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THE SYNTHESIS OF DAILY RIVER FLOW RATES

FROM DAILY RAINFALL RECORDS

by

Brian Archer, B.Sc., M.I.C.E., A.M.I.W.E.

Presented as a thesis for the degree of M.Sc.

Faculty of Science University of Durham



October, 1968

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Abstract of the thesis

THE SYNTHESIS OF DAILY RIVERFLOW RATES

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The work commences with an explanation of the need for synthesised riverflow values for reservoir yield calculations and a review of the factors which influence the rainfall-runoff relationship. This is followed by a critical résumé of the modern literature on mathematical models of hydrologic systems which involve the use of a digital computer for runoff synthesis. The problem of synthesis from limited data is discussed and a line of investigation is proposed.

The investigation is carried out using data from a catchment lying on the borders of Co. Durham and Northumberland. Details of this area and the recording instruments on the catchment are given.

The actual daily riverflow values are first processed and then plotted in the form of a cumulative deficiency diagram. The storage conditions revealed by this detailed curve are then compared with

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published values obtained by traditional methods. The distribution of daily riverflow is investigated for various calendar groupings and conclusions drawn. The seasonal correlation of rainfall-runoff values, grouped according to antecedent precipitation index values is then performed. Since the store capacity of the computer available for the analysis was only 8K the data had to be read in and out in groups and as a result the processing time was approximately 8 hours. An alternative method was devised, by which a large number of pieces of data could be held in a store of limited capacity. This system, which performed the analysis in less than a twentieth of the time, is then described.

The effect of varying the recession factor through a range of values from 0.85 to 0.95, in steps of 0.01, is investigated. As a result of this investigation a further analysis is attempted. In this instance runoff on dry days is correlated with A.P.I. whilst runoff on rainfall days is correlated with daily rainfall in moving, fifty day, A.P.I. groupings. Daily runoff values are then synthesised, from the equations derived from the last analysis, and diagrams of cumulative excess of runoff values are drawn. Finally, details are given for an alternative method of analysis using potential transpiration value in place of A.P.I. values for all but the winter months.

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The programmes written for this thesis are given in appendix I together with samples of the input data required to run them, and the output data derived from them. Copies of the fifty-three graphs drawn by the plotter are shown in appendix II.

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1. INTRODUCTION

1.1. Reservoir yield calculations

The reliable yield of an impounding reservoir in a temperate climate is mainly dependent on the run-off from the catchment, on the size of the reservoir and on the management of the draw off rate. Difficulties arise with the design of reservoirs because data on future run-off is of course unobtainable and the validity of using river flow records from the last 60 or 80 years to predict the required reservoir capacity is open to doubt. However, since we have no means of determining what will happen in the future we have no alternative but to base our design on past occurrences.

Numerous techniques have been developed to generate hydrologic data and to make sequences of hydrologic events from limited historical data. Two different concepts have been employed for the process of synthesis : the parametric and the stochastic. (Ref.1).

Parametric Hydrology is defined as the development and analysis of relationships among the physical parameters involved in hydrologic events and the use of these relationships to generate, or synthesise, hydrologic events. Historical data and known physical data generally are ultilized to develop the relationship.

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Stochastic Hydrology is defined as the manipulation of statistical characteristics of hydrologic variables to solve hydrologic problems, on the basis of the stochastic properties of the variables. One of the applications is the rearrangement of time sequences of historic hydrologic events and the generation of representative non-historical sequences.

Methods which estimate reservoir yield from available flow data depend on that data being representative of a long period. Collinge (Ref.2) considered that the statistical approaches have limitations in that the random processes used to generate fictitious data may differ from reality because they take insufficient account of serial correlation. They also suffer from the disadvantage that in order to obtain the required precision it may be necessary to generate an extremely long record.

In the design of small reservoirs which only utilize a fraction of the long term average run-off the sequence of wet and dry years is not particularly significant, for such reservoirs have critical periods of 8 months or 20 months. In the case of larger reservoirs with a high utilization of run-off, then the sequence is of great importance since the reservoirs may remain in a drawn down state for a number of years.

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Run-off records from gauging stations near proposed reservoirs rarely extend for more than 10 or 15 years and if we are to take into account the sequence of wet and dry years with their tendency to persist at given levels longer than might be expected, then it is necessary to correlate the run-off with carefully chosen causative parameters.

It is necessary to have a long period of parameter record for the area under consideration and this stipulation alone excludes most climatological and ground moisture data. Long term records of rainfall exist for many areas throughout the country. It is the obvious input and is therefore the most significant causative parameter.

1.2. Factors which influence the rainfall-runoff relationship

For any catchment, the relationship between rainfall and run-off is controlled by the physiographic characteristics of that catchment. The characteristics are inter-connected in their effects on the water loss for any period.

Geologic characteristics at the surface and sub-surface of the catchment, will determine the rate of percolation, according to the permeability of the material, and will thus effect evapotranspiration, according to the retention and occurrence of ponding and plant cover. Geologic factors influence the manner of drainage, the infiltration capacity and the drainage pattern.

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Climatic effects are related to the physical features of the catchment, especially orographic rainfall. Other climatic effects include general rainfall distribution, storm patterns and movements, prevailing winds, duration, intensity and type of precipitation. The type of precipitation, whether snow, or rain will influence the lag effect. Snow can have a high evaporation rate and may be almost totally lost to the atmosphere as evaporation or become direct run-off on a saturated catchment. Duration and intensity of rainfall are involved in lag effects as are distribution and storm movement. Climatic conditions also effect losses resulting from evaporation and transpiration since these are dependent on the intensity and duration of sunshine, on temperature, atmospheric humidity, wind speed and type of vegetation.

The vegetation of the catchment most certainly affects the disposal of the precipitation. In Vegetation and Hydrology, H.L.Penman (Ref. 3) has given an excellent survey of the relevant information on the subject. The vegetation influences the interception of precipitation, and the fate of the intercepted water. It also affects the evaporation, transpiration and the rates of infiltration, run-off and erosion.

The morphology of the catchment affects the run-off characteristics. The area of a catchment is normally delineated by surface topography but a more precise value is obtained if the sub-surface or phreatic boundary is measured. This boundary is not necessarily fixed, but may

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vary with ground water levels. Shape governs the efficiency of a catchment or watershed and various factors are used to represent linear, areal and gradient aspects. A. Gerard Boulton (Ref.4) has given a useful summary of these in a Water Resources Board pamphlet.

The slope of the catchment influences the rate of surface run-off, interflow and ground-water storage. The relative mean elevation of the catchment is a controlling factor in relation to temperature and thus precipitation, probable snowfall and hence potential losses.

The orientation of the catchment is another factor involved in the relationship. For example the orientation of the catchment slopes will control the amount of heat received from the sun and thus snow melting, plant growth and losses in the form of evaporation and transpiration. Also the orientation of the catchment with respect to the direction of rain-bearing winds is a consideration in the higher parts of the catchment.

For any one catchment, many of the factors described previously are fixed. To compensate for those that are variable, long time climatic records other than rainfall would be required. These are normally only available for one or two sites in each county and this

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limits their application. As a consequence, maximum use must be made of the rainfall data.

1.3. Mathematical models of catchments

The word "model" as applied to hydrological processes has been defined by Snyder and Stall (Ref. 5) as "An equation or formula, built by consideration of the pertinent physical principles, operated on by logic and modified by experimental judgement and plain intuition".

Work on modelling the rainfall runoff relationship has been mainly directed to either the prediction of runoff, for use in work involving the prediction of flood magnitude and frequency, or the calculation of weekly, monthly or annual runoff from a catchment for water resources studies. As work on mathematical models proceeds this dichotomy should disappear.

The ideal mathematical model would specify all the properties of a catchment and all the processes that occur in all the relative components of a catchment. The specification would be given in terms of physical parameters and would involve all behavioural relationships within the catchment. Given such a full specification, the hydrologic effects of rainfall events over a catchment could be determined objectively. Present day mathematical models only approximate to this idea.

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In recent years it has been recognised that catchment behaviour can be represented as a system from which an output occurs as a response of the system to an input. As a consequence certain computational techniques developed in systems engineering have been applied to riverflow analysis and synthesis. This systems engineering approach has led to a classification which divides mathematical models of catchment behaviour into two broad classes: linear systems and non-linear systems. O'Donnell (Ref.6) has given a clear explanation of this classification.

These two broad classes of linear and non-linear systems are further subdivided into (a) those in which the input to and output from a system can be treated by methods of analysis to yield information on the response characteristics of the system and (b) those in which synthesis or simulation techniques that in effect provide mathematical models of catchment behaviour are used.

1.4. Linear system synthesis

The unit hydrograph theory, which applies to storm runoff, is based on the premise of a linear input-output relationship. This theory states that if the input to a catchment is in the form of a rainfall excess which is uniformly distributed over the area, then the catchment will modify the input by passing it through a linear system and will produce an output which is in the form of a hydrograph of surface runoff.

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All linear systems obey the principle of superposition and therefore with the unit hydrograph method the runoff hydrograph produced from a sequence of rainfall events can be obtained by adding the separate runoff hydrographs which would be produced if each of the rainfall events were applied to the catchment individually. The theory assumes that the catchment is a time invariant linear system and therefore no matter when the rainfall excess is applied it will always produce the same surface runoff hydrograph.

Sherman (Ref. 7) used the word "unit" in the hydrograph theory to refer to a specified period of time. A T-hour unit hydrograph for a catchment may be defined as the surface runoff hydrograph due to a unit volume of rainfall excess falling uniformly over the catchment in a period of T-hours. If the duration of rainfall excess becomes infinitesimally small the resulting unit hydrograph is called an instantaneous unit hydrograph.

By the principle of superposition in the linear unit hydrograph theory, the rainfall excess during a short period $d\tau$ at a time τ will produce a runoff at a time t equal to the volume of the rainfall excess in the period $d\tau$ multiplied by the ordinate of the instantaneous unit hydrograph at time t- τ . Thus the ordinate of the surface runoff hydrograph at time t is

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$$Q(t) = \int_{0}^{t} i(\tau)u(t-\tau)d\tau \qquad (1a)$$

where

- Q(t) = the ordinate of the surface runoff hydrograph at a time t.
- $i(\tau)$ = the rainfall excess at a time τ
- $u(t-\tau) = the ordinate of the instantaneous unit hydrograph at a time <math>(t-\tau)$.

Equation (1a) is called the convolution integral, it is also known as Duhamel's integral in which $u(t-\tau)$ is termed a kernel function.

The unit hydrograph theory has proved to be an effective, simple tool for determining the surface runoff hydrograph from storm rainfall. Within the limitations of a fixed duration and a similar rate and distribution of rainfall, the hydrographs of various storms are substantially similar in shape with ordinates approximately proportional to the surface runoff volumes. The theory assumes that the time bases of all floods caused by rainfalls of equal duration are the same. Recession curves show that the time required for flows to recede to some fixed value increases with the initial flow. Since however recession curves approach zero asymptotically a practical compromise is possible without excessive error.

There are definite limitations to the use of unit hydrographs. Reasonably similar rainfall distribution from storm to storm over very large areas is rare, hence unit hydrographs are not suited to

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large catchments. Odd-shaped catchments, particularly those which are long and narrow, commonly have very uneven rainfall distribution, and hence unit hydrographs are not well adapted to such catchments. In mountainous areas subject to orographic rainfall, the areal distribution is very uneven. It is almost impossible to identify typical intensity patterns from storm to storm, and uniform rainfall rates over an extended period of time are uncommon. This is not so serious as might seem at first. Much of the variation in rainfall intensity is smoothed out in the course of surface detention during overland flow and it is further levelled by channel storage. Hence short period variations in rainfall intensity have little effect on the accuracy of the unit hydrograph method. Relatively long period variations such as the successive bursts of rainfall accompanying a series of frontal passages can be handled by treating each burst as an individual storm and applying the correct T-hour unit hydrograph.

The major advantage of the instantaneous unit hydrograph in comparison with a unit hydrograph is that the instantaneous unit hydrograph is independent of the duration of effective precipitation, thereby eliminating one of the variables in hydrograph analysis. Furthermore, the use of the instantaneous unit hydrograph is better suited for the needs of theoretical investigations on the rainfall and runoff relationships in catchments.

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In 1958 Nash (Ref. 8) reviewed the various methods which had been used to date, to relate the impulse response of a catchment system to the characteristics of a catchment. As a result of this work, he concluded that one parameter of the impulse response from each of several catchments should be correlated with the characteristics of the catchment. He suggested that the instantaneous unit hydrograph would be a suitable choice for the impulse response and that consideration might be given to using the time from the instant of effective rainfall to the centre of the instantaneous unit hydrograph (the first moment of area of the instantan eous unit hydrograph about its origin) as the parameter. He went on to suggest that a second order refinement would be to find correlations between the second moment of areas of dimensionless unit hydrographs and a second characteristic of the catchments.

In 1960 Nash (Ref. 9) showed how the moments of the instantaneous unit hydrograph could be determined without deriving the actual instantaneous unit hydrograph. For a time invariant linear system the first moment (about the origin) of the instantaneous unit hydrograph (U_1) is related to the first moment of the input (I_1) and the output of the system (O_1) by the equation

This means that the "lag" or "mean delay time" of the instantaneous unit hydrograph (U_1) is equal to the lag between the centres of area of

 $U_1 = O_1 - I_1$ (2a)

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effective rainfall and storm runoff even in a complex storm.

The second moments about the centres of areas of the input (I_2) , the instantaneous unit hydrograph (U_2) and the output (O_2) are related by a similar equation.

$$U_2 = Q_2 - I_2$$
 (3a)

If this second moment U_2 is divided by the first moment squared $(U_1)^2$ a dimensionless coefficient m_2 is obtained.

$$m_2 = U_2 / (U_1)^2$$
 (4a)

The parameters which Nash had suggested as being suitable for correlation with catchment characteristics in 1958 could therefore be simply determined.

Nash then went on to attempt a correlation between these parameters and the topographical characteristics of ninety gauged catchments in the British Isles. He obtained good correlation between m_1 , the catchment area (A), and the overland slope (OLS). The equation being of the form $m_1 = 27.6 \text{ A}^{0.3} \text{ OLS}^{-0.3} \dots (5a)$

where m 1 is measured in hours

A is measured in square miles

and OLS is measured in parts per 10,000

The correlation between m_{γ} and the length of the longest stream to

the catchment boundary L was not so good. This equation was of the form

 $m_2 = 0.41 L^{-0.1}$ (6a)

where m₂ is dimensionless

and L is measured in miles.

In order to estimate the shape of the instantaneous unit hydrograph, which has to be integrated to yield any T-hour unit hydrograph, Nash set out to develop a linear model of catchment behaviour. Such a model has an impulse response for which some general analytical equation, expressed in terms of the parameters of the model, can be derived. From this equation, expressions for the moments of area of the model impulse response can also be derived, again in terms of the postulated model. For any given actual catchment data, numerical values of the instantaneous unit hydrograph moments got from equations such as (2a) and (3a) when equated to the general expressions for the moments of the model impulse response will yield numerical estimates of the model parameter. These estimates if substituted back into the general analytical equation for the model impulse response, will give a specific version of that equation. This is taken to be an approximation to the instantaneous unit hydrograph of the catchment in question.

After considerable investigation Nash chose to model the catchment

by routing the inflow through a series of identical linear reservoirs (n in number) all having storage directly proportional to discharge.

A model system of this form was shown to have an impulse response

$$U(0, t) = \frac{1}{K[n]} \cdot e^{-t/K} (t/K)^{n-1} \cdots 7a$$

where n is the gamma function.

For equation (7a) the first moment about the origin can be shown to be nK whilst the second moment about the centre of area is nK^2 .

Equating the moments given by equations (2a) and (3a) with the moments from the model gives

$$K = \frac{nK^2}{nK} = \frac{0_2 - I_2}{0_1 - I_1} \qquad (8a)$$

and

n =
$$\frac{nK}{K}$$
 = $\frac{O_1 - I_1}{1} \times \frac{O_1 - I_1}{O_2 - I_2} = \frac{(O_1 - I_1)^2}{O_2 - I_2} \dots (9a)$

These values of n and K when substituted back into the model will give an approximation to the instantaneous unit hydrograph of the catchment, the approximation depending on how closely the linear model of n reservours in series, represents the actual response of the catchment. From a consideration of the S-curve, Nash showed that the general equation of the unit hydrograph of period T is given by,

u(T, t) = 1/T(I(n, t/K) - I(n, (t-T)/K)) (10a) where I (n, t/K) is the incomplete gamma function of order n at t/K. Tables of I (n, t) are available which enable the ordinates of u(T, t) to be written down very easily, so that a tabular form of solution for a series of T-hour storms can be simply drawn up. If adequate records of streamflow are not available for a particular catchment the equations (5a) and (6a) can be used to determine m₁ and m₂ and since m₁ = nK and m₂ = 1/n then the instantaneous unit hydrograph and the T-hour unit hydrograph can be obtained.

The "Nash" model makes allowance for storage effects on a catchment but it takes no account of the translation effects present in any catchment. In 1959 Dooge (Ref. 10) presented a general theory of the unit hydrograph based on the single physical assumption that the reservoir action which takes place on a catchment, can be separated from the translatory action and lumped in a number of reservoirs unrestricted in number, size or distribution. The general equation which he developed proved to be rather unwieldly but he showed that it could be greatly simplified by assuming that above each confluence in the catchment the reservoirs are equally distributed for equal lengths of tributary and by further assuming that the idealised

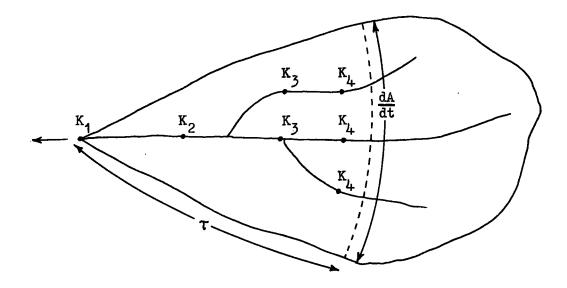


Fig. 1 Uniform distribution of reservoirs

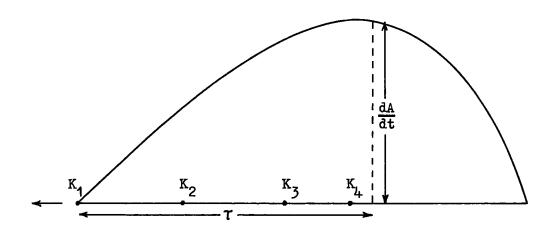


Fig. 2 Folding of tributaries onto main river

reservoirs in the catchment are equal. These assumptions yield a model system that can be represented diagrammatically as in figure (1) p.16.

The output from an element of catchment incurs a translation delay time, τ . Each element of area separated from the outlet by the same translation time lies on an isochrone. All the tributaries can be folded on to the main river to give a single chain of reservoirs as shown in figure (2), p.16, the inflow at any point being proportional to the length of isochrone (dA/d τ)cutting the river at that point. The output from the element also passes through n linear reservoirs on its passage to the catchment outlet, n being dependent on τ .

Dooge's simplified equation for the impulse response assuming that all the reservoirs are equal in size is:-

$$u(0,t) = \frac{V}{T} \int_{0}^{t/K} P(m,(n-1)) w(\tau) dm \dots (11a)$$

where V = volume of rainfall excess

$$T = \text{time of concentration}$$

$$P(m, (n-1)) = \text{poissons probability function}$$

$$\tau = \text{translation time}$$

$$n(\tau) = \text{number of linear reservoirs downstream of } \tau$$

$$K = \text{size of linear reservoirs}$$

t = time elapsed since occurrence of rainfall excess

 $m = (t-\tau)/K$, i.e. a dimensionless time factor

 $w(\tau)$ = ordinate of dimensionless time-area-concentration curve. In order to complete the hydrograph from this equation the size and distribution of the linear reservoirs (as a function of τ) must be known or assumed. Dooge presents a procedure for evaluating equation (11a) in his paper.

The methods of instantaneous unit hydrograph derivation, described by Nash and Dooge, depend on an assumed linear model of the catchment. These methods can therefore only give as good a representation of the real equation response, as the model itself is a good representation of reality. Methods of linear system analysis bypass the need for a model and give an accurate mathematical solution.

1.5. Linear system analysis.

In a paper presented in 1960, O'Donnell (Ref. 11) demonstrated a method of determining the shape of the instantaneous unit hydrograph analytically by harmonic analysis. He showed that the curve of rainfall excess, the instantaneous unit hydrograph and the hydrograph of surface runoff could each be represented by the sum of a harmonic series, each series having the same fundamental time period, equal to or greater than the storm runoff. The coefficients of the nth harmonics of the three series are related. The harmonic coefficients of an instantaneous unit hydrograph can therefore be derived from the curve of rainfall excess and its resultant runoff hydrograph. The ordinates of the instantaneous unit hydrograph can then be calculated. The evaluation of these coefficients involves a tremendous amount of repetetive computation but solutions can be obtained by use of a digital computer with programmes which are available.

In a review paper O'Donnell (Ref. 6) has reported on work by Dooge (Ref. 12) who has studied the use of Laguerre functions in deriving the impulse response of a time invariant linear system and has described a technique which uses these functions to derive the instantaneous unit hydrograph and the T-hour unit hydrograph.

Newton and Vinyard (Ref. 13) have described a procedure which makes use of a high capacity digital computer to compute a T-hour unit hydrograph directly from rainfall excess and recorded complex floods. The mathematics involved makes use of matrix algebra which is particularly well suited to the employment of digital electronic computing, since a set of matrix subroutines is almost invariably a major part of the programme libraries associated with a digital computer. The solution involves the solving of (i + j - 1) equations where i is the number of periods of rainfall excess and j is the number of T-hour unit hydrograph ordinates. Since real data is bound to yield a set of incomplete equations the solution involves a least squares fitting procedure.

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It must be noted that these methods of system analysis provide a mathematical solution to a mathematical problem. A mathematically accurate solution can be developed which will be highly illogical from a hydrologic standpoint.

Although the linear methods of synthesis and analysis have found world wide application in the form of the unit hydrograph, all hydrologists would agree that a strictly linear and time invariant relation between rainfall and runoff cannot exist. Basically the unit hydrograph attempts to deal with a complex non-uniform input, rainfall, which varies in time and area, by considering it to be constant in time and uniform over area. This simplified input is assumed to be acted on by an invariant linear system of storages. Actually, we know that the storage is the result of surface detention in overland flow, subsurface delay, interflow and groundwater and channel storage. None of these storages are linear and their relative role in each runoff event varies.

In the routine application of unit hydrograph procedures the time variability of system response is usually handled by introducing certain assumptions regarding antecedent catchment conditions, whereby the input is modified by variable time distribution of infiltration to yield a rainfall excess. The surface elements of the catchment operate on the rainfall excess to produce a surface runoff which is modified in turn by the base flow in order to yield ultimately the flood runoff,

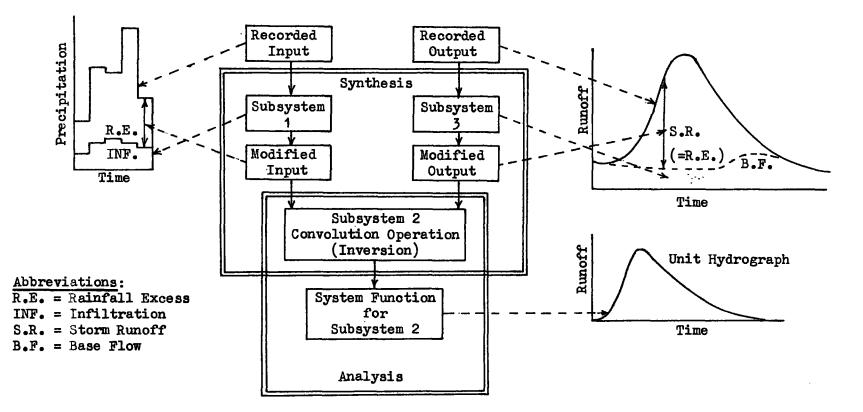


Fig. 3. Flow chart of a partial system synthesis with linear analysis.

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recorded as output at a gauging station. Amorocho and Hart (Ref. 14) have described the complete unit hydrograph procedure as being one involving partial system synthesis with linear analysis. The operations involved in the method can be represented by a flow chart (Figure 3). The synthesis operation involves three subsystems, designated in the figure by numbers 1, 2 and 3, whose combined effect is assumed to be equivalent to the operation of the catchment. Subsystem 1 performs the operation of subtracting the values of the infiltration function from the recorded input. This is determined by empirical procedures. Subsystem 3 separates the surface runoff hydrograph from the recorded output. This is usually done by judgement and a semisubjective procedure involving the determination of the regression curve for the catchment. Subsystem 2 is a linear convolution which can be analysed by one of the numerical methods of inversion in order to determine the unit hydrograph which is assumed to be the invariant system function of subsystem 2.

In the prediction process the input has to pass through the three subsystems before the output can be obtained.

Since the analysis operation has been performed on the basis of modified inputs and outputs, the unit hydrograph obtained fits specifically the modified inputs and outputs. If the assumptions for these modifications vary, the unit hydrograph will also. Since hydrologic systems in general are non-linear the method assumes that the overall

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nonlinearity of the system is modified by proper nonlinear operations on the gross input and output so that the operation of the catchment on the rainfall excess can be represented by linear equations. The degree of approximation which the linearity assumption may yield depends on the degree of actual nonlinearity of the system.

1.6. Nonlinear system analysis

The nonlinear response of a catchment under rainfall may be represented by means of a functional series incorporating mathematical operations equivalent to the physical actions of the hydrologic system. This functional representation involves the consideration of higher powers of the elements of inflow.

One of the outstanding characteristics of the process of analysis is that it permits direct operation with the recorded precipitation and recorded river runoff, without accounting for elements such as evapotranspiration and groundwaterflow which are usually difficult to evaluate. The process essentially consists of analyzing the recorded input and the recorded output in order to find an equivalent system function, which operates roughly as a nonlinear convolution. In principle this is parallel to the inversion problem of linear analysis used in the unit hydrograph procedure. However, general methods of direct nonlinear inversion are not yet available, and current procedures involve approximation of the inversion operation.

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One procedure is based on the assumption that hydrologic systems are equivalent to analytic systems, defined as a combination of systems of progressively higher order, whose outputs when added equal the output of the natural system.

$$Q(t) = \int_{-\alpha}^{\alpha} U_{1}(\tau_{1})i(t-\tau_{1}) d\tau_{1} + \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} U_{2}(\tau_{1}, \tau_{2})i(t-\tau_{1})i(t-\tau_{2}) d\tau_{1}, d\tau_{2} + \int \int \int \dots + \dots (12a)$$

where U_1 , U_2 are systems functions of progressively higher order, i is input and Q is output (all are functions of time). If the assumption of analyticity is valid, the task is to evaluate each of the non-linear kernels of the functions of this series. Current mathematical knowledge is not sufficient to effect this multiple inversion when complex inputs are involved.

In 1963 Amorocho (Ref. 15) presented a paper in which the theory of the functional series representation of hydrological systems was discussed and measures of the linearity of hydrologic systems were developed. The degree of linearity of a system was measured in terms of "unit linearity" which was defined as the ratio between the first linear term of the functional series and the summation of the full series when the system is responding to a simple unit step input. Unit linearity does not measure the total non-linear departures under complex input sequences but it does indicate when these departures can be expected to be large and when they should become small. Amorocho reported on a series of laboratory experiments which were conducted for the purpose of testing the procedures of non-linear analysis on the basis of functional representation, and to show the nature of the departures from linearity which a system, having the essential attributes of a hydrologic system could exhibit. It was shown that the unit hydrograph method tended to underestimate flood episodes which were longer than those used for its analysis and to overestimate small floods. Experimental results also showed that systems which exhibit only moderate departures from linearity under simple unit step inputs, may respond in a grossly non-linear manner under complex excitation.

1.7. Correlation methods

Hydrologists studying techniques for flood forecasting found that the unit hydrograph concept was a reasonably adequate technique for dealing with small and medium sized catchments. For areas of 1,000 square miles or greater, and for areas where the unit hydrograph theory did not appear valid, alternative methods were sought. Research into multiple regression relationships ultimately led to multiple correlations involving a parameter to indicate antecedent conditions, the duration of the storm and the amount of rainfall as being the key factors in predicting storm runoff. Linsley (Ref. 16) in a review paper, states that the problem of finding a suitable index of antecedent conditions proved to be the most difficult one. The most successful antecedent

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parameter which was developed was a combination of calendar date and antecedent precipitation. The antecedent precipitation index was initially conceived as a series of the form

 $I = P_{1} + P_{2}/2 + P_{3}/3 + \dots P_{t}/t \dots (13a)$ where I = antecedent precipitation index $P_{t} =$ the precipitation which occurred on a day t days before the day in question

Ultimately, the exponential form

 $I = P_t K^t$ (14a) where K = a constant less than unit proved to be the most effective antecedent index.

The antecedent precipitation index was combined with other parameters in a multiple, graphical correlation by a technique known as coaxial correlation. Linsley et al (Ref. 17). This proved to be a reasonably successful way of dealing with the clearly non-linear relationships involved. The antecedent precipitation index is of course only an approximation of soil moisture conditions. The depletion coefficient should not be constant for it should allow for a more rapid depletion during periods of high evaporation. The use of calendar date in the coaxial correlation makes allowance for normal variations in the evapor/transpiration throughout theyear. Workers interested in water resources studies have also used correlation methods to relate long period rainfall values to corresponding long period riverflow. Law (Ref. 18) used 42 years of record from the Yorkshire Derwent catchment and correlated summer rainfall (April to September) with summer riverflow and winter precipitation (October to March) with winter runoff. For the rainfall parameter he used the readings of three chosen gauges in turn. The correlation coefficients were found to be similar for the summer and winter seasons but the regression equation coefficients varied somewhat. This was to be expected as may be seen from equation (15a).

Q_n = m.P_n - C (15a) where Q = winter or summer runoff in year n P = winter or summer rainfall in year n m = regression coefficient C = regression constant

If m is regarded as the proportion of rainfall which appears as runoff, then it will have a higher value in the winter. C can be thought of as losses and these will tend to be higher in summer when evaporation losses are much greater.

In 1962 Andrews (Ref. 19) presented a paper in which three graphical methods of estimating groundwater discharge were employed. Using monthly data presented in this paper Rodda (Ref. 20) carried out a multiple regression analysis to correlate groundwater discharge with rainfall for

comparison of both the method and results. To determine the runoff for a given month the rainfall in the nine preceding months was considered and an equation was obtained of the form

$$Q_{m} = 100 P_{1} + 103 P_{2} + 76 P_{3} + 98 P_{4} + 81 P_{5} + 125 P_{6} + 48 P_{7}$$

+ 28 P₈ + 10 P₉ - 589 (16a)
where Q_m = daily mean of groundwater in M.G.D. during March.
P₁ to P₉ = general rainfall, in inches, for the months from
February to June preceding March

The multiple correlation coefficient obtained = + 0.97 and the standard error of estimate of Q = + 14.

The high degree of correlation points to a satisfactory relationship, while at the same time the relative sizes of the coefficients of P show no diminution with time except during the summer months. Barring unusual storage conditions this would appear illogical when considered in a hydrological context.

Collinge (Ref. 2) using 38 years of data from the Derwent at Yorkshire Bridge, derived regression equations relating monthly rainfall and runoff. Typical equations for a wet month, January; equation (17a) and a dry month, June; equation (18a), are given below:

 $Q_i = 0.88 P_i + 0.12 P_{(i-1)} - 0.79 \dots (17a)$ $Q_i = 0.47 P_i + 0.19 P_{(i-1)} - 0.61 \dots (18a)$ where $Q_i = runoff$ in i'th month $P_i = rainfall$ in i'th month. Collinge went on to use the twelve monthly rainfall- runoff regression equations with random generated monthly rainfall data to give a synthetic runoff record. The mean and standard deviation of monthly runoff was calculated for each month for both the actual and synthetic runoff and comparisons were made. These could not be regarded as satisfactory.

Sharp et al., (Ref. 21) examined the efficacy of the use of the multiple correlation and regression approach in evaluating parameters affecting water yields of catchments in the U.S.A. The parameters were chosen by judgement for a number of models on the basis of a general knowledge of the hydrology of a catchment. Each individual model yielded both a series of regression coefficients and a set of error estimates and measures of correlation as determined by analysis of variance.. By examining the error estimates and the measures of correlation the best model was selected.

It was found that almost equally good prediction equations could be obtained with sets of different parameters and that careful judgement had to be exercised to avoid using physically irrelevant parameters or parameters which possessed strong interdependence. Unfortunately reliable non-subjective procedures to eliminate these personal factors do not exist at the moment.

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Inherent in the application of the multiple regression method of analysis to hydrologic problems are three tacit assumptions,

1) there are no errors in the independent variables : errors only occur in the dependent variable, (runoff)

2) the variance of the dependent variable does not depend on the values of the independent variables,

3) the observed values of runoff are uncorrelated random variables.

In the application of tests of significance, a fourth assumption is made. The population of the dependent variable (runoff) is normally distributed about the regression line for any fixed level of the independent variables, (precipitation for instance) under consideration.

The first assumption is obviously violated by hydrologic data. Precipitation on the catchment, soil moisture, vegetative conditions and other variables pertaining to catchments all contain certain amounts of error.

Considering the second assumption, the variance of runoff values is not entirely unaffected by the values of the independent variables. Small values of precipitation for instance are associated with low values and variance of runoff, but great precipitation events tend to generate runoff events with large variance.

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In the case of riverflow, the third assumption is suspect. Riverflow, whether it be on an annual, monthly, storm or daily basis, is not unrelated to preceding events. What occurred yesterday affects what happens today.

With regard to significance tests, the only information available about the distribution of the population is that provided by the sample. In hydrologic data there simply are not enough large events in the relatively few years of record of riverflow, precipitation and other factors, to afford reliable information about the distribution of the dependent variable about the regression line. This fourth assumption may therefore be suspect, and high coefficients of correlation and high t-test values may be misleading.

1.8. General system synthesis.

The basis for the construction of synthetic models in hydrology is a statement of continuity, which can be expressed in the form:total inflow = total outflow + change in internal storage.

The simplest form of accounting assumes that the catchment is represented by a single reservoir with a maximum moisture capacity, s. The quantity of water present in the reservoir at a certain instant is s-d where d is the moisture deficiency. Evaporation draws moisture from the reservoir at a specified rate. Starting with the assumption that runoff will occur only when the reservoir is full, then a precipitation P will produce a runoff Q = P-d, if P exceeds d. The value of d at a particular instant can be calculated by assuming that evaporation takes place at the potential rate until the storage capacity s is exhausted.

In 1957 Kohler (Ref. 22) proposed a refinement to this simple accounting procedure. In this method the storage capacity is divided into two levels. The upper level of storage capacity s_u represents the upper layer of the soil profile from which evaporation takes place at the potential rate, E_p until the storage capacity s_u is exhausted. Evaporation from the lower level of storage s_1 occurs only when there is no water left in the upper level and it is then assumed to be proportional to the amount of water left in the lower reservoir.

Actual evaporation $E_a = (s_1 - d) E_p/s_1$ (19a) The lower level reservoir is replenished only after the moisture deficiency in the upper level has been filled up completely.

The rate at which moisture is depleted from an initially saturated catchment decreases with time and approximates to a logarithmic recession. This function could be used in an accounting procedure and would be satisfactory if each storm saturated the catchment. Unfortunately, it would not provide for the increased evaporation immediately following a moderate storm on a dry catchment. The arbitrary separation of the moisture capacity into two categories does not make such an allowance.

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If one considers a catchment area as a whole then there is probably within the catchment a great variety of soil moisture conditions. This concept has been represented by Kohler and Richards (Ref. 23) in their method of multi-capacity accounting. Instead of representing the catchment by a single reservoir or a two level reservoir, different parts of the area are introduced as separate reservoirs, each refervoir having its own maximum capacity, e.g. 2, 5, 10, 20 inches. The evaporation for each reservoir is put equal to the potential evaporation until the storage is exhausted. The day by day accounting computations are carried forward independently for each of the several selected capacities without regard to the relative portions of the area for which each capacity is applicable. The mean moisture deficiency for the catchment is then derived by weighting the simultaneous values thus obtained. The weights applicable to the several deficiencies are determined through multiple correlation with observed preceipitation and riverflow.

This multi-capacity accounting technique is equivalent to approximating the recession curve of soil moisture depletion by a series of straight lines. A better fit could be obtained by using a nonlinear function or by increasing the number of selected capacities. Considering the reliability of estimated potential evapotranspiration it may be that the use of a complex function for each of the selected capacities is unwarranted.

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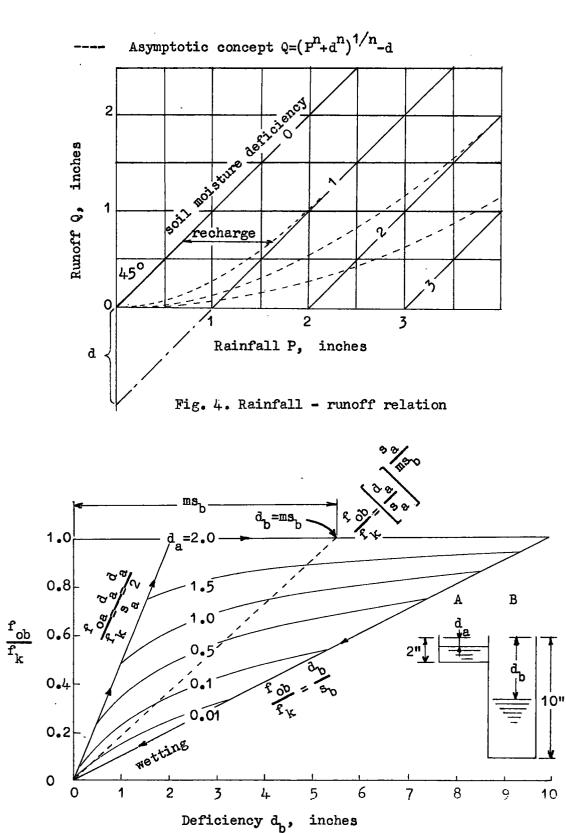


Fig. 5. Threshold concept and infiltration approach

Kohler and Richards (Ref. 23) proposed further refinements for use in the multi-capacity accounting technique. With the data available to them they found that generally the relationship between rainfall and runoff closely approximated to the following expression.

$$Q = (P^{n} + d^{n})^{1/n} - d \qquad (20a)$$

where Q = runoff

- P = precipitation
- d = moisture deficiency

and where the exponent n is always greater than unity and may be expressed as a linear function of d

They commented that limited studies indicated that c = 2.0 and k = 0.5 could be used as a first approximation in multicapacity accounting.

This equation for the runoff Q, is a departure from the threshold concept which assumes that no runoff can occur until the soil moisture deficiency is satisfied after which all rainfall becomes runoff, i.e. the equation Q = P-d if P > d. In proposing the more complex equation they reasoned that the catchment recharge r, which is equal to precipitation P, minus runoff Q, i.e. r = P-Q, is equal to precipitation as a storm begins and that the recharge approaches the deficiency asymptotically as precipitation continues as shown by the dashed lines in figure (4). In 1963 Kohler (Ref. 24) described a further model for determining storm runoff. In this instance he related the initial infiltration capacity to the existing soil moisture deficit and predicted the soil moisture deficit with the aid of multi-capacity accounting techniques. Starting with the infiltration concept:-

$$f = f_{c} + (f_{o} - f_{c})e^{-kt} \qquad \dots \qquad (22a)$$
where $f = infiltration$ capacity
$$f_{o} = initial \text{ value of } f$$

$$f_{c} = minimum \text{ infiltration capacity (saturated soil profile)}$$

$$t = time \text{ from beginning of rainfall}$$

$$k = a \text{ constant}$$

he proposed that the infiltration capacity must approach zero as precipitation and storm duration increase while the total amount of soil moisture is limited.

or
$$f = f_0 e^{-kt}$$
 (23a)

As a consequence f represents the capacity rate of absorption. From the above formula it may be deduced that at any moment the capacity rate of absorption is proportional to the then existing moisture deficiency d:

f = kd or with t = 0: $f_0 = kd_0$ (24a) The equation furthermore shows that the recharge of soil moisture r, equals

 $r = d (1-e^{-kt})$ (25a)

Kohler went on to propose the following equation for the recharge r, of a portion of the catchment with moisture capacity s.

$$r_{s} = d_{s} \left[1 - e^{-(f_{o}/d_{s})(a+T+bP)} \right] \dots (26a)$$

where T = duration of an idealised storm

- P = rainfall amount
- d = moisture deficiency
- f_= initial absorption capacity
 - a and b are constants.

Equation (26a) yields the amount of recharge to soil moisture for a storm of duration T hours during which the rainfall intensity exceeds the capacity rate of absorption. As a rule, rainfall intensity does not exceed the capacity rate of absorption throughout the storm and the application of equation (26a) requires that a value of T be estimated such that the computed recharge will be the same as for an idealized storm. This must be done subjectively. Using the above equation in conjunction with the multi-capacity accounting technique it is possible to calculate the soil moisture recharge for every storm period and consequently the runoff over that period since Q = P-rwhere Q is the runoff and P is the precipitation.

It is interesting to note that equation (26a) indicates that the soil moisture deficiency is filled asymptotically with increasing precipitation and duration whereas equation (20a) assumed that it was filled asymptotically as precipitation increases indefinitely. There would not appear to be any justification for assuming a simple weighting of T and P in the exponent of equation (26a). The constant a, will allow an appreciable time for absorption even when the duration of T approaches zero.

The value of the initial absorption capacity f_0 in equation (26a) depends on the moisture distribution within the soil profile. In order to take this phenomenon into account Kohler made the initial absorption capacity of an area b, with moisture capacity s_b not only depend on the initial moisture deficiency d_b , of this area, but also on the moisture deficiency of an area a, where the moisture capacity s_a is considerably less than s_b . The procedure is illustrated with the aid of figure (5), page 34.

In this example $s_a = 2$ inches and $s_b = 10$ inches. For both areas the maximum absorption capacity is equal to f_k . For area a, the following equation is always applicable

value of d_b cannot be used in equation (28a) to determine f_{ob} . Kohler related the value of f_{ob} to both the value of d_a and d_b by assuming that the existing values of d_a and d_b have been reached by,

- a) starting at a saturated condition when $d_a = d_b = 0$
- b) evaporation without precipitation until $d_a = d_b = s_a$

c) continued evaporation without precipitation until $d_b = ms_b$. where m is a proportion factor such that ms can take any value between 2 inches and s_b .

d) precipitation without evaporation until existing values of d_a and d_b have been reached.

It is possible to demonstrate that, starting from such a situation at any moment of the storm period, the following equation applies:

$$\frac{f_{ob}}{f_k} = \begin{bmatrix} \frac{d_a}{s_a} \end{bmatrix}^{\frac{s_a}{ms_b}} = \frac{d_b}{ms_b} \qquad (29a)$$

Starting with certain values of d_a and d_b it is possible to reconstruct the hypothetical original situation, i.e. to calculate the value of ms_b. In view of the above equation it is suggested that this should be done graphically as in figure (5), page 34.

Entering the graph with the values of d_a and d_b the value of f_{ob}/f_k is obtained and hence f_{ob} . Since runoff is equal to precipitation minus recharge, equations (26a) and (29a) provide the means of computing runoff for each of several portions of a catchment with assumed moisture capacities. To obtain the runoff for the whole catchment Q_w , these

these values must be weighted, that is

 $Q_w = \leq k Q$ where $\leq k = 1$ (30a) Kohler applied the above procedure to small intervals during a storm period and determined values of d_a and d_b for longer periods without rainfall by multi-capacity accounting.

This method, developed by Kohler, is, despite a number of assumptions, a very interesting one. On the one hand it profits by the advantages offered by the accounting concept with regard to predicting the moisture deficit and on the other hand it attempts to use the advantages of the infiltration approach to predict the runoff for parts of a storm period.

At Stanford University a non-linear mathematical model has been progressively developed since 1959. The original reason for undertaking the work was to develop a model which could be used as a device to synthesise riverflow data to supplement short records of observed flow, but it has been found that the model can also be used to evaluate the effects of artificial changes in the hydrological regime of a catchment. A further potential use of the model, perhaps its most important use, is as a means of exploring the runoff process for an improved understanding of hydrology.

The model attempts to simulate the hydrologic cycle using a moisture accounting procedure to derive the riverflow.

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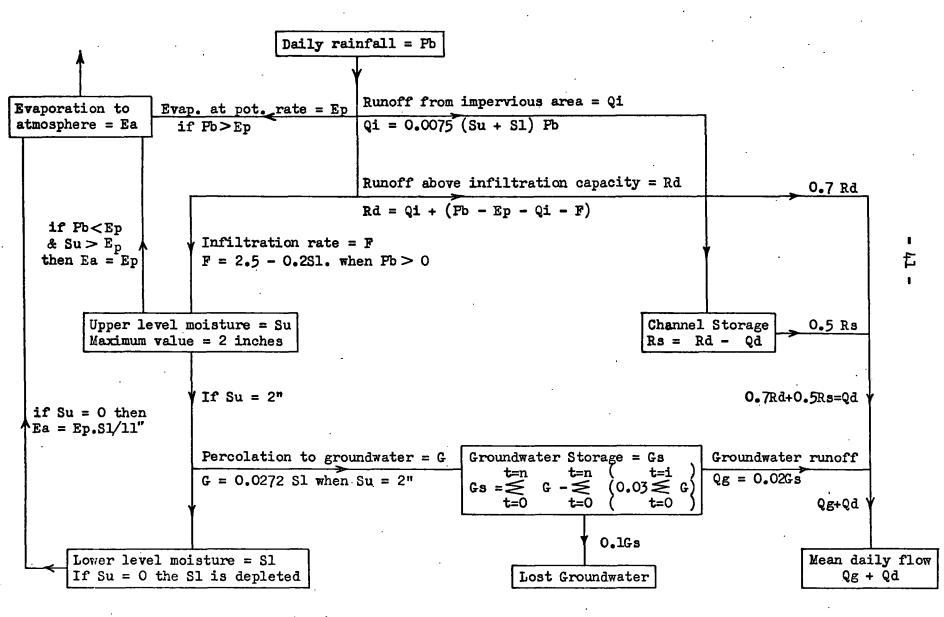


Fig. 6. Flow chart for Stanford Watershed Model Mark I.

The first model was described in a paper presented by Linsley and Crawford (Ref. 25). A flow diagram showing the runoff processes that they assumed together with the equations which they used for the calculations is shown in figure (6). In order to fit the model to the actual catchment they used data from a short period when daily rainfall, potential evapotranspiration and runoff values were available. This data was used to develop estimates of the model parameters that would give the best fit of the general model to the actual catchment. The initial values of the model parameters were selected on the basis of previous experience. These were later adjusted using a combination of experience and intuition, clues being provided by the timing and magnitude of the differences between the synthesised and the recorded riverflow hydrograph.

With the first model the daily distribution of runoff from large storms was not particularly accurate. This may be because the daily values of rainfall which were used tend to divide major storms between two days although the total duration of such storms may be considerably less than twentyfour hours. Further errors would result because the daily precipitation values which were used were read at 17.00 hours each day whilst the streamflow observations were made at midnight.

The Mark II Stanford Watershed Model of Crawford and Linsley (Ref. 26) was more complex than that proposed in 1960. The Mark II model calculated a continuous hydrograph based on average hourly

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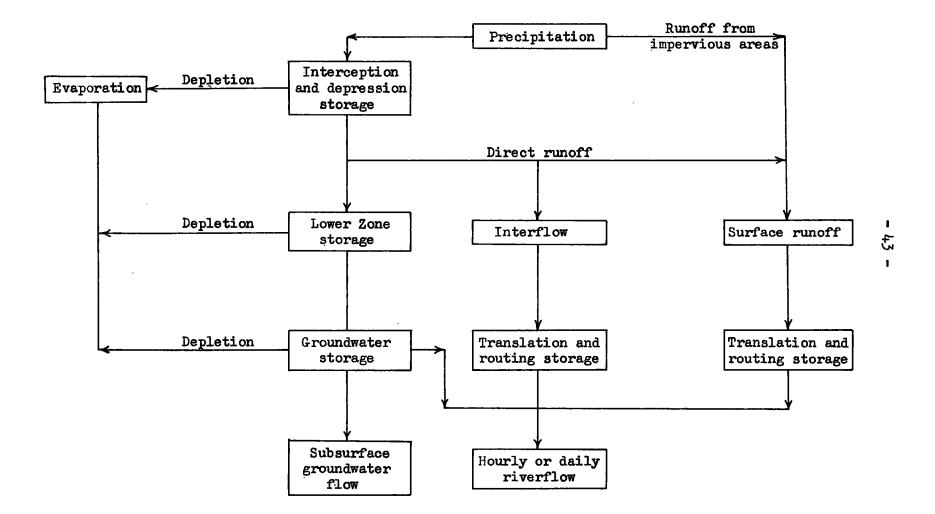


Fig. 7. Flow chart for Stanford Watershed Model Mark II.B.

rainfall and daily transpiration on the catchment. A flow diagram is shown in figure (7). The computer programme for this model incorporated over 500 Algol statements and required many preset constants and functions to represent all the processes under simulation. A subroutine was incorporated in the programme so that the major constants and functions could be determined semi-automatically. When this subroutine was in use riverflow data and selected values of daily groundwater flow had to be provided as additional input data. The computer commenced the calculations with a set of constants and functions which had been assumed by the operator on the basis of experience and when a months values had been computed the machine compared the computed monthly total riverflow and the groundwater flows with the observed. If these values were not within a preset tolerance the machine automatically selected new constants following rules incorporated in the subroutine and repeated the computation for the month. This was continued until the computed values were acceptable. The constants determined by the machine for each month were not completely consistent and some smoothing was required to give a single set of values for use in riverflow synthesis. After the volumetric constants had been established detailed hydrographs for major runoff events were compared with the computed values and the assumed routine constants were then adjusted to give the best fit. Five years of concurrent riverflow and rainfall record were used to derive catchment constants and an

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additional five years of record were used for an independent check on the results.

The model was tested on eight catchments ranging in size from 22 to 88 square miles and in mean annual rainfall from 23 to 54 inches. The correlation of observed and computed values was not given in statistical terms. Errors in the values of peak flows on the two flood hydrographs which were shown in the paper amounted to approximately 20% whilst errors in the hydrograph of mean daily flow at times of flood in some instances exceeded 100%. Nevertheless, the errors appeared to be random and the frequency characteristics of the derived series agreed very well with the observed data. In evaluating the method it must be noted that only one recording raingauge was used on each catchment, hence the rainfall data may not always be completely representative of the whole area.

Morgali and Linsley (Ref. 27) reported that studies leading to the development of the Stanford Watershed models had indicated that for small catchments the key storage element is the storage of overland flow. As catchment size increases, overland flow storage diminishes in its importance and probably becomes relatively unimportant on catchments in excess of 20 square miles. They developed a mathematical model to synthesise overland flow using flow equations derived from continuity and momentum principles. The hydrographs produced from the computed results were compared with experimental results obtained from laboratory

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flow planes with different surface finishes. Parameters such as slope, rate of uniform rainfall and length of plane were varied one at a time and the effect of each of these parameters on the hydrograph was noted. The hydrographs from the computed results were found to compare satisfactorily with those obtained in the laboratory.

In the discussion following the presentation of the paper the finite difference form of solution which was adopted by the authors was criticised on the grounds that it was subject to instabilities and it was suggested that the authors were only able to obtain significant results because they considered cases with steep slopes. It is noted that Linsley did not choose to make use of this finite difference method in the Mark IV Watershed model.

A detailed report on the mark IV Watershed model was presented in 1966; Crawford and Linsley (Ref. 28). The basic simulation model is designed to accept input from any number of recording and storage rain gauges and to calculate riverflow values at several sites in the river channel, called flowpoints. These flowpoints are usually at river gauging stations but they may be placed at any other point in the river system. The area above each flowpoint is divided into segments so that there are one or more segments for each recording raingauge. The segments are selected from topographical considerations or by constructing a Thiessen network. The general model continuously

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calculates the riverflow at each flowpoint from rainfall in each successive catchment segment, and from flows measured or calculated at upstream flowpoints. All calculations are carried out independently for each catchment segment so that areal variations in rainfall and topographic features are represented. The major data inputs to the model are precipitation and potential evapotranspiration. If snowfall is significant, temperature and radiation values are required.

The major elements of the model are similar to those shown in the flowchart for the mark IIB model, figure (7) although refinements have been added. The operation of the mark IV model in a small catchment or a catchment segment is as follows.

Precipitation is stored in three soil moisture storages and in the snowpack if it exists. The upper and lower zone storages together with the groundwater storage, combine to represent variable soil moisture profiles and groundwater conditions. The upper and lower zone storages control overland flow, infiltration, interflow and inflow to groundwater storage. The upper zone simulates the initial catchment response to rainfall and is of major importance for smaller storms and for the first few hours of larger storms. The lower zone controls the catchment response to major storms by controlling longer term infiltration rates. Groundwater storage supplies the base flow

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to river channels. Evaporation and transpiration may occur from all these storages. The total channel inflow from overland flow, interflow and groundwater enters the channel system simulation and emerges as synthesised riverflow.

The Stanford Algol programme for the Mark IV model (Appendix C of the report) requires the basic data listed below to simulate riverflow in a small catchment or a segment of a catchment.

1. Hourly rainfall (inches).

- 2. Total watershed area above flowpoint (square miles).
- 3. Flowtime from upstream flowpoint (hours).
- 4. Initial groundwater storage (inches).
- 5. Initial upper zone storage (inches).
- 6. Initial lower zone storage (inches).
- 7. Initial groundwater slope index.
- 8. Daily mean potential evapotranspiration (inches).
- 9. Routing interval in hours.
- 10. Number of time delay elements.
- 11. Elements of time-delay histogram.
- 12. Ratio of average segment rainfall to average gauge rainfall.
- 13. Segment area (square miles).
- 14. Impervious area (fraction).
- 15. Interception storage : maximum value (inches).

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- 16. Nominal upper zone storage (inches).
- 17. Nominal lower zone storage (inches).
- 18. Actual evaporation loss index.
- 19. Portion of groundwater recharge assigned to deep percolation (fraction).
- 20. Evapotranspiration from groundwater (fraction of area).
- 21. Infiltration index.
- 22. Interflow index.
- 23. Overland flow length (feet).
- 24. Overland flow slope (feet per foot).
- 25. Manning's n for overland flow.
- 26. Stream channel storage recession parameter (hourly).
- 27. Interflow recession (daily).
- 28. Groundwater recession : variable component.
- 29. Groundwater recession : basic rate (daily).
- 30. Evaporation from stream surfaces (fraction of area).

If snowfall is significant additional daily temperature and radiation values are required together with information on nine snowmelt parameters.

With reference to this data list of 30 items, those numbered 1 to 8 inclusive can be found from hydrological or meteorological records and topographical maps. Numbers 9, 10 and 11 refer to the channel time delay histogram which is used in the simulation of the time delay of channel inflow as it moves in the channel system. This histogram can be constructed from estimates of time of flow in channels using the equation

t =
$$\frac{n.^{3/5} L. W.^{2/5}}{4560. S^{3/10} Q^{2/5}}$$
 (31a)

where t = flow time in hours for steady flow in a reach of wide channel

- n = Manning's n
- W = channel width
- S = channel slope
- Q = discharge

The land surface parameters are numbered 15 to 25 inclusive and the channel system and groundwater parameters are numbered 26 to 30. Of these, the overland flow length and slope can be found from topographic maps, as can an estimate of the fraction of area from which evaporation should occur at the potential rate. Manning's n can be estimated from tables. Procedures are given in the report for estimating values for eight of the remaining twelve items of data, but for the remaining four i.e. the upper zone storage, lower zone storage, infiltration index and interflow index, the report recommends a simulation procedure, using a period of recorded riverflow data to determine the combination of values that will most satisfactorily reduce the long term groundwater and surface runoff volumes and short term response to individual storms.

In an ideal mathematical model the parameters would represent quantities that are physically measureable. This is important for two reasons. First, the hydrologist can have an immediate feeling of the realism of any fitted parameter values and can check them against field data. Second, once a model is shown to be adequate, parameter values can be derived from field data in order to synthesise runoff data at ungauged sites.

The formidable amount of data required by the Mark IV Watershed Model limits its usefulness at the present time. The size of the catchment governs the time increment required for successful modelling. The smaller the catchment, the shorter the time interval. This in turn increases the running time of the model on the computer and the labour required to prepare data for input. Hourly rainfall data has been found necessary for medium sized catchments, but provision has been made in the Stanford Algol programme for 15-minute rainfall values. At the hourly rate, one year's rainfall data from one gauge amounts to 8,760 values and for the similation of flow on the Russian river

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catchment in California rainfall data from ten recording rain gauges was used. There can be few, if any, upland catchments in Great Britain of this size (362 square miles) on which ten recording gauges have been installed, and fewer still where the rainfall data is logged on magnetic tape or punched tape. Where the rainfall has been recorded on charts, values must be read visually and punched up manually and this takes a great deal of time.

The Stanford Watershed Model is an excellent research tool. It leads to the suggestion that it will eventually be possible to establish a completely general simulation model which is applicable to all catchments by the insertion of appropriate parameter values defining the physical characteristics of the catchment. Because the model is capable of dealing with short term rainfall efficiently, it also solves the problem of long-term runoff which is merely the summation of the short-term runoff.

The successful operation of a Stanford type digital computer catchment model in which the parameter values are adjusted by the operator, relies, to a considerable extent, on the skilled experience and personal judgement of the operator. Dawdy and O'Donnell (Ref. 29) considered that as detailed knowledge of the elements of the hydrologic cycle increases the resulting more precise specification of their behavioural

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relations will lead to more sophisticated but inevitably more complicated models. It is likely that adjustment of the larger number of parameters of these more complex models by subjective trial and error procedures will become impracticable. With this in mind they explored automatic objective methods of finding numerical values of the parameters with a view to gaining experience and know-how. For their studies they used an over-all catchment model, similar to, but simpler than the Stanford models. The model was composed of four storage elements whose behaviour was controlled by nine parameters. By assuming a long dry period prior to the start of a synthesis all four initial storage values could be taken as zero. In order to free the initial studies from the effects of error in the data flow values were generated by allotting a set of values to the model parameters and calculating the output generated by the model from an arbitrary input. The speed and effectiveness of the optimisation technique was tested by deliberately choosing wrong sets of parameters for the model at the beginning of the test and then noting the progress towards a known set of correct parameters. The optimisation procedure which was used was a modification of a method developed by Rosenbrock (Ref. 30) to find the greatest and least value of a function U. in an arbitrarily restricted region, when partial derivatives of the dependent function (which is being optimised) cannot be found. In this hydrological application the minimum value of U was sought. This was taken to be the sum of the squares of the differences between

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the initial generated flow values and the flow values synthesised from the current set of parameter values. The same quantity was used to examine the sensitivity of the model response to each of nine parameters by finding the U value computed with eight correct parameters but with the ninth displaced by one per cent from its correct value.

Optimization trials were carried out starting with parameter values set 50% above or below their correct values. Seven of the nine parameters were optimised to within 15% of their correct value (five to within 3%) but two parameters ended up about 400% out.

Studies on the sensitivity of the model response indicated that the greater the sensitivity of the model response to a parameter the closer and sooner will that parameter be optimised. It was also observed that the less sensitive parameters have insignificant influence on the fitting of a record.

This work of Dawdy and O'Donnell shows that it is not sufficient to take the minimised value of the sum of the squares of differences between correct and calculated values as the sole criterion in interpreting the fit of any model. A further criterion of response sensitivity or its equivalent must be used when selecting what can be considered adequately optimised parameters.

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1.9. Comments on mathematical models

The output of hydrologic systems, containing storage elements, depends on processes which are not purely stochastic. Monte Carlo techniques may be used if the data is first grouped to minimise persistence effects or probability studies of streamflow may be undertaken under the assumption that the output sequences represent time series in which each successive value of the variable depends on its present value plus a random component i.e. a Markov process. These methods have been applied to annual runoff values and with grouped annual values. Stochastic methods require that the size of the historical data records, which is a sample of the universe of natural events, be sufficiently large to represent the true distribution of the variables. In many instances where stochastic studies of runoff are needed, the records are too short to fulfill this condition. The Monte Carlo method is not applicable to daily riverflow values because of strong persistence effects and the Markov process would appear illogical in the context of daily flows if the riverflow values on a long recession curve are considered. Methods of linear system analysis used in conjunction with partial system synthesis lead to approximations of the hydrologic process which may be grossly in error. The degree of approximation which the linearity assumption may yield depends on the degree of actual non-linearity of the system. When the method is used to approximate non-linear cases great caution should be exercised in interpreting the results as grossly erroneous predictions may result, without any

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foreknowledge of the magnitude or sign of the error. In general, since linear analysis is much simpler and much better known than nonlinear analysis it offers more appeal to the investigator.

The methods of non-linear analysis are still in the development stage. Rigorous methods of non-linear inversion are wanting. Approximations have been proposed and tested in simplified hydrological situations but they are not suitable for general application.

The elaborate synthetic models based on qualitative and semiquantitative knowledge of the phenomena involved in the hydrologic cycle give results which are most encouraging but the performance of the models is not sufficiently reliable so that complete confidence can be placed in extended reconstruction of runoff histories. This lack of reliability is the result of weaknesses in our present state of knowledge and capability with regard to mathematical models of catchment behaviour. O'Donnell (Ref. 31) considered the most significant points to be

a) Errors in the recorded date. Small errors may generate large errors in those model parameters for which the response sensitivity of the model is low, even when an observed record has been fitted extremely closely.

b) The wide variability over a catchment of the many factors controlling its behaviour. These have to be drastically averaged when constructing catchment models and distributed effects are lumped in the various components of a model.

c) Whilst optimisation of parameter values for a given model in a completely objective way is feasible, the choice of structure of the model is largely subjective. A technique for objectively optimising structure will be difficult to find.

The basic underlying assumption of all the procedures of system studies discussed in this chapter is that hydrologic systems are time invariant. If system variability is likely, or possible, then the use of any of these methods, for the reconstruction of past records or the prediction of future events, is suspect.

1.10. Runoff prediction with limited data

Adequate riverflow and meteorological data must be available to develop and operate the mathematical models which have been described in this chapter. On medium sized and small catchments hourly values of meteorological and riverflow data may be required for successful modelling and prediction. Unfortunately at the present time such data is not generally available for British catchments, nor for catchments in developing countries. Riverflow levels are recorded continuously at most rivergauging stations. Over the course of the last seven years a number of these stations in Great Britain have been fitted with digital water level recorders and where this has been done discharge values, at fifteen minute intervals, can be calculated with the aid of a digital computer with the minimum of effort (Clay Ref. 32). At the present time the only meteorological data recorded on most gauged British upland catchments is the daily rainfall value, which is observed at 9.0 am. G.M.T. each day. Low priced automatic hydrometeorological stations which will record data in digital form at fifteen minute intervals, have been developed (Strangeways and McCullock Ref. 33) but one or two decades will elapse before adequate data from these stations is available for all rivergauged catchments.

Where the available meteorological data has been limited to daily, weekly or monthly rainfall values and when the interest in the rainfallrunoff relationship has been for the purpose of estimating the yield of a catchment for which only ten or fifteen years runoff data is available, workers have usually attempted to obtain regression equations relating seasonal, monthly or weekly rainfall on a catchment to the short period riverflow.values. When data is so limited, maximum use should be made of what little is available. Summation of daily values results in much detailed information being lost, for a once a month observation

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of a raingauge will give the same rainfall value as the sum of a months daily values. A further objection to this method is that summation of variable daily riverflow and rainfall into weekly, monthly or seasonal values attenuates the peaks and troughs that occur when successive daily values are used. Furthermore, the use of calculated regression equations to synthesise riverflow from rainfall data leads to a further reduction in extreme values.

Correlation of daily rainfall and run-off will eliminate the attenuation due to summation of daily values but the greater variations that occur with daily values may produce a low degree of correlation, so that any one calculated daily run-off value may differ considerably from the actual run-off on that day. If however, the runoff which has been calculated from daily rainfall values is used to build up a mass curve or a cummulative deficiency diagram, the overestimates on one day might well be cancelled out by the underestimates on the preceding or subsequent days.

As has been stated previously, sufficiently adequate data is not normally available from upland catchments to allow the calculation of evaporation, however it is possible to make an estimate of the soil moisture conditions by using an antecedent precipitation index, for the date associated with daily rainfall value is itself data which

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should not be ignored or devalued. In pervious areas it may also be worthwhile to consider the use of the antecedent conditions index, as proposed by Andrews (Ref. 19) to express the state of saturation of the deeper strata.

If a method could be developed whereby a mass diagram of daily riverflow could be built up from dated daily rainfall and short period runoff values it would be of great use in reservoir yield calculations. The correlation of daily rainfall-runoff values using daily antecedent precipitation index as a measure of soil moisture conditions and the application of the results to build up a cumulative deficiency diagram to determine the yield of an upland catchment, would appear to be worth investigation.

1.11. Computer and graph plotter

The use of daily values to calculate regression equations and the calculation of run-off from these equations and daily rainfall would lead to massive calculations. The drawing of mass curves or cululative deficiency diagrams from daily values could involve the plotting of tens of thousands of points (e.g. 50 years would require 18,250 points). A modern high speed digital computer coupled with a graph plotter would carry out these massive calculations in a relatively short time and it could plot the graphs automatically, once the programme had been developed and written. It was possible to carry out an investigation to determine the value of this method, of correlating daily values, since a small medium speed electronic digital computer is available in the College, for use by trained staff and students, on the open shop principle.

This machine is a National-Elliott 803 computer with a solid state store of 8,192 words, each word being equivalent to a twelve decimal digit number or 39 bits. The installation has an automatic floating point unit and computer input is by five track punched paper tape through a photo electric cell reader reading up to 500 characters per second (in binary form). Output is by five track paper tape punched at 100 characters per second by a teletype punch.

The graph plotter is a Benson-Lehner incremental machine working off line. It is driven by five track paper tape which runs through a photo electric cell reader. It will plot 200 increments per second each increment being 0.1 m.m. The available plotting width is 320 m.m. and the length of paper on the roll is 50 meters.

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2. THE DERWENT CATCHMENT

2.1. Description of area

The river Derwent is a southern tributary of the river Tyne and in part it forms the boundary of County Durham and Northumberland.

The possibility of siting a large impounding reservoir scheme in the valley was assessed before World War II and in the late 1940's the increasing water consumption and the lack of suitable resources within their own statutory areas, prompted the two main undertakings in County Durham to investigate fully the potential of the area as a source of supply. Thus in 1949 a joint decision of the Durham County Water Board and the Sunderland and South Shields Water Company was made to carry out exploratory work, in the nature of aerial surveys of the catchment, trial borings, the installation of a grid of rain gauges, and the construction of a river gauge.

Gibberd (Ref. 34.) describes the landscape of the Derwent valley as a lush, green and gentle parkland. The river is bordered by pasture and arable land which gives way to rough grazing near the highest point of the catchment.

The area of the catchment delineated by surface contours is 29,150 acres (45.6 square miles). The catchment area delineated topographically should be representative of the phreatic boundary, as no non-contributory areas are recognised and geologically the catchment is assumed to be water tight. The main vegetative cover over the upper part of the catchment is heather and bracken moorland, lower down this gives way to grass covered farmland with extensive wooded areas.

Borings undertaken to investigate the foundations of the embankment of the dam, which lies about 1 mile upstream of the river gauge, showed that the rock was composed of alternating bands of sandstone and shale. This was overlain by morainic, lacustrine, and alluvial deposits, including thick layers of laminated clay formed by deposition in a fresh water lake.

The mean elevation of the catchment is 1,175 ft. 0.D. and the total rise is some 1,250 ft., the highest point being 1,838 ft. 0.D. The aspect of the upper catchment is easterly, the general axis being east-west.

2.2. Raingauges on the catchment

Nineteen rain-gauges were installed on the catchment in 1952 and observers were recruited from local residents. Readings were commenced in July, 1952 and have been continuous to date. Originally it was

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intended that eleven of the gauges should be read daily and the remaining eight should be read monthly as the latter were in remote parts of the upper catchment, but due to observer difficulties Cowyers gauge, originally intended to be read daily, was read monthly from 1953 onwards.

These ten daily and nine monthly gauges operated until September 1961, when Belmont gauge was converted to monthly readings. In November 1962 Penny Pie gauge was converted into a monthly gauge and a new daily gauge at Penny Pie was installed.

2.3. Rain-gauges off the catchment

Durham County Water Board maintains five rain-gauges which are sited in a fairly compact group just over the crest of the ridge on the south side of the Derwent Catchment. Three of these are monthly gauges and two are daily gauges. One of these daily gauges is at Tunstall reservoir Grid Reference NZ (45) 063407. The altitude of this gauge is 724 ft. 0.D. and the average annual rainfall for the period 1916 to 1950 is 34.5 inches. Reliable daily records for this gauge are available from 1911. There are two other daily gauges reasonably near the catchment, one at Allenheads Grid Reference NY (35) 860453 and the other at Hexham Reservoir Grid Reference NY (35) 931632.

2.4. Long average rainfall

The long average rainfall for the Derwent Catchment was originally assessed by reference to the older gauges outside the catchment and was found to be 37.5 inches. Ruffle (Ref.35) has analysed the results, collected over the period 1952 to 1963, from the 19 gauges sited on the catchment and his calculations give a similar figure for the long average rainfall.

2.5. River gauge

The Derwent river gauge is sited at Grid Reference NZ (45) 041508 about one mile downstream of the dam. Daily river level readings were instituted at the site in September 1952, and the construction of the river gauge was completed in 1954. A stage discharge relationship was then established by model tests and earlier level readings were then converted to river discharge. The gauge is a 50 ft. wide reinforced concrete venturi flume. A 6 ft. wide central notch measures flow up to 40 m.g.d. and the whole flume accommodates 2,400 m.g.d. which is half the unreservoired normal maximum flood. The highest recorded rate of run-off from the 29,150 acre catchment has been 1,200 m.g.d. in August, 1956, and the lowest over one day was 2 m.g.d. in September, 1959.

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3. ANALYSIS OF RIVER FLOW RECORDS

3.1. Data from the Derwent catchment

Previous experience with statistical work on the computer had shown that a tremendous amount of time is required to punch up and check dated daily river flow data. This consideration had a bearing on the choice of catchment to be used for the analysis.

Dated river flow records from the river gauge on the Derwent at Eddysbridge were available in punched tape form for the years October, 1953 to September, 1965. A search through British Rainfall (Ref.36.) showed that there were no long term records available from rain gauges on this catchment but that records were available from the year 1911 for a raingauge which was sited approximately 6 miles south of the river gauge, on an adjacent catchment.

The Derwent catchment is conveniently close to the College as it lies about 30 miles West of Sunderland. It is being developed as a further source of water supply by the Sunderland and South Shields Water Company and Durham County Water Board.

It was hoped that the results of the analysis would prove to be useful to these two water Undertakings. Taking all these considerations into account it was decided that the analysis should be carried out on data relating to this catchment.

3.2. Cumulative deficiency diagram for the Derwent Reservoir

The results of the daily analysis of rainfall and run-off from the Derwent catchment were to be plotted in a form similar to that of a cumulative deficiency diagram. It was thought advisable to develop a programme to plot a diagram from the actual river flow data, collected during the years 1953 to 1965.

The Derwent reservoir receives the run-off from an area of 21,550 acres, whilst the river gauge (which is sited about one mile downstream of the dam site) measures the flow from 29,150 acres. Most of this extra area of 7,600 acres, is drained by Burnhope Burn and the characteristics of this small burn catchment are similar to those of the whole catchment area. As a consequence 2155/2915 of the flow at the river gauge was assumed to flow into the reservoir.

The compensation water supplied from the reservoir will be 5.5 m.g.d. from April to September 5 m.g.d. from October to March, and 182 m.g. may be released each year as freshets at a rate not exceeding 20 m.g.d. If this 182 m.g. is called for in the six months April to September it will result in a greater reduction in reservoir yield than if it were called for during the other six months, or if it were spread over the year as a whole.

Ruffle (Ref.35.) has calculated that the gross yield of a reservoir with a capacity of 8,400 m.g. would be $26\frac{1}{2}$ m.g.d. (1% probability). Using this figure, and assuming that a flow (say Q) is to pass out of the reservoir down the pipe line and through the filter plant at a steady rate throughout the year, then

 $(Q + 5)183 + (Q + 5\frac{1}{2})182 + 182 = 26\frac{1}{2} \times 365$

hence $Q = 20\frac{3}{4}$ m.g.d.

The draw off from the reservoir during the months October - March will be

 $20\frac{3}{4}$ + 5 m.g.d. = $25\frac{3}{4}$ m.g.d.

and during the months April to September the draw off will be

 $20\frac{3}{4} + 5\frac{1}{2} + 182/182 = 27\frac{1}{4} \text{ m.g.d.}$

A programme was written to calculate the excess water which would have to be stored in the reservoir on any date. It was assumed that there was neither excess nor deficiency of water at the starting date. The programme allows for a yield which varies according to calendar date and will give a numerical print out of the storage required every day. The answer can also be obtained in a graphical form with daily storage plotted on the vertical axis and calendar date on the horizontal axis. If need be both print-up and graph can be produced from the same calculation. The graphical output uses less computer time.

The programme is shown in appendix I, pp. 1-3. A sample of the input data for the programme can be found in appendix I, p.45 and a sample of the numerical print up output by the computer on pp. 46-48. The graph drawn by the plotter is shown in appendix II, p.1.

The graph shows that the maximum draw down occurred between April 1955 and November 1959. An inspection of the print-up shows that the peak came on 4th April 1955 when the excess equalled 6816 m.g. The trough occurred on the 13th November 1959 when the excess equalled -1163 m.g. Thus the critical period of the reservoir exceeds four and a half years.

The minimum reservoir capacity which would be required to supply a draw-off rate of $25\frac{3}{4}$ m.g.d. in winter and $27\frac{1}{4}$ m.g.d. in summer during the twelve year period September, 1953 to September, 1965 will be

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6816 + 1163 = 7979 m.g.

This is approximately 95% of Ruffle's 1% probability figure of 8,400 m.g. storage.

3.3. Histograms of river flow

Since it was proposed to carry out the correlation on a statistical basis, it was thought desirable to investigate the distribution of the rate of run-off throughout the year.

Twelve years of river flow data were to be analysed i.e. 9 x 365 + 3 x 366 = 4383 pieces of data. If a histogram was to be plotted for each month this would approximate to 365 blocks each month. This should be sufficient to give a reasonably smooth curve.

Since the data for each year was on a separate tape, the programme was written so that the computer was instructed to wait after each years data was read in. It could then be instructed to read a further years data, or to plot a histogram from the data which it had already read into its store.

The width of each block of the histogram represents a flow range. e.g. 0 to 2 m.g.d., 2 to 4 m.g.d., 4 to 6 m.g.d., etc. In the example above the increment of flow in each range, E, is 2 m.g.d. but a suitable value can only be found by inspection. The programme was written to read a range increment, E, initially. The scales of the Y and X ordinates were then read (L and M) and these were followed by the first year's data.

For each months plot, the number of steps making up the base of the histogram will be the maximum flow divided by the range increment. A visual inspection of the data showed that on most days the flow was less than 40 m.g.d. and on no day did the flow exceed 700 m.g.d. The base of each histogram was fixed at 700 m.g.d. and the number of steps was therefore 700/E. The number of store locations required to count the total number of blocks in any one column would be (700/E)x 12. If the minimum E used is 2 m.g.d. then the maximum number of store locations required will not exceed (700/2)x 12 = 4200. This was within the capacity of the computer.

A copy of the programme is shown in appendix I, pp. 4-5; the input data being of the form shown in appendix I, p.45. The twelve histograms drawn by the plotter are to be found in appendix II, pp. 3-4.

From the histograms it can be seen that in the three winter months of January, February and March the flow was above 10 m.g.d. on all but

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6 days out of the 1083 analysed. For about half of the days the flow lay between 10 and 40 m.g.d. and there was a fairly uniform distribution over this range. Also in this period there were a number of days when the flow exceeded 100 m.g.d.

Considering the three summer months July, August and September, on about one third of the days the flow was less than 10 m.g.d. and there were a number of days when the flow lay between 2 and 4 m.g.d. There were few large flows but there were occasional days in August when flood flows occurred.

The histograms show that there are seasonal differences in flow patterns. Before the form of calendar grouping was finally decided it was thought worthwhile to plot running averages of groups of daily flow in the hope that a more definite pattern would emerge. The grouping of the data will attenuate the peaks and troughs which occur when daily values are plotted and smooth curves showing seasonal trends should result.

3.4. Data presentation

Experience gained during the running of the previous two programmes showed that a large amount of computer time was being used to read in data containing dates. Not only was time lost in reading: complications produced by leap years meant that calculations had to be performed on the dates each time a programme was run in order to determine the position of the month endings. It was therefore decided that the original data tapes should be modified.

The data tape which is fed into the computer can control the programme if there are triggers punched on it. A trigger is a group of figures of the form n(where n is an integer. If such a group of figures is encountered on the data tape while the computer is obeying a READ instruction, the computer will jump to the instruction in the programme with the reference number n. The READ instruction written at that point in the programme will not be obeyed: that is, the value of the symbol following the word READ will not be obeyed.

The daily rainfall values from the Tunstall gauge were all teleprinted up in the dated form (shown in appendix I, p.49). The river gauge data was also dated. A programme was written to convert both of these original forms of data presentation, into an undated form with triggers marking the end of each month. In the case of rainfall the monthly totals were also printed up at the end of each month, as this was found to facilitate the checking of the teleprinted values. The programme for the conversion is shown in appendix I, pp. 6-7 and a sample of the converted form of output follows the original data and is shown in appendix I, pp. 50-53.

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When the new style rainfall and run-off tapes had been output a further programme was written so that these undated rainfall-runoff values could be paired together and combined in one large tape containing the twelve years data in consecutive order. The programme was designed to check that the rainfall tapes were put into the tape reader in the correct order and that the number of days of rainfall and run-off in each month agreed. As a further check against programming errors or misreading by the tape reader, the total annual rainfall and the total annual run-off for each water year were printed up at the end of each year. The programme is shown in appendix I, pp. 8-10 and a sample of the combined output is to be found in appendix I, p.54.

3.5. Graphs of grouped daily flows

The number of consecutive run-off values which have to be averaged to obtain a reasonable smoothing of the curves was unknown. In order to save computing time in reading in data the programme was written so that up to 12 years of the modified run-off data could be read into separate locations in the computer store. The computer was programmed to WAIT for a key instruction after it had read in each year's data. When the required data was read into the store the number of the appropriate reference in the programme was pressed on the keyboard and a further piece of data tape with information on the number of values to be added together and the scales of the X and Y co-ordinates was then

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read in. After completing the calculations the computer punched out the data tape required to drive the graph plotter and followed this with a print up giving the number of days data read in, the value of the last run-off read, the sum of all the run-offs and the average run-off over the period. The horizontal line marking the average run-off throughout the period was drawn on the finished graph by hand.

The programme is shown in appendix I, pp. 11-12. The input data was of the form shown in appendix I, pp. 52-54 and the graphical output is shown in appendix II, p.5.

3.6. Observations on histograms and grouped data graphs

The histograms and the graphs of average run-off over 15,30 and 91 day periods do not show a close relationship between calendar date and run-off rate. As a consequence it was thought that there was nothing to be gained by grouping the data into calendar months. There could even be a loss because there would be a smaller number of values to correlate in each group.

It was therefore decided that the data should be grouped into the quarters shown below:-

Autumn quarter to run from 1st October, to 31st December. Winter quarter to run from 1st January, to 31st March.

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As dated rainfall is the only significant long term data available to correlate with run-off, it was thought that it could be used most effectively if the quarterly rainfall-runoff values were correlated in groups, according to their daily anticedent precipitation index values.

4. CORRELATION OF RAINFALL-RUNOFF VALUES

4.1. Antecedent precipitation index

The antecedent precipitation index is an index of ground moisture conditions preceding the onset of rainfall. It is most commonly used with a coaxial correlation technique for forecasting run-off from storm-rainfall. (Ref. 17). It has been suggested that whilst it satisfactorily represents soil moisture conditions at the surface it does not allow sufficiently for the moisture content of the deeper strata. Andrews (Ref. 19) has proposed that an additional index should be used in pervious areas, which he calls an antecedent conditions index.

The antecedent precipitation index, I, may be expressed in the form:

 $I_0 = A_1 P_1 + A_2 P_2 + \dots A_t P_t \dots I$

where P_t is the precipitation which occured on a day t days before the day in question, and the A's are constant coefficients which are always less than unity.

As time passes the effect of a rainfall on the catchment moisture conditions will diminish, so that $A_1 > A_2 > A_3$ etc. For the proposed application a day-by-day value of the index is required. There is considerable advantage in assuming that A_t decreases with t according to a logarithmic recession.

i.e.
$$A_t = K^t$$

where K is a constant which is always less than unity.

The index may then be defined as

 $I_{o} = K^{1}P_{1} + K^{2}P_{2} + K^{3}P_{3} \dots K^{t}P_{t} \dots 2$

An examination of equation 2 shows that I_0 depends most strongly on P_1 and P_2 and is not very sensitive to slight variations of K. By the nature of the exponential function, errors in I due to incorrect assessment of K, are not cumulative.

Theoretically the value of the recession factor K should be a function of season and should vary from one region to another. The figure for the daily recession factor may vary between 0.85 and 0.95.

The choice of starting value of A.P.I. is not of great importance as the portion of any one day's index due to conditions more than thirty days before is extremely small. Provided a not unreasonable starting value is used, the index will be independent of this value within a month of commencement. Using a computer the calculations of the initial A.P.I. value from some assumed value, for say, one year beforehand would be completed in seconds and the effect of varying the assumed value could be noted.

The programme which was written to carry out this calculation is shown in appendix I, p.13. The operation was as follows:-

Three numbers were punched on a short length of data tape, they were:-

a recession factor (K) say 0.9

an assumed value for the A.P.I. on the last day of the previous water year (I) say 2.0

the actual recorded value of rainfall which fell on the last day of the previous water year (P) 0.66

This short length of data tape was placed in the reader and the three numbers were read in. The computer then waited for further instructions. The complete rainfall data tape for the water year prior to the starting date was then entered into the tape reader and the wait instruction cleared. The computer then read the rainfall tape day by day, calculating the A.P.I. for each day immediately after reading the rainfall. When it had read in a complete years data it printed out the K value used for the calculation, the last A.P.I. value calculated and the rainfall which fell on the last day. These three values will be used as input data for the main programme.

The rainfall data for the water year commencing 1st October, 1952 was used to calculate the A.P.I. on the 30th September, 1953. The programme was run three times using a recession factor of 0.9. The first time the A.P.I. on September 30th, 1952 was assumed to be 2.0; the second time 1.5, and the third time 1.0. Each time the answer for the A.P.I. on September 30th, 1953 was the same 1.14.

The result confirms the assumption that, within limits, the A.P.I. after one year is independent of the assumed starting value.

The value of the recession factor K, which is used in the calculation, obviously effects the rate of change of A.P.I. from day to day. It was therefore thought worthwhile to run through the calculation another three times, this time using a higher value for K; 0.95.

Again, each time the answer for the A.P.I. on September, 30th 1953 was the same, but in this case it was higher and equalled 2.22.

4.2. The first analysis

The procedure for the first analysis of the rainfall run-off data was written out in the form shown below:-

- a) Using an assumed value for K, calculate the A.P.I. value of each calendar day of the eleven years of record, i.e. 11 x 365 days
 + 3 days for leap years = 4018 days.
- b) Associate each days A.P.I. with that days rainfall and run-off.
- c) Sort the three daily values of A.P.I., rainfall and run-off, into groups according to season, i.e. Autumn Oct., Nov., Dec. (92 x 11) days = 1012 x 3 values Winter Jan., Feb., March. (90 x 8 + 91 x 3) days = 993 x 3 values Spring April, May, June. (91 x 11) days = 1001 x 3 values Summer July, Aug., Sept. (92 x 11) days = 1012 x 3 values
- i.e. $1012 \times 3 + 993 \times 3 + 1001 \times 3 + 1012 \times 3 = 12,054$ values in all.
 - d) For each season, sort the rainfall run-off values into further groups according to the range in which their associated A.P.I. falls.
 - e) Carry out a correlation of the rainfall-runoff values in each of, say ten, A.P.I. ranges. Determine the regression equations and the standard error of estimate.
 - f) Repeat all of the above work using various values for the recession factor K. The recession factor giving the minimum standard error of estimate will be the most suitable for that season.

- g) Using the set of regression equations associated with the most suitable K value for a season, generate synthetic run-off values from actual rainfall and calculated A.P.I.
- h) Determine the mean of all the calculated run-off values and the deviation of each of the calculated daily run-off values from this mean.
- i) Repeat 'h' using actual run-off figures.
- j) Repeat 'h' using actual rainfall figures.
- k) Using the graph plotter, draw graphs on a common base of time, of the accumulation of the deviations from the mean calculated run-off, the mean actual run-off and the mean rainfall.
- Using the 1964-65 rainfall record, which is not included in the eleven year period to be analysed, repeat the calculations and graph plotting specified in h, i, j, and k.

4.3. Computer storage limitations

When a programme is being translated and run the Autocode Translator and the Functions and Ancillaries are always in the store, along with the programme. There must also be room for each of the variables and constants which are required for the calculation. The capacity of the store of the computer is limited, and this imposes limitations by which the lengths of programmes and/or amounts of data which can be handled at one time are restricted. The capacity of the computers store is measured in words, i.e. one number or two instructions. Each word occupies a location. The number of locations which a programme, and its associated data, may occupy in the computer store depends on the method of running and on the Autocode language being used. The maximum store capacity available on College Elliott 803 computer using the 803 A 103 Autocode is 7588 words and this capacity is only available if the programme is translated first into binary form and then run separately.

The analysis of the rainfall-runoff data involves the calculation of 4018 A.P.I's and then the sorting of groups of A.P.I., rainfall and runoff i.e. $3 \times 4018 = 12,054$ pieces in all.

This is clearly well above the capacity of the computer store and a method will have to be evolved for handling and sorting the data in parts.

4.4. Grouping of data according to A.P.I.

The A.P.I's must be calculated in chronological order since each day's value is dependent on the previous day. Once the A.P.I. is calculated it must be recorded in such a way that it can be associated with its days rainfall and runoff. Each calculated A.P.I. value could be dated, but previous experience has shown that the punching and reading of dates is time consuming and in any case each date is another word to be stored. If the A.P.I's are punched out day by day as they are calculated and a trigger is punched at the end of each month the order in which they appear on the tape will serve to identify them and the print up of A.P.I.s could be arranged, to look like the printed up page of the associated rainfall. All the 12,054 pieces of data i.e. rainfall, runoff and A.P.I.,would then have to be read in again however, in order to sort the rainfall-runoff values into groups according to the range in which their associated A.P.I. falls and again this is above the store capacity of the computer.

The solution to this problem would be to feed only two or three years data into the computer at a time. It could then hold this in its store while it calculated the A.P.I.s and sorted the rainfallrunoff into A.P.I. ranges. Then it could punch out the rainfall-runoff pairs in groups according to A.P.I. range, each group being separated by blank tape, so that at a later date the lengths of the tape containing rainfall-runoff data associated with say the summer season and having A.P.I.s within a range of say 0.6 to 0.8, could be spliced together to form one input tape suitable for statistical analysis. This method however produces its own problems, since there is no way of knowing how many rainfall-runoff pairs will fall in any one A.P.I. group before the A.P.I. calculation, yet store locations must be allocated to contain the rainfall-runoff pairs associated with each A.P.I. group in advance of the calculation.

Obviously this problem will become more acute as the number of

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A.P.I. groups is increased. So consideration should be given at this stage to the number of A.P.I. groups to be used. The Handbook of Statistical Methods in Meteorology (Ref. 37.) gives a guide on group numbers. It recommends that the number of classes be not more than five times the logarithm of the number of observations. The number of observations per season for eleven years of data lies between 993 and 1012. For 100 observations the number of classes should not exceed $5 \times 2^{\circ} = 10$. For 1000 observations the number of classes should not exceed $5 \times 3 = 15$.

The data is to be grouped into four seasons. If 10 A.P.I. groups are allocated to a season this accounts for 40 groups in all. Ideally, there should be an equal number of rainfall-runoff pairs in each group, but if group limits are assumed before the calculations of A.P.I. this is not likely to occur. The maximum store available for programme and data is 7588 words. If 6000 words are allocated for the storage of rainfall-runoff values i.e. 3000 for each and there are to be 40 groups in all then the number of values per group = 3000/40 = 75.

The maximum number of days in a single season is 92 and there are to be 10 A.P.I. groups per season. If the number of rainfallrunoff pairs in each group was equal this would mean that there were 9 or 10 values per group each year, but the distribution of A.P.I.s.

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values will differ from one year to another, so that in a dry year we will have many more values in the low A.P.I. groups than we have in high A.P.I. groups and vice versa. The answer to this problem would be to arrange for the computer to print out the number accumulated in each A.P.I. group at the end of each successive year's calculation.

A critical inspection of this print out will give an indication as to whether or not the next years data would cause one of the group stores to exceed 75. If it was thought that this might be so, then all the sorted rainfall-runoff values in the computer store could be output onto punched tape, suitably referenced so that they could be spliced into one complete tape when all the ll years data has been through the computer.

This would complete the calculations down to (d) on the list of procedure for the first analysis, see page 81.

The programme which was written to carry out the calculations described above, can be found in appendix I, pp. 14-17. The limits of the A.P.I. groups were written into the programme as 0 to 0.25, .25 to .4, .4 to .6, .6 to .8, 1.0 to 1.2, 1.2 to 1.4, 1.4 to 1.7, 1.7 to 2.2 and greater than 2.2. It was thought that with a K value of 0.9 these limits should give approximately equal numbers of rainfall-runoff in each group. Each group was allocated a reference number. The 10 autumn groups were given the references 0 to 9 inclusive, the winter groups 10 to 19, the spring groups 20 to 29 and the summer groups 30 to 39. The lowest reference number in each season represented the 0 to .25 group, the next lowest .25 to .4 group and so on, in sequence for the remaining 8 groups. The input data to this programme was in two parts. The first data tape was short and contained only three numbers, which were output data from the previous programme.

- 1) an assumed recession factor K, e.g. 0.9.
- 2) a calculated value of the A.P.I. on September, 30th 1953.
- 3) the actual rainfall on September, 30th 1953.

The second data tape carried the rainfall-runoff data, presented in the modified form shown in appendix I,p.5⁴, starting with the 1st October, 1953.

The computer printed out the daily A.P.I. values as they were calculated. A sample of this print out is shown in appendix I, p.p. 55-56.

The calendar date was printed at the head of the sheet as was the value of K used. At the end of each month a trigger symbol 1(was printed as this will simplify future calculations using this data. At the end of the years calculations and print up of A.P.I. a length of blank tape was output, then the K value was printed followed by four lines of ten numbers per line. These gave information on the number of store locations used to date, to hold the rainfall-runoff values associated with each of the ten A.P.I. groups and each of the four seasons. e.g. at the end of the 1953 calculations there were 34 rainfall-runoff values where the A.P.I. fell in the range .4 to .6 in the autumn season. Since there were 75 store locations allocated to each group it should be possible to input another years data in this instance without exceeding the store capacity. In fact it was found possible to input the three years data up to September 30th, 1956 without the store overflowing.

In order to check that there were no errors in the reading of the data the values of the total rainfall, and the total runoff in the water year were printed up at the end of each years calculations. In addition, the recession factor, the A.P.I. and the rainfall which are required as initial data for the following years calculations were also printed.

On September 30th, 1956 the maximum number of rainfall-runoff values in any one group had reached 64.

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It was most unlikely that another years calculations could be completed before the number of rainfall-runoff values in this group reached 75, at this stage the computer was instructed to jump to instruction 21 which was the reference at the head of the print out instructions for the sorted rainfall-runoff data. The computer then punched out tape which produced the print up in the form shown in appendix I, pp. 57-58. This is the tape which, in combination with the others to follow, will form the input data for the next programme which will carry out a statistical analysis of the sorted data. The print up starts with the recession factor used in the calculation of the A.P.I., the next number is the reference of the A.P.I. group, and then follows the rainfall-runoff values in pairs. The end of each group is marked by a 1(and this is followed by a length of blank tape to facilitate splicing.

4.5. The regression equations

A further programme, appendix I, pp. 18-22, was written to carry out a correlation of the rainfall-runoff values in each of the ten groups of A.P.I. which occur each quarter i.e. section 'e' of the procedure listed under the heading "The first Analysis". In order to obtain a visual impression of the nature of the scatter about the line of the regression equation, the programme was written so that a graph of all the rainfall-runoff values in a group could be plotted, by giving an

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appropriate keyboard instruction.

The input data tapes for this programme had been output tapes from the previous programme, so it was designed to check each tape as it was read in to ensure that the tape carried the correct data, i.e. recession factor (k), A.P.I. group (reference number T) and also that the correct number of tapes had been read in. It had proved possible to put the sorted rainfall-runoff data onto four tapes, each tape starting with a number representing the K value, followed by the reference number of the A.P.I. group, these two numbers being followed by the sorted rainfallrunoff pairs in that group. Each group on a tape was followed by a trigger 1(and a length of blank tape and then a new group of data preceeded by a K value and a reference number. Since there were only four data tapes to be handled each could be placed in a separate rack thus avoiding splicing.

The method of operation was as follows :-

A short length of data tape was prepared. The first number on this tape was the recession factor value to be used for this analysis. The second was the reference number of the A.P.I. group to be analysed and the last number was the number of data tapes which had to be read in i.e. $\frac{1}{4}$.

This short length of tape was entered into the reader and the computer was instructed to read. When the tape had been read in the computer came to a WAIT instruction. At this point one of the four data tapes was entered into the reader. This tape was read in and the checks for K value and A.P.I. group automatically carried out. If these two values were correct then the computer carried on reading in rainfall-runoff pairs into store until the trigger 1(was reached. This trigger brought the computer to the WAIT instruction again, ready for the next length of data tape. Another tape was placed in the reader and the computer checked this for K and A.P.I. value to ensure that it was the same as for the last tape before reading this new data into store. This procedure was repeated until the last of the four tapes was read in. At this stage the trigger 1(directs the computer to carry on with the statistical calculations. When these have been completed the machine reaches a WAIT instruction.

At this stage the computer could be instructed to start reading in the next group of rainfall-runoff data with a new A.P.I. reference number one greater than previously, or alternatively it could be instructed to graph the rainfall-runoff values already held in the store. When these had been plotted the line representing the regression equation was drawn out and short dashes were drawn on the runoff axis to mark the limits of one standard error on either side of this line.

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Red ink lines were drawn through these markers parallel to the line of the regression equation, at a later date. In order to keep the graph down to a reasonable size and yet still show individual points at the lower rainfall-runoff values it was necessary to draw the graph with two sets of scales. If a rainfall value exceeded 0.75 inches or a runoff value exceeded 150 m.g.d. then a point was plotted with scales 1/5th of the normal, the position of the point being surrounded with a small triangle.

The programme, suitably annotated can be found in appendix I, pp. 18-22. A sample of the statistical output is shown in appendix I, pp. 59-60, and a sample of ten autumn graphs of rainfall plotted against runoff with regression equations and one standard error lines can be found in appendix II, pp. 6-13. The vertical and horizontal axis were drawn on these graphs, by hand in red ink, so that those points which lay on the axis would not be obscured.

4.6. Observations on the statistical analysis

Referring to the printed statistical output page with recession factor K = 0.9. The numbers in each of the A.P.I. groups i.e. column N., vary considerably, for example, in the eleven spring quarters that were analysed there were only four days on which the A.P.I. value

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exceeded 2.2 inches yet there were one hundred and eighty eight days when the A.P.I. fell between 0.25 and 0.4 inches. In the eleven autumn quarters the balance was better, there were seventy six days when the A.P.I. exceeded 2.2 inches and there were ninety days when the A.P.I. fell between 0.25 and 0.4 inches. This wide variation is an obvious disadvantage since four values will not give a valid regression equation, but if another value of recession factor is used, say K = 0.92, and the limits of the A.P.I. groups are kept at the same values, the number in each group will differ again. In order to get some indication of this variation it was decided to run the last two programmes again, this time calculating A.P.I. values for K = 0.92. The result of this work is shown in appendix I, p.p. 61-62. The number in each of the A.P.I. ranges has naturally changed, but not in a uniform manner. The first of the autumn groups has now only two values in the range 0 to 0.25 whilst previously it contained forty five. New ranges could be guessed and tried but this would be most time consuming as about eight hours of computing time is required for each run.

Referring to the following columns on the printed statistical output. An inspection of the average value of rainfall P and runoff Q show that they follow the expected trend, increasing as the A.P.I. increases, but the numerical values representing the slope of the regression equation M do not appear to follow a pattern for any one season. The intercept of the line of regression equation with the runoff axis C does increase as the A.P.I. increases, but not uniformly.

It was thought that it might prove worthwhile to draw out all the ten regression equations for a season on one graph. If this was repeated for a range of K values a pattern may well be revealed.

4.7. Programme to draw regression equations

This programme which is shown in appendix I, pp. 23-24, was written so that the data tape output by the previous correlation programme could be fed directly into the computer as input for the graph plotting. In effect the computer read one line of data at a time. Each line contained the statistical information for one A.P.I. group within an A.P.I. range. The data was as follows:-

I = Lower limit of A.P.I. range	Stored	as	RO
N = Number of rainfall-runoff pairs	11	11	Rl
P = Average rainfall for group	11	17	R2
Q = Average runoff for group	11	11	R3
M = Slope of regression equation	**	11	R4
C = Intercept of regression equation	ŦŦ	11	R5
E = One standard error of estimate	11	11	r6
Reference number	11	11	С

The Scales chosen for the graph were:-

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Rainfall l inch = 1000 units i.e. 10 cm. Runoff l m.g.d. = 5 units i.e. 0.5 mm.

The origin was taken as the initial position of the pen on the graph plotter and the first instructions on the programme following the reading was the calculation of the number of unit steps through which the pen was to be moved to reach the position of the intercept C on the runoff axis. (Y axis). This was the point from which the regression equation line was to be drawn. The next step was to calculate the partial co-ordinates of a point on the regression equation line, say 750 units (7.5 cm.) to the right of the Y axis. That was simply (750, (M x 750/1000 x 5)). The instruction to move the pen in the down position to this point was then written, and the pen then raised.

In order that the lines of the various regression equations, drawn on the graph, could be easily identified a dot was marked at the righthand end of each line. If the reference number of a particular equation was 1 then the dot was marked 10 units (1 m.m.) to the right of the line, if the reference number was 2 then the dot was marked 20 units to the right, and so on, for each quarters results. After the pen had drawn the identification dot, the distance back from this position to position of the average values must of course lie on the line of the regression equation and so in order that their position could be easily

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seen, a triangle was drawn at this point. When this operation was completed the distances back to the origin were calculated and the pen was moved back to the origin ready to start the plotting of the next equation. At this point 200 blanks were output and the computer came to a WAIT instruction. All that was necessary to plot the next line of results was to clear the WAIT button. A sample of the graphical output can be found in appendix II, pp. 14-17.

4.8. Observations on graphs of regression equations

In general the slopes of the regression equations increase as the A.P.I. increases. The obvious exception to this is the spring quarter. With K = .9 and the A.P.I.s in the range 1.7 to 2.2 and greater than 2.2, the slope is negative.

In spring the A.P.I. values are naturally lower than in other quarters and high A.P.I. values only occur on rare occasions. In consequence the number of results analysed is small and the results of the analysis are not really significant. An inspection of the graph for the spring quarter with K = .92 shows that the equation for the range 1.7 to 2.2 is now positive. This is because the number of pairs of rainfall-runoff values which have been analysed has increased to 63 and the odd pairs of values now have less effect. The number of pairs

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analysed for A.P.I. greater than 2.2 is only 9 and again, one unusual high value could throw the whole line out of balance and this could account for the negative slope.

Considering the eight graphs drawn, it would appear that the pattern becomes badly broken up when one or two of the intercept values get out of step, but it is difficult to suggest ways of bringing these into line because each regression equation has been derived from a variable number of pairs of rainfall-runoff values and therefore each line should carry a different weight if adjustments are to be made.

At this stage it was felt that further progress could not be made until a high speed method could be found for handling masses of data in and out of the computer, at every change in recession factor value. It was also necessary to find another method for grouping the data according to A.P.I. range so that an approximately equal number of values could be placed in each A.P.I. grouping.

5. A MODIFIED METHOD OF CALCULATION

5.1. A scheme for working with integer values

Whilst working on a problem involving a pump storage scheme the author had written a programme to sort numerical data into order of magnitude. It occurred to him at that time that it might be useful to sort A.P.I. values into numerical order. The first job would be to calculate the eleven years A.P.I. values, when this had been completed they could be sorted into four seasonal groups. The A.P.I. values in each seasonal group could then be sorted into order of magnitude and when sorted, each seasons values could be divided into sub-groups with equal numbers in each sub-group. If this was done the limits of each A.P.I. group would not be equidistant from each other, but this is not a serious disadvantage.

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The drawback to this idea is that each A.P.I. value is associated with a particular rainfall and runoff and it is these two values which are to be used in the statistical analysis. These two values therefore must always be identified with their particular A.P.I. value. In the previous work this was done by using the same numerical suffix for each store location holding the A.P.I., rainfall and runoff value.

e.g. I1, P1, Q1 and I2, P2, Q2, etc.

Once the A.P.I. value is moved out of a store location for sorting into numerical order the identity is lost. This loss of identity made the sorting method inapplicable.

Earlier work with sorting routines had shown that integer numbers could be sorted at higher speeds than floating point numbers. In particular, one programme was found in the Elliott 803 library which would sort 4,000 integer numbers into order of magnitude in about four minutes. This programme was written in machine code and could be simply inserted as a block into an Autocode programme. Out of general interest a programme was written to convert the original runoff data into integer values. All that was required of this programme was that the original data tape of runoff values be read, each value multiplied by ten and the integer part of this new number punched out. The new runoff tape then contained values of runoff in 10⁵ gallons/day units instead of m.g.d. and there were no decimal digits on the tape. This tape was then used with the integer sorting programme and all the twelve years runoff data were output in order of magnitude. While this print up was being inspected an idea was born which solved two major problems; first that of reading and punching 16,000 pieces of data every time the recession factor was changed; second the problem of retaining identity between the three numbers; rainfall, runoff and A.P.I. when the A.P.I. values were being sorted.

The print up of sorted runoff showed that the highest runoff value in the twelve years of record was 7190 (10^5 gallons/day), i.e. a four digit integer number; yet the computer was capable of handling any integer value between + 274 877 906 943.

Now the daily rainfall values do not exceed 9.99 inches i.e. a three digit number and the A.P.I. values even with the highest practical recession factor would never exceed 99.99 inches, a four digit number. It is therefore possible to combine the rainfall, runoff and A.P.I. values all together to make one eleven digit integer which would occupy only one store location. If this is done the total number of values to be held in the computer store for processing, drops from 12,054 to 4,018; this last number is well within the store capacity of a medium size computer such as the 803, while the 12,054words were too great and as a result the work had to be fed in and out on lengths of tape, the assembly of the various tapes being carried out manually. If the A.P.I. is made the first four digits of the eleven digit number, then the A.P.I.s can be sorted into order of magnitude simply by sorting the eleven digit integers. Once sorted the rainfall and runoff parts of the number can be separated off by dividing the eleven digit number by 10⁷ and storing the fractional part. This fractional part can be further divided by 104 to separate rainfall, the integer part, from runoff the fractional part. For example, using a

recession factor of 0.90 the A.P.I. value on the morning of January, 2nd, 1955 was 0.77 inches, the rainfall that day was 0.33 inches and the runoff 26.3 m.g.d. These three values can be stored as one eleven digit integer. If the A.P.I. is put first and this is followed by rainfall and then runoff the eleven digit number would equal 00770330263. To obtain the A.P.I. for further computer calculations this eleven digit number would have to be divided by 10⁷ and the integer part used, i.e. 0077.0330263, if the rainfall and runoff were required then the fractional part would be multiplied by 10⁷ thus giving the 7 digit integer 0330263. If this number is divided by 10⁴ then the integer part is rainfall and the fractional part is runoff. If the runoff is required for a calculation, then the fractional part could be multiplied by 10⁴ to make an integer.

This method appears at first sight, to involve a great deal of computation and hence computing time, but this is not so. The computer can divide a twelve digit integer number and take off the fractional part in 12.096 milliseconds, so that the time involved in carrying out this operation on 4,000 numbers in only about 50 seconds. This is only a fraction of the time required for punching out numbers in the previous method, for the output punch can only punch out single digits at the rate of 100 per second.



5.2. Analysis of data presented in integer form

This programme, see appendix I, pp. 26-33, follows the basic procedure set out under the heading "First analysis a to f" (page 81).

Initially a short length of data tape, containing three numbers, was read into the computer store. The first number gave information on the placing of leap years. If the first years rainfall-runoff data contained a February of 29 days then the number read in was 4. If the rainfall-runoff data was for the first year after a leap year, then the initial number was 1, the second year after a leap year it was 2 and the third year after 3. The second number on the short data tape gave information on the number of years of rainfall-runoff record which was to be analysed. The maximum number of years data which could be handled by this programme was eleven, but for testing purposes only two or three years data was read in. The third number was the number of A.P.I. groups into which a seasons data was to be divided. For this analysis it was decided that ten A.P.I. groupings should be used.

The next step was to read in the integer values of rainfall and runoff in the form shown in appendix I, p.63. These integer values were obtained by processing the original rainfall-runoff data using a program which is shown in appendix I, p.25. They were read, in chronological order, into four blocks of store locations. The particular block chosen was dependent on the season. The block containing autumn values was labelled from Al to AlOl2, the block containing the winter values, Wl to W993, the spring block ran from Vl to VlOOl and the summer, Sl to SlOl2. When all the eleven years values had been read into these blocks the computer came to a WAIT instruction. Again a short length of data tape was read in. This contained information on the recession factor to be used for the analysis, the A.P.I. value on the morning of September 30th of the previous water year and the rainfall which fell on that day.

The computer then started to calculate the A.P.I. values, day by day, in chronological order, using the rainfall part of the 7 digit number. As each days A.P.I. value was calculated it was attached to the front part of the 7 digit number representing rainfall-runoff on that day, thus forming an 11 digit number. All of the eleven years data was processed in this way.

The next step was to sort all the values in a season according to the magnitude of the A.P.I. A separate block of locations labelled Bl to Bl012 was reserved for sorting, so that all the rainfall-runoff values could be retained in their original chronological order. Each of the seasons values were copied in turn, in 11 digit form, into the B locations, then the 11 digit number in the seasonal store location was

broken down into two parts, a 4 digit A.P.I. value, which was then discarded and a 7 digit rainfall-runoff value, which was left in its original store location for future use. All the ll digit numbers in the B locations were then sorted according to the magnitude of the A.P.I. The sorted numbers were then divided into ten approximately equal groups and a statistical analysis was carried out on all the rainfall-runoff pairs in each group (approx. 100 pairs). The results of this analysis were printed out in the form shown in appendix I, pp. 64-74. When the analysis of one seasons values was completed, the process of copying the next seasons values into the sorting store locations B1 to B1012 was commenced, the old values in these locations being wiped out by the new ones which replaced them. When the analysis of the four seasons data was complete, the computer jumped to the WAIT instruction, reference number 8, ready to repeat the analysis with a new value of K should this be required.

The time taken by this programme, to analyse eleven years data using one particular K value was 20 minutes, whereas with the previous method the time taken was nearer 8 hours. It was therefore possible to analyse the data using K values which ranged from 0.85 to 0.95 in steps of 0.01 i.e. eleven sets of analysis.

The tabulated results are shown in appendix I, pp. 64-74. Each

line contains the statistical information for one A.P.I. group. The number of rainfall-runoff pairs in each group is given under the heading N and the maximum A.P.I. value in the group is given under the heading I. There are ten groups per season, the lowest A.P.I. group in the autumn season being numbered 1 and the highest 10. The lowest A.P.I. group in the winter season is numbered 11 and the highest 20. In spring the numberings run from 21 to 30 and in summer from 31 to 40.

The highest A.P.I. values in a season are shown on the lines numbered 10, 20, 30 and 40. The lowest A.P.I. values are not printed. The nomenclature for the other columns is as follows:-

- P = Average rainfall group.
- Q = Average runoff for group.
- M = Slope of regression equation.
- C = Intercept of regression equation.
- E = One standard error of estimate.

The 3(which follows the data on each page is a trigger symbol which will be used in following programmes.

5.3. Graphs of results for a range of recession factor

It was difficult to interpret the results of the last programme because there were so many values to compare. It was thought that it would help if curves could be drawn for M, C and E for each of the four seasons; autumn, winter, spring and summer. If the reference numbers were to be marked out in uniform steps along the X axis and if the values of M, C or E were scaled in the Y direction then a curve could be plotted through each of the points representing the ten A.P.I. groups per season. Eleven such curves could be plotted on each graph, one for each of the K values from 0.95 to 0.85 inclusive and it should then be possible to see which is the smoothest and which has the lowest standard error of estimate. These curves could be drawn by the graph plotter and the tape which was data output from the previous programme. This would save time in punching out data and what is more important would eliminate the inevitable human errors which result when data is copied on a teleprinter. If this procedure was followed computing time would be used for reading figures which would not be required for the programme but the time lost in this process would be very small.

If a set of eleven curves was to be plotted on one graph then a great deal of computing time could be saved by arranging for the plotter pen to draw the first curve starting from reference number 1, then the curve for the next K value in the reverse direction, starting with reference number 10, and so on, the direction of the pen travel being reversed for each curve representing a K value.

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Such a programme was written, see appendix I, pp. 34-36. The computer first output a length of blank tape and then read in the statistical output data, see appendix I, pp. 64-74, line by line, storing all the values read. When all the values relating to a particular K value had been read the trigger 3(was reached and the computer waited until the next length of data tape was inserted and the WAIT cleared. When all the eleven sets of data for the K values, 0.95 to 0.85 inclusive, had been read into store a short length of data tape carrying three numbers was placed in the reader. The computer was then given a keyboard instruction to proceed from reference number 16 on the programme. The first number on the tape was the number of A.P.I. groups in each season. e.g. 10. The second number was 4,5,6,7 or 8, depending on whether P,Q,M,C or E was to be plotted and the last number was 1,11,21 or 31 depending on whether the autumn, winter, spring or summer season values were to be graphed.

The working store locations were given the labels Ul to U441 and subroutines, labelled 4,5,6,7 or 8, were used to place the particular values of P,M,Q,C or E into these working store locations. The count of the suffix reference number Z for each working store location UZ was quite complex because of the decision to graph each curve in the reverse order to the previous one. For example, the plotter pen had to travel from the origin of the graph to a point whose Y co-ordinate was Ul, with the pen raised. The pen was then lowered and the curve through the Y co-ordinates U2 to U10 was drawn with the pen down. The pen was raised and moved to the position of Y co-ordinate U50. The pen was lowered and the curve through the Y co-ordinate U49 to U41 drawn, in that order. The pen was then raised and moved to the position of Y co-ordinate U81, the pen was lowered and the curve drawn through the Y co-ordinates U82 to U90, and so on. In this way a set of curves were drawn for each season with a minimum of pen travel and thus computer time. The graphs showing curves of M, the regression coefficient, C, the intercept of the regression equation and E, the standard error of estimate, for each of the four seasons, are shown in appendix II, pp. 18-23.

5.4. Interpretation of the results

The graphs of the regression coefficient M are not smooth. Considering the curve for any one particular group, it was noticed that small changes in the recession factor K could bring about large changes in the regression coefficient M, but it was also noticed that where this had occurred, the value for M in the adjacent group had moved in the opposite direction. The explanation for this could be that when the recession factor K was changed in value, all the A.P.I. values throughout the seasons changed. As a consequence one or more rainfall runoff pairs could have their A.P.I. value changed so that they would fall into an adjacent grouping. If these pairs which have changed over, have rainfall or runoff values which are high in comparison with others in the group, then they would have a great effect on the slope of the regression equation and on moving over into the adjacent group they would modify its M value, thus producing a see-saw effect. An example of this is shown below.

Group reference number	12	13	14
Regression coef. M for K = .91	138.7	42.5	94.1
Regression coef. M for $K = .90$	25.5 (down)		89.9

The graphs for the winter season, reference numbers 11 to 20, were the most erratic but this was to be expected. The low temperatures and snow which occur in winter will obviously delay the runoff following rainfall and will consequently spoil the correlation between daily rainfall-runoff values.

The curves showing the variation of the intercept of the regression equation C, with various values of K are shown in appendix II, pp. 20-21. The curves are fairly smooth and lie close to each other. As a consequence, a change in K has little effect on the value of the intercept. It should be possible to develop an equation so that the value of C can be determined for any A.P.I. value and a given K value. Considering the graphs of the standard error of estimate E, appendix II, pp. 22-23. These were most erratic. There was no one curve obviously lower than another for any of the four seasons. It would appear that the choice of the recession factor K has little effect on the scatter of the rainfall-runoff values in any group.

A decision had to be made as to the choice of K for each season, and so an alternative approach was tried. It was thought that the most suitable K value would be the one which gave the lowest standard error of estimate for each of the ten A.P.I. groups in a season. Unfortunately that was not a straightforward choice. The graphs show that a particular K value can give a low E value in one group and a high E value in another. It was decided that the best K value would be the one which gave the lowest mean standard error of estimate for all ten values in a season, but consideration would be given to the scatter of the E values within this group of ten. The mean E and the standard deviation of E for each group of ten were therefore calculated for all eleven K values and each of the four seasons. The results are shown on the table given on page 111. Each season's values were also plotted, by hand, see appendix II, p.24. An inspection of these graphs shows that so far as the spring and summer seasons are concerned, the recession factor K, had little effect on the mean E or the standard error of estimate of E. For the winter months, the results were erratic, the

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AUTUMN

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К	•950	•940	•930	•920	•910	•900	. 890	•880	.870	.860	.850
Mean E	34•7	33.4	32.8	32.2	31.7	31.2	31.1	30.7	30.4	31.2	31.2
σE	23.4	23.6	23.7	24.3	23.0	23.6	23.4	23.0	23.2	25.0	25.0

WINTER

К	•950	•940	•930	•920	•910	•900	•890	.880	. 870	•860	.850
Mean E	55.5	55.8	54.4	57.3	57.9	55•4	55.2	56.8	56.8	56.3	56.8
σΕ	22.2	21.0	22.3	15.9	14.5	21.5	21.9	17.1	17.5	18.1	17.8

SPRING

К	•950	•940	•930	•920	•910	•900	.890	.880	.870	.860	.850
Mean E	13.9	13.9	13.8	13.8	13.9	13.9	13.7	13.7	13.6	13.9	14.1
σE	11.0	10.9	11.2	11.2	11.3	11.3	11.4	11.4	11.5	11.6	11.5

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SUMMER

K .950 .940 .930 .920 .910 .900 .890 .880 .870 .860 .850
Mean E 20.0 19.5 19.2 19.3 19.1 19.2 19.2 19.3 19.2 19.2 19.2
G'E 21.6 21.4 21.7 22.0 22.1 22.0 21.8 21.8 21.8 21.9 21.8

standard deviation of E came down as the mean came up and vice versa. It was not possible to discern a trend. For the autumn there was a low mean E at K = 0.87 and the standard deviation was also low at this K value.

The 40 regression equations obtained by using a recession factor of 0.87, were drawn out by the plotter using the programme described earlier on page 42. The graphs of the ten regression equations for each of the four seasons are shown in appendix II, p.25. The arrangement of the ten equations for the summer season follows the expected pattern and is good. The results for the winter season are again shown to be the worst.

After giving the matter due thought it was decided that a K value of 0.87 should be used for all seasons in future calculations. It was obviously the best for autumn and the choice of K seemed to make little difference for the other three seasons.

6. THE FINAL SCHEME OF WORK

6.1. The analysis of values on dry days

Consideration of the graphs shown in appendix II, pp. 20-21, which show the variation of the values of the intercept of the regression equation C, for each of the ten A.P.I. groups, had led to the conclusion that an equation could be developed between A.P.I. and C. Since C is the intercept of the regression equation it represents the daily runoff on days when there has been no rainfall. This being so, it would seem reasonable to assume that this relationship between A.P.I. and C could be best determined by making an independent analysis of all the days on which there was no rainfall, instead of just considering the ten group values.

A new programme was written to do this (appendix I, p. 37). It followed the previous programme, for the sorting and analysis of all rainfall-runoff values, up to the point where the data was grouped according to season and then each seasons values sorted in order of A.P.I. magnitude. The rest of the programme, which followed the Elliott machine code sorting routine, was discarded and instead of carrying out a statistical analysis on groups of data a new page was written to separate the rainfall days from those on which no rainfall fell. When a day was found on which there had been no rainfall, the A.P.I. value on that day and the runoff value on that day were printed out. This procedure was followed until a whole seasons values were processed, a trigger 1(was then printed and this was followed by a number representing the number of days, in that season, on which no rainfall had fallen. A length of blank tape was then output and calculations for the next season started. Since all the values being processed were in integer form, the print out of A.P.I. was in the units of, inches x 100, and the units of runoff were M.G.D. x 10. The piece of programme which followed on after the Elliott sorting routine was fully annotated and is shown in appendix I, p. 37. An example of the print out of A.P.I. and runoff Q, when the rainfall on a day is zero, is shown in appendix I, pp. 75-76.

A statistical analysis could have been carried out on the results, without printing them all out, as had been done in the previous programme, but since nothing was known of the distribution of the A.P.I. against Q values it was thought that a clearer picture would be obtained if a scatter diagram of all A.P.I. against Q values was produced for each of the four seasons. A calculated regression equation could then be super imposed on each of these diagrams. A small, simple programme was written to produce this plot. This can be found in appendix I, p. 38. The data input to this programme was the pairs of A.P.I. and Q values output by the previous programme. As each pair of these values

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was read into the computer the point representing these pairs was plotted. This procedure was followed until the trigger 1(was read. The computer then came to a WAIT instruction ready for the next seasons values to be fed in. When the scatter diagrams had been produced each seasons A.P.I. and Q values were fed into the computer with a programme to analyse an ungrouped bivariate distribution. The regression equations which were output are shown on the following page and the scatter diagrams can be seen in appendix II, pp. 26-27. Each regression equation was drawn on the appropriate scatter diagram, by hand.

The number of points on each of the vertical lines of the scatter diagram gives an indication of the distribution of A.P.I. values. It would appear that the distribution of these values is skew, the spring season being farthest from a normal distribution. The distribution of runoff on dry days is indicated by the number of points lying between pairs of horizontal lines on the scatter diagrams. The distribution of these runoff values is also skew and again, Spring, would appear to be farthest from the normal.

An investigation was carried out on sample values of A.P.I. and runoff to see if transformations of log.A.P.I., log.Q, square root or log. arithmetic type, would force the data into a better approximation

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RELATIONSHIP BETWEEN A.P.I. AND RUNOFF ON DRY DAYS

Units:- Q in m.g.d. A.P.I. in inches.

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Season	Number of pairs	Values	Mean	Standard Deviation	Regression equation	Correlation Coefficient	Standard Error of Estimate Q on A.P.I.
Autumn	429	A.P.I.	.613	•474	Q = 40.1(A.P.I) + 2.55	•758	16.4
Autumi	429	ୡ	27.1	25.1	w = +0.1(A.1.1) / 2.))	•100	10.4
Winter	442	A.P.I.	.644	.446	Q = 29.7(A.P.I) + 17.2	•432	27.6
WINCEL	442	ୡ	36.4	30.7	₩ - 29• (N•1•1) + 1 •2	•+52	21.0
Comina	551	A.P.I.	•450	•373	Q = 30.9(A.P.I) + 2.64	.684	12.3
Spring	551	Q	16.6	16.8	W = 30.9(H.1.1) + 2.04	.004	L2•3
Summer	504	A.P.I.	.608	.517	Q = 23.3(A.P.I) - 0.95	•740	10.9
Summer	7 04	ୟ	13.2	16.3	w = 23.3(A.r.) = 0.95	•140	10.9

of the normal distribution but only the spring data proved to be amenable to this kind of treatment, it was obviously much too severe for the others as they all showed pronounced curvature after the transformation had been carried out. The best straight line for spring was obtained when the logarithm of runoff on dry days was plotted against arithmetic A.P.I. values. The amount of calculation required to synthesise runoff values would obviously be less if use is made of the regression equations which have been obtained on the assumption that the distribution is near normal. In the summer season this could lead to negative values of river flow. The equation Q = 23.3 (A.P.I.) -0.95 shows that this could occur when the A.P.I. value falls to 0.04 inches or less. Such low values of A.P.I. would only occur at times of severe drought and then only for a short period of time, and it was thought that the few negative daily values would not effect the accummulated values to any great extent.

6.2. Graphs of A.P.I. and runoff on wet days

The modification of a single line in the penultimate programme, i.e. a change from JUMP 1F P = 0 to JUMP 1F P greater than 0 would enable data to be output which could be used to draw scatter diagrams of runoff on rainfall days against A.P.I. These would be of interest, if not of direct use. The diagrams which were produced are shown in appendix II, pp. 28-31. The scatter of points in each of these four

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diagrams follow curves and these curves might well be straightened if logarthms of runoff on rainfall days were plotted against arithmetic A.P.I.

6.3. Recapitulation

The procedure described under the heading "First analysis" (see page 81) has been completed up to paragraph f. Paragraph g reads,

g) Using the set of regression equations associated with the most suitable K value for a season, generate synthetic runoff values from actual rainfall data and calculated A.P.I. values.

In a previous programme sets of regression equations for various values of the recession factor, K, had been determined and a value of K = 0.87 had been chosen as the best of all four seasons (see page 112).

There were ten regression equations for each of four seasons and these equations could be used for the generation of synthetic runoff values if the rainfall data was first sorted into the A.P.I. groups, shown in the left hand column under the heading I. The objection to this procedure was that some of the graphs of M, C and E plotted according to A.P.I. groups (see appendix II, pp. 18-23) were erratic and it appeared unlikely that a reasonably accurate value of runoff could be calculated from these equations as they stood. The graph of C against A.P.I. group number had led to the idea of correlating the runoff on dry days against A.P.I. Scatter diagrams were produced using these values (see appendix II, pp. 26-27) and regression equations were calculated for each of the four seasons. Using a similar programme, scatter diagrams were also produced of runoff on rainfall days against A.P.I. (see appendix II, pp. 28-31).

Although the values in the column headed C (appendix I, pp. 64-74) represent the intercept of the regression equations and will therefore give the theoretical value of runoff on a day when the rainfall equals zero, the value of the runoff on a dry day calculated from one of the four equations given on page 116, could not be used in place of C. This is because all values of runoff were used in the calculation of the original C. If the data for the runoff on dry days had been omitted from the original calculation then different values from M and C would have been obtained.

If the four equations, relating runoff on dry days to A.P.I. are to be used for the generation of synthetic runoff values then another correlation of rainfall-runoff values will have to be carried out. This time the data will have to be further sorted after all the A.P.I. values have been calculated so that runoff values on dry days are excluded from the analysis.

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6.4. Correlation of values on wet days, in moving groups

In the previous analysis, the value of the regression coefficient M varied considerably in adjacent A.P.I. groups. It was thought that smoother transitions may be obtained if a different system of grouping was used. Instead of sorting the rainfall-runoff values into ten groups of approximately a hundred pairs, according to the range of their A.P.I. values, the rainfall-runoff pairs would be sorted into order of magnitude of A.P.I. values, lowest first. Then the first fifty of the rainfall-runoff values would be analysed. When the results of this analysis had been printed out on a line, the fifty rainfall-runoff pairs lying between the 5th and 56th lowest A.P.I. values would be analysed. The statistical analysis would be repeated on each group of fifty values formed by moving through the full range of A.P.I. values in steps of five.

The programme which had been used previously to sort and analyse the eleven digit integer numbers was applicable up to the end of the sorting routine of 11 digit integers. A new piece of programme was written to follow on from this point, the setting instructions of the old programme being modified to suit. This replacement part is shown in appendix I, pp. 39-41. This piece of programme first broke down the eleven digit integer into A.P.I., P and Q. The value of P was then inspected to see if it was greater than zero. If it was not greater, then the value of A.P.I., P and Q, were overwritten by new values derived from the next eleven digit integer. If P was greater than zero then the computer jumped to instruction which directed it to store the values A.P.I., P and Q in a numbered location, to calculate and store P^2 , Q^2 and P x Q in separate locations with the same suffix number, and to sum A.P.I., P,Q, P^2 , Q^2 and P x Q values as they were calculated, until fifty values of each had been summed. When these fifty sets of values had been summed the computer jumped to a block of instructions to carry out a statistical analysis using the sum of these six values as initial data. The results of this analysis being printed out on a single line under the following headings (see appendix I, pp. 77-79 for sample).

I = Average A.P.I. for these 50 values.

- N = Number of values analysed (first line). Steps of moving average (subsequent lines).
- P = Average rainfall of these 50 values.

Q = Average runoff of these 50 values.

- M = Slope of regression equation for these 50 values.
- C = Intercept of the above equation on the runoff axis.

E = One standard error of estimate.

Reference number of group.

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When the results of the analysis of the first fifty values had been printed out, a jump instruction sent the computer back to the end of the sorting routine to break down the next eleven digit integer and to test to see if there was rainfall on that day. If so, the value of A.P.I., P, Q, P^2 , Q^2 and P x Q fifty days before was subtracted from the current values of A.P.I., P, Q, P^2 , Q^2 and P x Q. The difference, which could be positive or negative in value, was then added on to the fifty day totals of A.P.I., P, Q, P^2 , Q^2 and P x Q.

This process was repeated five times and then the computer was instructed to jump to the block of instructions which carried out the statistical analysis, using the six new values of fifty day totals as data.

This procedure was repeated throughout a season. If the number of days on which rainfall fell was not exactly divisible by five, then the last fifty values analysed would not have moved five steps from the previous one, but only by the number of steps in the remainder. This movement is printed out in the column headed N.

In order to keep the requirement for computer storage down to workable limits the programme had to be written so that each value, fifty days before the one being currently used, was overwritten. This

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made the programme a little more complicated, but using this method fifty values of each A.P.I., P, Q, P^2 , Q^2 and P x Q could be stored in three hundred locations, instead of using more than the three thousand, which would have been required if cyclic overwriting had not been employed. An additional three thousand values could not have been stored in our Elliott 803 computer and the analysis could not have been performed on this machine.

A sample of the results of the statistical analysis is shown in appendix I, pp. 77-79. These results were graphed on the plotter using a programme similar to that shown in appendix I, pp. 34-36. The only difference in the new programme was that instead of moving along the horizontal axis in uniform steps of 1 cm. at each change in A.P.I. group, the movement was made proportional to the difference between the A.P.I. values of each group. The graphs of M against A.P.I. and C against A.P.I. are shown in appendix II, pp. 32-37. The best straight line through these curves was determined by correlating the M and A.P.I. values and by correlating the C and A.P.I. values. The regression equations obtained are shown over.

Season	No. of groups analysed	Regression equation of M on A. P. I.	Regression equation of C on A. P. I.
Autumn	108	M = 93.3(A.P.I) + 0.2	C = 55.6(A.P.I) - 5.6
Winter	102	M = 45.3(A.P.I) +30.0	C = 45.8(A.P.I) + 21.0
Spring	82	M = 72.4(A.P.I) - 1.0	C = 31.2(A.P.I) + 0.5
Summer	93 [.]	M = 105.6(A.P.I)-31.1	C = 27.1(A.P.I) - 3.0

The line representing each equation was drawn on the appropriate curve by hand.

6.5. Observations on the results of the moving group analysis

The curves of M against A.P.I. are still erratic. If a larger number of pairs of rainfall and runoff had been grouped together for analysis (say 100 pairs instead of 50) a smoother curve would have been obtained, but the lowest of the average A.P.I. values would have been considerably larger and the highest of the average A.P.I. values would have been lower. Since these extreme values are important it was felt that the best compromise had been adopted and that nothing would be gained by repeating the analysis with groups of hundred pairs.

The regression equations of M on A.P.I. and of C on A.P.I. have a negative intercept in some instances. This means that at low A.P.I. values negative runoff will be obtained from these equations. It

would appear that the skew distribution of rainfall and runoff is responsible for this effect. A closer approximation to a straight line may be obtained for some seasons if the logarithm of runoff values were analysed.

6.6. Calculation of runoff from rainfall and equations

The analysis of the run off values on days when there had been no rainfall gave four equations, one for each season, each of which related Q to A.P.I. for a particular value of the recession factor K. With a K of 0.87 the equations were as follows:-

> Autumn Q = 40.1 (A.P.I.) + 2.5Winter Q = 29.7 (A.P.I.) + 17.2Spring Q = 30.9 (A.P.I.) + 2.6Summer Q = 23.3 (A.P.I.) - 1.0

The runoff on wet days was related to the rainfall by an equation of the form Q = PM + C where P is the rainfall,M is the slope of a straight line and C is the intercept of this line of the Q axis. This equation was applied to a group of rainfall-runoff values occurring on days when the A.P.I. fell within certain limits. The values of M and C were found to be related to the A.P.I. calculated from a recession factor of 0.87 by the following equations:- Autumn M = 93.3 (A.P.I.) + 0.2 C = 55.6 (A.P.I.) - 5.6 Winter M = 45.3 (A.P.I.) + 30.0 C = 45.8 (A.P.I.) + 21.0 Spring M = 72.4 (A.P.I.) - 1.0 C = 31.2 (A.P.I.) + 0.6 Summer M = 105.6(A.P.I.) - 31.1 C = 27.1 (A.P.I.) - 3.0

Before these equations can be used a daily A.P.I. value must be calculated for a recession factor of 0.87. This is a simple calculation which requires only a knowledge of the previous days A.P.I. value and the previous days rainfall.

The programme which was written to calculate the runoff is shown in appendix I, pp. 42-43. First the slopes of and the intercepts of the three equations, for each of the four seasons, were read into store. The computer then waited for information on K, A.P.I., precipitation and date from the previous water year. When this had been read in it again waited for the first years data on rainfall to be inserted. The rainfall data was presented in the form shown in appendix I, p.50 and was complete with date, daily rainfall values, monthly totals and triggers. The date was first checked to ensure that the rainfall data which had been inserted followed on in chronological order from the last date read. The first rainfall value was then read and a count of days and months started. A running total of rainfall was also kept so that a check could be made at the end of the water year, to see that rainfall values had not been misread. The A.P.I. was then calculated for this day and if the rainfall, P, was zero the appropriate equation for calculating runoff from this A.P.I. value was called out of store and used. If the rainfall value was greater than zero the two equations for calculating M and C for a particular A.P.I. value were first used and then this calculated value of M and C was used to determine the runoff using the days rainfall value. The calculated runoff value was then printed out in a similar format to that used for actual runoff values.

This procedure was followed until a whole years rainfall data had been processed. At the end of the year the following values were printed out after a run of forty blanks:-

Annual rainfall, annual runoff (calculated), recession factor, A.P.I. on September, 30th of current year, rainfall on September, 30th of current year and date of commencement of the water year which has just been processed. These last four values were required as data for the start of the next years calculations. They did not need to be read into the computer if the next years calculations was to follow on immediately, for the programme was written so that a WAIT instruction could be cleared immediately after inserting a new years rainfall data, without having to re-read all the preliminary data.

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The twelve years of rainfall record from 1st October, 1953 to 30th September, 1965 were processed as described. A sample of the output can be seen in appendix I, pp. 80-81. The calculated results gave negative daily flows on some days, as was to be expected from the formula used, but this did not cause undue concern because it was realised at the outset that the calculated daily value could differ considerably from the actual runoff value on any one day. If however, the calculated runoff is used to build up a diagram of accummulated deficiencies or excesses from a mean value, then the overestimates of one days calculated runoff may well be cancelled out by under-estimates on preceding or subsequent days.

6.7. Graph of accummulation of daily runoff deficiencies

The programme to plot the accummulation of daily runoff deficiencies or excesses from an eleven year mean runoff is shown in appendix I, p.44. It is similar in form to the first programme in this thesis but the re-arrangement of the form of presentation of the runoff data, and the inclusion of the trigger 1(at the end of each month has allowed a great simplification.

Before the calculations could be started it was necessary to sum up the total runoff over the eleven year period. This need had been anticipated and all that was required was to add up the eleven

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yearly totals, that had been printed up by the computer at the bottom of each page of calculated runoff. The first step was to read in this total runoff value which was immediately divided by 4018 days to give the average daily runoff. The computer then read in the plotting scales in the X and Y directions. At the WAIT instruction the calculated runoff data for the first water year was inserted and the computer then plotted the accummulated deficiency or excess from the mean, day by day as it was read in. At the end of each month, when the trigger 1(was read, the computer was instructed to draw a vertical line 1 cm. long and at the beginning of the calendar year, to draw a vertical line 2 cm. long. When a full years data had been read in the computer returned to the WAIT instructions and the next years data could then be inserted.

Calculated runoff values for twelve years were read in to be plotted. The twelfth years values were outside the period of record used for the correlation analysis.

Using the same programme, actual runoff values were plotted onto the same graph. The programme was then modified and daily rainfall values from the rain gauge which lay outside the catchment, but which had been used for the correlation of rainfall-runoff values, were plotted onto the same sheet of graph paper. These graphs are shown in appendix II, p.38.

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7. COMMENTS AND CONCLUSIONS

7.1 Cumulative deficiency diagrams

These can be quickly and accurately plotted by computer. Daily runoff records from a river gauge close to the dam site can be adjusted and used with variable draw off rates to produce a graph or a numerical print out of quantities to be stored in a reservoir at any given date. The graphical output, shows at a glance, the critical period, the maximum storage and the dates and magnitudes of overflows for a given initial reservoir condition. Variable evaporation rates can be included with the variable draw off and compensation water These were omitted from the Derwent data because evaporation data. rates are low in South Northumberland. A check was made using figures for potential transpiration and evaporation from free water surfaces which are given in a Ministry of Agriculture bulletin (Ref. 38). These results showed that the difference between the evaporation from a free water surface of 1000 acres and the potential evaporation from the original vegetation on this area only amounted to 1 m.g. per year.

An inspection of the Cumulative deficiency diagram for the Derwent, appendix II, p.l, shows a very high utilization of runoff. If it is assumed that the reservoir was just full on April, 4th, 1955 then the maximum draw down would occur on the 13th November, 1959 i.e. in approximately four and a half years. It would not fill up again until February 5th, 1961 i.e. taking almost six years to refill. It would then overflow up to February 15th; during this ten day period 483 m.g. would be lost. In April, 1962 it would overflow again, 173 m.g. being lost, and in April, 1964 the loss would be 225 m.g. It would not overflow again in the period under review which ends in September, 1965. Hence in the ten and a half years from April 1955 to September 1965 only

483 + 173 + 225 = 881 million gallons would be lost.

The total inflow over the ten and a half year period (i.e. 3833 days) would be

 $(3833 \times 26.5) + 881 = 101575 + 881 = 102456 m.g.$

The actual yield will be 101575/102456 x 100% of the runoff over this period, which is more than 99%.

Ruffle, in his I.W.E. paper (ref. 35) gives the effective available storage as 8,400 m.g. If it is assumed that the condition of the reservoir on October, 1st 1953 was such that it would lead to a draw down of 8,400 m.g. on November 13th, 1959 then 62 m.g. would overflow between 11th and 15th of February, 1962, a further 173 m.g. would overflow in 1962 and 225 m.g. would overflow in 1964. This makes the total overflow over the twelve year period of record from October 1st 1953 to September 30th 1965 equal to

 $62 + 173 + 225 = 460 \text{ m} \cdot \text{g} \cdot \text{g}$

The total inflow over this period (4383 days) would be

 $(4383 \times 26.5) + 460 = 116150 + 460 = 116610 \text{ m.g.}$

The actual yield would be

116150/116610 x 100%

of the runoff, which equals 99.6% of the runoff.

With regard to the critical period of reservoir depletion R.W.S. Thompson (Ref. 39) states that the length of the critical period will be greater where,

- a) the storage is large in relation to the mean annual runoff
- b) the variability of the runoff, both annually and seasonally is the greatest.

As was stated previously, the critical period for a reservoir of 7,979 m.g. capacity exceeds four and a half years. If the reservoir is assumed to be drawn down 8,400 m.g. on November 13th, 1959 then the reservoir would be expected to have a critical period exceeding this. An inspection of the cumulative deficiency curve, (Appendix II, p.1) shows that the reservoir would not fill up in the six years prior to November 1959 and even if it is assumed that the years preceding October, 1953 were dry and followed a pattern similar to the 1955-59 period, then the reservoir would not pass from full to a draw down of 8,400 m.g. in a period of ten years.

7.2. Histograms of Runoff

The graphs, appendix II, pp. 2-4, show that frequency distributions of daily runoff are positively skewed. The months of December and January are nearest to normal with few extremely high flows and few low flows. At the other extreme we have the example of August where there are the highest daily flows and a large number of days when the flow is in the minimum flow range.

Positive skewness is a characteristic of the distribution of daily rainfall values, for these are limited by zero values on the left, but are unlimited on the right. Since rainfall is the largest input to the runoff cycle it is to be expected that the distribution of the runoff will have similarities to that of rainfall distribution.

7.3. Moving average of a grouped period of runoff

The graphs of 15,30 and 91 day periods, (appendix II, p.5) show the attenuation of the peaks and troughs and also the smoothing effect, which occurs when daily values are averaged over a period. The graph of the 91 day values is easiest to follow.

There is normally one peak and one trough each year on this graph but the dates on which they occur vary from year to year. In 1954 there was a very small trough followed by a very large runoff in the latter part of the year. This was fortunate when one considers the low total runoff which occurred in 1955. Three and a half years later there is a further period of eighteen months with very low runoff and a low peak over the winter period. It is the proximity of these two low runoffs that results in the critical period of four and a half years for the reservoir of 7979 m.g. capacity. 1956 was an unusual year in that two peaks and one deep trough occurred in a ten months period.

The graphs show that there is not a close relationship between calendar date and runoff rate on the Derwent catchment. As a consequence, it was decided that it would be best to correlate a large number of events in four seasonal groups rather than a smaller number of events in monthly groups.

7.4. Graphs of regression equations

In the original analysis the ten straight lines, representing the

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regression equations for each of the ten A.P.I. groups, showed a definite pattern, (appendix II, pp. 14-17). The equations for the values in the lower A.P.I. groups had the smallest intercepts and slopes whilst in the higher A.P.I. groups the regression equations had higher intercept values and steeper slopes. This was promising. However, in the first analysis, the number of pairs of values, in each of the ten A.P.I. groups, was variable and in some instances the number of pairs was so small that the result was not valid. This difficulty, together with the problem of handling the thousands of values in and out of the computer led to the abandonment of this method.

The new method of analysis using sorted integer values allowed a more complete analysis to be made, with a predetermined number of values in each A.P.I. group and using various recession factor values, all in a fraction of the time of the original method. This however brought its own problems in that with the eleven K values used, there were now four hundred and forty equations to assess. Once again, use was made of the graph plotter. Previous work had shown that the slope of the regression equation could be expected to increase in the higher A.P.I. groups, as could the value of the intercept. It was hoped that the values of the slope and the intercept would increase in a smooth fashion. In order to view the effect of the recession factors on this rate of increase the new graphs were plotted in the form shown in appendix II, pp. 18-23.

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An inspection of these graphs reveals the following features:-

- a) The seasonal effects on correlation are apparent. In every case the curves for spring and summer are smoother than those for winter.
- b) The correlation between the intercept of the regression equation
 C, and the A.P.I. grouping, is sufficiently good to give a reasonably smooth curve for all four seasons.
- c) The recession factor K, appears to have little effect on the correlation.

7.5. Choice of recession factor

The marginal effect of the recession factor on the correlation was confirmed by the graphs shown in appendix II, p. 24. These were plotted on a false origin so that the scale could be made large enough to show the slight differences which did occur. After studying these graphs it was decided that a recession factor of 0.87 would be used for the remainder of this work.

7.6. Runoff on Dry Days

The intercept of the regression equation is the runoff value obtained from the equation when the rainfall is zero. The correlation between the intercept of the regression equation and the A.P.I. group value was good. This suggested that it would be worthwhile to separate the data referring to runoff and A.P.I. on dry days from the total data. The runoff on dry days could then be correlated with the A.P.I. and the runoff on wet days correlated with rainfall according to the A.P.I. group. The scatter diagrams of the A.P.I. and runoff on dry days are shown in appendix II, pp. 26-27. These show that the distribution of both values is positively skewed and that once again, the correlation for the winter quarter is worst. On each of the four seasonal graphs there is a lower line of dots which is almost horizontal. On the autumn and spring graphs there are further distinct lines of dots where the A.P.I. is less than 0.1 inches. These are possibly due to rain falling on dry ground, where it would have little effect on the runoff on subsequent dry days. The A.P.I. value would rise at 9.0 a.m. on the day following the rainfall. A better correlation may be obtained if anticedent conditions index values were plotted against runoff on dry days.

A statistical analysis was carried out on these values, assuming a normal distribution, and the regression equations were drawn on each of the graphs. These equations give runoff values which do not accord with those actually recorded at the lower end of the A.P.I. range. This may be due to the skewed distribution of both the A.P.I. and runoff values.

7.7. Runoff on Wet Days

The scatter diagrams of A.P.I. and runoff on wet days are shown in appendix II, pp. 28-31. The scales differ from those used to plot the runoff on dry days because the range of both A.P.I. and runoff values was much greater on wet days.

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The correlation coefficients for each of the four seasons for both the dry day and wet day analyses are set out below.

	Autumn	Winter	Spring	Summer
A.P.I. and dry days	.76	•43	.68	•74
A.P.I. and wet days	.68	•32	•63	. 68

As can be seen, there is a higher correlation between A.P.I. and runoff on dry days than there is between A.P.I. and runoff on wet days, when rainfall is obviously a causative parameter. Since the A.P.I. is in part derived from rainfall figures for the previous day and since it is known that the probability that rain will fall on a given day is much greater if it fell the preceding day, (ref. 37) it is difficult to know what value to place on the coefficient obtained by correlating runoff on wet days with A.P.I.

The correlation of runoff on wet days with rainfall on those days was carried out on groups of fifty pairs of values, the values being grouped according to A.P.I. range. The regression equations obtained were therefore associated with a certain mean A.P.I. value for the group. The regression coefficients and the intercepts of the regression equation were plotted against mean A.P.I. for each of the four seasons and these are shown in appendix II, pp. 32-37. The slopes and intercepts of the regression equations did not increase smoothly with increasing A.P.I. and so the equation for the best fit straight line was calculated for each of the eight graphs. These lines gave the relationship between slope and A.P.I., and between intercept and A.P.I. for the regression equations relating runoff on wet days to rainfall on that day.

The equations for the summer season could give a negative value for the runoff when the A.P.I. falls below 0.3 inches, but there are only a few days when low A.P.I. is associated with heavy rainfall and so these negative quantities would not be expected to have a great influence on the final graph.

7.8. Calculation of Runoff

The daily runoff values were calculated from the equations relating runoff to A.P.I. on dry days, when there was no rainfall. On days when rain fell, the eight equations relating A.P.I. to the slope and intercept of the regression equations for rainfall and runoff were used. As expected, a few of the values were negative, but these were used with all the others to plot the curve showing the accumulation of daily runoff, (deficiencies or excesses), from the daily mean runoff, which was calculated from the first eleven years of derived run-

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off. Similar curves were plotted for actual runoff and for rainfall at the Tunstall gauge.

The actual runoff over the eleven year period amounted to 150622 m.g. whilst the calculated runoff over this eleven year period amounted to 155226 m.g. The percentage error equals

$$\frac{155226 - 150622}{150622} \times 100 = + 3\%$$

Considering the water year 1964-65, which was outside the period of analysis, the actual runoff for the year amounted to 14275 m.g. and the calculated total amounted to 14094 m.g. The percentage error equals

$$\frac{14275 - 14094}{14275} \times 100 = -1.3\%$$

7.9. Cumulative Curves

The following points were borne in mind when studying the curves (see appendix II, p.38).

a) Since for each of the three curves the excess or deficiency has been calculated from their individual eleven year means, then all three curves should coincide initially and the end of the eleven year period.

- b) At the date when a curve crosses the horizontal axis representing the mean, the total rainfall or runoff between October 1st, 1953 and that date is equal to the daily average multiplied by the number of days from October 1st, 1953.
- c) If a part of a curve is parallel to the horizontal axis representing the mean, for any period of time, then during that period the rainfall or runoff is at the average rate.
- d) On days when there is no rainfall, the rainfall curve will be a straight line of negative slope, which is equal in magnitude to minus the average daily rainfall.

The curves derived from the calculated and the actual runoff show that all the major peaks and troughs occur in the correct sequence and that many of the smaller irregularities on the actual curve are reproduced on the calculated one.

Unfortunately, the magnitude of the deficiencies and excesses do not agree.

A detailed inspection of the three curves, season by season, reveals the following.

Winter.

For eight of the twelve winters, the calculated runoffs are less

than the actual, and two are in close agreement. The winters of 1962 and 1963 show different patterns for the actual and calculated values. It is believed that this is due to precipitation being retained on the catchment in the form of snow and ice.

Spring.

There is fair agreement between actual and calculated values for this quarter but it is noticed that where the first month of the season was dry the calculated response to rainfall is too high.

Summer.

Five seasons gave good agreement, for the others it was noticed that if rainfall was preceded by drought the calculated values responded immediately to the rainfall, whilst the actual values did not. The opposite effect occurred if heavy rainfalls followed a period of average rainfall, the actual runoff increased much more than the calculated.

Autumn.

This seasons calculated values responded to rainfall in the same manner as the summers. If the ground is dry due to a low rainfall then the calculated values respond too quickly to rainfall. If the rainfall in the preceding months has been sufficient to saturate the ground then the actual response to rainfall is greater than the calculated.

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During the winter, the correlation between daily rainfall and runoff values is not particularly good, but an inspection of the cumulative runoff values at the beginning and end of each winter season shows that the daily errors have not accumulated. It may be that frost and snow have held up the water on the catchment for a limited period, so that the runoff on any one day cannot be directly related to the rainfall on that day. Over the whole of the winter period however, the daily errors are averaged out and the majority of rainfall falling in the winter season ultimately appears as runoff.

The major errors in spring, summer and the early part of autumn are possibly due to rainfall replenishing a groundwater deficiency.

Between 80 - 85% of transpiration losses occur in the spring and summer season, the losses possibly exceeding the rainfall during this period. When this occurs, the deficit has got to be replaced by rainfall before there is any appreciable increase in runoff rate.

The antecedent precipitation index is an index of ground moisture conditions preceding the onset of rainfall. It is most commonly used with a coaxial correlation technique for forecasting runoff from storm rainfall. Since the A.P.I. rises on the day following a rainfall day by an amount equal to that day's rainfall and decreases daily according to a logarithmic recession, it is a good index of soil moisture conditions at the surface, but it does not allow sufficiently for moisture content below the surface. Runoff in spring and summer is possibly more closely related to groundwater deficiency, than it is to surface moisture conditions and the A.P.I. may not be the most suitable index to use for correlation during these months.

The Ministry of Agriculture, Fisheries and Food, Technical bulletin Number 4 (Ref. 38) describes a method for calculating the groundwater deficit, using monthly averages of potential transpiration, sunshine hours and weighting factors. Tables are supplied giving values for England and Wales, Scotland and Northern Ireland based on the years 1930 to 1949.

A refinement is described whereby the average monthly potential transpiration figures can be adjusted to allow for actual sunshine hours, if these are available. Since the actual sunshine hours do not vary greatly from place to place the same hourly sunshine figures can be used over quite large areas, possibly a county or half a county.

7.10. An alternative approach

Bearing in mind the foregoing observations, the following alternative method is suggested as a means of synthesising daily

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runoff from daily rainfall, using a correlation technique.

First group the data into four quarters, Spring, Summer, Autumn and Winter and then determine the regression equations for each quarter following the procedure described below.

Spring and Summer.

- a) Calculate the daily potential transpiration, making adjustments for sunshine hours.
- b) Subtract the daily potential transpiration value from the daily rainfall value.
- c) Correlate the modified daily rainfall value with the actual runoff value. (Some of the modified daily rainfall values will be negative, in which case they represent groundwater deficit.)
- d) If the modified daily rainfall value is negative then add an equal positive value to the next days potential transpiration when it has been calculated

Repeat the above procedure each day for all spring and summer seasons. A negative, modified daily rainfall value on September 30th should be carried forward into the Autumn season where it will be steadily reduced, until there is no groundwater deficit. Plot a scatter diagram of all rainfall and runoff values. If it appears necessary, modify rainfall or runoff values, or both, to allow for a skew distribution, then determine the regression equation.

Winter, dry days

If a dry day is preceded by a wet day, correlate the runoff on the dry day with the previous days rainfall, in a group of pairs of values with a reference number 0. If a dry day is preceded by one dry day, correlate the runoff on the last dry day with the rainfall two days previously, in a group of pairs of values with a reference number 1.

et seq.

Autumn

If the modified rainfall figure for September, 30th is negative then modify the daily rainfall on October, 1st by this amount and correlate the modified rainfall value on October, 1st with the runoff on that day. If the new modified rainfall is still negative repeat the above procedure for each subsequent day until the modified rainfall is positive. When this occurs, carry out the procedure as described for the winter quarter. The calculation of daily runoff from the regression equations, the daily rainfall data and daily sun hour data, in spring and summer would then be performed as follows

Spring and Summer

Calculate the modified daily rainfall day by day and then use this modified value and the appropriate regression equation to calculate the daily runoff.

Winter, rainfall days

Use rainfall value and appropriate regression equation to calculate runoff.

Winter, dry days

Count the number of preceding dry days and then use the value of rainfall on the last wet day with the regression equation, derived from the data in the group with the number equal to the count, to calculate runoff.

Autumn

If the modified rainfall figure for September 30th is negative use the procedure for spring and summer. When the modified rainfall becomes positive use the procedure for winter.

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The calculation of monthly potential transpiration is fully described in Ref. 3⁸ (Page 16 table 10). A simple modification to the equation given in this publication allows the calculation of an approximate daily potential transpiration value.

A modified equation is given below.

P.d. = 1/N (P.m. + W(H.d. - H.m.))

where P.d. is the approximate daily potential transpiration.

P.m. is the average monthly potential transpiration (Table 10).W. is the weighting factor (Table 10).H.m. is the average sun hours per day (Table 10).H.d. is the actual sun hours per day.N. is the number of days per month.

7.11. Summary of Conclusions

The 53 graphs which have been produced for this thesis demonstrate that a graph plotter, coupled to a digital computer, is a most useful tool for the analysis of hydrological data.

The distribution of daily runoff values is positively skewed. The magnitude of the skew is dependent on the season, the distribution in winter months being nearer to normal. The correlation between A.P.I. and runoff is better in spring and summer than it is in winter.

The choice of recession factor, within the range 0.85 to 0.95, does not materially effect the correlation with runoff values.

The curves of cumulative excess, produced from the calculated runoff values, show that a much closer relationship exists between these values and actual runoff than with the original rainfall and actual runoff.

A close inspection of the curves of cumulative excess has led the author to suggest that calculation based on potential transpiration values would lead to a more accurate curve than that which was obtained from the analysis using daily A.P.I. values. Minor modifications could be made to the programmes given in appendix I which would allow them to be used for an analysis based on potential transpiration.

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APPENDIX I

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PROGRAMM	IES .	•				Pages	l	to	44.
SAMPLES	OF	TNPIIT	ΔND	ດເຫຼົອນກ	ከልሞል .	Pages	45	to	81.

::GRAPH PLOTTER AUTOCODE ::CUMULATIVE DEFICIENCY DIAGRAM ::DERWENT RESERVOIR SETV-T(3)X(3)Y(2)LD(366)B(366)R(2)C(366)EFMNW(3)S(2)HZA SETS QUIK SETF FRAC INT MOD GRAPH PLOT SETR 17 1)READ H:: H=-1PRINT H=+1PRINT AND GRAPH READ X:: MILL GALS PER .1MM STEP READ X3:: NO OF 0.1MM STEPS PER DAY READ S1:: RESERVOIR YIELD OCT - MARCH READ S2:: RESERVOIR YIELD APR - SEPT READ CO:: CAPACITY OF RESERVOIR ON SEPTEMBER 30TH. X1=0 X2=0 INSERT ANNUAL DATA 2)WAIT:: CYCLE Q=1:1:100 OUTPUT O:: 100 BLANKS REPEAT O JUMP IF H\$0@17:: GRAPH NOT REQUIRED ₩1=1 · W2 = 0E=PLOT W1, W2:: MOVE ON ONE DAY. TO 1 ST. OF CCT. E=0 E=CRAPH O.E:: PEN DOWN W1=0 W2=50 F=GRAPH W1.W2:: DRAW VERTICAL LINE AT START OF MONTH W2 = -100F=GRAPH V1.W2 W2=50 F=GRAPH W1.W2 E=0 E=PLOT 0, E:: PEN UP W1=-1:: MOVE BACK ONE DAY W2=0 E=PLOT W1,W2 17) READ Y1:: YEAR 1 YEAR 2 READ Y2:: READ N Y1=Y1-1900 Y1=Y1+10000 Y2=Y2-1900 Y2=Y2•10000 T2=0 J=0

3)J=J+1 TO = T2MONTH DAY READ DJ:: JUMP IF DJ\$1000@4 BJ=Y1+DJ:: YEAR MONTH DAY JUMP @5 4)BJ=Y2+DJ:: YEAR MONTH DAY 5)T3=DJ/100 T2=FRAC T3 T2=100•T2:: · DAY T1=INT T3:: MONTH READ R:: RAINFALL READ R1:: RUNOFF R1=R1+21550 R1=R1/29150 JUMP IF T1\$10@6:: BEFORE SEPTEMBER 30 TH. JUMP @8 6)JUMP IF T1%3@7::AFTER MARCH 31 ST. 82R1=R1-S1:: WINTER JUMP @9 - 7)R1=R1-S2:: SUMMER 9)CJ=C(J-1)+R1:: CUMULATIVE EXCESS JUMP IF H\$0@12:: GRAPH NOT REQUIRED R2=R1/XX2=X2+R2:: PREVIOUS RESIDUAL +SCALED EXCESS X 1 = X 1 + X 3E =0 E=CRAPH 0,E:: PEN DOWN E=GRAPH X1.X2M=DJ-101 M=MOD M JUMP IF M\$0.1@10::JUMP AT JANUARY 1 ST. OM=DJ-930 M=MOD M JUMP IF M\$0.1@11::JUMP AT END OF SEPT/TAPE JUMP IF T2%T003:: JUMP AT START OF MONTH W1=0 ··· W2=50 F=GRAPH W1.W2:: DRAW VERTICAL LINE AT START OF MONTH W2 = -100F=GRAPH W1.W2 W2=50 F=GRAPH W1.W2 JUMP @3 $100 \times 1 = 0$ W2=100 F=CRAPH W1.W2:: DRAW VERTICAL LINE AT START OF YEAR ∴w2=-200

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F=GRAPH W1.W2 W2 = 100F=GRAPH W1,W2 JUMP @3 11)CYCLE 0=1:1:100 OUTPUT O:: 100 BLANKS REPEAT Q WAIT:: JUMP @2 FOR GRAPH PLOT ONLY JUMP @13 12)M=DJ-930 M=MOD M JUMP IF M\$0.1@13::JUMP AT END OF SEPTEMBER TAPE JUMP @3 CAPACITY OF RESERVOIR ON SEPT. 30 TH. 13)C0=CJ:: WAIT:: JUMP AT 2 IF NO PRINT UP REQUIRED TITLE DATE EXCESS DATE EXCESS DATE EXCESS T2=0 L=0 1=0 1501 = 1 + 1T0=T2 $T_{3}=D_{1}/100$ T2 = FRAC T3T2=100+T2:: DAY JUMP IF T2%TO@14::JUMP UNLESS END OF MONTH LINES · 2 L =0 14) PRINT BI, 6:0:: YEAR MONTH DAY PRINT CI, 4:1:: CUMULATIVE EXCESS K=J-1 K = MOD KZ=STAND K JUMP IF Z\$0.1016::END OF WATER YEAR SPACES 2 L=L+1 N=L/3N=FRAC N JUMP IF N%0.1@15::3 DAYS DATA PER LINE OF PRINT UP LINE 🗄 JUMP @15 163LINES 10 CYCLE **Q=1:1:100** OUTPUT 0 REPEAT Q JUMP @2 START 1

:: RIVERFLOW INTO DERWENT RESERVOIR ::HISTOGRAM SHOWING NUMBER OF DAYS OF FLOW IN GIVEN RANGE SETV ABC(2)DEF(4200)G(2)HIJK(2)LMN SETS ZYXWVUTS SETF INT FRAC GRAPH PLOT SETR 6 1)READ E:: INCREMENT OF FLOW M.G.D. E.G.2 NUMBER OF 0.1MM STEPS/UNIT VERTICALLY READ L:: NUMBER OF Q.1MM STEPS/UNIT HORIZONTALLY READ M:: N=700/E:: 700 M.G.D. EXCEEDS MAX.FLOW EVER RECORDED N=N+.01:: ADD SMALL NUMBER TO ENSURE CORRECT INTEGER T=INT N:: NUMBER OF STEPS ALONG HORIZONTAL AXIS EACH MONTH WILL REQUIRE T STORE LOCATIONS S=12•T:: CYCLE Z=0:1:S:: RESERVE S STORE LOCATIONS FOR COUNTING FZ=0:: SET INITIAL VALUES IN STORE LOCATIONS TO O REPEAT Z INSERT NEW DATA TAPE OR 2) WAIT:: X=0:: PRESS READ BUTTON THEN KEY IN 6 C2=0 READ A:: YEAR DATE. NOT REQUIRED READ A:: YEAR DATE. NOT REQUIRED YEAR DATE. NOT REQUIRED READ A:: Y = 03)READ B:: MONTH-DAY E.G. 1001 C=C2C1 = B / 100C2=FRAC C1:: DAY E.G. .01 DAY E.G. 1 C2=100+C2:: JUMP IF C2%C@4:: (IF TODAYS DATE IS GREATER THAN YESTERDAYS (IF NOT THEN FIRST DAY OF NEW MONTH Y = Y + 1JUMP IF Y=12@2:: END OF YEAR NEXT MONTHS STORE LOCATIONS START AT FX X = X + T : :

RAINFALL. NOT REQUIRED 4) READ A:: RUNOFF I.E.RATE OF FLOW M.G.D. READ D:: D=D/E:: (NUMBER OF STEPS ALONG BASE (I.E.FLOW RANGE Z=INT D:: STORE LOCATION FOR THIS RANGE AND MONTH Z=Z+X:: FZ=FZ+1:: SUM NUMBER OF DAYS IN EACH FLOW RANGE JUMP @3 6) CYCLE U=1:1:100:: (OUTPUT 100 BLANKS OUTPUT O:: -REPEAT U:: · .. GRAPH DAYS IN EACH RANGE 1 MONTH AT A TIME VARY W=0:T:12:: G2=0 H=0 1=0 I=GRAPH 0.1:: LOWER PEN VARY V=0:1:T:: START OF EACH MONTHS GRAPH PLOT Z = W + VG1=0 RESIDUAL AND NUMBER OF DAYS IN RANGE Z G2=G2+FZ:: H=NUMBER OF DAYS IN PREVIOUS FLOW RANGE G2=G2-H::-NUMBER OF DAYS .SCALE G2=G2•L:: J=GRAPH G1,G2:: DRAW VERTICAL LINE NUMBER OF .1 MM. STEPS HORIZONTALLY K1=M:: $K_{2}=0$ J=GRAPH K1, K2:: DRAW HORIZONTAL LINE H=FZ:: NUMBER OF DAYS IN PREVIOUS FLOW RANGE REPEAT V I = 0RAISE PEN I=PLOT 0, I:: CYCLE U=1:1:400:: OUTPUT O:: -OUTPUT 40 BLANKS REPEAT U:: WAIT REPEAT W STOP START 1

::CHECKING OF R	RAINFALL DATA FORMAT AINFALL DATA MONTHLY TOTALS
SETS ZDN SETV RC1) SETR 8	
8)SUBR 6:: OUTPUT 27:: LINE	OUTPUT 30 BLA NKS AT START OF WATER YEAR OUTPUT FIGURE SHIFT
OUTPUT 19:: OUTPUT 17:: LINE	OUTPUT 3 OUTPUT C
READ D::	DATE PRINT YEAR MONTH DAY. E.C.551001
OUTPUT 2:: OUTPUT 17:: LINE	OUTPUT 2 OUTPUT C
	OUTPUT 30 BLANKS AFTER YEAR MONTH DAY
5)\$UBR 6:: Z=0 R=0 N=0	OUTPUT 30 BLANKS AT START OF NEW MONTH
4)READ D::	DATE
7)READ R1:: R=R+R1:: PRINT R1,1:2:: N=N+1 Z=Z+1	ACCUMULATE RAINFALL
JUMP IF N=28@2:	WHEN 28 VALUES OF RAINFALL HAVE BEEN READ THERE ARE 10 VALUES PRINTED ON EACH LINE
	WHEN 30 VALUES HAVE BEEN READ IN

2)WAIT:: JUMP @4	IF 28 DAY MONTH AT FIRST WAIT READ 9 IF 29 DAY MONTH AT FIRST WAIT READ 1 IF 30 DAY MONTH CLEAR. © SECOND WAIT READ 9 IF 31 DAY MONTH CLEAR. © SECOND WAIT READ 1
1)READ D:: READ R1:: R=R+R1:: LINE PRINT R1,1:2::	DATE RAINFALL ACCUMULATE RAINFALL RAINFALL
3)LINE OUTPUT 1:: OUTPUT 17:: LINE PRINT R,2:2:: LINE OUTPUT 2:: OUTPUT 17:: LINE JUMP @5	OUTPUT 1 OUTPUT C ACCUMULATED MONTHLY TOTAL OUTPUT 2 OUTPUT C
6)CYCLE Z=1:1:30 OUTPUT 0:: REPEAT Z EXIT) OUTPUT 30 BLANKS

START 8

:: PRODUCE DATA TAPE OF DAILY RAINFALL - RUNOFF WITH TRÍGGERS • . • SETS D(1)ZYXWVUSPC(13) SETV AR(601)Q(601) SETR 19 17) READ D:: READ IN DATE OF PREVIOUS YEAR E.C. 541001 WAIT:: INSERT RAINFALL TAPE IN READER 15) READ A:: TRIGGER TO REFERENCE 3) 3)SUBR 6:: OUTPUT 30 BLANKS OUTPUT FIGURE SHIFT OUTPUT 27:: LINE OUTPUT 19:: OUTPUT 3 OUTPUT (OUTPUT 17:: LINE READ D1:: DATE PRINT D1; 7:: PRINT DATE D=D+10000:: CHANGE DATE TO THIS YEAR JUMP UNLESS D1=D@18::DATES MUST CHECK IF CORRECT TAPE INSERTED. IF D1 DOES NOT EQUAL 551001 JUMP TO 18 LINE:: OUTPUT 2:: OUTPUT 2 OUTPUT 17:: OUTPUT (LINE SUBR 6:: OUTPUT 30 BLANKS Y=-50 X =0 P=0R=0 READ A:: TRIGGER TO REFERENCE 2) 8)Z=Z+Y 16)READ RZ:: READ RAINFALL AND STORE IN LOCATION RZ ACCUMULATE RAINFALL R=R+RZ:: Z=Z+1 JUMP @16:: UNTIL TRIGGER IS READ

IF X=1 RUNOFF DATA IS BEING READ IN, SEE: 7) 1)JUMP IF X=1@4:: MONTHLY TOTAL NOT REQUIRED READ A:: P=P+1:: NUMBER OF MONTH E.G.FEBRUARY =5 DAYS IN MONTH E.G. C5=29 CP=Z-1:: TRIGGER TO REFERENCE 2 READ A:: 2)Y=Y+50:: 50 STORE LOCATIONS ALLOCATED EACH MONTH Z=1 JUMP IF P=1207:: WHEN 12 MONTHS DATA HAS BEEN READ IN JUMP @8 • 7)X = 1 Y =0 Q=0 P=0 · INSERT RUNOFF TAPE AT APPROPRIATE POINT WAIT:: 11)Z=Z+Y READ RUNOFF AND STORE IN LOCATION QZ 5)READ OZ:: ACCUMULATE RUNOFF Q=Q+QZ:: Z = Z + 1JUMP @5:: UNTIL TRIGGER IS READ . . 4)P=P+1Z=Z-1 JUMP UNLESS CP=Z@18::CHECK WITH NUMBER OF RAINFALLS DAYS READ Y=Y+50 Z=1 JUMP IF P=12010:: END OF YEARS DATA JUMP @11 10)Z=1:: SET TO 1 FOR START OF PRINT INSTRUCTIONS Y = 0 P=0

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12)P=P+1V=0 Z=Z+Y CYCLE S=Z:1:CP:: E.C. FOR MONTH 1. S VARIES FROM 1 TO 31 V = V + 1PRINT RS.2:2:: PRINT RAINFALL PRINT RUNOFF PRINT QS, 5:: JUMP IF V\$4@13:: THERE ARE 4 PAIRS OF DATA PRINTED/LINE LINE · V=0 13)REPEAT S JUMP IF CP=28@19:: 28 IS DIVISIBLE BY 4. LINE NOT REQUIRED LINE 19)OUTPUT 1:: OUTPUT 1 OUTPUT 17:: OUTPUT (LINES 2 Y = Y + 50Z=1 X=1 SUBR 6:: 30 BLANKS JUMP IF F=12@14:: END OF YEARS PRINTING JUMP @12 14)TITLE TOTAL R TOTAL O LINE PRINT R, 3:2:: ACCUBULATED ANNUAL RAINFALL SPACES 5 PRINT Q, 6:1:: ACCUMULATED ANNUAL RUNOFF LINES 5 SUBR 6:: 30 BLANKS WAIT:: ENTER NEW RAINFALL TAPE AND CLEAR WAIT JUMP @15 6)CYCLE U=1:1:30 OUTPUT 0:: OUTPUT 30 BLANKS REPEAT U EXIT 18)CYCLE U=1:1:300 OUTPUT 0:: OUTPUT 300 BLANKS REPEAT U WAIT:: ERRORS. RESET AND ENTER NEW LEADER TAPE START 17

10.

SETS ZYXWYUTSRP SETY AB(4400)CD(3)E(2)FHI(2)JK(4)LMN SETF STAND MOD GRAPH PLOT SETR 6 5)B=0 Z=1 4) WAIT:: CENTER TRIGGERED RUNOFF VALUES AND CLEAR Y=0:: -(WHEN LAST TAPE ENTERED READ 6 ON KEYBOARD 3)READ BZ:: DAILY RUNOFF Z=Z+1:: -COUNT OF DAYS JUMP @3 COUNT OF MONTHS **1**)Y=Y+1:: WHEN ONE YEARS DATA HAS BEEN STORED JUMP-IF-Y=12@4:: JUMP @3 · ۰. 6)READ W:: NUMBER OF EVENTS TO BE ADDED E.G.30 NUMBER OF .1M.M. STEPI/DAY IN X DIRECTION READ V:: · READ J:: NUMBER OF .1M.M. STEPS/M.G.D. IN Y DIRECTION CYCLE S=1:1:100:: C COUTPUT 100 BLANKS OUTPUT 0:: REPEAT S:: C E=0 · E≈GRAPH 0,E:: PEN DOWN D=0 D1=0 Z=Z-1 F=STAND W:: F=FLOATING POINT FORM OF W EVENTS TO BE ADDED/SCALE F=F/J:: H=0 L=FLOATING POINT FORM OF V L=STAND V:: 12 = 0۰... K1=0 11 = 0k2 ≈0 K3=0 R=W-1:: 1 LESS THAN NUMBER OF EVENTS TO BE ADDED

:: GRAPHS OF GROUPED DAILY FLOWS

CYCLE T=1:1:R:: D=SUM OF FIRST W VALUES OF RUNOFF D=D+BT:: REPEAT T:: Z=TOTAL NUMBER OF VALUES READ IN CYCLE T=W: 1:Z:: P=T-WSUM OF T VALUES OF RUNOFF D=D+BT::SUM OF (T-W)VALUES OF RUNOFF H=H+BP:: MOVING SUM OF W VALUES OF RUNOFF K=D-H:: W DAY AVERAGE TO SCALE K=K/F:: K4=K-K1:: .1 M.M. STEPS FROM LAST PEN POSITION NUMBER OF .1 M.M.STEPS/DAY IN X DIRECTION 11=L+11:: 12=K4+12:: NUMBER OF .1 M.M.STEPS IN Y DIRECTION N=GRAPH 11,12 POSITION RELATIVE TO ORIGIN IN Y DIRECTION K3=K3+K4:: K1 = KREPEAT T:: UNTIL ALL Z VALUES HAVE BEEN PROCESSED E=0RAISE PEN E=PLOT 0,E:: $K_{3} = -K_{3}$ I=PLOT K2,K3:: MOVE WITH PEN UP, TO X AXIS CYCLE S=1:1:500:: OUTPUT O:: OUTPUT 500 BLANKS REPEAT S:: M=STAND Z:: M=FLOATING POINT FORM OF Z D3=D/M:: AVERAGE DAILY RUNOFF FROM Z VALUES D2=D3+J:: AVERAGE DAILY RUNOFF TO SCALE I=PLOT D1, D2:: MOVE WITH PEN UP TO AVERAGE VALUE E=0 E=GRAPH O,E:: LOWER PEN E1=100 E2=0 E=GRAPH E1,E2:: DRAW HORIZONTAL LINE 1 CM.LONG CYCLE S=1:1:100:: OUTPUT 100 BLANKS OUTPUT O:: REPEAT S:: LINE PRINT Z,5:: NUMBER OF VALUES USED PRINT BZ,5:: LAST VALUE READ IN PRINT D,7:: TOTAL RUNOFF (DAYS +M.G.D.) PRINT D3,5:: AVERAGE DAILY RUNOFF FROM Z VALUES WAIT START 5

:: PROGRAMME TO DETERMINE INITIAL A.P.I. FROM TRIGGERED ::RAINFALL DATA · · · · SETS Z SETV KI(1)P(1)D SETR 5 5)READ K:: RECESSION FACTOR ASSUMED VALUE OF A.P.I. ON SEPT.30TH, 1952 READ I:: RAINFALL ON SEPTEMBER 30TH, 1952 READ P:: Z=0 WAIT:: INSERT RAINFALL TAPE FOR 1952-53 READ D:: TRIGGER TO REFERENCE 3) DATE NOT REQUIRED 3)READ D:: TRIGGER TO REFERENCE 2) READ D:: . - -RAINFALL 2)READ P1:: **11=1**●K · . . 11=11+P:: A.P.I. ON MORNING OF OCTOBER 1ST, 1952 I= 11 - ---P=P1JUMP @2:: UNTIL TRIGGER 1(IS READ - --1)READ D:: MONTHLY TOTAL OF RAINFALL. NOT REQD. Z=Z+1 JUMP IF Z=1204:: END OF YEARS DATA - TRIGGER TO REFERENCE 2) READ D:: · . - --4) V ARY Z=0:1:20 OUTPUT 20 BLANKS CUTPUT O:: -REPEAT Z WTFUT 27:: **CUT PUT** FIGURE SHIFT LINE PR INT K, 1:2:: RECESSION FACTOR PRINT 1,1:2:: A.P.I. ON SEPTEMBER 30TH, 1953 PR INT P, 1:2:: RAINFALL ON SEPTEMBER 30TH, 1953 LINE . . WAIT JUMP @5 START 5

:: TO CALCULATE A.P.I.VALUES AND PRINT OUT GROUPS OF RAINFALL ::RUNOFF ACCORDING TO SEASON AND A.P.I.RANGE. SETS SBLM(40)D(2)NETFG(40)HJ(1)ZU(40)YV(2) SETV K(1)I(1)P(2)Q(2)X(2)A(6000)C(1)R(2) SETF FRAC INT SETR 22 19)READ K:: ASSUMED RECESSION FACTOR READ IØ:: A.P.I. ON SEPTEMBER 30TH READ PO:: PRECIPITATION ON SEPTEMBER 30TH WAIT:: ENTER RAINFALL RUNOFF TAPE WITH TRIGGERS 9)TITLE SORTING OF API VALUES FROM READ X1:: TRIGGER 30 3)READ V1:: DATE PRINT V1,7:: PRINT DATE READ V2:: TRIGGER 2(2)LINE CYCLE L=0:1:4000:: ((SET STORE LOCATIONS A@ TO A4000=0 AL=0:: REPEAT L:: (TITLE VALUE OF K = :: RECESSION FACTOR PRINT K,1:2 SUBR 13:: OUTPUT 20 BLANKS S=0 CYCLE L=0:1:40:: ((SET STORE LOCATIONS M0 TO M40=-1 ML=-1:: REPEAT L:: V = -18)J=0 J1 = 0D=0 D1 = 002=0 P2=0 Z=0 LINE

11)Z=Z+1READ P1:: PRECIPITATION READ Q1:: / RUNOFF ACCUMULATE PRECIPITATION P2=P2+P1:: 02=02+01:: ACCUMULATE RUNOFF 11=10*K I1=I1+P0:: A.P.I. ASSOCIATED WITH P1 AND Q1 PRINT 11,1:2 $I \emptyset = I I$ P0=P1JUMP 1F 11&2.205:: IF A.P.1. IS GREATER THAN 2.2 N=9 N=9 JUMP @7 5) JUMP IF I1&1.7@6:: IF A.0.1. IS GREATER THAN 1.7 N=8 N= 8 JUMP 07 6) JUMP IF I1&1.4@4:: IF A.P.1. IS GREATER THAN 1.4 N=7 N=7 JUMP @7 4) JUMP IF I1%0.25@22::1F A.P.1. IS GREATER THAN 0.25 N=0 N = 0JUMP @7 22)C1=I1*5:: E.G. IF A.P.1.=1.21 THEN C1=6.05 N=INT C1:: E.G. IF C1=6.05 N=6 FLOATING PUINT FORM FOR N 7)R=STAND N:: AUTUMN J=0 0&N&10. WINTER J=10 10&N&20 N=N+J:: R=R*1.5:: (START OF CALCULATION TO ALLOCATE STORE (LOCATIONS TO EACH GROUP OF RAINFALL R2=STAND J1:: (RUNOFF VALUES. 150 LOCATIONS ARE R=R+R2:: (AVAILABLE FOR EACH GROUP DESIGNATED R1=R*100:: (BY AN N VALUE. 'I.E. 75 FOR RAINFALL R1=R1+•01:: E=INT R1:: (AND 75 FUR RUNOFF. COUNT NUMBER OF DAYS IN EACH GROUP MN=MN+1:: STORE LUCATION REFERENCE NUMBER E = E + MN:: AE=P1:: PUT P1 INTO APPROPRIATE STORE LOCATION PUT Q1 INTO APPROPRIATE STORE LOCATION A(E+75)=Q1:: JUMP UNLESS Z=10@11:: (START NEW LINE WHEN 10 VALUES HAVE LINE: (BEEN PRINTED UUT Z=0 15) JUMP @11 1)D=D+1:: COUNT NUMBER OF MONTHS FROM OCTOBER JUMP 15 2=1010 LINE

10)OUTPUT 1:: OUTPUT 1 OUTPUT 17:: OUTPUT (LINE Z=Ø JUMP IF D&3011:: AT START OF NEXT SEASON J = J + 10J1 = J1 + 15D=0 D1=D1+1:: COUNT NUMBER OF SEASONS FROM AUTUMN JUMP IF D1&4@11:: IF D&4 REPEAT CALCULATIONS T=0 $\mathbf{F} = \mathbf{O}$ H=0 LINES 8 SUBR 14:: OUTPUT 40 BLANKS PRINT K,1:2 SUBR 13:: OUTPUT 20 BLANKS Z=0 12)VARY Y=0:1:40 Z = Z + 1UY=MY+1:: NUMBER OF DAYS IN EACH GROUP PRINT UY,4 JUMP UNLESS Z=10018::(START NEW LINE WHEN 10 VALUES HAVE LINE:: (BEEN PRINTED OUT Z=0 18)REPEAT Y LINES 8 TITLE VALUES ON SEPTEMBER 30 TH. TOTAL P TOTAL Q K DØ PØ PRINT P2,2:2:: TOTAL PRECIPITATION IN WATER YEAR SPACES 2 PRINT 02,5:1:: TOTAL RUNOFF IN WATER YEAR SPACES 4 PRINT K,1:2:: THE ASSUMED RECESSION FACTOR PRINT 10,1:2:: THE A.P.I. UN LAST DAY OF WATER YEAR PRINT P0,1:2:: PRECIPITATION ON LAST DAY OF WATER YEAR LINES 8 Y=0 Z=0 (IF STORE ADEQUATE ENTER NEXT TAPE AND WAIT:: (CLEAR IF NOT READ 21' I.E. PRINT OUT JUMP @8:: (RAINFALL AND KUNDEF VALUES IN GROUPS JUMP 08::

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OUTPUT 40 BLANKS 21)SUBR 14:: PRINT K,1:2:: K VALUE PRINT T,2:: GROUP REFERENCE NUMBER T=T+1OUTPUT 20 BLANKS SUBR 13:: JUMP IF UY&1@17:: IF NO VALUES IN THIS GROUP JUMP @17 L=0 S=0 RAINFALL VALUE 16)PRINT AZ,1:2:: PRINT A(Z+75),4:: ASSOCIATED RUNOFF VALUE L=L+1 JUMP IF L&4@20:: (START NEW LINE WHEN 4 PAIRS OF LINE:: (VALUES HAVE BEEN PRINTED L=0 20)2=Z+1S=S+1JUMP IF S&UY@16:: (IF S=UY THE LAST PAIR OF VALUES IN GROU LINE:: (HAS BEEN PRINTED, SO START NEW LINE 17)Y = Y + 1(THE NEXT GROUPS STORE LOCATIONS START F=F+150:: Z=F:: (AT F+150 CYCLE L=1:1:10:: **COUTPUT 10 BLANKS** OUTPUT Ø:: REPEAT L:: (OUTPUT 1:: OUTPUT 1 OUTPUT 17:: OUTPUT (LINE JUMP IF T&40021:: REPEAT UNTIL ALL 39 GROUPS PRINTED WAIT:: ENTER NEXT TAPE AND CLEAR WAIT. JUMP @9 13)CYCLE L=1:1:20:: (SUB-ROUTINE TO OUTPUT 20 BLANKS OUTPUT Ø:: (FOLLOWED BY A FIGURE SHIFT AND REPEAT L:: (NEW LINE OUTPUT 27:: (LINE EXIT 14)CYCLE L=1:1:40:: (SUB-ROUTINE TO OUTPUT 40 BLANKS (FOLLOWED BY A FIGURE SHIFT AND OUTPUT Ø:: REPEAT L:: (NEW LINE OUTPUT 27:: (LINE:: (EXIT START 19

::TO CORRELATE RAINFALL AND RUNOFF VALUES IN EACH OF TEN **::**A.P.I. GROUPS/QUARTER AND TO GRAPH EACH REGRESSION EQUATION

SETS ZT(1)YXNWILE SETV AK(1)M(39)P(400)Q(400)R(31)F(1)S(2)VU(2)B(2)CD SETF SORT GRAPH PLOT TRI SETR 21 10)LINES 10 SUBR 19:: OUTPUT 20 BLANKS TITLE Ι N P Q С M E SUBR 19:: OUTPUT 20 BLANKS READ K1:: A.P.I. VALUE FOR THIS CALCULATION READ T1:: REFERENCE NUMBER OF A.P.I. GROUP READ Y:: NUMBER OF DATA TAPES IN 11 YEAR PERIOD CYCLE X=0:10:30 E = XCYCLE C=0,.25,.4,.6,.8,1.0,1.2,1.4,1.7,2.2 ME = CE = E + 1 : :SET M0=0 M1=.25 M2=.4 M3=.6 ETC. REPEAT C REPEAT X:: SIMILARLY MID=0 MI1=.25 ETC. N=Ø W=0 WAIT:: ENTER 1ST SORTED RAINFALL RUNOFF TAPE THIS SHOULD CHECK WITH KI EACH TIME 8) READ K:: JUMP IF K=K109 WAIT:: INCORRECT TAPE. ENTER CORRECT TAPE JUMP @8:: AND CLEAR 9)READ T:: THIS SHOULD CHECK WITH T1 JUMP IF T=T102 WAIT:: INCORRECT POSITION ON TAPE. CORRECT JUMP @8:: POSITION AND CLEAR WAIT 2)READ PN:: RAINFALL READ QN:: ASSOCIATED RUNOFF N=N+1:: COUNT NUMBER OF PAIRS OF VALUES READ IN JUMP @2 1)W=W+1 CHECK NUMBER OF DATA TAPES READ IN JUMP IF W=Y@3:: ENTER NEXT TAFE AND CLEAR WAIT:: JUMP @8

3)CYCLE Z=0:2:8:: ((SET SUMMING VARIABLES R0,R2,R4,R6,R8=0 RZ=0:: REPEAT Z:: (VARY I=0:1:N SUM OF P R=PI+R:: FQ1+R2:: SUM OF Q R3=PI*PI:: P SQRD . R4=R3+R4:: SUM OF P SORD. R5=01*01:: Q SORD. R6=R6+R5:: SUM OF Q SQRD. R7=PI*QI:: P * Q R8=R8+R7:: SUM OF P * Q REPEAT 1 D=STAND NR9=R/D:: AV .P R10=R2/D:: AV • Q R11=R4/D:: 1/N (SUM OF P SQRD.) R12=R6/D:: 1/N (SUM OF Q SQRD.) (AV.P)SORD. R13=R9*R9:: R14=R10*R10:: (AV.Q)SORD. (STANDARD DEVIATION OF P)SQRD. R15=R11-R13:: (STANDARD DEVIATION OF P) R16=SQRT R15:: (STANDARD DEVIATION OF Q) SQRD. R17=R12-R14:: R18=SQRT R17:: (STANDARD DEVIATION OF Q STAND P * STAND Q R19=R16*R18:: 1/N (SUM OF (P*0)) R20=R8/D R21=R9*R10:: $(AV \cdot P) * (AV \cdot Q)$ 1/N (SUM OF (P*Q))-(AV.P*AV.Q) R22=R20-R21:: R23=R22/R19:: R=CORRELATION COEFFICIENT STAND Q/STAND P R24=R18/R16:: R25=R23*R24:: M=REGRESSION COEFFICIENT R26=R25*R9:: M*(AV.P) R27=R10-R26:: $C=(AV \cdot Q) - M(AV \cdot P)$ R28=R23*R23:: R SORD. R29=1-R28:: 1-(R SQRD.) R30=SQRT R29:: SORT (1-(R SORD.)) STANDARD ERROR OF ESTIMATE R31=R18*R30::

LINE

JUMP IF T&30@4:: SUMMER QUARTER IF T%29 TITLE SUMMER SUBR 19:: OUTPUT 20 BLANKS JUMP 07 4) JUMP IF T&20@5:: TEST FOR SPRING TITLE SPRING SUBR 19:: OUTPUT 20 BLANKS JUMP 07 5) JUMP IF T&1006:: TEST FOR WINTER TITLE WINTER SUBR 19:: OUTPUT 20 BLANKS JUMP 07 6)TITLE AUTUMN OUTPUT 20 BLANKS SUBR 19:: 7)SPACES 2 PRINT MT,1:2:: I=LOWEST VALUE OF A.P.I. GROUP PRINT N,3:: N=NUMBER OF PAIRS IN GROUP P=MEAN DAILY PRECIPITATION FOR GROUP PRINT R9,1:2:: PRINT R10,3:1:: Q=MEAN DAILY RUNOFF FOR GROUP PRINT R25,3:2:: M=SLOPE OF REGRESSION EQUATION C=INTERCEPT OF REGRESSION EQUATION PRINT R27,3:1:: PRINT R31,3:1:: E=STANDARD ERROR OF ESTIMATE OF Q L=T+1PRINT L,2:: REFERENCE NUMBER SUBR 18 READ 21 IF GRAPHS ARE REQUIRED WAIT:: JUMP @20 21)B1=0 B2=0 R=Ø F=0 S1=Ø S2=0

VARY I=0:1:N JUMP IF PI%.75@11:: JUMP IF RAINFALL %0.75 INCHES/DAY, JUMP IF Q1%150@11:: JUMP IF RUNOFF %150 M.G.D./DAY 12)R1=PI*2000:: PLOTTER INCREMENTS 1 INCH=200 M.M. F1=QI*10:: PLOTTER INCREMENTS $1 M \cdot G \cdot D \cdot = 1 M \cdot M \cdot$ Z=1 JUMP @13 11)R1=PI*400:: PLOTTER INCREMENTS 1INCH=40 M.M. F1=QI*2:: PLOTTER INCREMENTS 5 M.G.D.=1 M.M. Z=0 JUMP @13 17)REPEAT I S1=S1-R:: UNITS TO ORIGIN IN X DIRECTION S2=S2-F:: UNITS TO ORIGIN IN Y DIRECTION SUBR 16:: DRAW A SHORT HORIZONTAL LINE V=R27-R31:: C - EV = V * 10S2=S2+V:: UNITS FROM ORIGIN TO LOWER E LINE S1=0 SUBR 16:: DRAW A SHORT HORIZONTAL LINE 2E V=R31+R31:: * V=V*10 S2=S2+V SUBR 16:: DRAW A SHORT HORIZONTAL LINE V=R31*10:: E*10 S2=S2-V SUBR 16:: DRAW SHORT HORIZONTAL LINE S1=S1+1500:: 1500 UNITS IN HORIZONTAL DIRECTION V=R25*•75:: SLOPE *.75 INCHES OF RAINFALL V=V*10S2=S2+V:: UNITS VERTICALLY FROM C TO TOP OF SLOPE A=0 A=GRAPH Ø,A:: LOWER PEN A=GRAPH S1,S2:: DRAW SLOPE OF REGRESSION EQUATION A=0 A=PLOT Ø,A:: LIFT UP PEN SUBR 18:: OUTPUT 40 BLANKS

20)T1=T+1:: MAKE T1=TO NEXT REFERENCE NUMBER . N=0 W = 0JUMP @8:: READ IN NEXT LENGTH OF DATA TAPE PARTIAL COORDINATE OF RAINFALL 13)R=R1-R:: PARTIAL COORDINATE OF RUNOFF F=F1-F:: ADD IN RESIDUAL OF RAINFALL S1=S1+R:: ADD IN RESIDUAL OF RUNOFF S2=S2+F:: MOVE PEN TO PLOTTING POINT A=PLOT S1,S2:: JUMP IF Z=1@14A=TRI 3:: LOWER PEN DRAW TRIANGLE AND RAISE PEN-JUMP @15 14)A=GRAPH B1,B2:: LOWER PEN LIFT UP PEN A=PLOT B1,B2:: 15)R=R1:: REPLACE WITH NEW VALUE F=F1:: REPLACE WITH NEW VALUE JUMP @17 16)A=PLOT S1,S2:: MOVE PEN TO PLOTTING POINT A=GRAPH B1,B2:: LOWER PEN U2=Ø U1 = -10A=GRAPH U1,U2:: DRAW HORIZONTAL LINE 1 M.M.LONG U1 = 10RETURN ALONG SAME LINE A=GRAPH U1,U2:: LIFT UP PEN A=PLOT B1,B2:: EXIT 18)CYCLE Z=1:1:40:: (OUTPUT Ø:: (OUTPUT 40 BLANKS REPEAT Z:: (EXIT:: (19)CYCLE Z=1:1:20:: (OUTPUT Ø:: (OUTPUT 20 BLANKS REPEAT Z:: (EXIT:: (START 10

::TO DRAW REGRESSION EQUATIONS FROM DATA OUTPUT BY ::CORRELATION PROGRAMME.

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SETS ZYOUXW SETV S(2)R(6)AVDBE SETF GRAPH PLOT TRI SETR 5 5)₩=1 U=0 Y=0 1) VARY Z=0:1:7 READ I, N, P, Q, M, C, E READ RZ:: REPEAT Z-READ C:: REFERENCE NUMBER - -S1=0 POSITION OF C TO SCALE S2=R5 •5 :: A=0 . . A=PLOT 0,A:: PEN UP A=PLOT S1,S2:: MOVE TO INTERCEPT ON Y AXIS S1=S1+750 ::MOVE750 UNITS IN X DIRECTIONV=R4•3•75::Y MOVEMENT FROM POSITION C V=R4•3•75:: S2=S2+V A=0 A=GRAPH 0,A:: PEN DOWN A=GRAPH S1,S2:: DRAW REGRESSION EQUATION . . . A=0 A=PLOT 0,A:: PEN UP C=C-U C=C+10 JUMP IF C\$10002:: TEST FOR END OF SEASON U=U+10:: U=10 AT START OF WINTER QUARTER ₩=1 2) E=STAND C S1=S1+E A=PLOT S1,S2:: MARK COUNT BY DOT POSITION A=0 A=GRAPH 0,A:: PEN DOWN A=0 A=PLOT 0,A:: PEN UP

UNITS FROM RUNOFF AXIS B=E+750:: R2=R2+1000:: R2=P AND 1 INCH OF RAINFALL = 1000 UNITS X MOVEMENT FROM LAST POSITION B=R2-B:: D=R3-R5:: POSITION OF Q ABOVE C D=D+5::--1 $M \bullet G \bullet D \bullet = 5$ UNITS Y MOVEMENT FROM LAST POSITION D=D-V::S1=S1+B S2=S2+D A=PLOT S1,S2:: MOVE TO POSITION OF AVERAGE P. AND Q. A=TRI 3:: DRAW A SMALL TRIANGLE R2=-R2:: DISTANCE TO ORIGIN IN X DIRECTION R3=-R3•5:: DISTANCE TO ORIGIN IN Y DIRECTION S 1=S 1+R2 S2=S2+R3 A=PLOT S1,S2:: MOVEMENT TO ORIGIN A=0A=GRAPH 0,A:: PEN DOWN A=0 A=PLOT 0,A:: PEN UP OUTPUT 200 BLANKS SUBR 4:: JUMP UNLESS W=1@1::END OF SEASON W=0 WAIT:: CLEAR TO GRAPH NEXT SEASONS EQUATIONS JUMP @1 4) CYCLE X=1: 1: 200 OUTPUT 200 BLANKS OUTPUT O:: -REPEAT X EX IT START 5

:: PROGRAMME TO PRODUCE 7 DIGIT INTEGER OF RAINFALL-RUNOFF FROM MODIFIED RAINFALL-RUNOFF DATA :: WITH 1(TRIGGER SETS WXYZBC SETV A SETR 5 5) VARY W = 1: 1: 40OUTPUT 40 BLANKS OUTPUT 0 REPEAT W LINE X=0 4)Y = 02)LINE Z=0 · E.G. 1.02 RAINFALL IN FLOATING POINT FORM 3)READ A A=A + 100 $E \cdot G \cdot 1 \cdot 01999 = 101 \cdot 999$ ADD SMALL NUMBER TO ENSURE CORRECT INTEGER A = A + 01B=INT AE.G. 102 E.G. 1020000 B=B+10000 E.G. 236.5 RUNOFF IN FLOATING POINT FORM READ A E.G. 2364.999 A=A • 10 E.G. 2365.009 A = A + 01C=INT A E.G. 2365 B=B+CE.G. 1022365 Z=Z+1 PRINT B,7 7 DIGIT INTEGER REPRESENTING RAINFALL-RUNOFF JUMP UNLESS Z=7@3 SEVEN NUMBERS PER LINE OF PRINT Z=0 LINE JUMP @3 RETURN TO READ IN NEXT DAYS VALUES (1)Y = Y + 1COUNT OF MONTHS JUMP UNLESS Y=302 AT END OF EACH QUARTER LINE OUTPUT 1 OUTPUT 1 CUTPUT 17 OUTPUT (LINE X = X + 1COUNT OF QUARTERS JUMP UNLESS X=4@4 AT END OF FOUR QUARTERS WAIT INSERT NEW YEARS DATA AND CLEAR JUMP @5 START 5

::SORTING AND STATISTICAL ANALYSIS OF RAINFALL-RUNOFF DATA PRESENTED IN INTEGER FORM. DATA TO END WITH 80 SETS AC10123WC9933VC10013SC10123BC10123NC93FC73HC53EC83CC13 ZYXUPOTK(2) SETV R(32)D(2) SETF SORT INT SETR 15 15)READ F7:: COUNT. F7=4 AT LEAP YEAR. READ N1:: NUMBER OF YEARS TO BE ANALYSED E.G.11 NUMBER OF API GROUPS/SEASON E.G.10 READ NA:: WAIT:: CENTER TAPE CONTAINING 11 YEAR VALUES AT Z=1:: (RAINFALL AND RUNOFF IN INTEGER FORM Y=1 X=1 U=1 T=1 H1 = 1000RUNOFF REPRESENTED BY LAST A DIGITS H2 =10000:: H3=10000000:: RAINFALL-RUNOFF REPRESENTED BY 7 DIGITS COUNT FOR SEASONS. FOR AUTUMN T=2 1)T=T+1:: JUMP UNLESS T=6@T:: JUMP TO CORRECT SEASONAL GROUP T=2 JUMP @T:: WINTER T=3, SPRING T=4, SUMMER T=5 2)READ AZ:: AUTUMN INTEGERS IN CHRONOLOGICAL ORDER. Z=Z+1:: ... COUNT OF AUTUMN NUMBERS JUMP 02 WINTER INTEGERS IN CHRONOLOGICAL ORDER 3)READ WY:: Y=Y+1:: -COUNT OF WINTER NUMBERS JUMP @3 SPRING INTEGERS IN CHRONOLOGICAL ORDER 4)READ VX:: COUNT OF SPRING NUMBERS X=X+1:: -JUMP @4 SUMMER INTEGERS IN CHRONOLOGICAL OPDER 5)READ SU:: U=U+1:: -COUNT OF SUMMER NUMBERS JUMP @5

8)WAIT:: ENTER DATA TAPE AND CLEAR RECESSION FACTOR K*1000 READ F1:: READ F2:: (A.P.I. ON SEPT. 30TH 1953 (PRECIPITATION ON SEPT 30TH 1953)*100 READ C:: F = F7N6=0 Z=1 Y = 1X ≈1 U=1 N1=NUMBER OF YEARS TO BE ANALYSED VARY N2=1:1:N1:: VARY H=Z:1:92:: 92 DAYS IN AUTUMN SEASON @00H/30A:: PUT THE CONTENTS OF AH INTO ACCUMULATOR * BY 2**-38: DIVIDE BY (10,000*2**-38) 5038:56H2:: PUT CONTENTS OF ACCUMULATOR INTO C1. THE 2001:000:: ACCUM. HOLDS INTEGER PART I.E. RAINFALL):: CALCULATION OF A.P.I. IN INTEGER FORM SUBR 6:: AH=AH+F3:: RAINFALL RUNOFF INTEGER +(A.P.I.*10000000) REPEAT H:: UNTIL ONE AUTUMNS INTEGERS ARE COMPLETED VALUE OF Z AT START OF NEXT AUTUMN CYCLE Z=H+1:: F5=90:: 90 DAYS IN WINTER SEASON (NOT LEAP YEAR) COUNT F=F+1:: IF F=5 THEN LEAP YEAR JUMP UNLESS F=5@9:: F5=91:: 91 DAYS IN WINTER SEASON IN LEAP YEAR F=1::-START NEW COUNT-F5=NUMBER OF DAYS IN WINTER SEASON 9)VARY H=Y:1:F5:: @OOH/30W::-PUT THE CONTENTS OF WH'INTO ACCUMULATOR * BY 2**-38: DIVIDE BY (10,000*2**-38) 5038:56H2:: PUT CONTENTS OF ACCUMULATOR INTO C1. THE 2001:000:: ACCUM. HOLDS INTEGER PART I.E. RAINFALL):: SUBR 6:: CALCULATION OF A.P.I. IN INTEGER FORM RAINFALL- RUNOFF INTEGER+(A.P.I.*1000,000) WH=WH+F3:: UNTIL ONE WINTERS INTEGERS ARE COMPLETED REPEAT H:: VALUE OF Y AT START OF NEXT WINTER CYCLE Y=H+1::

PUT THE CONTENTS OF VH INTO ACCUMULATOR DIVIDE DOUBLE LENGTH NUMBER BY 10,000 PUT RAINFALL INTEGER INTO LOCATION C1 CALCULATE A.P.I. IN INTEGER FORM MAKE UP 11 DIGIT INTEGER OF A.P.I.P.Q. UNTIL ONE SPRINGS INTEGERS ARE COMPLETED VALUE OF X AT START OF NEXT SPRING SEASON VARY H=U:1:92:: 92 DAYS IN THE SUMMER SEASON PUT THE CONTENTS OF SH INTO ACCUMULAPOR DIVIDE DOUBLE LENGTH NUMBERS OF 10,000 PUT RAINFALL INTEGER INTO LOCATION C1 CALCULATION OF A.P.I. IN INTEGER FORM MAKE UP 11 DIGIT INTEGER OF A.P.I.P.Q. UNTIL ONE SUMMERS INTEGERS ARE COMPLETE VALUE OF U AT START OF NEXT SUMMER SEASON-RETURN TO WORK THROUGH THE NEXT 4 SEASONS 3 C OUTPUT 20 BLANKS FLOATING POINT FORM OF RECESSION FACTOR K R1=STAND F1:: DECIMAL FORM OF K PRINT R1,0:3:: PRINT K VALUE Р E I Ы Ø М С OUTPUT 20 BLANKS FIGURE SHIFT SYMBOL TOTAL NUMBER OF VALUES IN AUTUMN SEASONS COPY 11 DIGIT INTEGERS INTO SORTING STORE PUT THE CONTENTS OF AH INTO ACCUMULATOR *BY 2**-38:DIVIDE BY (10,000,000*2**-38) * BY (10,000,000*2**-38: PUT IN ACCUM. SUBTRACT ACCUMULATOR FROM AH (11DIGITS) PUT 7 DIGIT RAINFALL-RUNOFF INTEGER INTO AH UNTIL ALL AUTUMN SEASONS VALUES TREATED SORTING AND STATISTICAL ANALYSIS SUBR 10::

91 DAYS IN SPRING SEASON

VARY H=X:1:91:: ©00H/30V:: 5038:56H2:: 2001:000::) SUBR 6:: VH=VH+F3:: REPEAT H:: X=H+1:: @00H/30S:: 5038:56H2:: 2001:000::) SUBR 6:: SH=SH+F3:: REPEAT H:: U=H+1:: REPEAT N2:: LINE OUTPUT 19:: OUTPUT 17:: SUBR 13::--LINES 10 R1=R1/1000:: TITLE K = LINES 2 TITLE SUBR 13:: OUTPUT 27:: N=Z-1:: VARY H=1:1:N BH=AH:: @00H/30A:: 5038:56H3:: 52H3:570:: 00H/07A:: 00H/20A:: 2 REPEAT H::

TOTAL NUMBER OF VALUES IN WINTER SEASON N=Y-1:: VARY H=1:1:N COPY 11 DIGIT INTEGERS INTO SORTING STORE BH=WH:: PUT THE CONTENTS OF WH INTO ACCUMULATOR @00H/30W:: *BY 2**-38: DIVIDE BY 1(10,000,000*2**-38) 5038:56H3:: *BY(10,000;000*2**-38):PUT IN ACCUM. 52H3:570:: SUBTRACT ACCUMULATOR FROM WH(11 DIGITS) 00H/07W:: PUT 7 DIGIT RAINFALL RUNOFF INTEGER INTO WH. 00H/20W::) PEPEAT H:: UNTIL ALL WINTER SEASONS VALUES TREATED SUBR 10:: SORTING AND STATISTICAL ANALYSIS TOTAL NUMBER OF VALUES IN SPRING SEASON N=X-1:: VARY H=1:1:N COPY 11 DIGIT INTEGERS INTO SORTING STORE BH=VH:: PUT THE CONTENTS OF VH INTO THE ACCUMULATOR @00H/30V:: 5038:56H3:: DIVIDE THE DOUBLE LENGTH NUMBER BY 10,000,000 52H3:570:: PUT THE INTEGER PART INTO ACCUM. AS 11 DIGITS SUBTRACT ACCUMULATOR FROM VHC11 DIGITS) 00H/07V:: PUT 7 DIGIT RAINFALL-RUNOFF INTEGER INTO VH 00H/20V:: 2 REPEAT H:: UNTIL ALL SPRING SEASONS VALUES TREATED SORTING AND STATISTICAL ANALYSIS SUBR 10:: TOTAL NUMBER OF VALUES IN SUMMER SEASON N=U-1:: VARY H=1:1:N COPY 11 DIGIT INTEGERS INTO SORTING STORE BH=SH:: PUT THE CONTENTS OF SH INTO ACCUMULATOR @00H/30S:: DIVIDE THE DOUBLE LENGTH NUMBER BY 10,000,000 5038:56H3:: 52H3:570:: PUT INTEGER PART INTO ACCUM. AS 11 DIGITS SUBTRACT ACCUMULATOR FROM SH (11 DIGITS) 00H/07S:: PUT 7 DIGIT RAINFALL- RUNOFF INTEGER INTO SH 00H/20S:: ٦ REPEAT H:: UNTIL ALL SUMMERS SEASONS VALUES TREATED SUBR 10:: SORTING AND STATISTICAL AMALYSIS JUMP @8:: END OF CALCULATIONS WITH GIVEN K VALUE JUMP @8:: SUBROUTINES FOLLOW ON FROM HERE

10)@73K:4014):: 3030,:442,:: 3032,:2025,:: 3031,:2319,-2320,:2323, 2324,:2635, 000,/302 2419,:2420, 2 42 3, :2 42 4, 000,/301 2033,:2419, 2 12 3, :2034, 4712,:3025, 0530,:4626, 020:1733, 2420,:2424, 2734,:2734, 2735,:3035, 2036,:4425, 0037,/270 0037,/240 3037,:0534, 4525,:0435, 2037,/300 0037,/050 4119,:3236, 4123,:3033, 501:4229, 4414,:000 000,/403 4119,:3236, 778191/770 4129,:3236, +0 +0 +0 +0 +0) 14)[@]402,:000 73K:4014) 30N:207, 73K1:22K1 22K1:02K1 00K/200 00K/401 000:000 000:00B1 301,:000 201,/200::)::

START OF ELLIOTTS SORTING ROUTINE FOR 11 DIGIT INTEGER VALUES STORED IN LOCATIONS B1 TO BN

SORTED ACCORDING TO A.P.I. MAGNITUDE

N5=1

D=STAND N:: D=NUMBER OF VALUES TO BE ANALYSED D2=NUMBER OF A.P.I. GROUPS /SEASON D2 -STAND N4:: ADD SMALL NUMBER TO FLOATING POINT FORM D=D+.001:; · NUMBER OF VALUES IN A.P.I. GROUP D=D/D2::-N3=INTEGER PART OF D. N3=1NT D:: N7=N3:: -(NUMBER OF VALUES IN A.P.I. GROUP N9=N4-1:: (NUMBER OF A.P.I. GROUPS/SEASON)-1 N8 ≕0 11)VARY H=0:2:5:: ((SET SUMMING VARIABLES E0, E2, E4, E6, E8, =0 EH=0:: REPEAT H:: C N7=NUMBER OF VALUES IN EACH A.P.I. GROUP VARY H=N5:1:N7:: @OOH/30B:: PUT THE CONTENTS OF BH INTO THE ACCUMULATOR *BY 2**-38: DIVIDE BY(10,000,000*2**-38) 5038:56H3:: PUT INTEGER PART INTO K2. I.E. A.P.I. 20K2:000:: PUT INTEGER PART *H3 INTO ACCUM.(11 DIGITS) 52H3:570:: 00H/07B:: SUBTRACT ACCUMULATOR FROM BH (11 DIGITS) STORE 7 DIGITS IN LOCATION 12:*BY 2**-38 2012:5038:: 56H2:20P:: DIVIDE BY (10,000+2**-38): PUT ACCUM. IN P PUT INTEGER PART * H2 INTO ACCUM.(7 DIGITS) P 52H2:570:: SUBTRACT ACCUN. P FROM 7 DIGIT NUMBER IN 0712:200:: LOCATION 12. PUT 4 DIGIT RESULT INTO Q.):: SUM OF P E=P+E:: SUM OF Q. E2 =Q+E2 :: P SQRD E3=P*P:: SUM OF P SQRD E4=E3+E4:: E5=ର୍∗ର::-Q SQRD E6=E6+E5:: SUM OF Q SORD E7=P*Q::-P≉Q E8=E8+E7:: SUM OF P*Q REPEAT H:: REPEAT WITH NEXT RAINFALL RUNOFF VALUE R≔STAND E:: SUM OF RAINFALL TO FLOATING POINT FORM SUM OF RAINFALL P INCHES R=R/100:: SUM OF RUNOFF TO FLOATING POINT FORM R2=STAND E2:: SUM OF RUHOFF Q. M.G. R2=R2/10:: SUM OF P SQRD TO FLOATING POINT FORM R4=STAND-E4:: SUM OF P SQRD(INCHES SQRD) R4=R4/10000:: SUM OF Q SQRD TO FLOATING POINT FORM R6=STAND E6:: SUM OF Q SQRD (M.G.D. SQRD) R6=R6/100:: SUM OF P . Q TO FLOATING POINT FORM R8-STAND E8:: SUM OF P * Q (INCHES *M.G.D.) R8=R8/1000:: NUMBER OF VALUES IN A.P.I. GROUP D=STAND N7::

R9=R/D:: R10=R2/D:: R11=R4/D:: P12=R6/D:: R13=R9*R9:: R14=R10*R10:: R15=R11-R13:: R16=SQRT R15:: R17=R12-R14: R18=SQRT R17:: R19=R16+R18:: R20=R8/D:: ~ R21=R9*R10:: R22=R20-R21:: R23=R22/R19:: R2 4=R18/R16:: R25=R23*R24:: R26=R25*R9:: R27=R10-R26:: R28=R23*R23:: R29=1-R28:: R30=SQRT R29:: R31=R18*R30::

AVERAGE P AVERAGE Q 1/N(SUM OF P SQRD) 1/N(SUM OF Q SQRD) (AVERAGE P)SQRD (AVERAGE Q)SQRD (STANDARD DEVIATION OF P)SQRD STANDARD DEVIATION OF P (STANDARD DEVIATION OF Q)SQRD STANDARD DEVIATION OF Q STANDARD DEVIATION OF Q STANDARD DEVIATION OF Q 1/N(SUM OF (P*Q))-(AV.P*AV.Q)

R=CORRELATION COEFFICIENT STAND.DEV.Q/STAND DEV.P M=REGRESSION COEFFICIENT H*(AV.P) C=AV.Q-M*(AV.P) R.SQRD 1 -(R.SQRD) SQRT(1-(R SQRD)) E=STANDARD ERROR OF ESTIMATE.

LINE @00H/30B:: 5038:56H3:: 20F6:000:: >

D1=STAND F6:: D1=D1+.0001:: D1=D1=D1/100:: PUT THE CONTENTS OF BH INTO ACCUMULATOR

HIGHEST A.P.I.IN GROUP (INCHES)

PRINT D1, 1:2:: PRINT N7.3::-PRINT R9,1:2:: PRINT R10,3:1:: PRINT R25,3:1:: PRINT R27, 3:1:: PRINT R31,3:1:: N6 = N6+1:: PRINT N6,2 N8=N8+1:: N5=N5+N7:: JUMP IF N8\$N9@11:: JUMP IF N8=N4@12:: N7=N9*N7:: N7=N-N7:: JUMP @11:: 12)SUBR 13 EXIT:: 133LI NE VARY H=1:1:20:: OUTPUT O:: REPEAT H:: EXIT 6)F4=F2*F1:: @30F4:5038:: 56H1:20F4:: F4=F4+C:: F2=F4:: C=C1:: F3=F4*1000000:: EX1 T

START 15

I=HIGHEST A.P.I.VALUE IN THIS GROUP N=NUMBER OF PAIRS IN GROUP P=MEAN DAILY PRECIPITATION FOR GROUP Q=MEAN DAILY RUNOFF FOR GROUP M=SLOPE OF REGRESSION EQUATION C=INTERCEPT OF REGRESSION EQUATION E=STANDARD ERROR OF ESTIMATE OF Q REFERENCE NUMBER

- NUMBER OF A.P.I.GROUPS ANALYSED SO FAR (NUMBER OF VALUES ANALYSED SO FAR)+1 UNTIL ONLY ONE GROUP IS LEFT FOR ANALYSIS LAST OF THIS SEASONS GROUPS ANALYSED ((NUMBER OF GROUPS)-1)*NUMBER IN GROUP NUMBER OF VALUES IN LAST A.P.I.GROUP TO ANALYSE THE VALUES IN THE LAST GROUP

RETURN TO START ON NEXT SEASONS CALCS

COUTPUT 20 BLANKS

A.P.I.*(K*1000) PUT F4 INTO ACCUM.:*BY 2**-38 DIVIDE BY 1000*2**-38:PUT ANS.INTO F4

A.P.I.*K/1000+YESTERDAYS RAINFALL A.P.I.AT 9.0 A.M. THIS HORNING MAKE C.=TODAYS RAINFALL *A.P.I. BY 10,000,000 E.G.770000000

::PROGRAMME TO PLOT	POR Q OR M OR C OR E AGAINST N
SETS ZLNSTVRDWK(2)C SETV AI(441)U(441)F SETF GRAPH PLOT SETR 16) ?(440)Q(440)M(440)C(440)E(440)FG(440)HJX(2)BY
14)SUBR 13:: Z=1 ^{O=} 1	OUTPUT 160 BLANKS
15) READ IZ:: READ GZ:: READ PZ:: READ QZ:: READ MZ:: READ CZ:: READ CZ:: READ EZ:: READ V::- Z=Z+1 JU MP @15	HIGHEST A.P.I. VALUE IN GROUP NUMBER OF RAINFALL-RUNOFF PAIRS IN GROUP AVERAGE RAINFALL FOR GROUP AVERAGE RUNOFF FOR GROUP SLOPE OF REGRESSION EQUATION INTERCEPT OF REGRESSION EQUATION STANDARD ERROR OF ESTIMATE REFERENCE NUMBER OF GROUP
3)WAIT:: 0=0+1:: JUMP @15	3(TRIGGER AT END OF SHEET OF DATA/K VALUE COUNT NUMBER OF SETS OF K VALUES READ IN
16)READ D:: W=D-1 K1=3•D K1=K1+2:: K2=5•D K2=K2-2:: K=4•D:: K=K•11:: Z=K+1 IZ=0 UZ=0	NUMBER OF INCREMENTS PER SEASON COUNT TO FIND FIRST VALUE IN EACH SEASON COUNT TO FIND LAST VALUE IN EACH SEASON TOTAL NUMBER OF GROUPS PER K VALUE TOTAL NUMBER OF GROUPS IN ALL SETS OF K
2)READ L:: SUBR L::	P=4 Q=5 M=6 C=7 E=8 SET UZ=PZ. OR QZ. OR MZ. OR CZ OR EZ.
1)READ N:: S=N T=1 V=0 G=0 H=0 X1=0 X2=0	A=1 W=11 V=21 S=31

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O=NUMBER OF SETS OF K VALUES VARY R=1:1:0:: A=0 A=PLOT 0,A:: RAISE PEN -Z=S SU BR 12:: READY TO START DRAWING NEW LINE MOVE PEN. S=S+TA=0A=GRAPH 0.A:: LOWER PEN VARY Z=S:T:W SUBR 10:: ··· DRAW LINE BETWEEN PRESENT VALUE AND LAST REPEAT Z-JUMP IF T=109::REVERSE DIRECTION OF PLOTTING. SEE SUBR 10 FIRST VALUE IN EACH SEASON S=S+K1:: T = 1 JU MP @11 9)S=S+K2:: LAST VALUE IN EACH SEASON T =-1 11) REPEAT R A=0A=PLOT 0,A:: RAISE PEN IF Z=K+1 THEN UZ=0 Z=K+1:: SUBR 12:: RETURN-PEN TO X AXIS OUTPUT 160 BLANKS . SUBR 13:: WAIT JUMP @1:: START FOR NEW SEASON 10) F = UZ::VALUE TO BE PLOTTED F=F-H:: VALUE TO PLOTTED LESS PREVIOUS VALUE X MOVEMENT. POSITIVE OR NEGATIVE X=STAND T:: X=100 +X:: 100 • 1 M.M. STEPS IN X DIRECTION Y=F+B:: B=SCALE. SEE SUBR 4,5,6,7,8 FOR VALUES ADD ON RESIDUAL X1=X1+X:: ADD ON RESIDUAL X2=X2+Y:: A=GRAPH X1, X2:: DRAW LINE BETWEEN PRESENT VALUE AND LAST SET H= TO PRESENT VALUE H=UZ:: EXIT

12)F=UZ:: VALUE AT START OF NEW LINE VALUE TO BE PLOTTED LESS PREVIOUS VALUE F=F-H:: X=0 ··· Y=F+B:: B=SCALE. SEE SUBR 4,5,6,7,8 FOR VALUES X1=X1+X:: ADD ON RESIDUAL X2=X2+Y:: ADD ON RESIDUAL A=PLOT X1,X2:: MOVE PEN READY TO START DRAWING NEW LINE H=JZ:: SET H= TO PRESENT VALUE EX IT 13) VARY Z=1:1:160:: OUTPUT 160 BLANKS OUTPUT O:: REPEAT Z:: EX IT SCALE FOR MOVEMENT IN Y DIRECTION 4) B=1000:: VARY Z=1:1:K:: UZ=PZ:: (UZ= AVERAGE RAINFALL VALUE FOR EACH GROUP REPEAT Z:: EX IT SCALE FOR MOVEMENT IN Y DIRECTION 5) B=10:: VARY Z=1:1:K:: UZ=QZ:: (UZ=AVERAGE RUNOFF VALUE FOR EACH GROUP REPEAT Z:: EXIT 6)B=5:: SCALE FOR MOVEMENT IN Y DIRECTION VARY Z=1:1:K:: UZ = MZ: UZ=SLOPE OF EACH REGRESSION EQUATION REPEAT Z:: EX IT SCALE FOR MOVEMENT IN Y DIRECTION 7)B=10:: VARY Z=1:1:K:: UZ=CZ:: ~ · (UZ=INTERCEPT OF EACH REGRESSION EQUATION REPEAT Z:: EX IT 8)B=10:: SCALE FOR MOVEMENT IN Y DIRECTION VARY Z=1:1:K:: UZ=EZ:: (UZ=EACH STANDARD ERROR OF ESTIMATE REPEAT Z:: EX IT START 14

LINES 8 N3=0 N5 = 0N=TOTAL NUMBER OF VALUES PER SEASON **VARY** H= 1:1:N:: PUT THE CONTENTS OF BH INTO THE ACCUMULATOR @00H/30B:: 5038:56H3:: •BY 2••-38:DIVIDE BY(10,000,000+2••-38) PUT INTEGER PART INTO K2 1.E. A.P.I. 20K2:000:: PUT INTEGER PART +H3 INTO ACCUM. (11 DIGITS) 52H3:570:: SUBTRACT ACCUMULATOR FROM BH (11 DIGITS) 00H/07B:: STORE 7 DIGITS IN LOCATION 12: •BY 2 • • - 38 2012:5038:: DIVIDE BY (10,000+2++-38): PUT ACCUM. IN P 56H2:20P:: PUT INTEGER PART +H2 INTO ACCUM. (7 DIGITS)P 52H2:570:: 0712:20Q:: SUBTRACT ACCUM. P FROM 7 DIGIT NUMBER IN LOCATION 12. PUT 4 DIGIT RESULT IN Q):: JUMP UNLESS P=0012 COUNT. 5 A.P.I.-RUNOFF PAIRS/LINE N3=N3+1:: N5 = N5 +1:: COUNT OF NUMBER OF DAYS WHEN P=0 PRINT K2, 3:: A.P.I. VALUE PRINT Q, 4::-ASSOCIATED RUNOFF VALUE JUMP UNLESS N3=5012 LINE N3=0 12) REPEAT H LINE OUTPUT 1:: 1 OUTPUT 17:: LINE PRINT N5,3:: PRINT NUMBER OF DAYS WHEN P=O EACH SESSION SUBR 13:: OUTPUT 20 BLANKS RETURN TO START NEXT SEASONS CALCULATIONS EXIT:: --13) LINE **VARY H=1:1:20::** OUTPUT 20 BLANKS OUTPUT O:: REPEAT H:: EXIT 6)F4=F2+F1:: A.P.J. (K+1000) PUT F4 INTO ACCUMULATOR: • BY 2 • • - 38 @30F4:5038:: DIVIDE BY 1000+2++-38: PUT ANSWER INTO F4 56H1:20F4:: A.P.I.*(K*1000)+ YESTERDAYS RAINFALL F4=F4+C:: F2=F4:: A.P.I. AT 9.0 A.M. THIS MORNING MAKE C=TODAYS RAINFALL C=C1::· F3=F4+10000000::A.P.I. +10,000,000 E.G. 770000000 EXIT

START 15

SETS NIZ SETV RCDFS(2)B(2)AP(1000)Q(1000) SETF GRAPH PLOT SETR 4 4)R=0F=0S1=0 · S2=0 B1=0 B2=0 CYCLE Z=1:1:160:: C COUTPUT 160 BLANKS OUTPUT O:: REPEAT Z:: (2)A=PLOT B1, B2:: RAISE PEN 3)READ PI:: A.P.I. VALUE ON DAY WITH NO RAINFALL RUNOFF ON SAME DAY READ OI:: R=PI-R:: A P.I. AP.I. ON PREVIOUS DAY F=01-F:: RUNOFF-RUNOFF ON PREVIOUS DAY C=R+10:: • PARTIAL A P.I. • SCALE D=F/2:: PARTIAL RUNOFF . SCALE S1=S1+C:: PREVIOUS RESIDUAL + PARTIAL A.P.I. PREVIOUS RESIDUAL + PARTIAL RUNOFF S2=S2+D:: A=PLOT S1.S2:: MOVE THROUGH DISTANCE S1:S2 A=GRAPH B1, B2:: LOWER PEN. MARK AIDOT A≠PLOT B1.B2:: RAISE PEN R=P1:: PREVIOUS DAYS A P.I. F=01:: PREVIOUS DAYS RUNOFF JUMP @3:: RETURN TO PLOT NEXT VALUES 1)C=-R+10:: SCALED DISTANCE TO ORIGIN IN X DIRECTION D=-F/2:: SCALED DISTANCE TO ORIGIN IN Y DIRECTION S1=S1+C:: ADD ON RESIDUAL S2=S2+D:: ADD ON RESIDUAL A≑PLOT S1,S2:: MOVE THROUGH DISTANCE S1, S2 -A=GRAPH B1, B2:: LOWER PEN, MARK AIDOT A=PLOT B1.B2:: RAISE PEN WAIT:: CLEAR WAIT TO PLOT NEXT SEASONS VALUES READ N:: NUMBER OF DAYS WHEN P=0 JUMP @4 START 4

API - C WHEN RAINFALL IS ZERO

::GRAPH POINTS

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11)H=0
T=0
G=21
R 32=0
R=0 ·
R 4=0
R_{2=0}
R6=0
R8=0
N7=0
17)H=H+1
JUMP IF H%N@18:: N=NUMBER OF VALUES/SEASON
                 PUT THE CONTENTS OF BH INTO THE ACCUMULATOR
@ 00H/30B::
                 • BY 2**-38: DI VI DE BY (10,000,000*2**-38)
5038:56H3::
                 PUT INTEGER PART INTO K2 I.E. A.P.I.
20K2:000::
                 PUT INTEGER PART +H3 INTO ACCUM. (11 DIGITS)
52H3:570::
0 OH /07B::
                  SUBTRACT ACCUMULATOR FROM BH (11 DIGITS)
                  STORE 7 DIGITS IN LOCATION 12:*BY 2**-38
2012:5038::
                  DIVIDE BY: (10,000+2**-38): PUT ACCUM. IN P.
56H2:20P::
                 PUT INTEGER PART #H2 INTO ACCUM. (7 DIGITS) P+
52H2:570::
                  SUBTRACT ACCUM .P FROM 7 DIGIT NUMBER IN
0712:20<sup>Q</sup>::
                  LOCATION 12. PUT 4 DIGIT RESULT IN Q
)::-
JUMP (IF P=0017:: CALCULATIONS ON RAINFALL DAYS ONLY
                  FLOATING POINT FORM OF DAILY A.P.I. 1.
D1=STAND K2::
                 FLOATING POINT FORM OF DAILY RUNOFF Q
R1=STAND Q::
                 COUNT UP TO 5 1.E.STEPS OF MOVING AV.
N7=N7+1::
                 COUNT UP TO N4
T=T+1::
                  DAILY RUNOFF Q
R3=R1::
                  FLOATING POINT FORM OF DAILY RAINFALL P
R7=STAND P::
JUMP @G::
                  JUMP TO 21 FOR FIRST N4 VALUES THEN TO 22
                  DAILY RUNOFF - DAILY RUNOFF N4 DAYS BEFORE
22)D5=R3-OT::
                  DAILY RUNOFF. N4 DAYS BEFORE, OVERWRITTEN
OT=R3::
                  DAILY RAINFALL - DAILY RAINFALL N4 DAYS BEFORE
D6=R7-ET::
                  DAILY RAINFALL. N4 DAYS BEFORE, OVERWRITTEN
ET=R7::
                  DAILY A.P.I. -DAILY A.P.I. N4 DAYS BEFORE
D7=D1@-LT:: ·
                  DAILY A.P.I. NA DAYS BEFORE, OVERWRITTEN
LT=D1::
R=D6+R::
                  SUM OF N4 P++ DIFFERENCE IN P++
                  SUM OF NA Q + DIFFERENCE IN Q
R2=D5+R2::
                  SUM OF NA A.P.I. + DIFFERENCE IN A.P.I.
R 32=D7+R32::
                  P : SQRD
D8=ET*ET::
                  P SQRD - P SQRD N4 DAYS BEFORE
D9=D8-MT::
                  PISQRD. NA DAYS BEFORE, OVERWRITTEN
MT=D8:: ·
                  SUM OF N4 (P SQRD) + DIFFERENCE IN P SQRD
R4=D9+R4::
                  Q SQRD
D8=0T*0T:: •
                  Q SQRD - Q SQRD N4 DAYS BEFORE
D9=D8-IT:: •
                  Q SQRD. N4 DAYS BEFORE, OVERWRITTEN
1 T=D8::
                  SUM OF N4 (Q SQRD)+ DIFFERENCE IN Q SQRD
R6=R6+D9::
D8=ET*OT::
                  P * * Q
                  P * Q- (P*Q) N4 DAYS BEFORE
D9=D8-JT::
                         N4 DAYS BEFORE, OVERWRITTEN
                  P:• Q
JT=D8::
                  SUM OF N4 (P + 2)+ DIFFERENCE IN P + 2
Ra=R3+D0::
JUMP UNLESS N7=5@17:: REPEAT 5 TIMES
JUMP UNLESS T=N4018:: CYCLE COMPLETE
                  SET T=0 TO START NEW CYCLE OF N4
T=0::
```

18)R40=R/100:: SUM OF NA DAYS RAINFALL IN INCHES R 42=R2/10:: SUM OF N4 DAYS RUNOFF IN M.G.D. R 44=R4/10000:: • (SUM OF N4 DAYS RAINFALL IN INCHES) SQRD R 46=R6/100:: (SUM OF N4 DAYS RUNOFE IN M.G.D.) SORD R 48=R8/1000:: SUM OF N4 DAYS P ** Q (INS* M.G.D.) NUMBER OF VALUES IN A.P.I. GROUP D=STAND N4:: R9=R40/D:: AVERAGE P AVERAGE Q R10=R42/D:: R 11=R44/D:: · 1/N (SUM OF P) SQRD 1/N (SUM OF Q) SQRD R12=R46/D:: R13=R9*R9:: (AVERAGE P) SQRD (AVERAGE Q) SQRD R14=R10*R10:: · R15=R11-R13:: (STANDARD DEVIATION OF P) SORD R16=SQRT R15:: STANDARD DEVIATION OF P (R17=R12-R14:: . (STANDARD DEVIATION OF Q) SORD R 18=SQRT R 17:: STANDARD DEVIATION OF Q R 19=R16*R18: STANDARD DEV. P . STANDARD DEV. O R 20=R48 /D:: 1/N (SUM OF (P*)) R21=R9+R10:: AVERAGE P . AVERAGE Q R22=R20-R21:: 1/N (SUM OF (P*Q))-(AV.P*AV.Q) R23=R22/R19:: CORRELATION COEFFICIENT R R24=R18/R16:: STANDARD DEV.Q/STANDARD DEV.P. R25=R23*R24:: REGRESSION COEFFICIENT. M. R 26=R25+R9:: Mo(AV.P) R27=R10-R26:: C=AV.Q-Ma(AV.P) R28=R23*R23:: R SQRD R 29=1-R28: 1-R. SQRD R 30=SQRT R29:: · SQRT (1-(R. SQRD) E=STANDARD ERROR OF ESTIMATE R 31=R13 *R 30:: R 40=R 32/100:: SUM OF I IN INCHES R 40=R 40 /D:: · AVERAGE I FOR GROUP LINE PRINT R40, 1:2:: AVERAGE I FOR GROUP (PRINT N7,2:: NUMBER IN A.P.I. GROUP (OR MOVEMENT OF GROUP (PRINT R9,1:2:: AVERAGE P PRINT R10, 3:1:: AVERAGE Q PRINT R25,4:1:: REGRESSION COEFFICIENT M PRINT R27, 3:1:: INTERSEPT OF REGRESSION EQUATION C PRINT R31, 3:1:: STANDARD ERROR OF ESTIMATE COUNT OF NUMBER OF GROUPS/SEASON N6=N6+1:: PRINT GROUP REFERENCE NUMBER PRINT N6, 3:: JUMP IF H%N@12:: END OF SEASON COUNT UP TO 5 N7=0:: JUMP @17:: · RETURN TO PROCESS ANOTHER GROUP OF VALUES

12)LINE **OUTPUT 19** 3 OUTPUT 17 C SUBR 13:: OUTPUT 40 BLANKS COUNT OF REF. NUMBERS EACH SEASON N6=0:: -LINES 10 EXIT 21)0T=R3:: DAILY RUNOFF Q . . DAILY RAINFALL P. ET=R7:: -DAILY A.P.I. LT=D1:: Ι. R=ET+R:: SUM OF P R2=0T+R2:: SUM OF Q SUM OF I R32 = L∓+R32 :: P SQRD -MT=ET+ET::-R 4=MT+R 4:: SUM OF P SQRD IT=OT+OT:: O SORD R6--R6+IT:: SUM OF Q SQRD P * Q JT=ET+OT:: SUM OF P * Q R8=R8+JT:: JUMP UNLESS T=N4@17:: N4=NUMBER OF A.P.I. GROUP DO NOT RETURN TO REF.21 AGAIN THIS SEASON G=22:: T=0 JUMP @18 13) LINE VARY H=1:1:40:: C C OUTPUT 40 BLANKS OUTPUT O:: -REPEAT H:: C EXIT A.P.I.*(K*1000) 6)F4=F2*F1:: @30F4:5038:: PUT F4 INTO ACCUM: BY 2 ** -38 56H1:20F4:: DIVIDE BY 1000*2**-38:PUT ANS.INTO F4 2 A.P.I./1000 + YESTERDAYS RAINFALL F4=F4+C:: A.P.I.AT 9.0-A.M. THIS MORNING F2=F4::-C=C1:: MAKE C=TODAYS RAINFALL F3=F4*10000000:: * A.P.I. BY 10,000,000 E.G. 770000000 EXIT

START 15

:: TO CALCULATE RUNOFF FROM EQUATIONS AND RAINFALL SETS ZXN(12)WVD(2)YUT SETV E(4)F(4)G(4)H(4)I(366)P(366)AR(366)CMKBJ(4)L(4) SETR 12 5) VARY Z=1:1:4:: READ IN EQUATIONS FOR FOUR SEASONS ŘEAD EZ:: ~ INTERCEPT OF C GRAPH WHEN P%O SLOPE OF C GRAPH WHEN P%O INTERCEPT OF M GRAPH WHEN P%O SLOPE OF M GRAPH WHEN P%O INTERCEPT OF C GRAPH WHEN P=O READ FZ:: READ GZ:: READ HZ:: READ JZ:: SLOPE OF C GRAPH WHEN P=0 READ LZ:: REPEAT Z ENTER NEXT DATA TAPE WAIT:: • . . READ K::RECESSION FACTOR E.G. 0.87READ I::VALUE OF SEPT 30TH OF PREVIOUS WATER YEARREAD P::VALUE ON SEPT 30TH OF PREVIOUS WATER YEARREAD D::DATE AT START OF PREVIOUS WATER YEAR ENTER TAPE OF RAINFALL DATA 4) WAIT:: 3)READ D2::DATE OF FIRST RAINFALL ON TAPED1=D+10000::LAST YEARS DATE +10000 JUMP-UNLESS D2-D1@4::CHECK THAT CORRECT TAPE HAS BEEN ENTERED D=D2:: SET VALUE OF D TO THIS YEARS VALUE SUBR-8:: OUTPUT 40 BLANKS OUTPUT-27:: FIGURE SHIFT SYMBOL LINES 6 -Z=1 X≈0 2)READ PZ:: READ RAINFALL COUNT OF DAYS FROM OCTOBER 1ST Z=Z+1:: -JUMP @2 COUNT OF MONTHS FROM OCTOBER 1)X=X+1:: NX=Z-1:: JUMP IF X=12@6:: NUMBER OF DAYS TO END OF MONTH X AT END OF WATER YEAR READ A:: TOTAL MONTHLY RAINFALL READ A:: TRIGGER 20 6)R=0U=0 T=1 B=0 X=1 Z-Z-1 W=0

VARY V=1:1:Z::Z=NUMBER OF DAYS IN YEARB=B+PV::SUM YEARS RAINFALLIV=I*K::I=I ON PREVIOUS DAY *KIV=IV+P::I + PREVIOUS DAYS RAINFALLI=IV::SET I EQUAL TO THIS DAYS VALUEP=PV::SET P EQUAL TO THIS DAYS VALUEJUMP-IF P\$.001::USE DIFFERENT EQUATIONS IF P=0C=FT*IV::SLOPE OF C EQUATION * A.P.I.C=C+ET::INTERCEPT OF REGRESSION OF Q ON PM=HT*IV::SLOPE OF M EQUATION * A.P.I.M=M+GT::SLOPE OF REGRESSION EQUATION OF Q ON PRV=PV*M::RAINFALL * SLOPERV=RV+C::CRAINFALL * SLOPE) + INTERSEPTJUMP @11::JUMP TO PRINT INSTRUCTION M=HT+IV:: M=M+GT:: RV=PV+M:: RV=RV+C:: JUMP @11:: 12)RV=LT+IV::EQUATION WHEN P =0RV=RV+JT::--RUNOFF WHEN P=011)W=W+1::COUNTPRINT RV;4::PRINT OUT CALCULATED RUNOFF VALUES JUMP UNLESS W=8@10::EIGHT NUMBERS PER LINE LINE V=0 10)R=R+RV:: SUM CALCULATED RUNOFF JUMP UNLESS V=NX@9::JUMP @9 UNLESS AT END OF MONTH ₩=0 · LINE OUTPUT 1:: 1 OUTPUT 17:: C LINES 2 -SUBR 8::OUTPUT 40 BLANKSX=X+1::COUNT OF MONTHSU=U+1::COUNT UP TO QUARTERS JUMP UNLESS U=3@9::JUMP @9 UNLESS START OF NEW SEASON T=T+1:: - COUNT FOR NEW SEASONS EQUATIONS U=0 ··· JUMP IF X=13@7:: END OF YEAR • PRINT OUT 9) REPEAT V:: RETURN TO READ NEW RAINFALL VALUE 7)SUBR 8:: OUTPUT 40 BLANKS LINEPRINT B,2:2::ANNUAL RAINFALLPRINT R,5:1::ANNUAL RUNOFF (CALCULATEDPRINT K,0:2::RECESSION FACTORPRINT 1,2:2::A.P.1. ON SEPTEMBER 30THPRINT P,1:2::RAINFALL ON SEPTEMBER 30THPRINT D,6::-DATE AT START OF WATER YEARJUMP @4::-RETURN FOR NEW RAINFALL DATA LINE -8)VARY Y=1:1:40:: (OUTPUT 0:: - (0 COUTPUT 40 DLAMMO REPEAT Y:: C EXIT

START 5

:: PROGRAMME TO PLOT THE ACCUMULATION OF DAILY RUNOFF ::DEFICIENCY OR EXCESSES FROM A DAILY MEAN OF AN ELEVEN ::YEAR RECORD SETS J SETV QX(3) ER(1) SETF GRAPH PLOT SETR 6 11 YEAR TOTAL E.G.155225.5 M.G.(CALCULATED) AVERAGE DAILY FLOW OVER 11 YEAR PERIOD 6)READ Q:: ିଢ≕ହ∕4018:: READ X:: MILLION GALLS/0.1 MM STEP E.G.10 READ X3:: NUMBER OF 0.1 MM STEPS/DAY E.G.1 X1=0 X2 = 05)WAIT:: INSERT ONE YEARS RUNOFF DATA VARY J=1:1:40::C OUTPUT 0:: - COUTPUT 40 BLANKS REPEAT J:: C J=0 E=0 E=GRAPH O, E:: LOWER PEN DAILY RUNOFF VALUE SUBTRACT AVERAGE D 2)READ R:: R=R-Q:: SUBTRACT AVERAGE DAILY FLOW R1=R/X:: ACCUMULATION OR DEFICIENCY TO SCALE X1=R/X1:ACCOMULATION ORX2=X2+R1::ADD ON RESIDUALX1=X1+X3::ADD ON RESIDUAL ADD ON RESIDUAL E=GRAPH X1, X2:: DRAW LINE BETWEEN PRESENT VALUE AND LAST JUMP @2:: - REPEAT FOR NEXT VALUE 1)J=J+1:: DATA HAS 1C TRIGGER AT END OF MONTH JUMP IF J=3@3::START OF CALENDAR YEAR X1≔0+X1 X2=50+X2:: MAKE $X_{2=5}$ M.M. E=GRAPH X1, X2:: DRAW VERTICAL LINE 5 M.M.LONG X2=-100+X2:: MAKE X2=-10M.M. E=GRAPH X1, X2::DRAW VERTICAL LINE -10 M.M.LONG X2=50+X2:: - MAKE X2=5 M.M. E=GRAPH X1, X2::DRAW VERTICAL LINE 5 M.M.LONG JUMP IF J=12@4::AT END OF WATER YEAR JUMP @2 3)X1=0+X1:: C X2=100+X2:: C E=GRAPH X1, X2::C X2=-200+X2:: CDRAW VERTICAL LINE 20 M.M.LONG TO E=GRAPH X1, X2:: (MARK START OF NEW CALENDAR YEAR X2=100+X2:: - (E=GRAPH X1, X2::C JUMP @2 4) E=0 E=PLOT O, E:: (RAISE PEN JUMP @5 44 START 6

1954 1955 54500 1001 1004 1007 1010 1013 1016 1019 1022 1025 1028 1031	000 0.09 0.03 0.07 0.05 0.34 0.39 0.01 0.07 0.00 1.06 0.21	28.0 150.0 28.0 24.0 19.0 130.0 138.0 62.0 92.0 221.0 110.0	1002 1005 1008 1011 1014 1017 1020 1023 1026 1029	0.00 0.11 0.00 0.29 0.58 0.00 0.83 0.49 0.06	24.0 46.0 28.0 20.0 80.0 190.0 98.0 93.0 65.0 201.0	1003 1006 1009 1012 1015 1018 1021 1024 1027 1030	0.29 0.00 0.06 0.31 0.30 0.00 0.10 0.34 0.01	19.0 40.0 24.0 20.0 100.0 162.0 75.0 216.0 149.0 112.0
1101 1104 1107 1110 1113 1116 1119 1122 1125 1128	0 • 00 0 • 00 0 • 48 0 • 22 0 • 00 0 • 00 0 • 00 0 • 03 0 • 06	98.0 42.0 112.0 85.0 87.0 42.0 36.0 28.0 135.0 320.0	1102 1105 1108 1111 1114 1117 1120 1123 1126 1129	0.05 0.90 0.19 0.00 0.00 0.00 0.00 0.29 0.62 0.54	75.0 46.0 100.0 120.0 62.0 42.0 33.0 62.0 120.0 180.0	1103 1106 1109 1112 1115 1118 1121 1124 1127 1130	0.06 0.22 0.05 0.07 0.00 0.09 0.12 0.30 0.91 0.63	46.0 166.0 90.0 100.0 56.0 38.0 28.0 71.0 359.0 320.0
120 1 120 4 120 7 12 10 12 13 12 16 12 19 1222 1225 1228 1231	0.66 0.33 0.00 0.00 0.00 0.07 0.08 0.28 0.28 0.34 0.09 0.00	275 • 0 290 • 0 58 • 0 129 • 0 95 • 0 87 • 0 69 • 0 85 • 0 56 • 0 31 • 0	1202 1205 1208 1211 1214 1217 1220 1223 1226 1229	0.34 0.00 0.96 0.12 0.31 0.08 0.07 0.00 0.03 0.00	310 • 0 200 • 0 99 • 0 327 • 0 76 • 0 65 • 0 45 • 0 67 • 0 52 • 0	1203 1206 1209 1212 1215 1218 1221 1224 1227 1230	0.59 0.00 0.08 0.29 0.00 0.07 0.09 0.12 0.10 0.00	275 • 0 70 • 0 137 • 0 73 • 0 144 • 0 66 • 0 58 • 0 25 • 0 50 • 0 34 • 0

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DATE	EXCESS	DATE	EXCESS	DATE	EXCESS
5 4 100 1, 5 4 100 4 5 4 100 7 5 4 10 10 5 4 10 13 5 4 10 16 5 4 10 19 5 4 10 22 5 4 10 25 5 4 10 28 5 4 10 3 1	903.7 969.2 976.2 955.1 921.5 1073.4 1358.4 1454.9 1674.1 1918.5 2153.9	541002 541005 541008 541011 541014 541017 541020 541023 541026 541029	895 • 7 977 • 4 971 • 2 944 • 2 95 4 • 9 1188 • 1 1405 • 1 1497 • 9 1696 • 4 2041 • 3	541003 541006 541009 541012 541015 541018 541021 541024 541027 541030	884.0 981.3 963.1 933.2 1003.1 1282.2 1434.8 1631.9 1780.8 2098.3
541101 541104 541107 541110 541113 541116 541119 541122 541125 541128	2200.6 2243.9 2406.1 2532.2 2681.9 2722.9 2731.4 2720.0 2840.9 3354.3	541102 541105 541108 541111 541114 541117 541120 541123 541126 541129	2230.3 2252.1 2454.3 2595.2 2702.0 2728.2 2730.1 2740.1 2903.8 3461.6	541103 541106 541109 541112 541115 541118 541121 541124 541127 541130	2238.6 2349.1 2495.1 2643.3 2717.6 2730.6 2725.0 2766.8 3143.5 3672.4
541201 541204 541207 541210 541213 541216 541219 541222 541225 541228 541231	3850 • 0 4419 • 6 4584 • 8 4777 • 4 4862 • 8 5204 • 0 5296 • 1 5360 • 7 5398 • 1 5448 • 7 5458 • 0	541202 541205 541208 541211 541214 541217 541220 541223 541226 541229	4053.4 4541.7 4632.3 4808.6 5078.8 5234.4 5318.4 5368.3 5421.9 5461.4	541203 541206 541209 541212 541215 541221 541221 541224 541227 541230	4231.0 4567.7 4707.8 4836.8 5159.5 5257.5 5335.5 5361.0 5433.1 5460.8
550101 550104 550107 550110 550113 550116 550119 550122 550125 550128 550131	5 45 2 • 3 5 441 • 4 5 425 • 7 5695 • 6 5 739 • 7 5 738 • 3 5 7 19 • 7 5 780 • 0 6042 • 8 6 128 • 4 6 199 • 2	550102 550105 550108 550111 550114 550120 550123 550126 550129	5446 • 1 5437 • 4 5418 • 4 5725 • 3 5747 • 2 5731 • 1 5711 • 3 5804 • 2 6080 • 8 6 159 • 0	550103 550106 550109 550112 550115 550121 550121 550124 550127 550130	5 444 • 2 5 432 • 1 5 45 1 • 5 5 7 36 • 9 5 7 47 • 5 5 7 25 • 9 5 7 12 • 5 5 9 47 • 6 6 102 • 4 6 185 • 0

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550201 550204 550210 550213 550216 550219 550222 550225 550228	6209.6 6253.7 6306.5 6372.7 6361.6 6335.1 6305.6 6270.7 6233.8 6191.4	550202 550205 550211 550214 550217 550220 550223 550226	6218.9 6270.9 6350.1 6370.8 6355.3 6326.9 6294.5 6258.5 6220.0	550203 550206 550212 550215 550218 550221 550221 550224 550227	6230.5 6272.8 6367.9 6366.8 6348.1 6318.6 6283.2 6246.2 6204.6
550301 550304 550310 550313 550316 550319 550322 550325 550328 550331	6 177 • 7 6 147 • 3 6 130 • 0 6 136 • 2 6 168 • 7 6 405 • 4 6 450 • 3 6 436 • 7 669 2 • 4 6 784 • 4 6 79 4 • 7	550302 550305 550311 550314 550317 550320 550323 550326 550329	6164.8 6143.3 6128.0 6151.6 6204.8 6435.1 6448.2 6439.1 6743.5 6791.7	550303 550306 550312 550315 550318 550321 550324 550327 550320	6 15 2 • 3 6 137 • 5 6 124 • 0 6 159 • 5 6 298 • 1 6 447 • 5 6 442 • 8 65 47 • 3 6 767 • 6 6 792 • 2
550401 550404 550410 550413 550416 550419 550422 550425 550428	6795 • 4 6816 • 0 6802 • 6 6771 • 4 6724 • 7 6670 • 7 6613 • 4 6553 • 8 6493 • 8 6431 • 5	550402 550405 550411 550414 550417 550420 550423 550426 550429	6799 • 2 6815 • 6 6794 • 4 6757 • 1 6707 • 2 665 1 • 8 6593 • 6 6534 • 3 6473 • 2 6410 • 5	550403 550406 550412 550415 550418 550421 550421 550427 550430	6807 • 7 6810 • 4 6782 • 5 6741 • 6 6689 • 2 6632 • 8 6573 • 8 6514 • 3 6452 • 5 6389 • 2
550501 550504 550507 550510 550513 550516 550522 550525 550528 550531	6369 • 2 6315 • 3 6289 • 4 625 2 • 0 6208 • 2 615 7 • 2 610 4 • 4 60 46 • 3 5992 • 2 5927 • 9 5860 • 9	55 05 0 2 55 05 05 55 05 08 55 05 11 55 05 14 55 05 20 55 05 23 55 05 26 55 05 29	6352•4 6322•8 6270•1 6237•7 6192•5 6138•2 6085•6 6029•6 5971•4 5905•8	55 05 03 55 05 06 55 05 09 55 05 12 55 05 15 55 05 21 55 05 24 55 05 27 55 05 30	6333•2 6307•2 6252•6 6219•9 6175•9 6121•6 6065•7 6012•0 5950•0 5883•5

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59 100 1 59 100 4 59 1007 59 10 10 59 10 13 59 10 13 59 10 19 59 10 22 59 10 25 59 10 28 59 10 28	-267.2 -337.8 -408.7 -479.3 -550.0 -620.6 -688.0 -755.8 -822.8 -822.8 -831.0 -893.0	591002 591005 591011 591014 591014 591020 591023 591023 591029	-290.7 -361.4 -432.2 -502.9 -573.5 -643.9 -710.9 -778.7 -818.6 -851.2	591003 591006 591009 591012 591015 591018 591021 591024 591027 591030	-314.2 -385.1 -455.7 -526.5 -597.1 -665.5 -733.7 -801.0 -814.2 -871.8
591101 591104 591107 591110 591113 591116 591119 591122 591125 591128	-915.0 -982.2 -1049.8 -1104.8 -1163.3 -966.5 -730.9 -658.4 -633.6 -593.1	591102 591105 591108 591111 591114 591117 591120 591123 591126 591129	-937.2 -1004.8 -1072.5 -1126.0 -1071.6 -961.3 -683.6 -660.9 -601.1 -593.4	591103 591106 591109 591112 591115 591118 591121 591124 591127 591130	-959.6 -1027.4 -1085.3 -1147.6 -1010.5 -851.1 -664.1 -667.0 -592.9 -593.6
59 120 1 59 120 4 59 120 7 59 1210 59 1213 59 1213 59 1219 59 1222 59 1225 59 1228 59 1221	-588.7 -596.4 -320.4 45.9 171.9 207.8 261.8 386.0 536.5 738.3 838.5	591202 591205 591211 591214 591217 591220 591223 591223 591226 591229	-591.7 -579.7 -212.2 106.0 188.7 232.9 303.5 437.0 645.6 788.3	591203 591206 591212 591212 591215 591221 591221 591224 591227 591230	-596.0 -521.3 -55.9 145.9 198.4 251.5 324.9 461.5 705.8 819.9
600 10 1 600 10 4 600 10 7 600 110 600 113 600 116 600 119 600 122 600 125 600 128 600 13 1	854.3 918.0 988.9 1008.2 1007.4 991.2 1138.9 1481.9 1688.7 1781.9 2080.1	600102 600105 600108 600111 600114 600117 600120 600123 600126 600129	864.6 958.8 1000.1 1007.7 1003.2 984.4 1171.4 1580.2 1711.4 1815.3	600 103 600 106 600 109 600 112 600 115 600 118 600 121 600 124 600 127 600 130	867.2 976.9 1008.2 1009.5 997.8 1029.7 1285.7 1653.4 1729.1 1844.5

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531101 531105 531109 531113 531117 531121 531125 531129、	0-35 0-06 0-02 0-15 0-00 0-00 0-00 0-00	531102 531106 531110 531114 531118 531122 531126 531130	0.03 0.03 0.28 0.00 0.00 0.12 0.00	531103 531107 531111 531115 531115 531123 5 31123	0 • 18 0 • 13 0 • 02 0 • 00 0 • 03 0 • 00 0 • 05	531104 531108 531112 531116 531120 531124 531128	0 • 00 0 • 33 0 • 12 0 • 00 0 • 00 0 • 16 0 • 00
531201 531205 531209 531213 531217 531221 531225 531229	0.00 0.02 0.05 0.12 0.00 0.02 0.00 0.02	531210 531214 531218 531222 531226	0•05 0•00 0•15 0•10	531203 531207 531211 531215 531219 531223 531227 531231	0•17 0•03 0•00 0•00 0•00 0•18 0•00 0•00	531204 531208 531212 531216 531220 531224 531228	0.03 0.00 0.02 0.00 0.00 0.00 0.00
540101 540105 540109 540113 540117 540121 540125 540129	0.00 0.15 0.00 0.21 0.01 0.04 0.00 0.22	540102 540106 540110 540114 540118 540122 540126 540130	0.09 0.00 0.18 0.15 0.08 0.00	540103 540107 540111 540115 540119 540123 540127 540131	0.08 0.00 0.20 0.20 0.00 0.00 0.00 0.08	530104 540108 540112 540116 540120 540124 540128	0 • 25 0 • 05 0 • 35 1 • 25 0 • 00 0 • 09
540201 540205 540209 540213 540217 540221 540225	0.00	540202 540206 540210 540214 540218 540222 540226	0•22 0•15 0•04 0•06	54 0 219	0.02	54020 ³ 540212 540216 540220 540224	0•55 0•07 0•05

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2(0.00 0.00 0.31 1(2.24	0.00 0.22 0.00	0•00 0•88 0•10	0.00 0.00 0.00	0.00 0.00 0.00	0•00 0•05 0•56	0.00 0.00 0.00	0.00 0.00 0.00	0.00 0.00 0.05	0.00 0.00 0.07
2(0•35 0•02 0•00 1(2•09	0.03 0.12 0.00	0•18 0•15 0•00	0•00 0•28 0•16	0.06 0.00 0.00	0.03 0.00 0.12	0•13 0•00 0•05	0•33 0•00 0•00	0.02 0.03 0.00	0.03 0.00 0.00
2(0.00 0.00 0.02 0.00 1(0.04 0.02 0.00	0•17 0•12 0•18	0•03 0•15 0•00	0.02 0.00 0.00	0•05 0•00 0•18	0.03 0.00 0.00	0•00 0•10 0•00	0•05 0•00 0•02	0•00 0•00 0•15
2(0.00 0.00 0.04 0.08 1(0•00 0•05 0•08	0.08 0.21 0.00	0•25 0•18 0•00	0•15 0•20 0•00	0•09 0•35 0•00	0.00 0.01 0.00	0•05 0•15 0•09	0.00 0.00 0.22	0•00 1•25 0•24
2(0•04 0•00 0•02 1(2•55	0•00 0•55 0•04	0.00 0.25 0.02	0.00 0.04 0.09	0.00 0.00 0.45	0•22 0•07 0•00	0•00 0•00 0•05	0.08 0.06 0.04	0•20 0•13	0 • 15 0 • 05
2(0.00 0.05 0.05 0.00 1(2.22 2(0•25 0•00 0•13	0.02 0.02 0.08	0•24 0•00 0•06	0.00 0.00 0.14	0.32 0.00 0.00	0•16 0•00 0•00	0.02 0.00 0.06	0.00 0.12 0.40	0.00 0.00 0.10

0.04 0.00 0.00	0•21 0•00 0•00	0.04 0.00 0.00	0•03 0•00 0•00	0.02 0.00 0.00	0•00 0•04 0•00	0.00 0.09 0.00	0•00 0•00 0•00	0 • 00 0 • 00 0 • 00	0•00 0•00 0•25
1(0.72 2(0.75 0.00 0.27 0.00 1(0•73 0•12 0•27	0•23 0•11 0•03	0•04 0•35 0•15	0.83 0.00 0.00	0.12 0.02 0.00	0.00 0.00 0.20	0.00 0.00 0.98	0-00 0-00 0-00	0•00 0•05 0•07
5 • 32 2(0 • 00 0 • 00 0 • 00 1(1 • 5 4	0.00 0.00 0.00	0•00 0•05 0•00	0.00 0.00 0.03	0.00 0.09 0.06	0•18 0•00 0•00	0.20 0.03 0.12	0•36 0•00 0•00	0•31 0•00 0•00	0 • 06 0 • 05 0 • 00
2(0.06 0.00 0.00 0.00 1(1.53	0.00 0.04 0.05	0•11 0•09 0•32	0.06 0.00 0.00	0.02 0.03 0.00	0•08 0•05 0•18	0.07 0.03 0.24	0.00 0.02 0.08	0.00 0.00 0.00	0•00 0•00 0•00
2(0.08 0.00 0.65 0.00 1(7.27	0.00 0.04 0.22	0•00 0•02 0•50	0•15 0•00 0•01	0•45 0•30 0•00	0•05 0•28 0•00	0•24 1•16 0•00	0.02 0.75 0.05	0•20 1•50 0•03	0•12 0•45 0•00
2(0.00 0.02 0.06 1(3.46 2(0.07 0.04 0.00	0•00 0•05 0•35	0.00 0.05 0.26	0.00 0.45 0.00	0•15 0•10 0•05	0.01 0.09 0.05	0.06 0.00 0.00	0•18 0•35 0•30	0•56 0•20 0•01

34.04

RUNOFF 1953-5		ITH TRIG	GER 1C				
24.00 12.00 28.00 11.00	21.00 12.00 25.00 115.0	18.00 12.00 23.00 132.0	15.00 12.00 21.00 25.00	15•00 45•00 18•00 22•00	13.00 50.00 14.00 19.00	13.00 28.00 12.00 31.00	12 •00 28 •00 12 •00
57•00 67•00 37•00 20•00 10	55.00 60.00 31.00 23.00	40 •00 46 •00 27 •00 28 •00	33.00 40.00 24.00 25.00	31.00 36.00 23.00 20.00	29.00 58.00 21.00 19.00	42 •00 66 •00 20 •00	60.00 43.00 21.00
18.00 17.00 16.00 40.00 1C	18.00 17.00 17.00 35.00	20.00 17.00 17.00 38.00	23.00 17.00 16.00 28.00	22.00 18.00 15.00 26.00	19.00 21.00 14.00 30.00	19.00 25.00 40.00 30.00	18.00 20.00 62.00
24.00 21.00 70.00 36.00 10	22•00 21•00 145•0 30•00	23.00 21.00 220.0 27.00	31.00 22.00 385.0 25.00	31.00 24.00 232.0 23.00	22.00 40.00 130.0 23.00	21.00 56.00 68.00 22.00	22.00 62.00 48.00
20.00 15.00 100.0 70.00 1c	19.00 17.00 80.00 52.00	19.00 21.00 72.00 42.00	20.00 22.00 63.00 42.00	20.00 22.00 70.00	20.00 54.00 100.0	16•00 82•00 92•00	14.00 90.00 84.00
43.00 86.00 26.00 28.00 10	40 • 00 90 • 00 26 • 00 27 • 00	33.00 80.00 26.00 24.00	31.00 77.00 25.00 22.00	29.00 70.00 24.00 22.00	159.0 50.00 35.00 19.00	145.0 36.00 39.00 19.00	90.00 29.00 31.00

19•00 16•00 15•00 10•00 1.c	19•00 15•00 15•00 10•00	40•00 15•00 13•00 10•00	38.00 15.00 12.00 9.000	31.00 15.00 11.00 9.000	2 4•00 15•00 10•00 9•000	22.00 15.00 10.00	19.00 15.00 10.00
14.00 34.00 23.00 27.00	70.00 30.00 22.00 23.00	110•0 28•00 22•00 22•00	55•00 30•00 22•00 170•0	112.0 35.00 25.00 145.0	231.0 60.00 31.00 97.00	58.00 40.00 32.00 75.00	35.00 29.00 30.00
56.00 80.00 23.00 15.00	37•00 96•00 23•00 15•00	26.00 70.00 21.00 16.00	23.00 48.00 18.00 16.00	22.00 25.00 17.00 16.00	22.00 23.00 16.00 16.00	24.00 24.00 16.00	34.00 24.00 16.00
15.00 12.00 11.00 7.000 1C	14.00 12.00 11.00 8.000	12.00 12.00 10.00 8.000	12.00 12.00 9.000 12.00	12.00 12.00 8.000 9.000	12.00 12.00 7.000 9.000	12•00 12•00 6•000 8•000	12.00 11.00 7.000
7•000 6•000 76•00 80•00 1c	7.000 6.000 318.0 60.00	7.000 6.000 288.0 58.00	6.000 6.000 314.0 46.00	6•000 6•000 473•0 38•00	9.000 6.000 300.0 26.00	7•000 6•000 165•0 17•00	6.000 31.00 180.0
16.00 14.00 70.00 58.00 1c	15.00 20.00 39.00 58.00	14.00 23.00 20.00 47.00	12.00 20.00 22.00 47.00	12.00 16.00 21.00 40.00	12.00 16.00 21.00 35.00	13.00 20.00 19.00	13.00 120.0 28.00

3C 531001							
2 (0 • 00 0 • 00 0 • 88 0 • 00 0 • 00 0 • 00 0 • 05 1 (24.000 15.000 12.000 45.000 28.000 18.000 11.000 22.000	0.00 0.00 0.00 0.00 0.00 0.00 0.56 0.07	21.000 13.000 12.000 50.000 25.000 14.000 115.00 19.000	0-00 0-00 0-00 0-00 0-10 0-10 0-31	18.000 13.000 12.000 28.000 23.000 12.000 132.00 31.000	0.00 0.22 0.05 0.00 0.00 0.00	15.000 12.000 12.000 2 ³ .000 21.000 12.000 25.000
0.35 0.06 0.02 0.15 0.00 0.00 0.00 0.00	57.000 31.000 67.000 36.000 37.000 23.000 20.000 20.000	0.03 0.03 0.28 0.00 0.00 0.00 0.12 0.00	55.000 29.000 60.000 58.000 31.000 21.000 23.000 19.000	0 • 18 0 • 13 0 • 02 0 • 00 0 • 03 0 • 00 0 • 05	40•000 42•000 46•000 66•000 27•000 29•000 28•000	0.00 0.33 0.12 0.00 0.00 0.15 0.00	33.000 60.000 40.000 43.000 24.000 21.000 25.000
0.00 0.02 0.05 0.12 0.00 0.02 0.00 0.02 10	18 • 000 22 • 000 17 • 000 18 • 000 16 • 000 15 • 000 40 • 000 26 • 000	0•04 0•05 0•00 0•15 0•10 0•00 0•18 0•15	18.000 19.000 17.000 21.000 17.000 14.000 35.000 30.000	0 • 17 0 • 03 0 • 00 0 • 00 0 • 00 0 • 18 0 • 00 0 • 00	20.000 19.000 17.000 25.000 17.000 40.000 38.000 30.000	0.03 0.00 0.02 0.00 0.00 0.00 0.00 0.00	23.000 13.000 17.000 20.000 16.000 62.000 28.000
0.00 0.15 0.00 0.21 0.01 0.04 0.00 0.22	24.000 31.000 21.000 24.000 70.000 232.00 36.000 23.000	0.00 0.09 0.18 0.15 0.08 0.00 0.24	22.000 22.000 21.000 40.000 145.00 130.00 30.000 23.000	0.08 0.00 0.20 0.00 0.00 0.00 0.00 0.08	23.000 21.000 21.000 56.000 220.00 68.000 27.000 22.000	0.25 0.05 0.35 1.25 0.00 0.09	31.000 22.000 22.000 62.000 335.00 43.000 25.000

SORTING	OF API	VALUES	FROM	53100	1				
VALUE O	FK = 0	.90							
1.09 0.38 0.68 0.77 10	0.98 0.34 0.61	0.88 0.53 0.55	0.79 1.35 0.60	0.71 1.22 0.54	0.64 1.10 0.48	0.58 1.04 0.99	0.52 0.93 0.90	0.47 0.84 0.81	0.42 0.76 0.78
1.00 1.05 0.67 1(1.25 0.96 0.61	1.16 0.99 0.54	1.22 1.04 0.49	1.10 1.21 0.60	1.05 1.09 0.54	0.97 0.93 0.61	1.01 0.39 0.60	1.24 0.80 0.54	1.13 0.75 0.48
0,43 0,38 0,36 0,50 10	0.39 0.34 0.34	0.39 0.33 0.31	0.52 0.41 0.46	0.50 0.52 0.41	0.47 0.47 0.37	0.47 0.42 0.51	0.46 0.38 0.46	0.41 0.44 0.42	0.42 0.40 0.39
0.45 0.52 2.12 1.30 1(0.41 0.47 1.95	0.37 0.47 1.84	0.41 0.63 1.65	0.62 [.] 0.75 1.49	0.71 0.87 1.34	0.73 1.14 1.21	0.65 1.03 1.03	0.64 1.08 1.07	0.58 0.97 1.18
1.25 0.99 0.98 1(1.17 0.89 0.91	1.05 1.35 0.86	0.94 1.47 0.79	0.85 1.36 0.80	0.77 1.22 1.17	0.91 1.17 1.05	0.82 1.05 1.00	0.82 1.01	0.93 1.04
0.94 0.91 0.46 0.90	0.84 0.87 0.46	1.01 0.79 0.54	0.93 0.73 0.57	1.08 0.65 0.57	0.97 0.59 0.66	1.19 0.53 0.59	1.23 0.48 0.53	1.13 0.43 0.54	1.02 0.51 0.88
1C 0.81 0.43 0.24 1C	0.77 0.39 0.22	0.90 0.35 0.20	0.85 0.32 0.18	0.79 0.28 0.16	0.73 0.26 0.14	0.66 0.27 0.13	0.60 0.33 0.12	0.54 0.30 0.10	0.48 0.27 0.09
0.33 1.42 0.85 1.62	1.05 1.28 1.03	1.68 1.27 1.20	1.74 1.25 1.11	1.60 1.48 1.15	2.27 1.33 1.03	2.17 1.22 0.93	1.95 1.10 1.04	1.76 0.99 1.91	1.58 0.89 1.72

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1.46 1.40 0.64 10 0.36 0.35 0.27 0.63 10 4.02 2.06 10 1.55 1.53	$ \begin{array}{r} 1.31\\ 1.26\\ 0.57\\ 0.39\\ 0.31\\ 0.24\\ 0.59\\ 0.99\\ 4.27\\ 1.67\\ 1.42\\ 1.44\\ \end{array} $	1.18 1.14 0.52 0.35 0.32 0.27 0.53 0.93 4.06 1.57 1.32 1.29	1.06 1.07 0.47 0.42 0.38 0.56 0.48 0.86 4.16 1.42 1.23 1.51	0.96 0.97 0.45 0.44 0.34 0.51 0.58 0.77 3.75 1.27 1.16 1.62	0.86 0.96 0.46 0.42 0.34 0.46 0.97 0.99 3.38 1.15 1.49 1.46	0.96 0.36 0.42 0.45 0.35 0.35 0.92 1.17 3.04 1.18 1.45 1.36	1.06 0.81 0.50 0.48 0.35 0.77 1.07 2.22 2.74 1.07 1.39 1.28	1.31 0.73 0.45 0.43 0.33 0.77 0.98 2.74 2.51 1.03 1.25 1.15	1.49 0.65 0.40 0.39 0.30 0.70 1.09 3.97 2.29 1.10 1.48 1.33
10									
0.90 0 0 10 1	14 1 10 16	34 20 13 15	14 13 7 5	12 23 14 7	12 19 12 11	6 8 9 9	0 3 9 13	0 3 6 2	0 0 1 13
TOTAL P 34.04	, то.	VALUES (TAL Q 314.0		TEMBER [(I()0 1;]) P(

0•90 0•03 0•37 1(0 3•020 3•350	0.05	3.000	0.00	3.000	0•04	3•050
0.90 0.00 0.00 0.00 0.18 0.02	1 12.00 17.00 16.00 35.00 3.000	0•22 0•02 0•02 0•15 0•04	12.00 17.00 15.00 30.00 3.000	0•04 0•12 0•00 0•00 0•20	18.00 18.00 14.00 3.100 3.400	0 • 17 0 • 10 0 • 18 0 • 00 0 • 14	20.00 17.00 40.00 3.000 5.550
1(
0.90 0.00 0.88 0.56 0.00 0.03 0.00 0.00 0.00 0.02 0.00 0.02 0.00 0.02 0.00 0.02 0.18 0.19 1(2 13.00 45.00 115.0 25.00 23.00 18.00 25.00 62.00 26.00 3.300 3.500 6.000 6.800 12.00	0.00 0.10 0.00 0.02 0.05 0.00 0.00 0.00 0.00 0.0	12.00 12.00 20.00 22.00 17.00 20.00 40.00 30.00 3.400 4.200 5.750 7.100	0.00 0.00 0.16 0.00 0.05 0.00 0.00 0.00 0.02 0.00 0.27 0.00 0.15	12.00 12.00 21.00 19.00 19.00 17.00 16.00 38.00 4.000 3.100 5.800 5.400 15.55	0.00 0.00 0.12 0.00 0.03 0.15 0.00 0.00 0.00 0.00 0.00 0.00 0.00	12.00 11.00 23.00 18.00 19.00 21.00 17.00 28.00 3.500 3.100 6.250 5.350 21.90
0.90 0.00 0.03 0.00 0.27 0.59 0.64 0.00 0.00	3 15 • 00 18 • 00 27 • 00 20 • 00 19 • 00 4 • 400 9 • 55 0 5 • 900 9 • 200	0.00 0.00 0.05 0.21 0.00 0.05 0.02 0.44	15.00 14.00 24.00 28.00 8.800 5.250 6.100 7.000 56.25	0.00 0.07 0.00 0.00 0.00 0.00 0.03 0.00	13.00 19.00 23.00 20.00 5.000 5.000 6.100 6.600	0.00 0.31 0.00 0.00 0.00 0.06 0.00 0.00	21.00 31.00 21.00 20.00 4.700 4.900 6.050 6.200
0.90 0.00 0.12 0.00 0.47 0.00 0.90 1(4 21.00 132.0 40.00 24.00 130.0 5.500 6.600	0.00 0.00 0.15 0.00 0.00 0.00 0.31	18.00 25.00 36.00 24.00 31.00 5.400 16.00	0.00 0.05 0.00 0.13 0.08 0.00	25 • 00 22 • 00 37 • 00 80 • 00 7 • 830 5 • 800	0.00 0.13 0.00 0.20 0.14 0.00	23.00 42.00 31.00 100.0 5.900 5.900

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0.90 0.00 0.18 0.03 0.06 0.30 0.14 0.12 0.21 0.21 0.00 0.49 1(5 24.00 40.00 28.00 28.00 28.00 35.70 12.12 7.500 89.35	0.05 0.06 0.02 0.00 0.19 0.06 0.02 0.11 0.22 0.55	28.00 31.00 46.00 28.00 69.00 56.00 11.80 20.25 67.50 18.50	0.00 0.03 0.28 0.00 0.09 0.01 0.02 0.12 0.00	28.00 29.00 58.00 33.00 25.00 52.00 8.000 14.75 26.00	0 • 35 0 • 33 0 • 00 0 • 15 0 • 29 0 • 02 0 • 06 0 • 30 0 • 00	57.00 60.00 43.00 28.00 85.00 34.00 10.63 18.83 13.38
0.90 0.02 0.26 0.45 0.02 0.00 0.05 0.08 1(6 50.00 67.00 19.00 190.0 36.00 45.00 9.300 53.75	0.00 0.00 0.05 0.24 0.30 0.01 0.00 0.18	28.00 66.00 150.0 75.00 62.00 67.00 8.600 75.00	0.03 0.25 0.11 0.81 0.11 0.00 0.00 0.41	55.00 28.00 46.00 93.00 65.00 24.63 16.50 92.50	0.00 0.00 1.13 0.04 0.51 0.27	33.00 24.00 40.00 46.00 58.00 26.00 137.5
0.90 0.17 0.00 0.33 0.00 0.15 0.08 0.13 1(7 162.0 42.00 71.00 87.00 41.57 32.60 42.50	0 • 00 0 • 00 0 • 15 0 • 00 0 • 05 0 • 00	138.0 42.00 135.0 13.75 42.75 80.00	0.00 0.04 0.75 0.00 0.00 0.25	98.00 42.00 120.0 11.00 21.13 156.2	0.00 0.00 0.05 0.16 0.00 0.00	62.00 38.00 66.00 42.25 15.00 52.50
0.90 0.13 0.65 0.00 0.28 0.00 0.04	8 216•0 221•0 46•00 120•0 58•00 95•00	0.00 0.16 0.24 0.18 1.40 0.00	92.00 110.0 100.0 87.00 99.00 76.00	0.62 0.00 0.05 0.03 0.10 0.06	65.00 98.00 90.00 62.00 327.0 24.80	0.08 0.05 0.40 0.00 0.00 0.03	149•0 75•00 85•00 56•00 144•0 86•25
1(
0.90 0.11 0.08 0.45 0.27 0.00	9 201.0 100.0 320.0 290.0 129.0	0•00 0•67 0•26 0•00 0•20	112•0 359•0 275•0 200•0 77•00	0•17 0•04 0•14 0•00 0•17	166•0 320•0 310•0 70•00 73•00	0•01 0•35 0•32 0•04 0•00	112•0 180•0 275•0 137•0 70•00

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AUTUMN	0.00	45	0.05	8.6	14.96	7.9	5.4	1
AUTUMN	0.25	90	0.06	13.7	54.77	. 10 . 2	8.0	2 ·
AUTUMN	0.40	151	0.07	18.8	22.11	17 . 2 ·	14.2	3
AUTUMN	0.60	1 64 ·	0.09	22.1	35.31.	19.1	16.6	. 4
AUTUMN	0.80	110	0.07	. 33.3	106.51	26.0	22.5	5
AUTÚMN	1.00	107	0 .12	43.6	164.53	24.6	32 2	6
AUTUMN	1.20	83	0.16	56.6	64.27	46.7	36.8	7
AUTUMN	1.40	104	0.13	72.8	147.19	53.4	43.7	3
AUTUMN	1.70	32 ·	0.19	98.0	64.50	86.0	68.1	9
AUTUMN	2.20	76	0.23	164.3	247.75	108.3	37.5	10
WINTER	0.00	48	0.05	14.6	0.48.	14.6	4.5	11
WINTER	0.25	65	0.05	26.0	195.67	16.9	45.2	12 ·
WINTER	0.40	129	0.05	34.7	90.03	29.8	63.9	13
WINTER	0.60	137	0.09	48.9	'74.06	42:0	66.1	14
WINTER	0.80	147	0.10	51.0	120.82	38.7	47.5	15
WINTER	1.00	143	0.12	51.3	100.78	39.2	45.3	16
WINTER	1.20	103	0.10	63.5	3 5.08	60.1	70.3	17
WINTER	1.40	114	0.14	77.0	133.91	58.5	58.1	18
WINTER	1.70	84	0.12	102.4	113.85	88 . 2 ·	82.8	19
WINTER	2.20	23	0.11	118.8	264.68	90.5	86.3	20

Ş	SPRING	0.00	122 ·	0.05	7.5	17.89	6.6	3. 8	21
9	SPRING	0.25	138	0.04	9.8	11.48	9.3	4.9	22 ·
Ş	SPRING	0.40	163	0.05	13.5	30.01	12.0	14.5	23
9	SPRING	0.60	124	0.06	16.0	24.99	14.7	8.2	24
	SPRING	0.80	136	0.09	21.8	27.93	19.2	15.0	25
S	SPRING	1.00	103	0.14	32.2	91.45	19.7	16.8	26
ŝ	SPRING	1.20	77	0.10	40.5	105.28	30.5	21.3	27
Ś	SPRING	1.40	58	0.09	53.1	79.38	45.7	27.4	28
Ş	SPRING	1.70	26	0.06	71.5	-126.35	79.1	53.4	29
ç	SPRÍNG	2.20	4	0.23	155.2	-34.60	163.4	46.2	30
						1			
	SUMMER	0.00	76	0.05	3.6	5.83	3.3	1.2	31
ç	SUMMER	0.25	87	0.06	6.2	3.61	6.0	3. 2 ·	3 2 ·
ç	SUMMER	0.40	150	0.08	7.6	15.02·	6.4	4.7	33
S	SUMMER	0.60	136	0.08	10 . 2 ·	21.01	8.6	7.7	34
Ş	SUMMER	0.80	152 ·	0.12	16.2	60.88	9.0	14.9	35
	SUMMER	1.00	155	0.11	22.1	47.62	17.0	17.0	36
ŝ	SUMMER	1.20	80 [°]	0.09	29.6	4.59	29.2	23.3	37
5	SUMMER	1.40	71	0.14	39.0	44.36	32.9	34.7	38
9	SUMMER	1.70	43	0.12·	45.6	197.27	22.3	34.4	39
Ş	SUMMER	2.20	62	0.21	131.4	275.03	73.3	89.5	40

	Ι	Ν	Ρ	ହ	М	C ·	E	
AUTUMN	0.00	2 ·	0•04	2•9	-1.25	3•0	0.0	1
AUTUMN	0•25	59	0.06	9•2·	34•53	7•2	6•5	2 ÷
AUTUMN	0•40	99	0•07	16•8	39.01	1 4•3	11•5	3
AUTUMN	0•60	128	0•08	17•3	21. 86	15-5	13•8	4
AUTUMN	0.80	144	0•0 8	22•9	42•15	19•4	16•9	5
AUTUMN	1.00	104	0•07	31•5	101 •89	24•3	23•7	6
AUTUMN	1•20	98	0•09	41•3	199•74	22•5	29•9	· 7
AUTUMN	1 • 40	107	0•14	52•9	93•24	39•4	35•0	3
AUTUMN	1•70	129	0•15	74•5	85•06	61.5	50•2	9
AUTUMN	2•20	142	0•19	132-9	204•70	93•3	94•7	10
WINTER	0•00	14	0•08	11•7	-0•38	11•7	1.7	11
WINTER	0•25	46	0•04	16 • 1	35•72 ·	14•7	5•5	12 ·
WINTER	0•40	74	0•04	25.8	217.54	16•3	42•1	13
WINTER	0•60	100	0•06	42•7	190•77	30•5	75•2	14
WINTER	0.80	109	0•10	46•9	64•66	40•5	66•9	15
WINTER	1-00	134	0•10	51•1	115.86	39•7	50•4	16
WINTER	1•20	135	0•11	48•8	117•12	35•3	45•4	17
WINTER	1 • 40	139	0•11	58•7	28•27	55•7	63•5	13
WINTER	1•70	177	0•12	79•1	139•72 -	62+5	67•0	19
WINTER	2•20	65	0•13	111•5	115•51	96•6	76•6	23

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	SPRING	0•00	45	0•04	6•9	-2•25	7•0	2•6	21
	SPRING	0•25	102 -	0•05	7•4	19•41	6•4	4•0	22 -
	SPRING	0•40	205	0•05	9•9	8•09	9•5	4.8	23
	SPRING	0•60	125	0•05	15•0	55•42	12•4	15•7	24
ı	SPRING	0•80	121	0•07	15•5	8•35	15•0	8•3	25
	SPRING	1.00	1 1 6	0•10	23•4	60-60	17•4	14•7	26
	SPRING	1•20	101	0•13	30•5	97•98	18•2	16•3	27
	SPRING	1•40	114	0•09	38•9	105•15	29•2	20•1	23
	SPRING	1•70	63	0•07	57•6	31•33	55•5	36•1	29
	SPRING	2•20	9	0•16	134•1	-72•52	145•7	57•4	30
•									
	SUMMER	0•00	50	0•05	3•3	8•60	2•9	0•7	A
	SUMMER	0•25	36	0•07	4•2	1.71	4•1	1.6	3 2
	SUMMER	0•40	104	0•05	6•3	9•87	5.8	3•4	33
	SUMMER	0•60	135	0•07	6•9	7•53	6•3	3•1	34
	SUMMER	0.80	112	0•09	10• 6	1 9•99	8.8	6.6	3 5
	SUMMER	1.00	123	0•11	16•7	67•86	9•3	15.7	36
	SUMMER	1•20	141	0•13	20•9	48•07	14.9	15•9	37
	SUMMER	1•40	130	0•11	27•4	30•12	24•0	21.9	33
	SUMMER	1•70	95	0•10	37•2÷	119•41	25•5	32•2	3 9
	SUMMER	2•20	86	<u>0•18</u>	107•9	279•77	57•5	80•9	40
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240 120 280 140 50220	210 120 50280 100120 70190	180 120 280 120 310310	150 120 250 110	150 220120 230 561150	130 880450 210 1320	1 30 500 130 250
350570 330600 660 210 200	30550 20670 430 200 190	180400 30600 370 160210	330 20460 310 200	60310 120400 30270 120230	,30290 150360 240 50280	130420 280580 230 250
180 180 250 140 20260 10	40180 50170 200 180400 150300	170200 170 160 620 300	30230 170 100170 400	20220 20170 170 1 ⁸ 0350	50190 120180 160 380	,30190 150210 20150 230
240 50220 200560 81300 220230	220 210 350620 680 240230	80230 210 10700 480 80220	250310 210 151450 360	150310 50220 2200 300	90220 210240 1253850 270	210 130400 42320 90250
40200 80140 820 41000	190 200150 70900 20920	190 150170 1000 90840	200 210 60 ⁸ 00 450700	200 550220 130720 520	220290 250229 50630 50420	150 40540 20700 40420
430 20900 360 130350 400220 10	250400 860 290 80390 100190	20330 900 260 60310 190	240310 50800 260 140280	290 770 120260 270	321590 20700 250 240	1 ⁶ 1450 500 50240 60220
40190 190 150 100 90	210190 160 40150 100 250090	40400 150 90150 100	30380 150 150 100	20310 150 130 100	240 150 120 100	220 150 110 90
750140 350 400 270310 1450	20000 730700 340 20290 30320 70970	231100 300 230 150300 750	40550 280 220 270	8 31120 120300 220 230	122310 110350 50220 200220	58 0 350600 270250 93 1700
560 360340 90240 160 160	70 370 310800 240 160 160	260 60960 30230 30150	230 700 230 60150	220 480 210 150	180220 50250 50180 120160	200240 230 170 160

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I	N	Р	່	М	C	E	
0 • 87 1 • 15 1 • 34 1 • 5 3 1 • 78 2 • 02 2 • 39 2 • 8 4 3 • 5 1 6 • 33	101 101 101 101 101 101 101 101 103	0.07 0.05 0.10 0.09 0.07 0.10 0.11 0.14 0.16 0.20	11.6 16.2 21.3 23.0 30.8 39.0 49.9 68.4 82.1 142.1	38.3 34.0 33.0 32.9 129.5 147.3 126.8 108.3 112.2 207.0	8.7 14.3 18.0 20.1 21.9 24.0 36.0 52.8 64.1 100.8	9.5 11.8 17.7 18.2 26.6 31.2 33.4 56.0 54.7 87.7	1234567890 10
0 • 85 1 • 30 1 • 57 1 • 79 1 • 97 2 • 17 2 • 42 2 • 68 2 • 93 4 • 58	99 99 99 99 99 99 99 99 102	0.05 0.06 0.10 0.09 0.09 0.12 0.12 0.12 0.12	17.8 43.0 52.6 49.8 43.2 48.1 52.1 73.9 69.9 102.3	14.3 160.6 155.5 17.6 108.0 32.4 95.9 93.3 150.8 146.4	17 • 1 32 • 8 36 • 7 48 • 0 33 • 2 45 • 3 40 • 4 63 • 0 51 • 3 86 • 4	8 • 3 78 • 9 77 • 5 50 • 3 36 • 2 42 • 3 47 • 9 83 • 0 58 • 3 72 • 0	11 12 13 14 15 16 17 18 20
0.61 0.81 0.96 1.14 1.30 1.53 1.76 1.98 2.22 3.27	100 100 100 100 100 100 100 100	0.06 0.07 0.04 0.05 0.06 0.07 0.10 0.10 0.11 0.07	6.6 8.1 9.7 11.1 15.8 17.8 24.7 29.2 35.5 59.9	20.9 8.6 11.0 19.5 26.3 56.4 104.9 91.9 85.6 57.0	5 • 4 7 • 5 9 • 2 10 • 2 14 • 3 13 • 7 14 • 5 20 • 4 26 • 1 55 • 9	3.7 3.6 5.2 4.1 18.2 9.0 16.3 17.3 20.4 40.7	21 22 23 24 25 26 27 29 30
0 • 73 1 • 01 1 • 24 1 • 48 1 • 6 8 1 • 9 3 2 • 15 2 • 45 2 • 95 5 • 8 3	101 101 101 101 101 101 101 101 103	0.05 0.05 0.08 0.09 0.10 0.14 0.14 0.09 0.11 0.15	3.6 5.5 7.3 9.7 12.4 23.2 23.8 28.5 34.5 92.1	5 • 8 11 • 6 5 • 1 11 • 6 35 • 1 68 • 5 30 • 9 76 • 8 138 • 9 279 • 2	3.3 4.9 6.9 8.7 8.8 13.9 19.5 21.8 18.6 50.7	1 • 3 2 • 9 2 • 8 6 • 4 10 • 3 16 • 2 22 • 6 23 • 0 38 • 6 75 • 4	31 32 334 35 36 37 39 40

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1	Ν.	Р	ହ	М	C	E	
0.65 0.91 1.07 1.24 1.46 1.67 2.01 2.41 2.98 5.62	101 101 101 101 101 101 101 101 103	0.07 0.06 0.09 0.06 0.10 0.11 0.16 0.16 0.20	11.9 16.1 19.4 25.1 27.3 39.5 49.1 69.1 82.0 144.8	43.9 14.6 32.8 42.1 81.3 194.6 109.7 78.9 151.5 206.3		9.8 11.2 16.0 19.3 22.6 28.3 36.6 55.7 44.5 90.2	1234567890 10
0.66 1.00 1.25 1.44 1.61 1.79 2.04 2.27 2.51 4.25	99 99 99 99 99 99 99 99 99 99 99 99 99	0.05 0.05 0.10 0.10 0.10 0.09 0.11 0.12 0.12 0.12	17 • 1 41 • 6 48 • 4 54 • 8 44 • 0 49 • 6 45 • 7 71 • 3 82 • 6 97 • 8	14.1 262.8 87.4 159.5 74.4 34.5 78.6 89.6 199.9 104.4	16 • 4 27 • 6 39 • 5 38 • 9 36 • 9 46 • 3 36 • 7 58 • 1 85 • 7	6.9 78.0 74.3 53.7 43.3 38.9 50.1 68.6 72.4 71.9	11 12 13 14 15 16 17 18 20
0 • 46 0 • 62 0 • 74 0 • 89 1 • 07 1 • 26 1 • 48 1 • 68 1 • 91 2 • 93	100 100 100 100 100 100 100 100 100	0.06 0.06 0.04 0.07 0.07 0.07 0.08 0.11 0.12 0.06	6.7 8.2 10.0 11.4 15.5 17.6 22.7 29.6 37.4 59.4	19.9 6.6 14.9 20.8 25.0 57.6 73.7 102.9 84.0 82.4	5.6 7.7 9.1 10.7 13.9 13.5 16.5 18.3 27.5 54.2	3.6 3.6 5.2 17.9 9.7 14.8 16.7 21.5 40.9	21 22 23 24 25 26 27 28 29 30
0.58 0.81 1.00 1.21 1.40 1.62 1.81 2.07 2.52 5.57	101 101 101 101 101 101 101 101 103	0.06 0.07 0.10 0.09 0.14 0.11 0.12 0.10 0.16	3.7 5.9 7.0 9.7 12.9 22.1 21.8 30.0 33.0 94.6	5.5 12.8 6.0 15.2 34.4 68.2 28.0 40.1 63.6 279.5	3.4 5.2 6.6 8.2 9.7 12.6 18.9 25.1 26.7 49.9	1.4 3.0 2.9 5.6 10.8 16.5 19.1 26.8 32.9 76.1	31 32 334 35 37 37 39 40

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I	N	Р	Q	М	Ç	E	
0.53 0.75 0.89 1.05 1.25 1.45 1.76 2.08 2.62 5.08	101 101 101 101 101 101 101 101 103	0.07 0.06 0.11 0.08 0.06 0.10 0.11 0.17 0.15 0.20	11.7 17.1 18.2 23.4 27.3 41.7 46.3 69.5 81.9 147.2	43.1 30.0 24.2 47.4 97.2 189.7 84.9 88.9 145.7 202.5	8.8 15.4 19.5 21.3 22.9 37.0 54.7 60.3 106.0	9.6 10.6 15.7 17.6 20.3 30.9 35.5 53.7 42.9 90.9	1234567890 10
0.53 0.81 1.02 1.20 1.36 1.51 1.74 1.96 2.21 4.01	99 99 99 99 99 99 99 99 99 99 102	0.05 0.05 0.08 0.12 0.10 0.10 0.11 0.13 0.12 0.11	17 • 1 39 • 8 48 • 5 49 • 2 51 • 0 43 • 1 70 • 5 82 • 0 102 • 4	11.8 263.9 65.8 137.0 12.0 178.5 9.0 71.6 210.4 109.4	16 • 5 25 • 5 42 • 9 33 • 2 48 • 1 33 • 9 42 • 1 61 • 4 56 • 9 89 • 9	6 • 9 77 • 5 74 • 7 41 • 1 50 • 9 45 • 6 30 • 8 71 • 1 70 • 0 75 • 5	11 12 13 14 15 16 17 18 20
0.35 0.49 0.58 0.72 0.91 1.08 1.26 1.47 1.70 2.72	100 100 100 100 100 100 100 100 100	0.06 0.05 0.04 0.05 0.07 0.10 0.11 0.11 0.08	6 • 9 8 • 1 10 • 0 11 • 8 15 • 8 15 • 6 24 • 6 27 • 9 37 • 9 59 • 8	19•4 8•3 10•0 21•0 63•1 9•3 74•0 96•6 93•4 61•4	5 • 8 7 • 7 9 • 4 11 • 0 12 • 7 15 • 0 17 • 1 17 • 2 27 • 5 55 • 1	3.6 3.5 5.3 17.3 8.9 14.9 15.9 21.1 42.0	21 22 23 25 26 27 28 29 30
0 • 46 0 • 66 0 • 83 1 • 01 1 • 20 1 • 38 1 • 5 4 1 • 81 2 • 25 5 • 37	101 101 101 101 101 101 101 101 101 103	0.06 0.05 0.10 0.08 0.15 0.10 0.12 0.10 0.17	3.8 6.4 9.6 12.6 21.3 21.4 28.5 34.6 95.5	5 • 4 15 • 4 7 • 1 13 • 5 32 • 5 61 • 8 41 • 4 32 • 0 87 • 3 276 • 8	3.5 5.6 6.3 9.9 12.1 17.3 24.7 25.8 48.9	1 • 4 3 • 4 2 • 8 5 • 8 10 • 7 17 • 5 16 • 2 23 • 4 32 • 6 77 • 7	31 32 334 35 37 37 39 40

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1	N	Р	ହ	М	C	Ε	
0 • 43 0 • 61 0 • 75 0 • 90 1 • 08 1 • 28 1 • 5 4 1 • 85 2 • 37 4 • 6 4	101 101 101 101 101 101 101 101 101 103	0.06 0.09 0.10 0.07 0.08 0.12 0.15 0.17 0.21	11.4 17.3 18.1 22.9 27.9 38.2 47.7 64.4 87.4 148.8	55.3 18.7 18.9 43.5 93.2 133.2 137.5 102.8 95.8 200.8	8.3 16.1 16.4 18.7 21.9 27.5 31.5 48.6 71.4 106.8	8.3 11.2 14.8 18.1 16.5 28.3 37.8 39.0 56.8 90.9	1 2 3 4 5 6 7 8 9 10
0 • 45 0 • 6 7 0 • 86 1 • 02 1 • 17 1 • 31 1 • 5 1 1 • 7 4 1 • 99 3 • 8 3	99 99 99 99 99 99 99 99 102	0.05 0.05 0.11 0.10 0.12 0.09 0.12 0.13 0.12	21.2 34.4 44.9 44.8 56.7 48.6 48.3 66.2 82.5 105.2	81.7 136.7 37.4 94.5 156.1 126.5 7.5 80.1 149.1 123.0	17 • 5 27 • 2 41 • 9 34 • 6 41 • 0 33 • 6 47 • 6 56 • 5 63 • 5 90 • 5	38.8 71.0 70.0 37.0 56.3 47.3 36.0 70.6 62.9 83.1	11 12 13 14 15 16 17 18 20
0.27 0.40 0.48 0.61 0.78 0.94 1.11 1.31 1.54 2.60	100 100 100 100 100 100 100 100 100	0.06 0.05 0.06 0.04 0.05 0.07 0.09 0.13 0.10 0.08	7 • 1 8 • 4 9 • 9 11 • 2 16 • 4 15 • 7 23 • 3 29 • 5 37 • 0 59 • 9	19.3 9.7 6.3 20.9 63.5 8.8 59.7 97.6 93.9 61.6	5 • 9 7 • 9 9 • 6 10 • 4 13 • 4 15 • 1 17 • 8 16 • 9 27 • 6 55 • 0	3.6 3.8 5.2 5.0 17.2 8.6 15.2 15.7 21.1 42.3	21 22 23 24 25 26 27 28 29 30
0.38 0.55 0.71 0.88 1.05 1.20 1.35 1.58 2.01 5.19	101 101 101 101 101 101 101 101 103	0.06 0.05 0.09 0.08 0.08 0.14 0.12 0.09 0.12 0.17	3.8 6.5 9.6 12.4 20.1 22.2 26.2 37.1 95.7	5.3 11.1 7.1 17.6 43.8 68.1 41.6 20.0 42.6 272.3	3.5 6.0 6.2 9.1 10.5 17.3 24.3 32.0 48.4	1.6 3.4 3.2 5.4 11.0 16.6 16.5 22.9 33.6 78.8	31 32 33 35 35 37 38 39 40

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I	N	Р	Q	Μ	С	E	
0 • 37 0 • 5 1 0 • 65 0 • 78 0 • 95 1 • 15 1 • 38 1 • 66 2 • 16 4 • 28	101 101 101 101 101 101 101 101 103	0.05 0.07 0.08 0.10 0.07 0.08 0.12 0.17 0.16 0.20	10.9 18.5 17.6 22.2 28.3 37.3 48.8 63.5 86.9 150.1	103•6 68•4	8 • 4 16 • 5 15 • 9 18 • 0 23 • 0 28 • 6 31 • 2 46 • 3 75 • 6 100 • 1	7 • 9 13 • 6 12 • 6 17 • 2 19 • 2 28 • 4 35 • 8 37 • 9 58 • 2 85 • 7	1234567890 10
0.37 0.57 0.72 0.88 1.02 1.16 1.33 1.54 1.81 3.68	99 99 99 99 99 99 99 99 99 10 2	0.04 0.05 0.09 0.10 0.10 0.13 0.10 0.11 0.13 0.12	21.3 34.0 45.5 41.6 55.0 51.4 53.4 62.4 83.0 105.1		17.6 26.9 41.8 32.7 41.8 36.0 49.8 54.6 63.8 90.8	38 • 8 7 1 • 0 7 0 • 7 35 • 0 5 4 • 3 50 • 5 47 • 9 6 4 • 2 6 2 • 8 8 3 • 6	11 12 14 15 16 17 18 20
0.21 0.32 0.40 0.50 0.68 0.83 0.99 1.18 1.41 2.49	100 100 100 100 100 100 100 100 100	0.06 0.04 0.05 0.04 0.05 0.07 0.09 0.15 0.09 0.09	7 • 4 8 • 3 10 • 4 11 • 1 16 • 1 16 • 3 22 • 4 30 • 3 35 • 5 60 • 8	19 • 1 9 • 2 8 • 1 5 • 2 55 • 6 20 • 2 32 • 3 93 • 0 96 • 8 63 • 7	6 • 2 7 • 9 9 • 9 10 • 9 13 • 4 14 • 9 19 • 4 16 • 7 27 • 1 55 • 2	3 • 7 3 • 6 5 • 4 5 • 1 17 • 4 8 • 4 16 • 3 16 • 1 19 • 8 42 • 7	21 22 23 25 26 27 28 29 30
0.30 0.47 0.62 0.77 0.94 1.07 1.20 1.40 1.83 5.03	101 101 101 101 101 101 101 101 103	0.06 0.05 0.08 0.09 0.13 0.11 0.10 0.12 0.18	4.0 6.6 7.0 9.7 13.0 18.6 19.8 28.2 36.4 97.0	6 • 7 9 • 4 5 • 5 15 • 5 44 • 4 65 • 2 46 • 2 32 • 5 43 • 7 269 • 3	3•6 6•1 6•6 8•4 9•9 14•8 25•1 31•3 49•6	1•7 3•5 3•3 5•7 12•1 15•5 14•7 23•7 31•1 79•7	31 32 334 35 36 37 39 40

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I	N	Р	Q	Μ	С	E	
0 • 32 0 • 45 0 • 57 0 • 69 0 • 85 1 • 04 1 • 25 1 • 5 1 2 • 00 3 • 9 8	101 101 101 101 101 101 101 101 103	0.06 0.07 0.06 0.10 0.07 0.09 0.11 0.16 0.18 0.20	11.0 18.9 16.8 22.1 27.6 37.7 46.7 63.9 89.6 149.7		48•1 75•0	7 • 8 15 • 1 8 • 4 17 • 4 18 • 3 28 • 1 33 • 1 37 • 7 60 • 6 85 • 5	
0 • 32 0 • 48 0 • 6 3 0 • 78 0 • 91 1 • 04 1 • 20 1 • 42 1 • 67 3 • 5 7	99 99 99 99 99 99 99 99 99	0.04 0.05 0.09 0.10 0.13 0.10 0.11 0.11 0.14 0.12	21.2 27.5 51.4 42.9 49.0 54.0 55.6 63.2 80.0 107.9		60•7	38.7 20.2 96.6 40.1 46.1 51.6 48.7 66.2 59.9 85.5	11 12 13 14 15 16 17 18 19 20
0 • 18 0 • 27 0 • 35 0 • 44 0 • 60 0 • 76 0 • 90 1 • 08 1 • 32 2 • 40	100 100 100 100 100 100 100 100 101	0.06 0.04 0.04 0.05 0.06 0.10 0.13 0.10 0.09	7.5 8.6 10.4 11.4 15.9 16.5 20.5 30.0 37.2 60.6	18.9 7.5 14.5 -3.9 52.9 24.4 25.9 80.2 105.7 65.0	6.3 8.2 9.8 11.5 13.1 15.0 18.0 19.5 26.2 54.8	3.8 3.8 5.1 6.3 17.2 8.5 14.6 16.6 19.7 43.1	21 22 23 24 25 26 27 28 29 30
0 • 26 0 • 40 0 • 5 4 0 • 69 0 • 84 0 • 96 1 • 09 1 • 27 1 • 6 7 4 • 9 1	101 101 101 101 101 101 101 101 103	0.06 0.05 0.09 0.08 0.10 0.12 0.10 0.11 0.12 0.18	4.0 6.7 7.3 9.7 13.1 17.8 19.7 28.8 35.6 97.8	6.6 9.9 6.5 17.1 43.2 66.6 45.6 33.5 43.9 268.3	3•6 6•2 6•8 8•4 9•5 15•0 25•2 30•3 50•6	1 • 7 3 • 5 5 • 7 12 • 6 15 • 0 15 • 9 23 • 1 31 • 1 79 • 5	31 32 33 35 36 37 38 39 40

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1	Ν	Р	Q	М	C	Ε	
0.25 0.38 0.50 0.62 0.77 0.94 1.13 1.39 1.83 3.85	101 101 101 101 101 101 101 101 103	0.06 0.07 0.06 0.08 0.08 0.10 0.12 0.16 0.16 0.21	10.9 18.4 17.6 22.3 26.8 37.6 48.1 62.5 87.8 152.0	43.4 58.1 6.0 59.8 20.1 109.1 134.3 92.7 80.1 237.8	8.2 14.6 17.2 17.4 25.3 26.9 32.1 47.3 74.9 101.9	7 • 8 13 • 1 11 • 4 16 • 3 18 • 8 28 • 2 33 • 1 36 • 3 59 • 3 86 • 2	1234567 8910
0.28 0.42 0.55 0.68 0.81 0.93 1.08 1.30 1.53 3.47	99 99 99 99 99 99 99 99 99 99 902	0.04 0.05 0.09 0.08 0.09 0.14 0.11 0.11 0.14 0.12	21.0 27.7 51.8 41.7 46.5 55.9 59.8 62.0 77.3 108.8	90.1 24.2 63.7 107.8 30.6 144.6 76.6 73.9 112.6 131.2		38.6 20.2 96.7 39.2 43.7 49.4 49.4 69.4 69.4 69.4 69.3	11 12 13 14 15 16 17 18 19 20
0.15 0.23 0.30 0.53 0.68 0.81 1.00 1.21 2.32	100 100 100 100 100 100 100 100 100	0.05 0.05 0.04 0.05 0.06 0.09 0.13 0.11 0.09	7.6 8.9 10.5 11.2 15.5 17.1 18.7 31.5 36.6 61.0	23.4 4.8 14.0 -3.3 53.6 34.5 0.5 82.3 97.6 69.5	6.5 8.6 9.9 11.3 12.6 15.0 18.6 20.6 26.3 54.4	3.7 4.2 5.3 6.2 17.2 8.5 10.0 18.7 20.2 43.1	21 22 23 24 25 26 27 28 29 30
0 • 21 0 • 35 0 • 48 0 • 62 0 • 75 0 • 87 0 • 99 1 • 16 1 • 5 3 4 • 81	101 101 101 101 101 101 101 101 103	0.06 0.05 0.09 0.07 0.10 0.11 0.12 0.10 0.13 0.18	4.0 6.8 8.0 9.0 13.6 15.8 21.0 28.8 34.7 98.8	7 • 1 7 • 8 15 • 8 8 • 0 43 • 7 52 • 1 66 • 9 35 • 3 42 • 1 267 • 7	3.6 6.4 8.4 9.2 10.2 13.3 25.3 29.2 51.4	1.7 3.5 5.3 4.6 14.8 12.1 18.1 21.3 31.3 79.0	31 32 33 35 36 37 38 39 40

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I	N	Р	Q	М	C	E	
0 • 22 0 • 34 0 • 45 0 • 5 7 0 • 71 0 • 86 1 • 05 1 • 29 1 • 71 3 • 74	101 101 101 101 101 101 101 101 101 103	0.06 0.07 0.06 0.08 0.08 0.11 0.12 0.16 0.17 0.21	10.9 17.9 18.5 21.0 28.0 37.8 49.5 59.6 87.1 153.6	42.6 72.5 2.9 49.6 20.3 112.3 133.1 92.3 77.8 247.0	8.5 13.2 18.3 17.1 26.4 25.8 33.7 45.2 74.0 102.4	7 • 7 11 • 5 12 • 1 14 • 8 22 • 0 23 • 7 35 • 6 35 • 8 59 • 9 83 • 5	1 2 3 4 5 6 7 8 9 0
0 • 25 0 • 38 0 • 49 0 • 61 0 • 74 0 • 84 0 • 99 1 • 19 1 • 43 3 • 39	99 99 99 99 99 99 99 99 99 102	0.04 0.05 0.09 0.10 0.14 0.10 0.11 0.15 0.12	20.9 28.8 43.9 42.1 49.9 58.0 56.8 63.7 81.4 107.1	87.7 43.6 92.1 10.7 69.0 151.6 18.6 92.0 121.9 129.6	17.0 26.5 36.1 41.3 37.4 54.9 53.6 62.7 91.8	38.6 23.4 75.8 66.0 49.1 50.1 66.0 53.0 60.8 85.4	11 12 13 14 15 16 17 18 19 20
0.12 0.20 0.26 0.35 0.48 0.62 0.74 0.92 1.15 2.26	100 100 100 100 100 100 100 100 100	0.05 0.05 0.05 0.06 0.07 0.09 0.12 0.11 0.10	7 • 7 9 • 0 10 • 4 11 • 5 15 • 3 16 • 3 19 • 8 30 • 1 36 • 9 61 • 4	23.2 5.3 3.8 5.7 51.8 35.7 -0.6 84.7 91.3 69.1	6.5 8.7 10.2 11.3 12.3 14.0 19.9 20.0 26.6 54.5	3.7 4.0 4.7 7.1 17.4 7.6 10.9 18.6 20.2 43.1	21 22 23 24 25 26 27 26 29 30
0.19 0.30 0.42 0.55 0.69 0.79 0.91 1.07 1.44 4.75	101 101 101 101 101 101 101 101 103	0.05 0.09 0.05 0.12 0.12 0.10 0.10 0.14 0.18	4.1 6.8 9.3 13.1 15.1 19.8 29.9 35.3 98.6	6.8 8.6 14.2 22.9 28.1 47.1 64.8 49.1 39.5 267.4	3•7 6•4 7•1 9•8 9•6 13•1 25•2 29•7 51•1	1.8 3.6 5.4 7.5 14.4 11.8 13.6 23.9 31.5 79.3	31 32 334 35 36 37 38 39 40

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I	N	P	ହ	M	C	E	
0 • 19 0 • 29 0 • 40 0 • 52 0 • 65 0 • 80 0 • 97 1 • 20 1 • 58 3 • 6 4	101 101 101 101 101 101 101 101 101 103	0.06 0.06 0.07 0.08 0.11 0.11 0.16 0.17 0.21	11.0 17.6 19.4 20.4 28.1 38.6 45.3 62.5 87.0 154.1	42.7 80.8 15.9 35.1 21.7 111.0 40.7 129.3 81.6 245.4	8.3 13.0 18.4 17.6 26.4 26.3 41.0 41.8 72.9 102.9	7 • 7 11 • 2 14 • 5 13 • 3 21 • 8 24 • 2 28 • 8 38 • 1 61 • 1 83 • 5	1 2 3 4 5 6 7 8 9 10
0 • 22 0 • 34 0 • 44 0 • 55 0 • 67 0 • 77 0 • 92 1 • 10 1 • 35 3 • 31	99 99 99 99 99 99 99 99 99 99 99 99	0.05 0.05 0.08 0.09 0.13 0.10 0.11 0.14 0.12	21.3 28.6 43.0 43.5 48.2 56.8 60.6 61.1 82.8 106.6	88.0 43.6 83.7 31.9 80.1 162.2 65.3 77.2 141.0 95.0	17 • 3 26 • 3 36 • 4 41 • 1 40 • 8 34 • 9 53 • 8 52 • 6 62 • 5 94 • 8	38.8 23.1 75.7 66.2 48.0 51.0 67.5 50.3 60.3 86.7	11 12 13 14 15 16 17 18 19 20
0 • 10 0 • 17 0 • 23 0 • 31 0 • 43 0 • 57 0 • 69 0 • 85 1 • 08 2 • 19	100 100 100 100 100 100 100 100 100	0.05 0.03 0.05 0.04 0.06 0.07 0.09 0.13 0.10 0.11	7.8 9.3 10.0 11.7 13.8 18.3 19.7 30.0 34.8 62.9	22.5 1.2 6.3 8.9 8.6 87.5 -1.2 82.1 78.9 79.1	6 • 7 9 • 3 9 • 7 11 • 4 13 • 3 12 • 6 19 • 8 19 • 5 26 • 6 54 • 4	3.9 4.6 4.3 7.0 8.3 16.5 10.2 19.2 18.2 43.8	21 22 23 24 25 26 27 28 29 30
0 • 16 0 • 27 0 • 38 0 • 50 0 • 6 3 0 • 7 3 0 • 8 3 1 • 00 1 • 3 6 4 • 70	101 101 101 101 101 101 101 101 103	0.05 0.06 0.08 0.12 0.11 0.11 0.11 0.13 0.13	4.1 6.7 9.3 12.7 15.8 19.8 28.4 36.3 98.8	6.7 7.4 16.7 20.1 21.3 58.8 62.2 45.6 40.3 267.1	3.8 6.3 7.3 8.1 10.2 9.3 13.1 23.7 30.9 51.3	1•9 3•5 5•4 7•6 13•9 11•5 14•3 22•4 32•3 79•2	31 32 33 35 35 37 38 39 40

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0.16 0.26 0.36 0.48 0.59 0.74 0.91 1.11 1.50 3.56	101 101 101 101 101 101 101 101 103	0.06 0.05 0.08 0.08 0.08 0.10 0.11 0.17 0.15 0.23	10.9 17.3 20.0 27.6 37.5 46.5 61.5 90.9 151.3	41.0 86.0 16.6 33.0 23.0 106.3 45.8 129.4 118.5 186.9	8.5 12.8 18.7 18.0 25.9 27.0 41.3 40.0 72.8 109.0	7 • 3 10 • 0 15 • 2 14 • 1 21 • 4 22 • 1 31 • 2 37 • 4 62 • 6 90 • 3	1234567890 10
0.19 0.30 0.39 0.49 0.61 0.72 0.85 1.02 1.28 3.25	99 99 99 99 99 99 99 99 102	0.04 0.05 0.07 0.08 0.10 0.12 0.12 0.12 0.10 0.15 0.13	21.9 28.1 41.5 45.7 43.4 57.9 62.7 61.3 84.1 106.1	86.4 45.4 78.7 47.3 65.3 206.4 33.1 100.0 117.2 104.5	18.0 25.7 36.1 41.8 36.8 33.4 58.7 51.4 66.3 93.0	38.9 23.3 72.4 69.6 37.8 53.0 69.0 50.1 63.6 85.2	11 12 13 14 15 16 17 18 20
0.08 0.15 0.20 0.28 0.39 0.53 0.64 0.79 1.02 2.14	100 100 100 100 100 100 100 100 101	0.05 0.03 0.05 0.04 0.06 0.06 0.09 0.13 0.11 0.11	8.0 9.1 10.4 11.5 13.6 18.3 20.4 29.5 35.0 62.6	22.7 1.5 4.8 16.7 1.4 87.3 18.0 62.9 79.2 79.4	6.8 9.0 10.2 10.9 13.5 12.8 18.7 21.6 26.5 54.0	3.9 4.5 4.4 6.9 8.3 16.2 11.5 21.0 18.1 43.9	21 22 23 25 26 27 29 30
0 • 14 0 • 24 0 • 34 0 • 46 0 • 57 0 • 68 0 • 78 0 • 93 1 • 29 4 • 6 4	101 101 101 101 101 101 101 101 103	0.04 0.07 0.06 0.10 0.12 0.12 0.12 0.10 0.14 0.14	4.1 7.0 8.5 9.3 12.0 15.6 20.2 28.6 36.0 99.1	6.6 6.3 17.2 16.1 22.4 40.3 64.6 55.1 39.8 266.4	3.9 6.5 7.2 8.3 9.9 10.8 12.7 23.3 30.5 51.8	2.0 3.6 5.4 7.7 12.3 11.7 14.2 23.0 32.4 79.2	31 32 334 35 36 37 39 30

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1	N	Р	Q	М	C	E	
0.14 0.24 0.33 0.45 0.55 0.69 0.85 1.04 1.42 3.48	101 101 101 101 101 101 101 101 103	0.05 0.06 0.08 0.09 0.10 0.11 0.17 0.16 0.22	10.9 17.0 20.0 20.5 28.8 36.4 47.2 61.2 91.0 151.3	47.8 68.5 17.1 24.8 26.4 101.7 38.9 130.8 118.1 187.4	8 • 4 12 • 8 18 • 7 18 • 9 26 • 4 26 • 5 42 • 7 39 • 5 72 • 5 109 • 3	6.9 10.7 15.2 13.3 21.5 22.9 30.8 37.7 62.6 90.2	1234567890 10
0 • 17 0 • 27 0 • 36 0 • 45 0 • 5 7 0 • 6 7 0 • 79 0 • 96 1 • 21 3 • 18	99 99 99 99 99 99 99 99 99 102	0.04 0.06 0.08 0.09 0.11 0.14 0.10 0.15 0.13	21.7 30.8 39.8 44.3 42.5 55.4 63.7 63.2 86.0 105.3	90.3 29.6 76.0 50.5 66.2 171.4 97.6 101.4 112.2 104.1	18 • 1 29 • 2 33 • 9 40 • 4 36 • 6 37 • 3 50 • 1 52 • 8 69 • 1 92 • 1	38.9 27.5 71.3 69.8 35.0 52.4 71.6 52.3 63.7 85.7	11 12 13 14 15 16 17 18 19 20
0.07 0.13 0.18 0.25 0.36 0.49 0.60 0.74 0.98 2.07	100 100 100 100 100 100 100 100 100	0.04 0.04 0.05 0.03 0.06 0.07 0.10 0.12 0.11 0.11	8.0 9.2 10.5 11.8 13.5 18.0 21.3 28.5 34.9 62.8	23.9 -0.1 5.7 21.8 2.9 75.8 29.8 59.7 75.8 83.1	6.9 9.2 10.2 11.1 13.4 12.8 18.4 21.2 26.8 53.6	3 • 9 4 • 7 4 • 4 7 • 0 8 • 4 16 • 7 13 • 0 19 • 5 19 • 8 43 • 7	21 22 23 24 25 26 27 28 29 30
0.12 0.21 0.31 0.42 0.53 0.63 0.73 0.88 1.21 4.61	101 101 101 101 101 101 101 101 103	0.05 0.06 0.07 0.10 0.12 0.11 0.10 0.14 0.18	4.1 7.0 8.8 9.5 11.7 16.2 19.5 28.7 35.9 99.1	6.3 7.7 17.3 12.5 23.1 38.9 64.9 54.1 39.8 266.5	3.8 6.5 7.6 9.4 11.6 12.2 23.3 30.2 52.0	1.9 3.6 5.5 7.8 12.3 11.8 14.0 23.3 32.5 79.0	31 32 33 35 36 37 38 39 40

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478912346790123579134812462368060480503001501
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MOVING AVERAGE P GREATER THAN ZERO.

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1.39 1.41 1.45 1.48 1.52 1.56 1.60 1.64 1.68 1.73 1.77 1.82 1.999 2.07 2.15 2.33 3(555555555555555555	0.26 0.27 0.28 0.29 0.25 0.23 0.24 0.24 0.24 0.22 0.22 0.23 0.24 0.22 0.23 0.23 0.24 0.22 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.24 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.32 0.32	104.3 105.3 115.1 123.5 114.4 115.3 112.4 120.3 125.7 138.3 133.6 145.1 144.4 161.5 167.9 176.4 185.8 187.8 193.6	47.7 47.1 42.7 38.9 46.4 72.9 71.4 91.0 110.3 159.3 264.6 283.6 281.7 287.1 284.4 261.3 247.1 217.5 210.4	91.9 92.6 103.0 112.1 101.3 97.2 96.0 98.4 99.2 99.5 76.6 86.1 80.4 91.4 93.0 104.0 113.8 118.9 130.2	77 • 4 76 • 7 86 • 8 86 • 1 81 • 4 79 • 8 80 • 3 86 • 5 87 • 9 65 • 6 73 • 4 73 • 3 80 • 0 96 • 4 100 • 7	90 91 93 93 95 96 97 99 101 102 103 105 106 108
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67.31 52.49 32.30 19.33 10	56.77 46.92 28.43 18.72	63.87 41.08 27.17 20.70	43.24 42.96 23.34 19.79	48.42 46.78 20.64 17.55	42.74 60.20 18.29 15.60	45.23 41.85 16.24	62.07 36.74 15.36
13.90 11.82 15.29 15.28 1(8.996 14.65 12.35 14.40	12.58 13.07 16.21 19.40	17.23 7.500 14.43 17.21	15.66 9.004 9.191 12.66	15.01 16.00 12.34 15.06	14.69 19.39 9.795 18.92	15.88 17.20 17.18
27.80 32.59 67.40 52.87 10	26.43 30.60 71.64 48.24	36.72 28.86 44.38 44.21	46.46 38.88 139.9 63.09	53.03 46.47 114.6 70.94	53.90 53.99 108.8 78.93	35.56 61.28 64.31 73.79	48.30 77.35 58.19
68.24 58.78 44.30 72.19 10	43.43 62.53 61.24 45.33	40.02 39.89 63.64 62.03	37.06 84.41 60.09 58.65	34.49 92.87 55.72	55.78 80.73 53.41	36.83 50.24 50.11	51.77 70.42 51.19
38.22 43.66 27.05 47.60 10	63.60 40.22 25.77 32.49	58.24 54.86 37.43 30.51	68.02 36.12 27.27 41.67	42.53 47.44 36.63 58.47	75.26 32.13 40.71 61.57	76.95 30.19 42.98 39.51	68.25 28.51 43.40
22.75 14.09 6.583 5.332 10	28.52 12.60 9.768 4.982	25.69 11.31 8.842 4.678	23.30 10.18 8.036 4.414	20.90 9.201 7.335 4.184	20.02 8.343 6.725 2.470	17.76 7.607 6.195	15.80 5.305 5.733
24.42 48.03 30.71 29.99 1(83.89 42.13 27.06 26.44	76.88 37.00 23.89 30.96	55•81 39•02 21•34 79•53	132.2 38.10 29.24 53.97	83.87 53.72 39.17 52.88	62.61 38.90 31.78 43.65	54.81 33.86 35.94

38.32 56.52 20.51 9.986 10	33.69 43.95 19.66 11.66	29.65 37.37 17.45 10.58	26.14 32.86 15.04 13.13	23.08 30.13 15.40 11.81	25.84 27.06 13.74 10.62	31.46 26.40 12.30	45.56 23.90 9.615
2.621 6.503 4.020 8.891 10	4.993 5.534 3.930 8.334	2 • 159 4 • 691 4 • 764 15 • 73	5.253 2.343 4.021 18.48	5.818 2.360 3.375 14.59	5.218 5.671 .6624 12.57	6.616 3.538 -1.367 10.81	7.617 3.368 10.36
10.11 30.45 106.7 71.88 10	9.813 29.68 187.1 62.41	8.413 20.41 413.5 54.18	7.336 20.72 257.0 62.08	16.88 17.80 325.7 51.76	22.79 14.39 182.4 37.06	30.70 21.33 264.6 32.12	23.32 33.52 97.50
27.82 19.88 34.75 29.11 10	31.86 41.45 23.61 31.77	22.46 30.94 44.86 28.67	19.41 28.63 46.11 21.02	16.77 26.30 35.18 35.89	20.73 23.90 24.74 25.21	17.10 43.83 47.56	16.56 37.52 53.52
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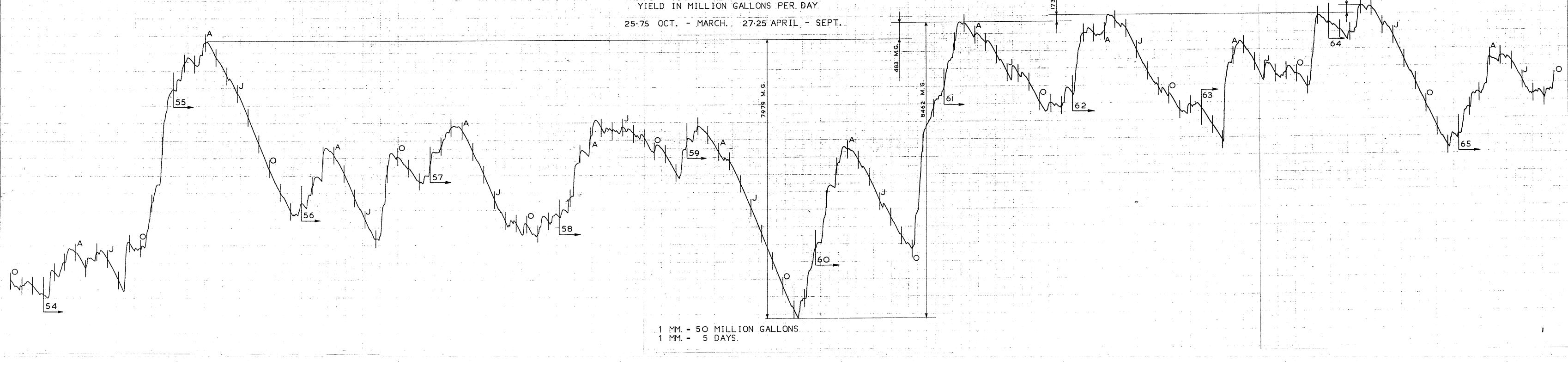
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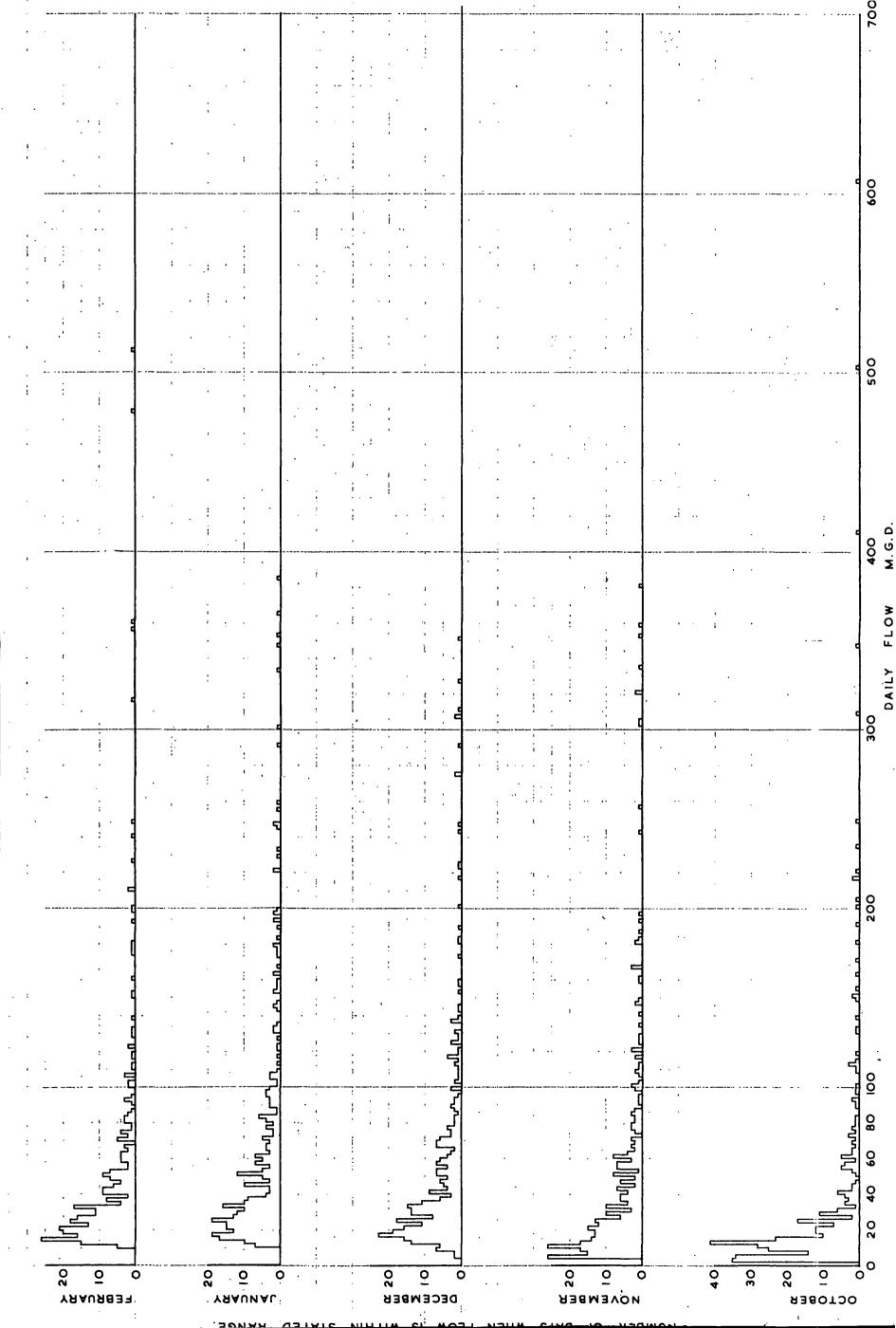
APPENDIX II

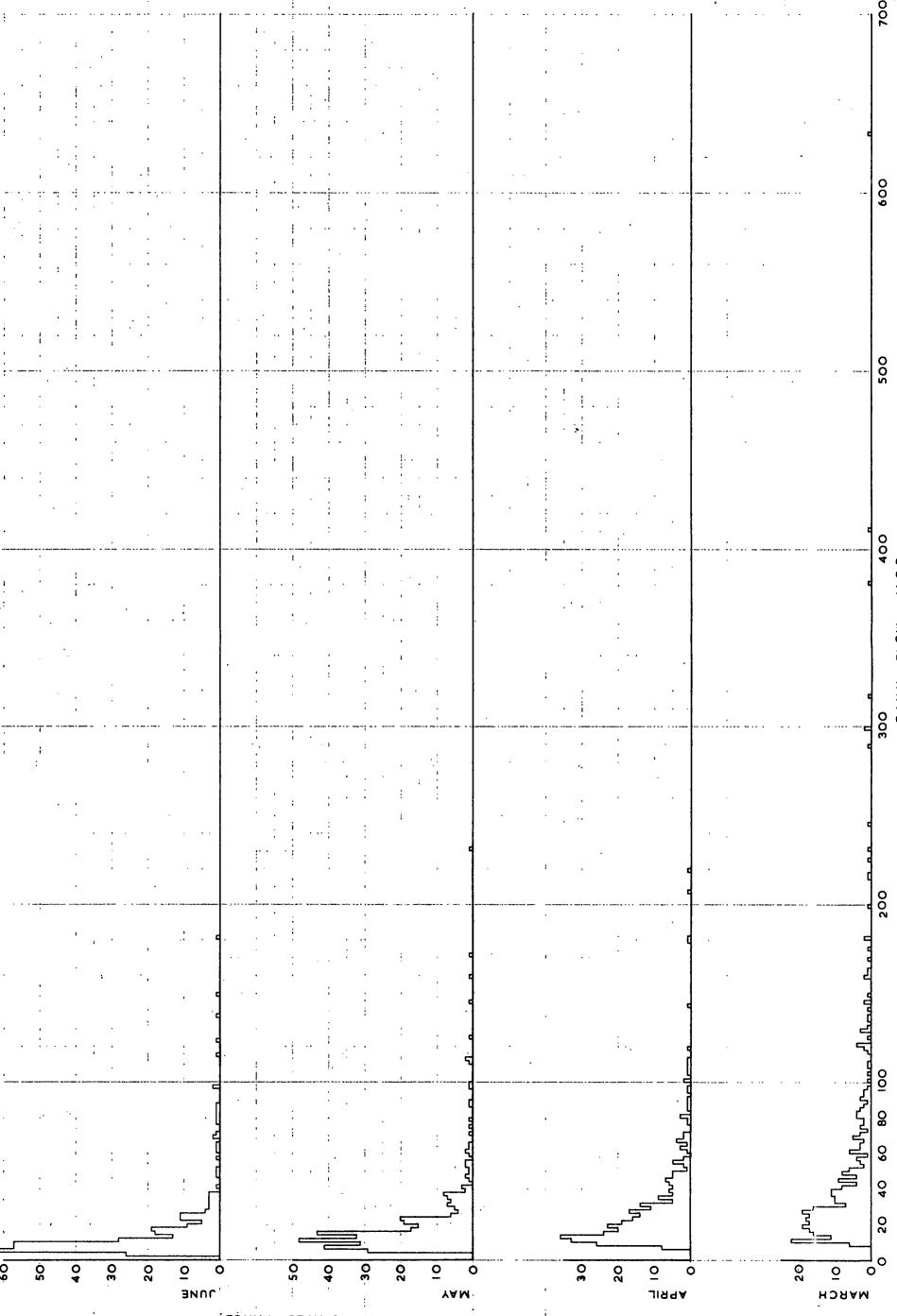
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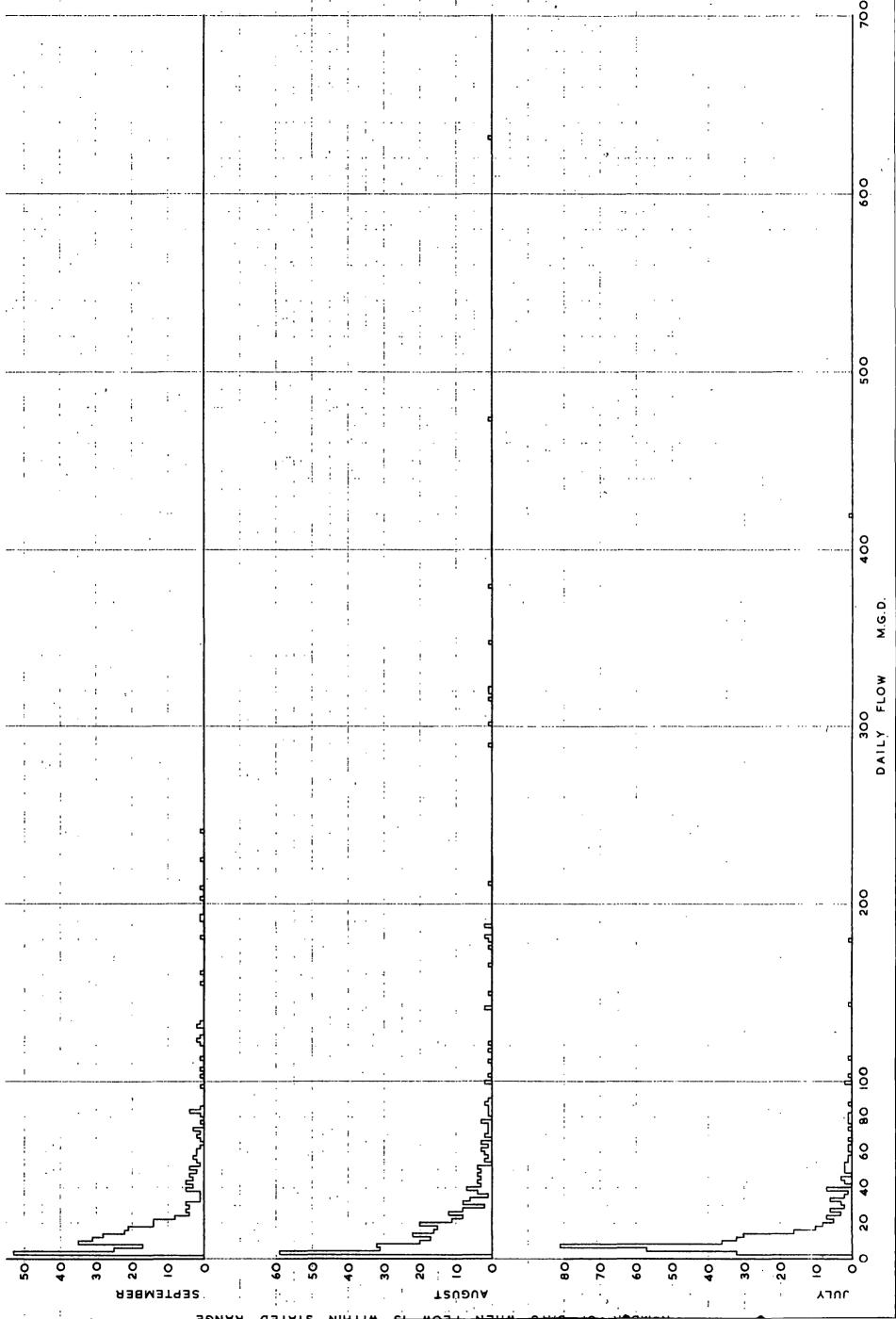
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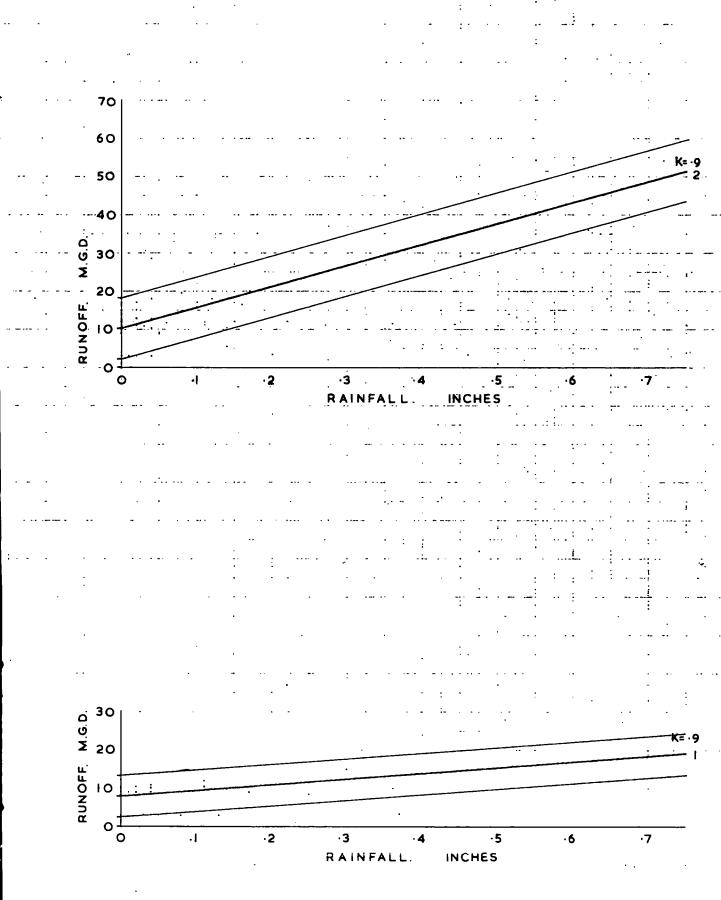
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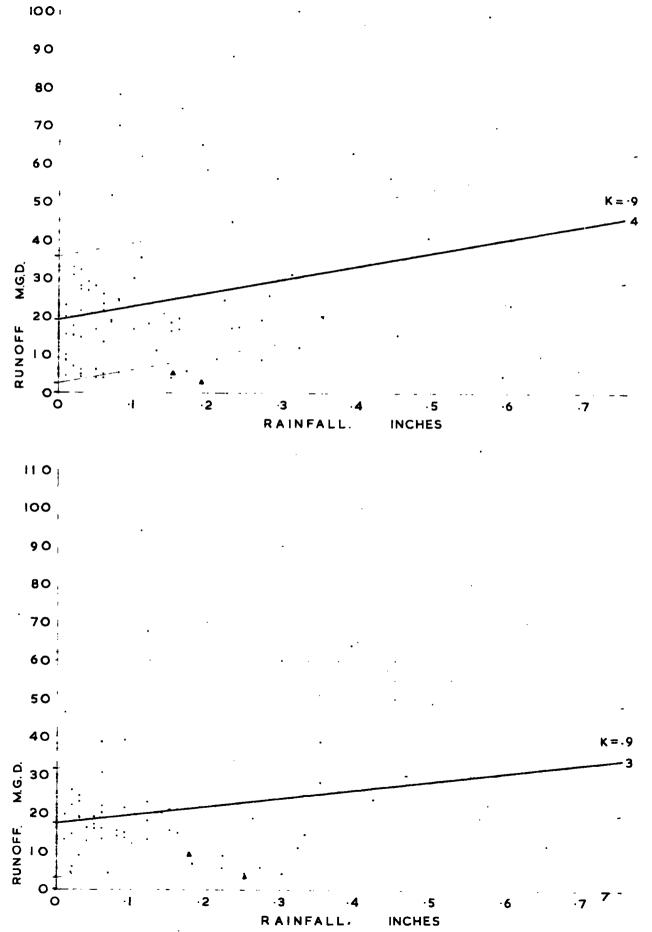
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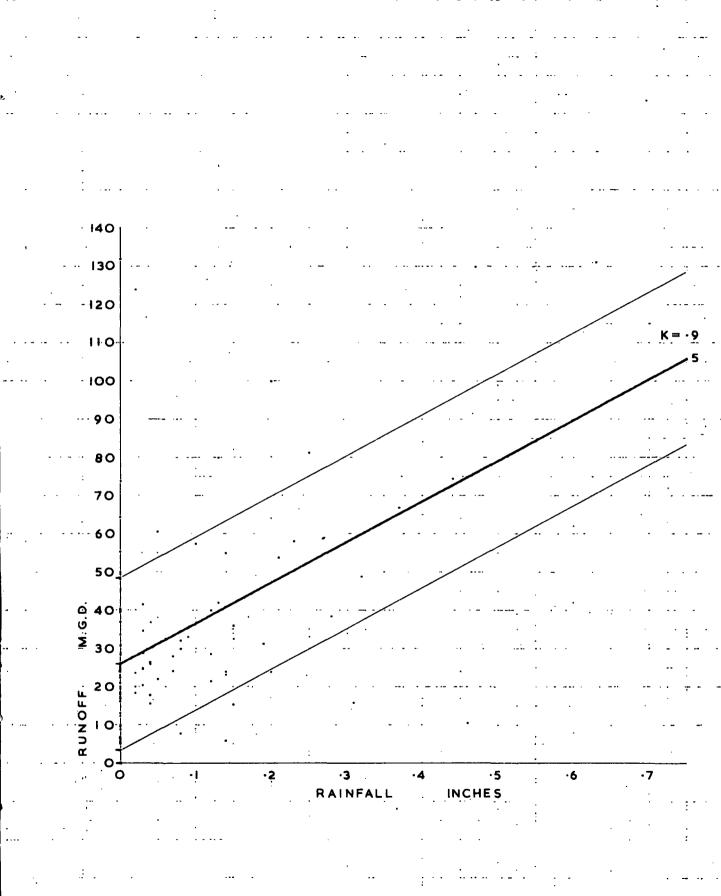
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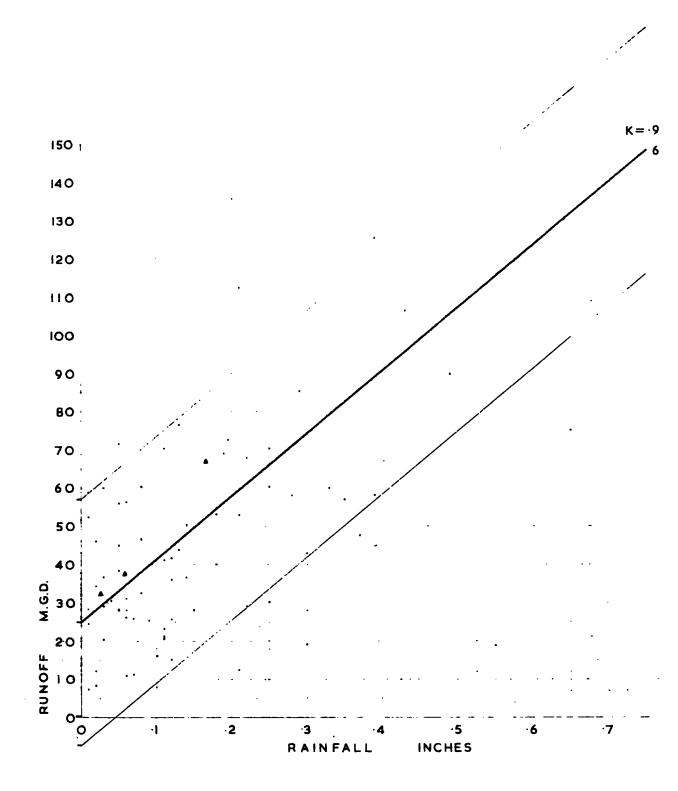
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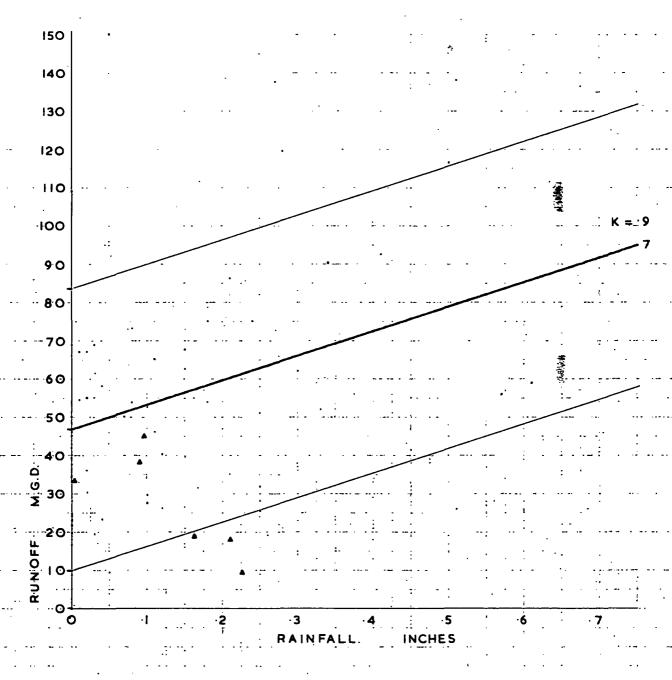
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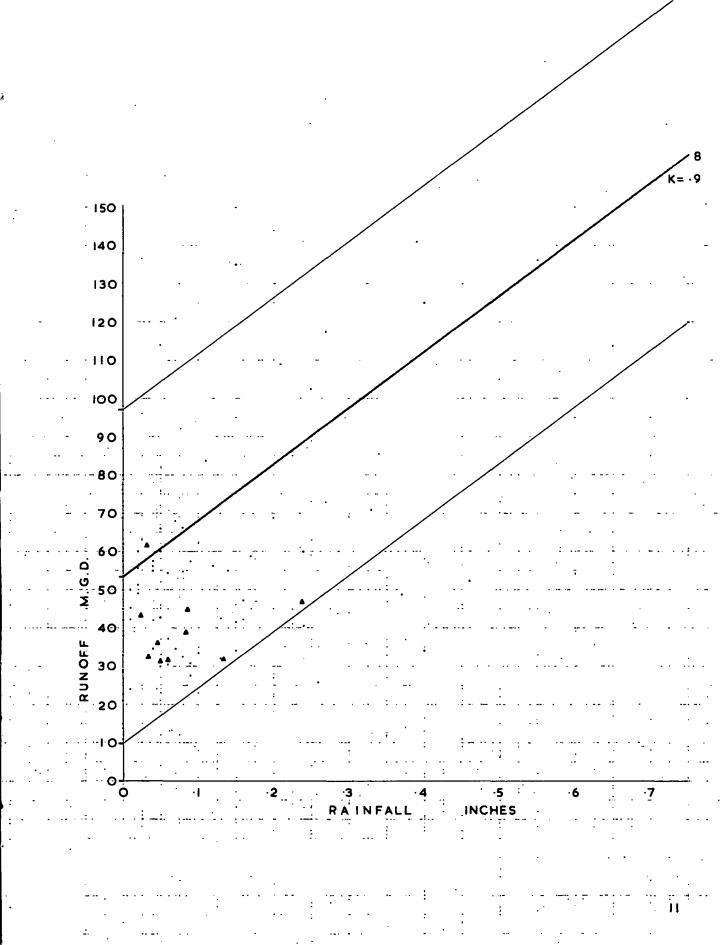


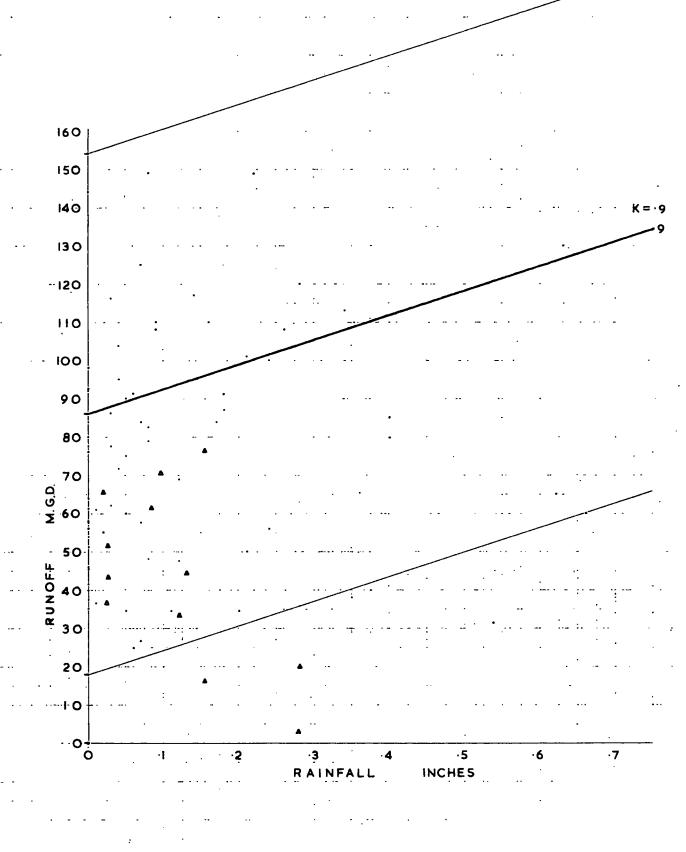


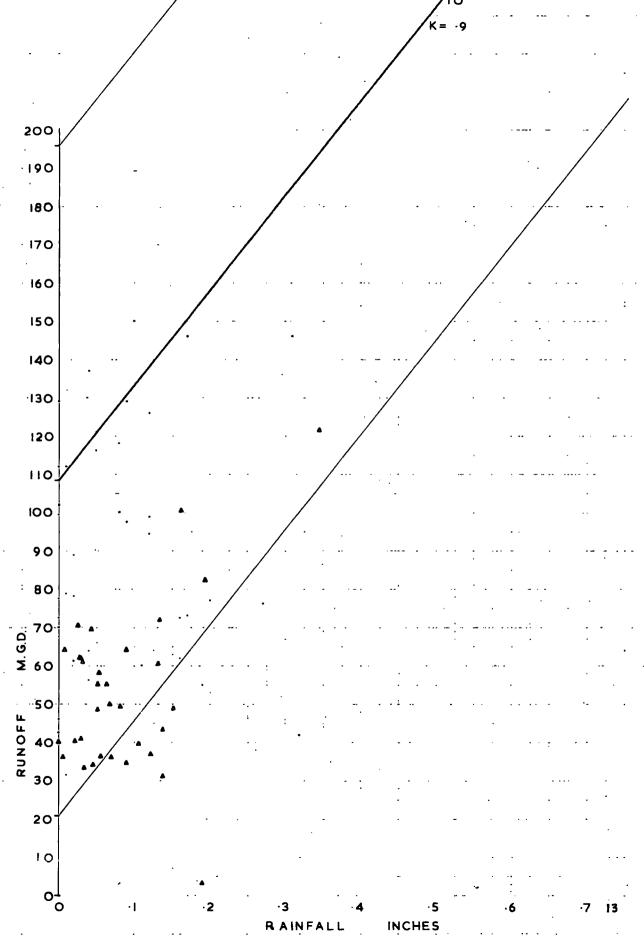
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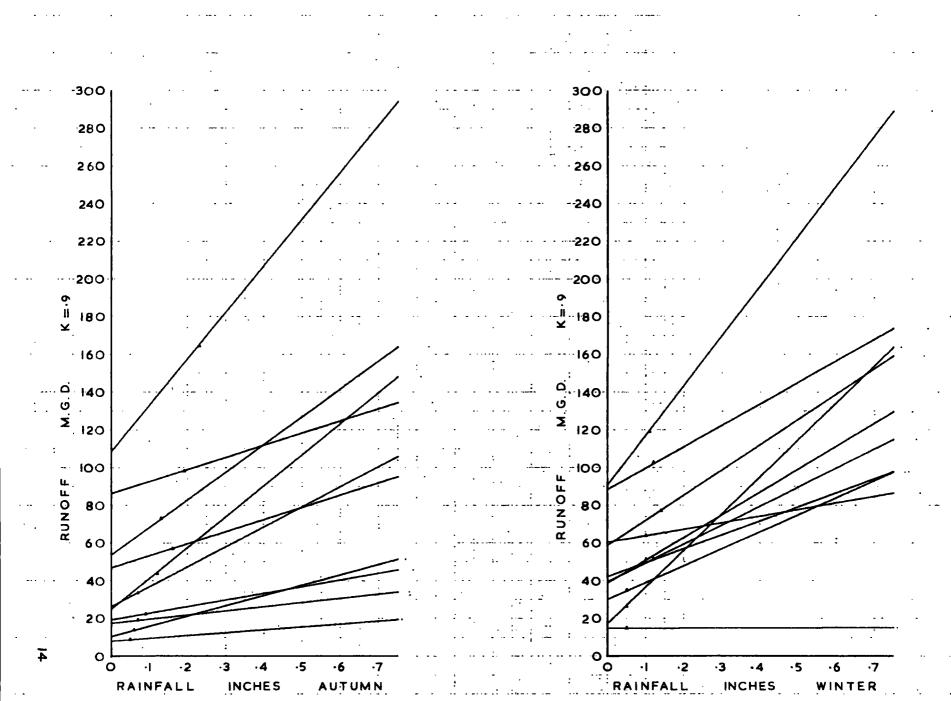


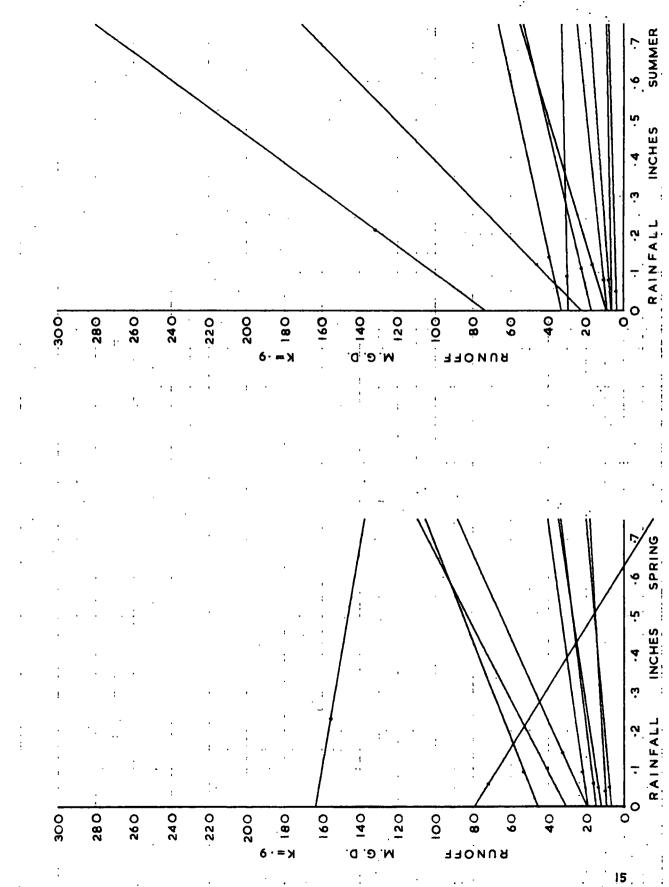
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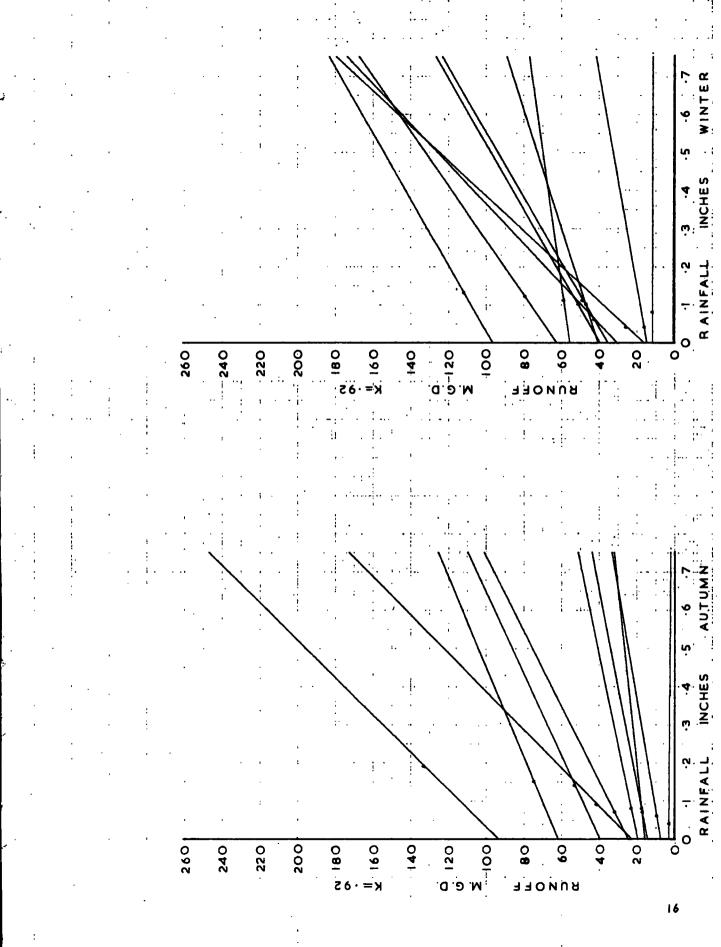


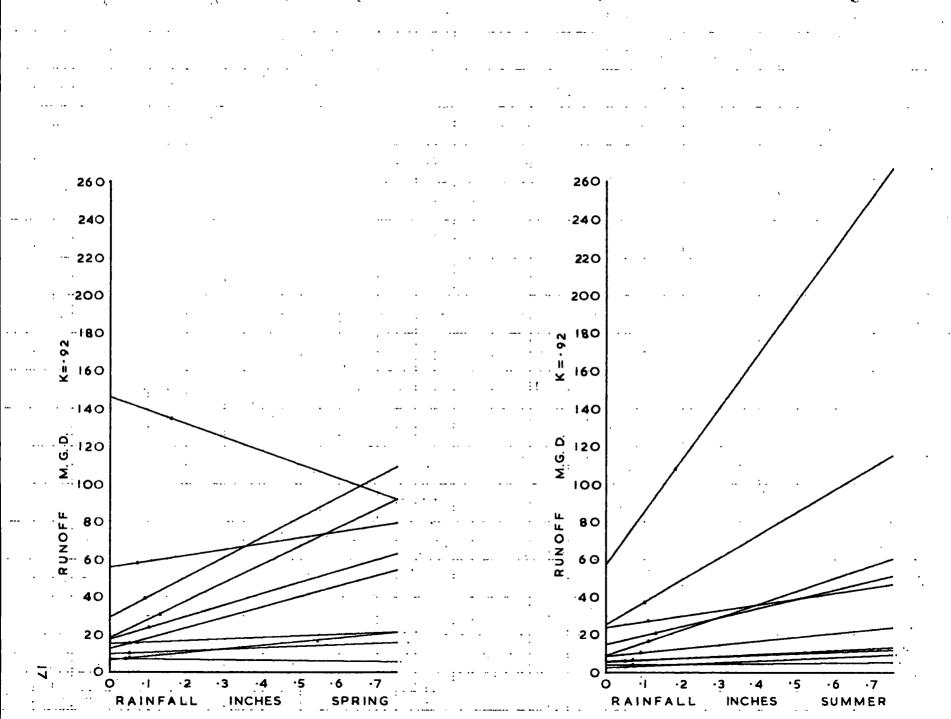


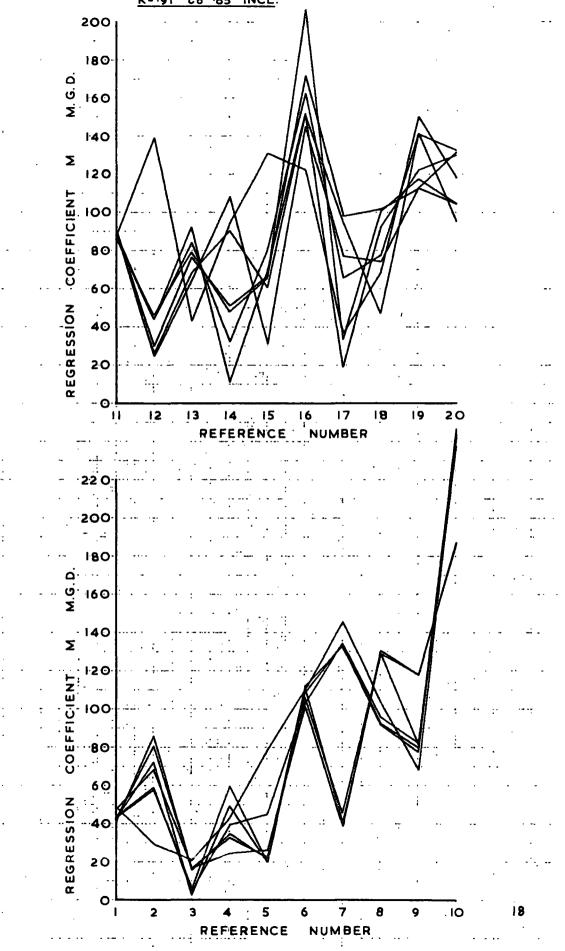


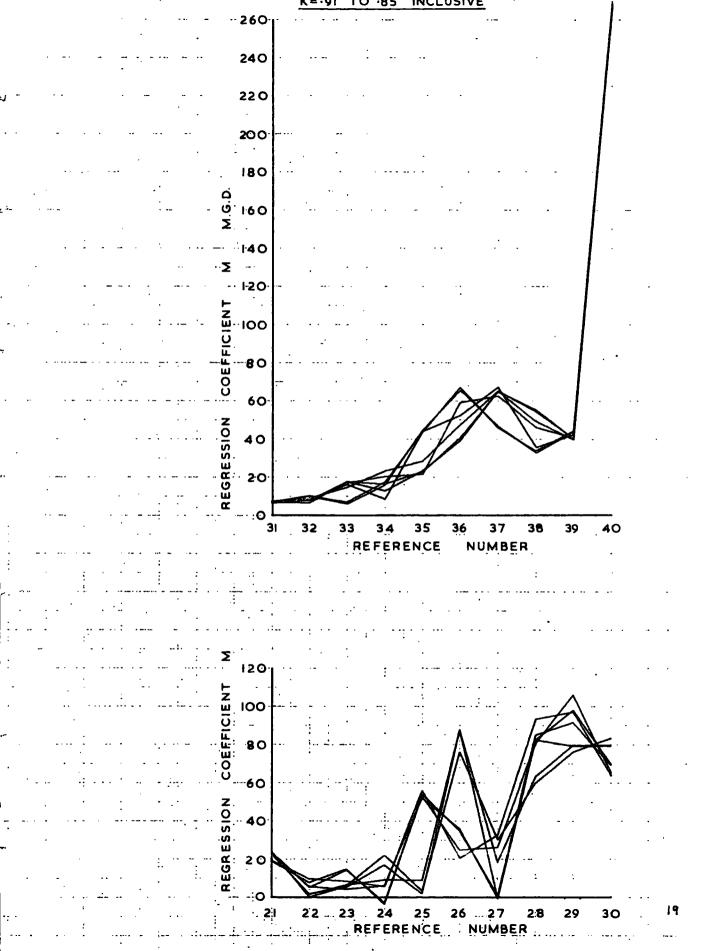


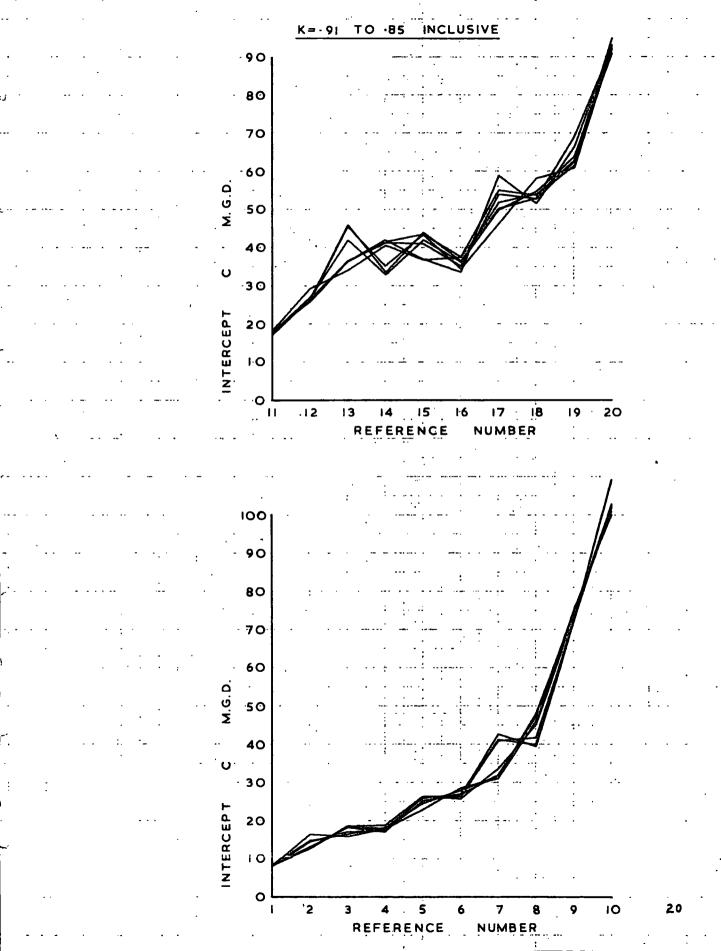




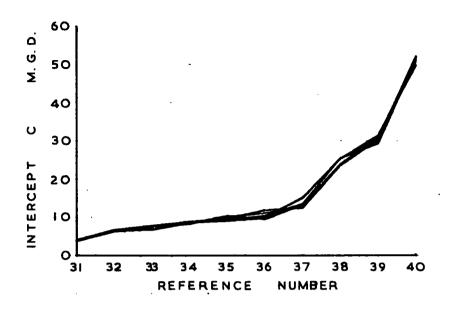


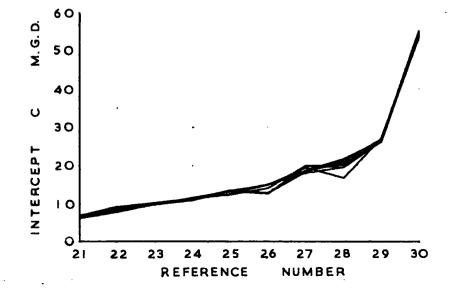


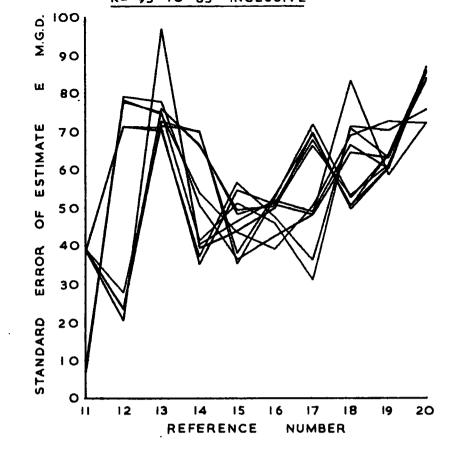


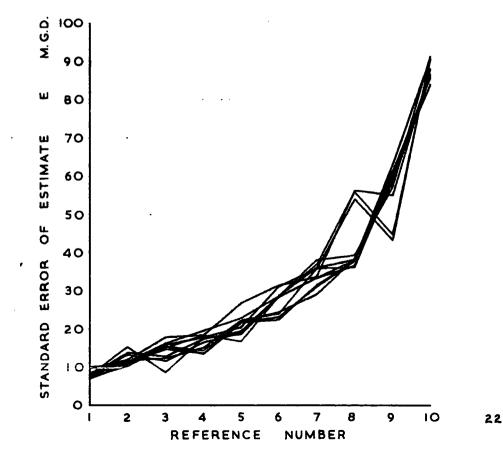


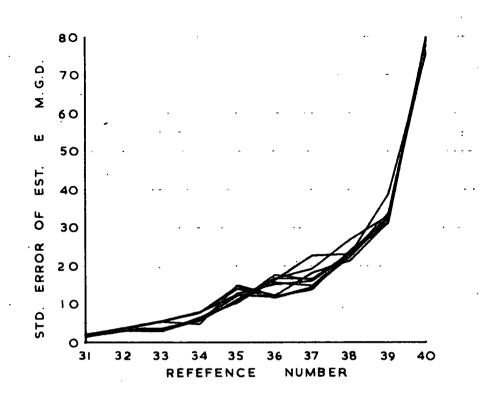
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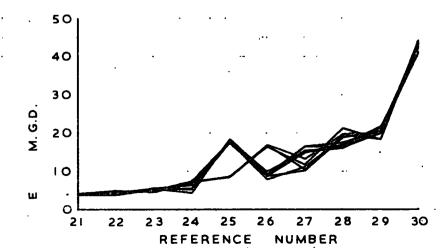


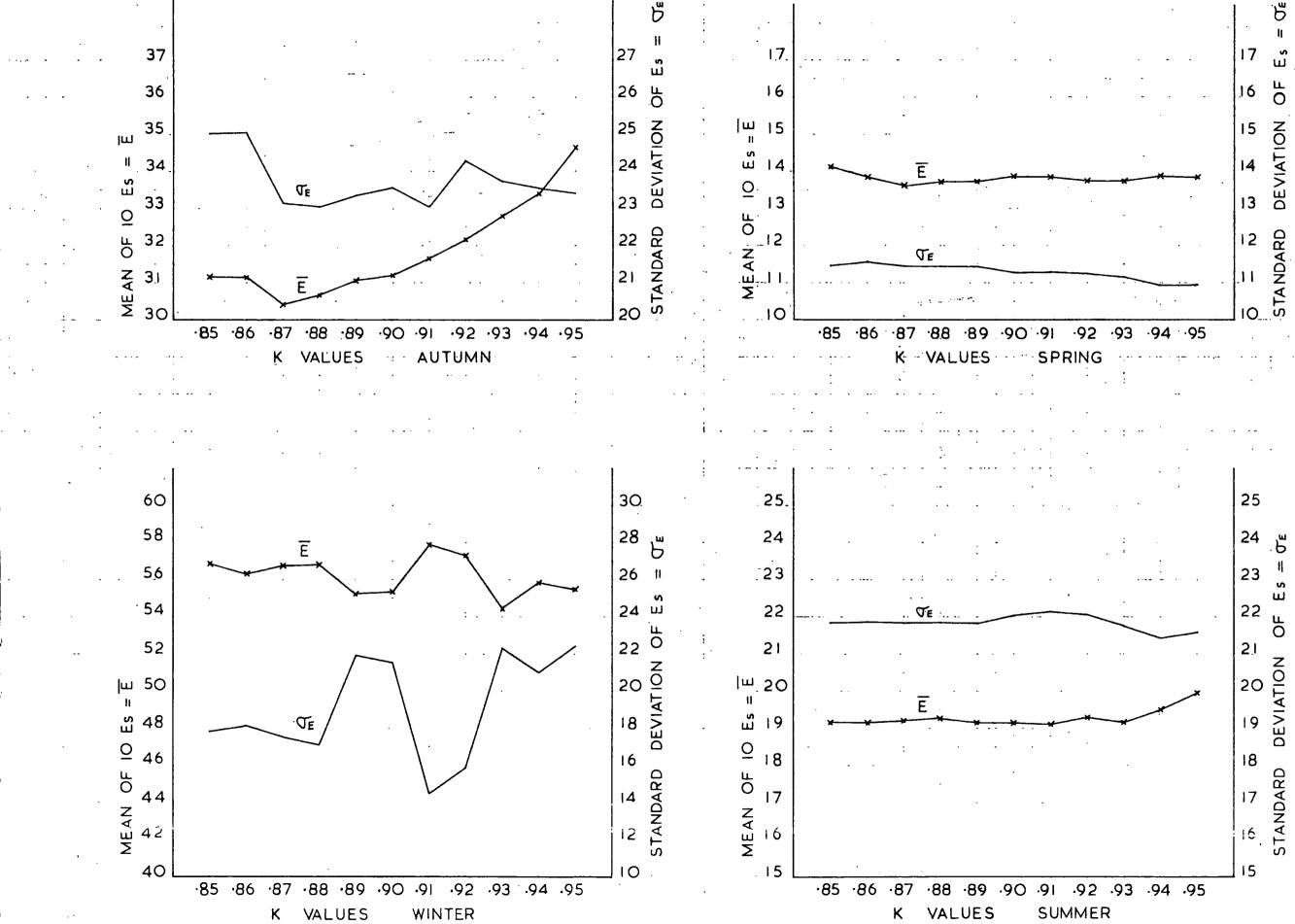


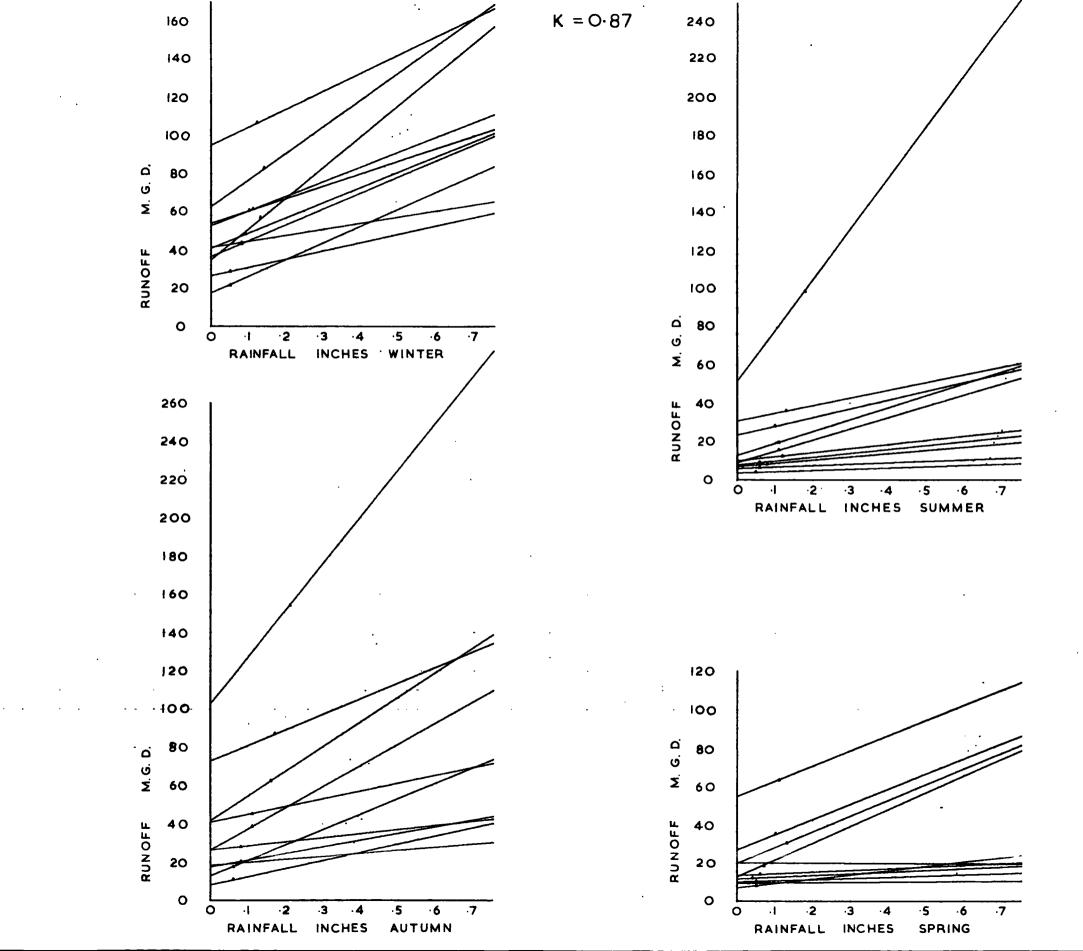


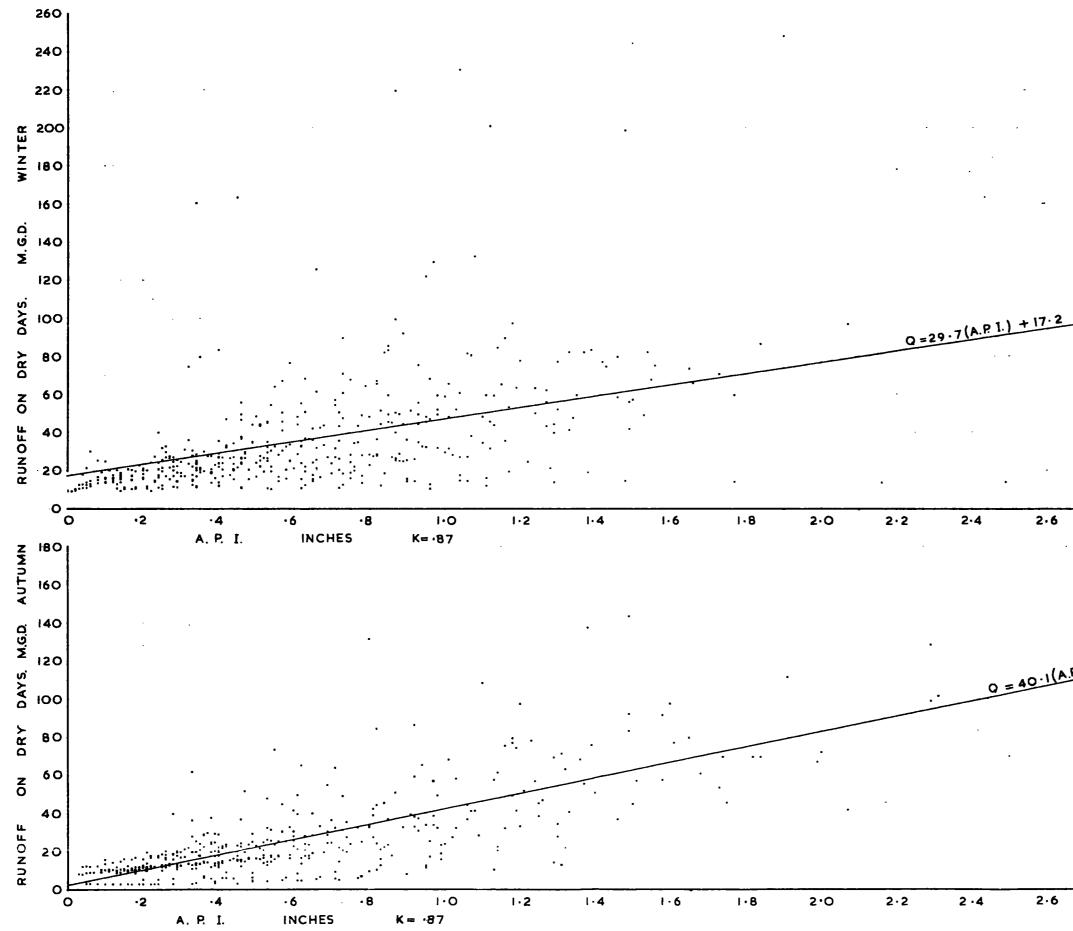








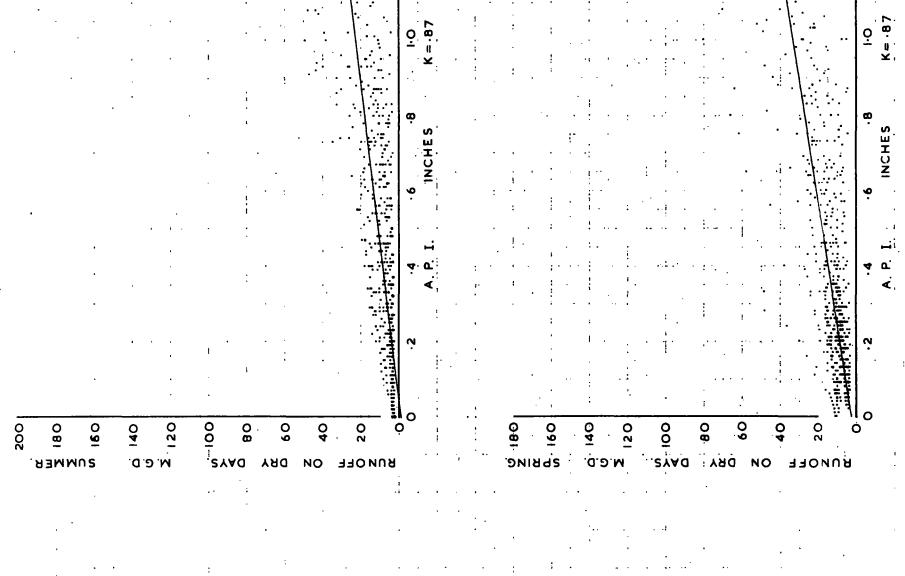




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