYDROLAPSE IN SOUTH-EAST ENGLAND, OCTOBER–NOVEMBER

Both hydrolapse and wind shear are important in the formation of fog, but in cases of widespread fog the absence of wind shear appears to be the dominating influence. On most radiation nights the square of the wind shear will not differ appreciably from the square of the gradient wind.

This diagram was transferred to the left-hand side of the prediction diagram (Fig. 2). The few exceptions were regarded as due to an unusually high or low initial water-vapour content of the air, and the depression of the dew point below the dry bulb was added to take account of this factor. The figure adopted is the average depression at 1800 G.M.T. on the evening in question at the synoptic stations over the area considered.

The stations used were Mildenhall, Cranwell, Croydon, Birmingham and Upper Heyford. By plotting the occurrence of widespread fog against mean depression of dew point as abscissa and hydrolapse as ordinate another curve could be drawn separating areas of fog from areas of no fog or only local and patchy occurrences. This is reproduced on the right-hand side of Fig. 2.

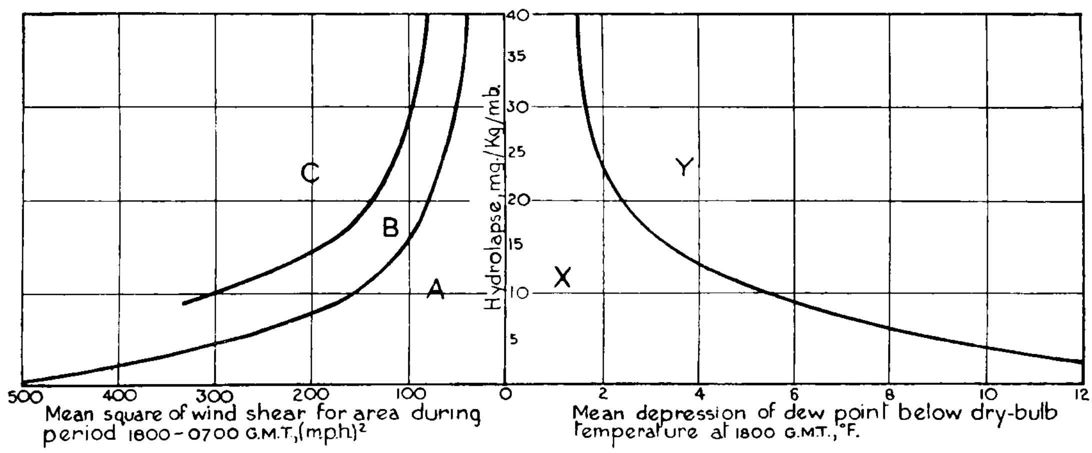

FIG. 2.—FOG PREDICTION DIAGRAM FOR OCTOBER AND NOVEMBER

England, south-east of line The Wash–Birmingham–Southampton

To use the diagram, the hydrolapse must first be determined from the afternoon upper air ascent. The depression of dew point for the area is measured at 1800 G.M.T. The wind shear for the period 1800 to 0700 has to be estimated at 1800.

These three values having been obtained, a representative point can be plotted on each part of the diagram, and an estimate made of the chance of radiation fog developing according to the following rules.

(i) If one point falls in the area X and the other in area A, then fog will develop ; and there is about an even chance that it will be widespread.

(ii) If one point falls in X, and the other in C, or if one point falls in A, and the other in Y then fog may or may not develop. If it does, the fog will be patchy or local.

(iii) If one point falls in the area C and the other in area Y, then fog will not develop.

(iv) The area of transition between A and C is B. If one point falls in area B and the other in area X local fog is more probable than if the point for this left-hand part of the diagram had fallen in C. If the one point falls in B and the other in Y even local fog is improbable.

A diagram on the same principles was afterwards constructed for October and November for Lincolnshire and Yorkshire. It is based on surface data from Driffield, Linton-on-Ouse, Catterick, Cranwell (1939–42), Waddington, Finningley, Leconfield (1940–42) and Leeming (1941–42), and upper air data from first Mildenhall and subsequently Bircham Newton. In calculating the hydrolapse, the mean screen-level dew point for all the stations at 1600 G.M.T. was used to obtain the humidity mixing ratio at the surface, instead of the value at Mildenhall or Bircham Newton only. The upper value of the humidity mixing ratio was obtained from aeroplane soundings at 1600 G.M.T. (except in 1939, when the ascents were made earlier in the afternoon). The 1600 G.M.T. upper value of the humidity mixing ratio usually differs little from that given by the midday ascent.

The result is shown in Fig. 3, but on the left-hand side only one curve is shown, separating the region of widespread fog from that of only local or patchy fog or no fog, *i.e.* it corresponds with the curve separating A and B in Fig. 2. It was not possible to recognise on this diagram an area corresponding to C in Figs. 1 and 2, but the data indicated that it is reasonable to regard the area B as one of gradually decreasing risk of fog with increasing distance from the origin. As was to be expected, the curves in Fig. 3 are otherwise similar to those in Fig. 2. The main difference is that in the diagram for East Anglia and south-east England (Fig. 2) the curves separating areas A and B and X and Y tended to be asymptotic to the abscissa through the zero ordinate, whereas in Fig. 3 they definitely cross these co-ordinates at higher values of the square of the wind shear and depression of dew point. The number of observations was sufficient to make the position of the curves in these regions fairly definite. This difference may be explained to some extent by the fact that in Fig. 3 the hydrolapse was measured at 1600 G.M.T. whereas Fig. 2 was based on an average time of measurement between 1300 and 1400 G.M.T.

The use of the diagram is similar to that of Fig. 2.

Plot the points P and Q on the right- and left-hand sides of the diagram. Then :—

(i) If P falls in area X and Q in area A widespread fog is likely.

(ii) If P falls in area Y and Q in area B fog is unlikely.

(iii) In other cases (*i.e.* P in X, Q in B or P in Y, Q in A) fog may occur but will be local or patchy.

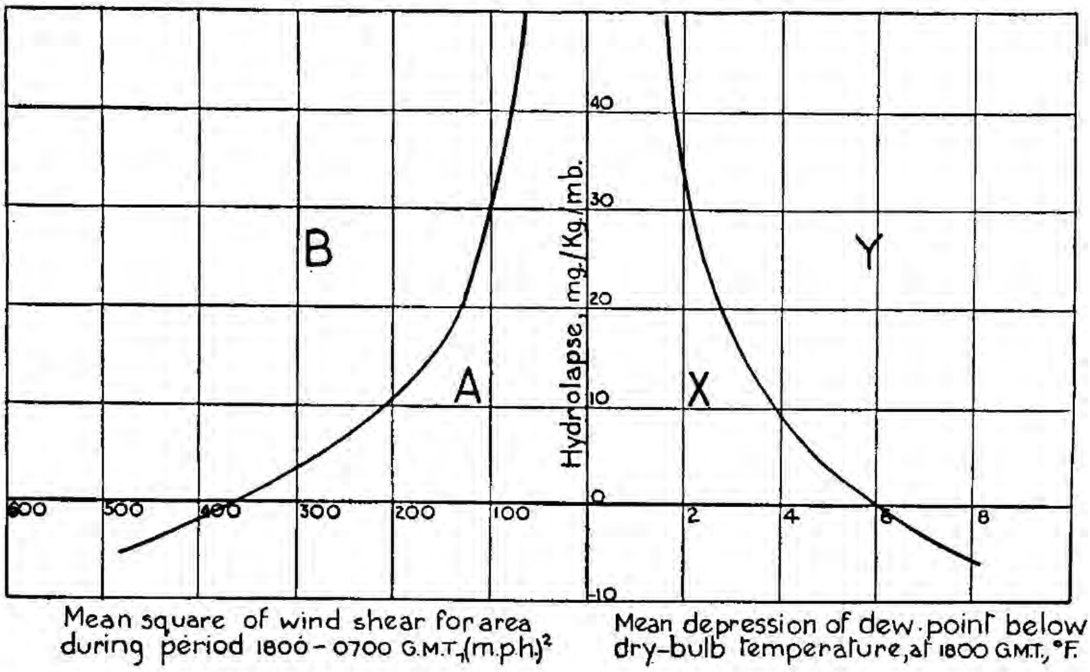

FIG. 3.—FOG PREDICTION DIAGRAM FOR OCTOBER AND NOVEMBER
Lincolnshire and Yorkshire

Prediction of time of formation of fog (Lincolnshire and Yorkshire).— In preparing Fig. 3, it was found that for given values of two of the three elements used in the construction of the diagram, the time of formation of fog was, in a general way, determined by the value of the third element. Accordingly the incidence of fog was plotted with square of wind shear as ordinate and depression of dew point at 1800 G.M.T. as abscissa, these two quantities being defined as before. Over a large area the time of formation of fog is difficult to define ; the convention adopted was that if the point fell in area A (widespread fog probable) the time was taken as that when fog had formed at half the stations. For points in area B (local fog) the time taken was the mean time of occurrence at stations reporting fog. All times were measured as hours since sunset.

Examination of individual points showed that a position on this diagram unfavourable to the formation of fog (*i.e.* well removed from the origin) could be counterbalanced by a small value of the hydrolapse, and the formation of fog accelerated. Conversely, a point favourably situated on the diagram could have the time of formation of fog retarded by a large hydrolapse. It was found that the diagram could be divided into zones so that in any one zone the combined effects of wind shear and depression of dew point on the time of formation of fog were approximately equal. The lines bounding these zones were roughly parallel to the lines separating the areas A, B and C. These zones were numbered 1 to 6 (Fig. 4).

It will be noticed that no zone has been drawn near the origin. A position so near the origin always resulted in fog very soon after 1800 G.M.T. irrespective of the value of the hydrolapse (which was always small on these occasions).

A subsidiary diagram (Fig. 5) was then prepared, the co-ordinates being time of formation of fog and hydrolapse. The appropriate values for the points in each zone of Fig. 4 were plotted against these co-ordinates and the best fitting curve was drawn through each group of points. These lines are marked 1 to 6 to correspond with zones 1 to 6 in Fig. 4 and refer to the middle of the corresponding zones.

To use these diagrams to obtain a broad indication of the time of formation of fog, the mean depression of the dew point below the dry-bulb temperature is measured and the mean wind shear throughout the night is estimated ; the representative point is then plotted on Fig. 4. This gives an indication whether the ensuing conditions will be widespread fog, local fog or no fog. If the point falls in areas A or B, the line in Fig. 5 appropriate to the zone in

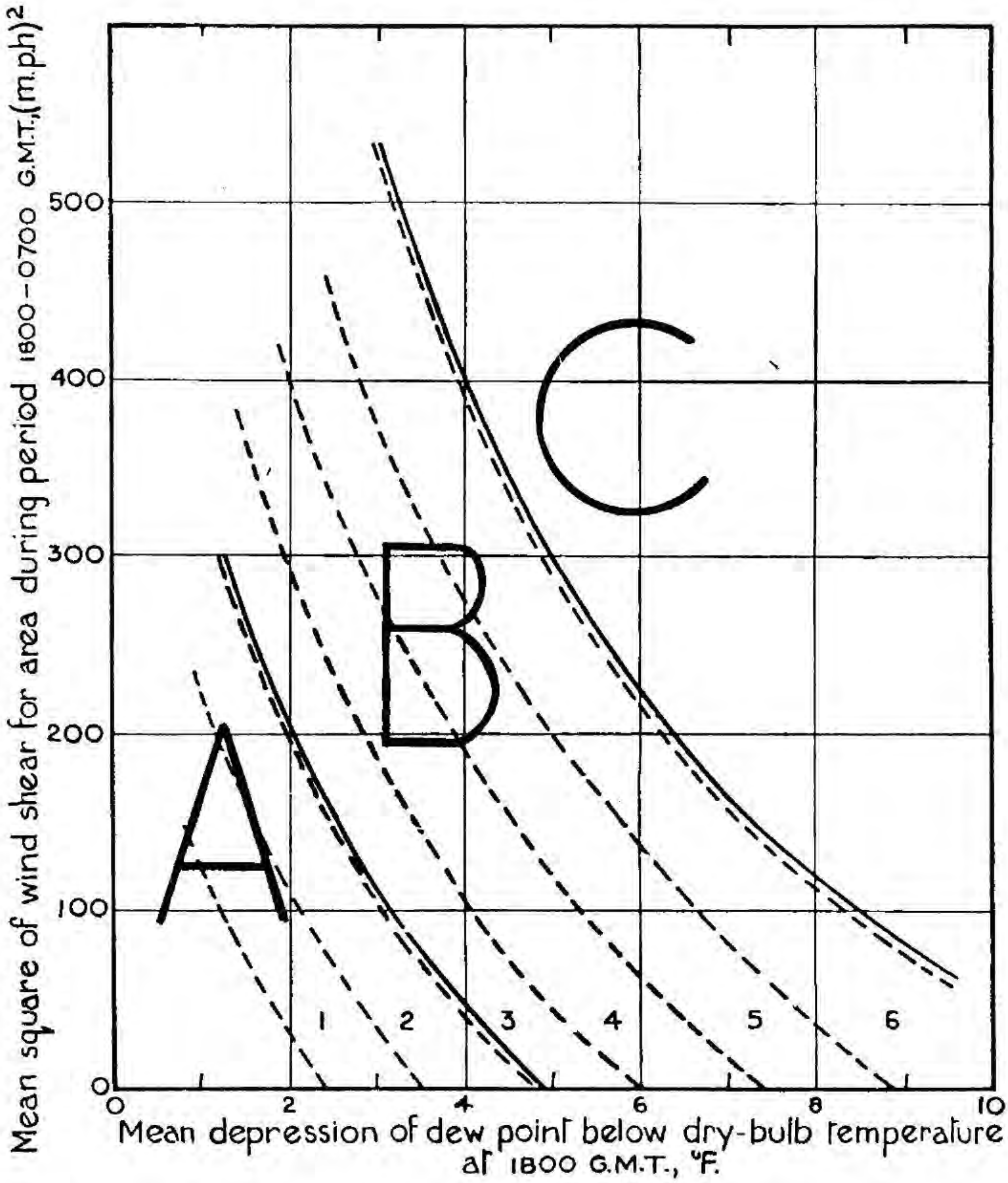

FIG. 4.—INCIDENCE OF FOG IN LINCOLNSHIRE AND YORKSHIRE

A = widespread fog, B = Patchy fog, C = No fog

The pecked lines divide the diagram into zones in which the combined effect of wind shear and depression of dew point on the time of formation of fog is practically constant.

which the representative point falls is used, in conjunction with the measured value of the afternoon hydrolapse, to give in hours after sunset an indication of the " time of formation " of fog as defined above. If the point falls near the edge of one of the zones in Fig. 4, it will be necessary to interpolate between the lines in Fig. 5 to obtain a more precise estimate.

It will be seen that if the representative point falls in area A of Fig. 4, fog forms at half the stations before midnight, even if the value of the hydrolapse is high. On the other hand, if only local fog is expected, even the most favourable value of the hydrolapse does not cause fog to form until some hours after 1800 G.M.T.

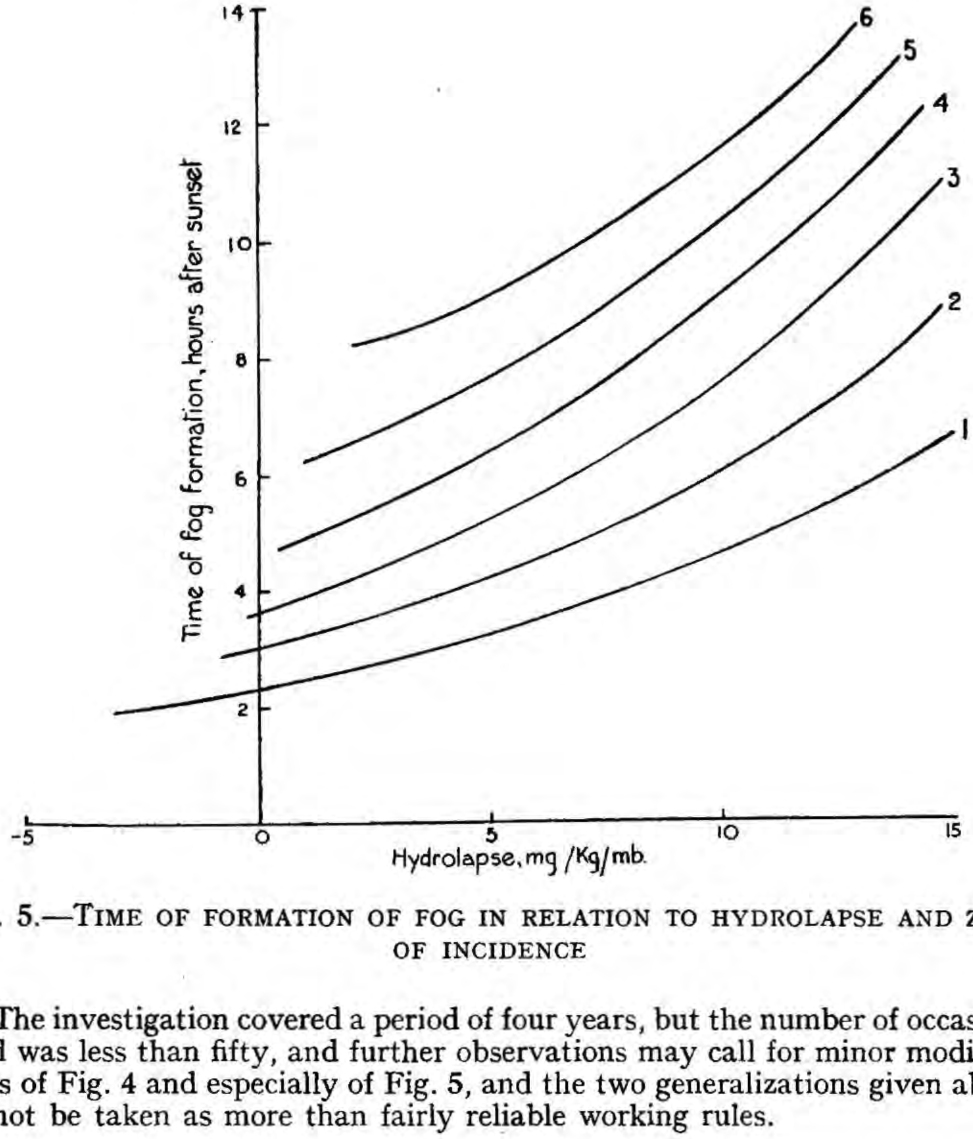

FIG. 5.—TIME OF FORMATION OF FOG IN RELATION TO HYDROLAPSE AND ZONE OF INCIDENCE

The investigation covered a period of four years, but the number of occasions used was less than fifty, and further observations may call for minor modifications of Fig. 4 and especially of Fig. 5, and the two generalizations given above cannot be taken as more than fairly reliable working rules.

(88913) Wt. 2018/M.3148 9/49 Hw. PRINTED IN GREAT BRITAIN

LONDON
PRINTED AND PUBLISHED BY HIS MAJESTY'S STATIONERY OFFICE
To be purchased directly from H.M. Stationery Office at the following addresses:
York House, Kingsway, London, W.C.2; 13a Castle Street, Edinburgh, 2;
39 King Street, Manchester, 2; 2 Edmund Street, Birmingham, 3;
1 St. Andrew's Crescent, Cardiff; Tower Lane, Bristol, 1;
80 Chichester Street, Belfast
OR THROUGH ANY BOOKSELLER
1949
Price 4*d.* net

4C-133