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THE PERFORMANCE AND BEHAVIOUR OF A NON-LINEAR  
RESONANT LINK FOR POWER SYSTEMS.

R. BARNES.

Thesis Submitted for the Degree of M.Sc., in the Faculty  
of Engineering Science, University of Durham.

Date: December, 1971.



Abstract of Thesis.

A Non-Linear Resonant Link for Power Systems.

The link is formed by a capacitor connected in series with a linear inductor and tuned to a state of resonance for normal load flow conditions so that the resulting impedance and voltage regulation are negligibly small.

Under fault conditions the state of resonance is destroyed by the saturation of a reactor in parallel with the series capacitor hence the fault throughput current is severely limited to a few times full load current. The limitation occurring in the first half cycle.

Certain sub-harmonic instabilities are investigated with a view to gaining insight into their cause and to remedy this response. A cause of the sub-harmonic response is put forward and a relationship between the sub-harmonic and the 50 Hz ferro-resonant response is suggested.

An analysis of the circuit by two methods is given and digital integration of the system equations yields  $1/3$  sub-harmonic solutions.

A double resonance link is introduced with load flow properties similar to those of a single resonance circuit, but which relies upon the high impedance of parallel resonance to limit the fault current to a little above full load value. It is shown that such a link actually reduces the fault level of a busbar to which other supplies are connected.

Spectrum analysis of the current waveforms show that the harmonics and sub-harmonics have basically different circulation paths in the link.

## II

The possibility of protecting the link against internal faults by the application of standard commercial systems is demonstrated and a complete protective scheme is presented.

Finally a discussion of previous papers is given and a summarised list of the salient points to emerge from the research.

### III

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List of Symbols used in the Text.

V	Supply voltage at 50 Hz.
$V_R$	Load voltage.
$V_S$	Saturable reactor voltage.
$V_C$	Parallel capacitor voltage.
$V_L$	Linear reactor voltage.
I	Current transmitted through the link.
$I_R$	Load current.
$I_S$	Current in the saturable reactor.
$I_C$	Current in the parallel capacitor.
$I_F$	Fault current.
$P_a$	Power transmitted by the link.
$P_b$	Power in a second supply working with the link.
$\psi$	Flux linkages.
B	Flux density.
t	Time.
f	Frequency Hz
$\omega$	Angular frequency $2\pi f$ . rad/sec.
$\mu$	Permeability.
R	Main damping resistance in series with the saturable reactor.
r	Auxiliary damping resistor.
C	Parallel capacitor capacitance.
$C'$	Load capacitance.
L	Inductance of linear series reactor.
$L'$	Inductance of linear balast reactor.
Z	Impedance.
$\theta$	Impedance angle.
$\Omega$	Ohms.
$\mu F$	Microfarads.

The Performance and Behaviour of a Non-Linear  
Resonant Link for Power Systems.

Introduction:

The interconnection of large power systems has several important advantages over the simple radial system:-

- 1) Improved security of supply to consumers and undertakings.
- 2) Plant operation can be rendered more economic.
- 3) Reduction in the amount of standby plant required.
- 4) Improves the systems stability.
- 5) Allows the use of larger, more efficient generator units.

There are, however, several disadvantages that arise from interconnection and one of the more important of these is the short circuit problem.

The effects of short circuit currents are both thermal and mechanical, and both increase as the square of the current.

Thermal effects arise from the excessive increase in temperature and may give rise to burning of insulation and excessive expansion of conductors leading to mechanical damage such as buckling at joints, end boxes and windings; annealing of overhead line conductors is also possible.

Some relief can be afforded by the reduction of fault time since the heating effect is a function of the energy input to a system,  $I^2R \times \text{time}$ . However, no such relief can be afforded in the case of the mechanical forces which arise from electromagnetic attraction or repulsion of current carrying conductors and which act in the first half cycle and may lead to



distortion and buckling of conductors e.g., bursting of cables etc.

It is thus evident that excessive fault currents within a system can lead to extensive damage to otherwise healthy equipment.

As an interconnected system increases in size, with the addition of generation to meet the load growth, the fault levels within the system increase and may ultimately exceed the short circuit ratings of the controlling circuit breakers and allied equipment. Most vulnerable are the circuit breakers controlling tail-end feeders to static loads.

An instance of this rise in fault level occurred on the 275 kV supergrid where the first circuit breakers had ratings of 7,500 MVA, which was thought to be adequate.

Very soon after the initial development of this system the fault levels in certain localities vastly exceeded this rating (Stella West 10,000 MVA) and new circuit breakers of 15,000 MVA are now standard. At Longannet, Scotland, where there are eight 330 MW machines, 20,000 MVA units are installed, with reactors in the 275 kV lines to Kincardine Power Station.

This tendency for the short circuit levels to rise and to exceed the ratings of circuit breakers in a system leads to the need for expensive uprating or replacement of circuit breakers.

In an effort to overcome the short circuit problem, methods have been developed in the past to curtail the rise in fault levels. These well known methods include the use of current limiting reactors and the judicious sectioning of the power system at the busbars. Both these methods have serious disadvantages.

The use of current limiting reactors has had some success in distribution and transmission systems, but it leads to excessive voltage regulation under load transference. Also since they increase the transfer reactance the stability margin of the system is reduced.

The disadvantage of system sectioning is that it reduces security and works against the advantages of interconnection. Sometimes sectioning is employed with automatic closing of the sectioning circuit breaker in the case of loss of plant. However, such automation has to be paid for and could under certain conditions lead to fault propagation.

Ideally what is required is an interconnecting link having the following operational properties:-

- 1) Under normal conditions, having a very low impedance to the transmission of load current thus leading to negligible voltage regulation.
- 2) In the event of a fault within either of the linked systems, the impedance should increase, so that the fault current transmitted is either negligible or small.

Three innovations satisfy these conditions to a good degree. These are:-

- 1) The transductor saturable reactor which requires a d.c. supply and control equipment.
- 2) The high voltage d.c. link.
- 3) The non-linear or conditionally resonant link.

The first two systems are expensive particularly the d.c. link, and require elaborate control.

This is not meant to be a condemnation of the high voltage d.c. link which it is believed has an established place in the field of power transmission, where its particular properties can be used to advantage.

To some extent the d.c. links ability to block potential fault current is a secondary advantage, although at one time it was seriously considered as a possible method of fault sectioning of the supergrid in the United Kingdom. It was never put into effect because of costs and security considerations.

The third system is the one studied in this work. It has the advantage of being cheaper than the first two systems and of being automatic without the necessity for elaborate control equipment.

However, as will be discussed later, the non-linear resonant link has certain ferro-resonant and sub-harmonic responses which if not arrested would seriously jeopardise its usefulness as a current limiting device.

The object of this work is, therefore, to study the performance and limitations of this system as a fault current limiting link for power systems.

The Resonant Link.

1.0            If the inductive reactance of the link is cancelled by the capacitive reactance of a series capacitor a resonant condition arises whereby the link becomes purely resistive, which resistance could be of a very low order. This is the ultimate of the series capacitor which is used occasionally in long transmission lines and which compensates for only part of the circuit inductive reactance and is thus not in resonance.

              Under normal conditions the resonant link should be ideal for the transfer of power, because the resulting regulation, dependent only on the resistance, could be exceedingly small. Two busbars linked by such a system would be virtually solidly coupled for the flow of power.

              However, under fault conditions the through-put fault current would be very large and the corresponding capacitor voltage would be enormous.

              The capacitor must thus be protected against these over-voltages and this could be achieved by arranging that the resonance state be destroyed under short circuit conditions. This in turn would limit the short circuit current which is the prime object of the link. The best way to achieve this would be to short circuit the capacitor when the voltage across it reached a predetermined value.

              This could be achieved by the arrangements shown in fig. 1 c and d. Circuit "c" operates by short circuiting the capacitor by flashover of the spark gap under fault conditions.

The gap current could then be used to close a parallel circuit breaker thus extinguishing the arc.

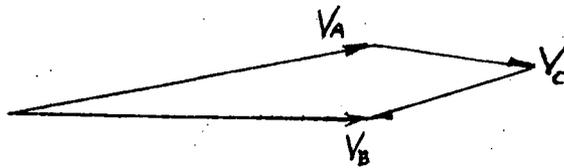
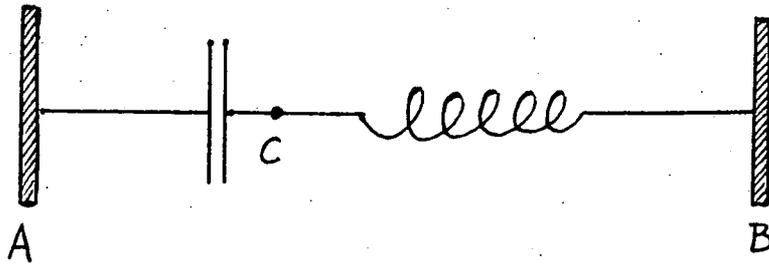
The advantages of this system over that of d is its simple operation and the complete absence of unstable response.

However, it has the following disadvantages:-

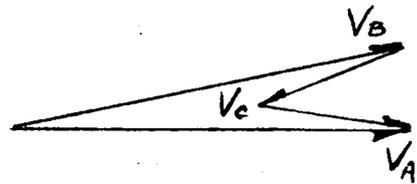
- 1) A circuit breaker is required with all its ramifications of cost and maintenance.
- 2) Control circuits are needed to close the circuit breaker and also to open it after the fault has cleared.
- 3) It is not really suitable for low voltage systems and systems over 33 kV because of difficulties in designing a suitable spark gap that would completely protect the capacitor.
- 4) After operation difficulty may be encountered during recovery if the voltage vectors at each end of the link were widely displaced in phase.

The alternative proposed system shown in fig. 1d is the system studied in this work. It has the advantage of being completely automatic and self recovering without the need for auxiliary parts and is cheaper than scheme (c) at Grid voltages and maintenance would be minimal.

In Scheme (d), S, is a saturable non-linear reactor which limits the voltage rise across the capacitor C and destroys the resonant state under external fault conditions. Under normal conditions S, will present a very high magnetising impedance across C and will thus not effect the resonant condition if  $x_s \gg x_c$ .

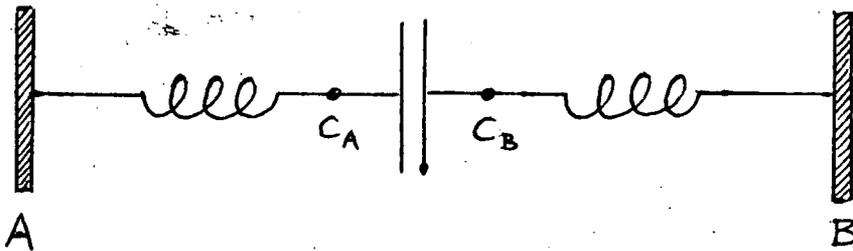


Reactive Power Flow  
from A - B.



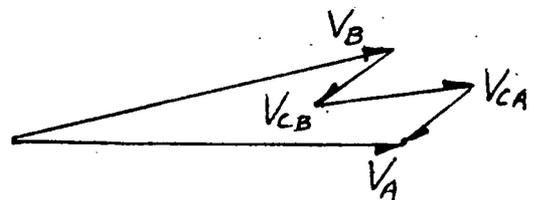
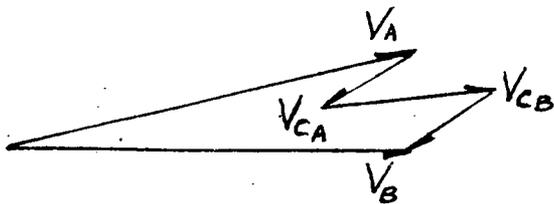
Reactive Power Flow  
from B - A

- a) Basic resonant link between two bus bars A and B. This is unsatisfactory because an internal fault could lead to excessive voltages in system A and under load flow conditions the voltages at point C could be high



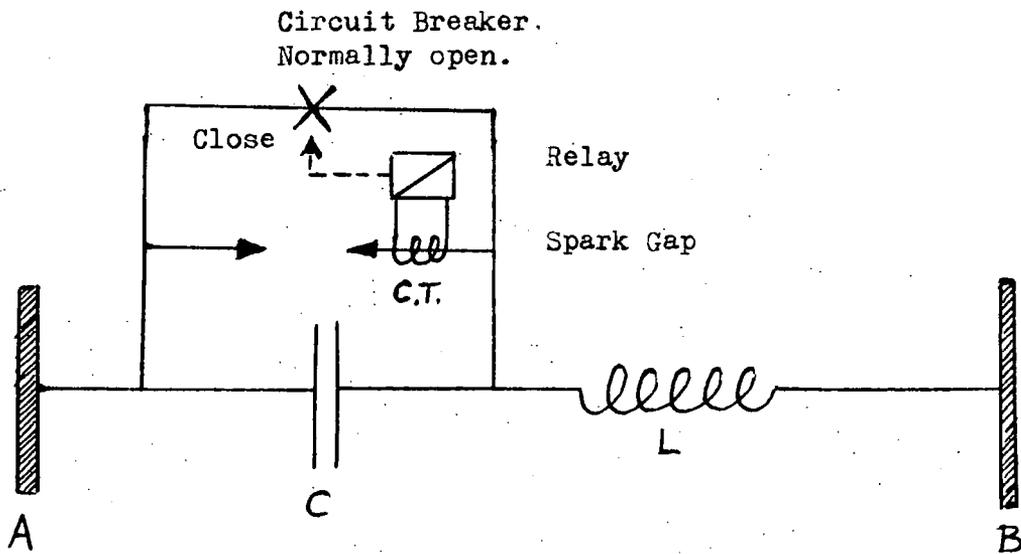
Reactive Power flow  
from A - B.

Reactive Power Flow  
from B - A.

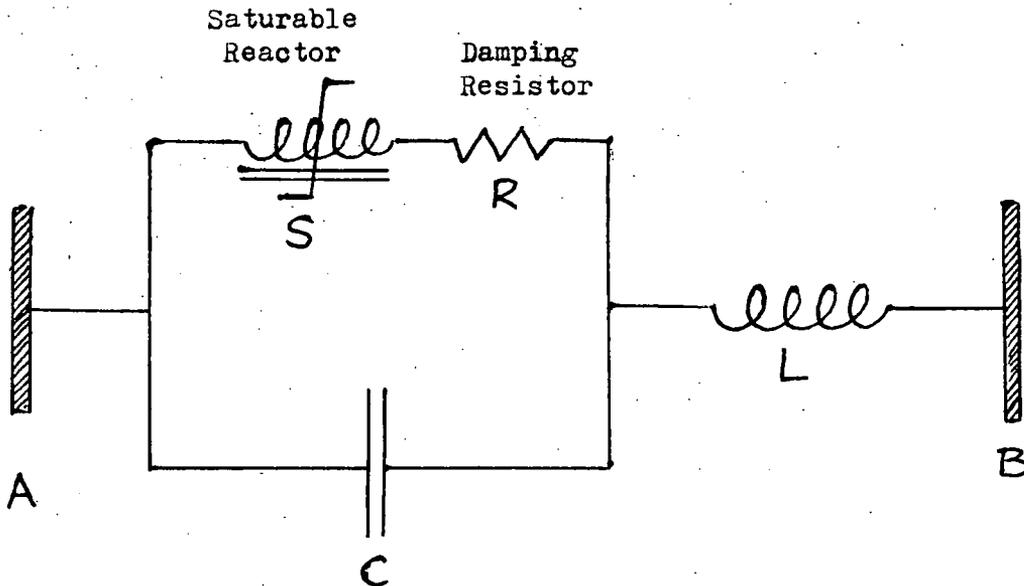


- b) Improved arrangement leading to a better internal voltage distribution

Fig. 1.



- c) Resonant link designed to short circuit the capacitor under fault conditions. see ref.12 E. Friedlander.



- d) Alternative proposed method of limiting the voltage across the capacitor and hence destroying the resonance state under fault conditions.

Fig. 1.

### 1.1 Design of the Model and Practical Considerations.

Complete details of the saturable reactor, including magnetic circuit properties are given in appendix I.

In brief, the reactor was built with several different windings ranging from 10 to 100 turns to give a great deal of flexibility in operation. It was so designed that a winding of 250 turns lead to saturation when the voltage applied just exceeded 44 volts R.M.S. H.C.R. square loop alloy was used for the core material since this gave a better switch action than stalloy on saturation. Furthermore, the initial tests showed that although both stalloy \* and H.C.R. alloy cored coils gave rise to sub-harmonics, these were more likely to arise with the latter material, and it was felt that this was a phenomena worth investigating with a view to finding methods to quench this response.

The difference between the two core materials with regard to sub-harmonic response is believed to lie in the amount of core loss exhibited. Stalloy being the more lossy material is less susceptible to sustained sub-harmonic response.

For this reason and for reasons of cost the core of a saturating reactor in a real link would probably be of stalloy or of stalloy-square loop material composite rather than of all square loop material. However, it must be stressed that the performance of a stalloy cored reactor would be much inferior to that of a reactor with a core of square loop material and some benefit of the resonant link would be sacrificed with the former. This nevertheless may be economically acceptable.

\* The name stalloy in this text means hot rolled silicon steel or cold rolled grain orientated silicon steel.

## 1.2 Scaling.

The model was designed per phase on the basis that 100 volts was equivalent to  $\frac{132}{\sqrt{3}}$  kV and that 1 ampere was equivalent to 438 amps giving a total 3 phase load of 100 MVA.

Hence the impedance scale follows:-

The linear inductor was set at a nominal  $\omega L = 20$  ohms at 50 Hz, and the main resonating capacitor to a nominal 160  $\mu$  F. Resonance being obtained by setting the capacitor value and adjusting the linear inductor core.

Impedance scaling:-

Model

Grid system.

$$\frac{100}{1} = 100 \text{ ohms}$$

$$\frac{76200}{438} = 173 \text{ ohms}$$

For the linear reactive components.

20 ohms is equivalent to 35 ohms at 132 kV.

so the main reactor would be

$$L = \frac{35}{2 \pi 50} = 0.112 \text{ Henry}$$

and rating

$$3 \times 438^2 \times 35 \times 10^{-6} = 20 \text{ MVA}$$

Main capacitor C

$$\frac{10^6}{2 \pi 50 \times 35} = 91 \mu \text{ F}$$

and rating = 20 MVA.

These being practically and economically reasonable values.

Initial Observations of Behaviour and Response  
of the Resonant Link.

2.0 Resonant Link feeding an isolated load.

With the following arrangement a full load current of 1 ampere gave a completely sinusoidal response and the voltage regulation was negligible with an inductive load. A similar resistive load gave a 3 volt, i.e., 3% regulation although this, being dependent solely upon the main linear reactor resistance, could be clearly reduced in practice.

Under short circuit conditions the fault throughput was limited to 3.65 amperes equivalent to 365 MVA.

This contrasted with the prospective fault throughput of 30 amps equivalent to 3000 MVA.

It is clear from this that the fault throughput is largely limited by the linear reactor and is not much effected by the potential fault level of a large source. This is an important practical aspect because it means that the throughput fault current can be controlled.

In the foregoing case the throughput current was in fact rather less than would be expected from the linear reactor limitation. In fact the fault impedance of the link is composed of the linear reactor impedance plus the residual impedance of the capacitor, saturable reactor parallel combination.

This residual impedance is non-linear being voltage or current dependent as the case may be. The overall link impedance is shown in fig.2 (a and b) as a function of applied voltage and current for the short circuit condition.

Resonant Link Impedance on Short Circuit as a function of R.M.S. Voltage

Link Resistance  $R = 2.6 \Omega$

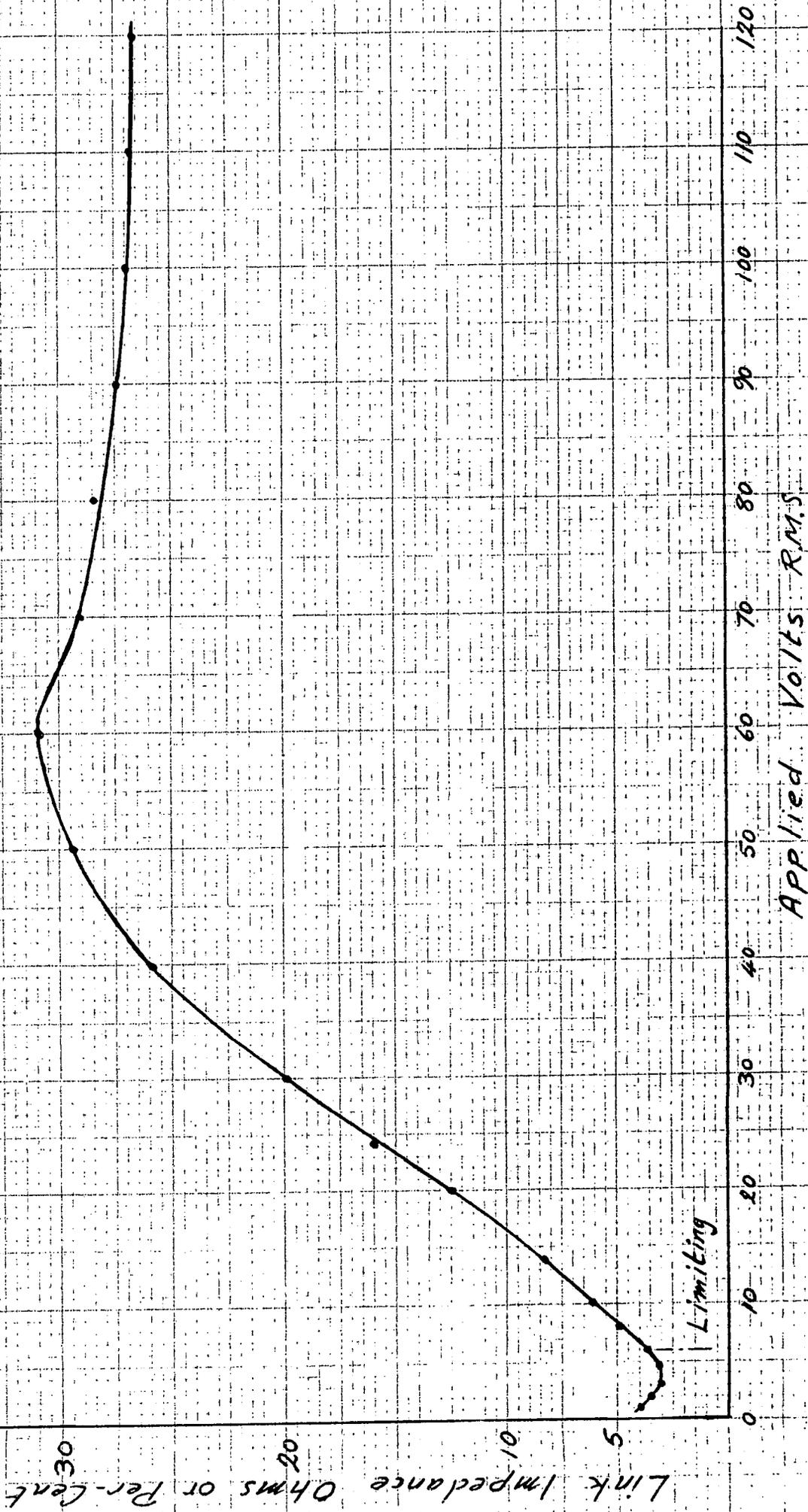
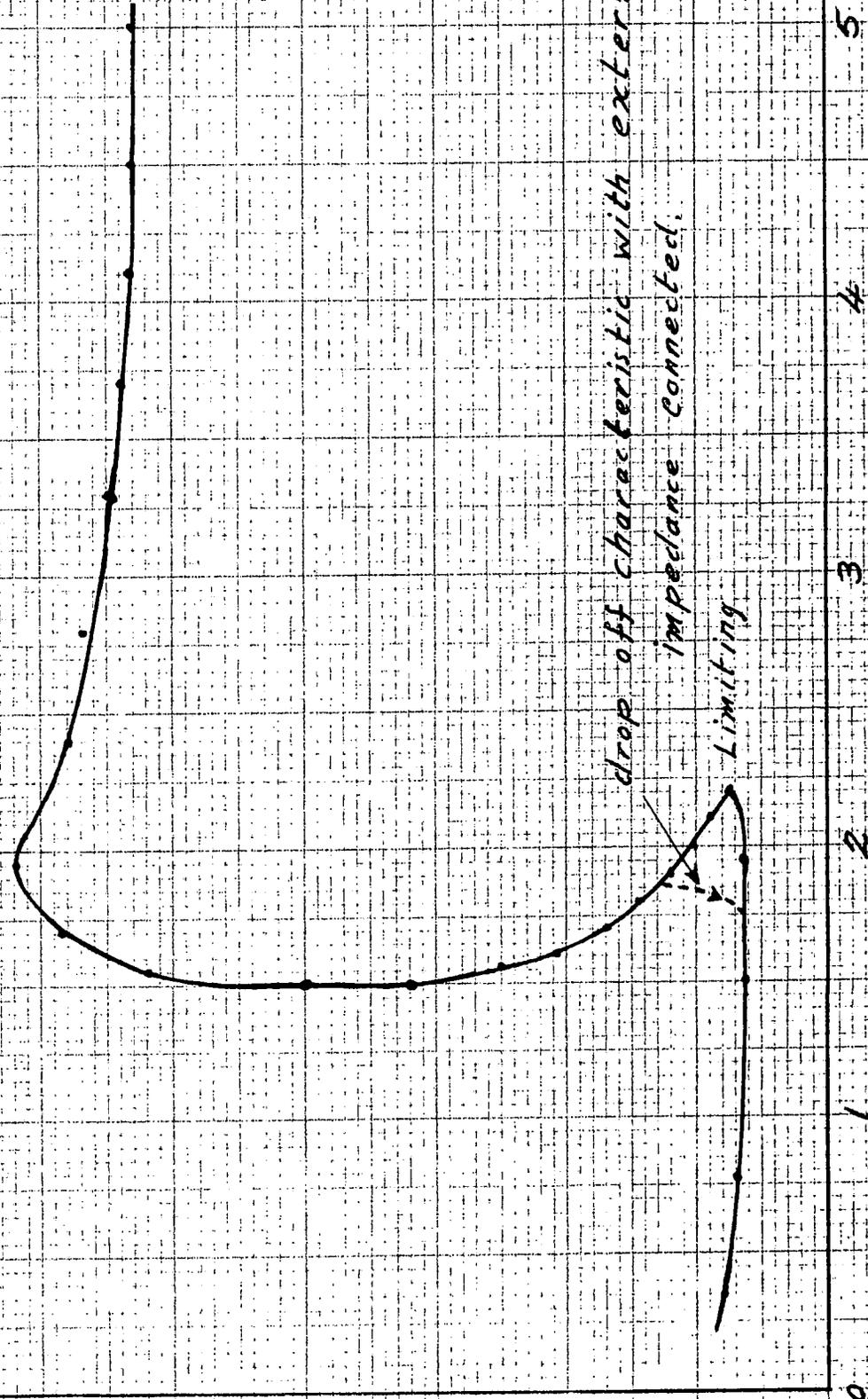


Fig 2a

Resonant Link Impedance on Short Circuit as a Function of R.M.S. Current

Link Resistance  $R = 2.6 \Omega$

Link Impedance Ohms or Per-cent



Current Through Link Amps R.M.S.

Fig 2b

At this juncture it is worth defining what is meant by impedance since the recordings of the wave forms of current and voltage under short circuit show that many are complex and may be far removed from a plain sinusoid.

In work of this nature where many of the waves of voltage and current are complex great care must be taken in deciding what instrument readings mean and what useful interpretation can be attached to say, a current. This further poses the question of the meaning of impedance derived from non-sinusoidal voltage and current. All that can be done in these cases is to approach the problem from a practical aspect by asking what factors of the voltage and current are important. In this respect the impedance derived say, from the fundamental components of a very non-sinusoidal current and voltage may be quite useless and as practically irrelevant as say the ratio of readings taken on rectifying instruments calibrated for sinusoidal quantities. In this work two aspects of voltage and current have been deemed of practical importance:-

- 1) Peak values since these determine stressing and magnetic forces.
- 2) R.M.S. values since these determine heating effects and are thus directly relevant to the rating of equipment.

Thus unless otherwise stated all currents recorded in tables and used in graphs are R.M.S. quantities as measured by precision moving iron or dynamometer instruments.

Furthermore the impedance exhibited between complex voltages and currents is defined as the ratio of the R.M.S. voltage

to the R.M.S. current. Plots of impedance thus defined and shown in Fig.2 may have use in analytical studies of networks containing resonant links particularly since in this case the departure of the current from a sinusoid was not great.

## 2.1 Ferro-resonance.

This is a phenomenon that can occur in any circuit containing non-linear reactors and capacitors. If the applied voltage is large enough to drive the reactor towards saturation its inductance falls and finally may resonate with the capacitance of the circuit. This results in the persistence of large spikes of current occurring at fundamental frequency.

The onset of ferro-resonance in a circuit is characterised by a sudden increase in the current flowing in the circuit often described as jump phenomena in the literature.

Once the condition has been started in a circuit it usually persists even when the voltage has been reduced below that required for its initiation. Finally at some lower voltage the current suddenly falls as the state of ferro-resonance is extinguished. See Fig.7.

In the initial short circuit studies it was found that the circuit was unstable, in that, when the fault was removed a ferro-resonant current persisted in the capacitor saturable reactor loop and gave rise to excessive regulation.

This failure of the circuit to recover was cured by the simple device of inserting a resistance in series with the saturable reactor. This resistor labelled R on the diagrams turned out to be a very important component of the circuit and

moreover had no effect on the normal resonant condition.

An important aspect of the short circuit behaviour of the circuit was that the through-put current showed very little distortion whilst the internal loop current and voltages were very non-sinusoidal. See fig. 38., Appendix II.

## 2.2 Sub-harmonics.

About the time of the initial investigations it was noticed that certain capacitive loads gave rise to a peculiar response different from the ferro-resonant condition mentioned above. This response was characterised by sub-harmonic frequencies appearing in current and voltage wave forms and causing considerable distortion at the load terminals.

Adjustment of the resistor R had no effect upon this response unless its value was made very large which in turn would lead to large throughput currents see fig.3, plot of throughput current against R.

The fascination of this peculiar response ultimately led to its study forming a major part of this work. In any case it was clear that some method had to be found to quench the sub-harmonics because such a response would be quite unacceptable on a power system.

## 2.3 The significance of the resistor R.

A programme of tests was embarked upon to demonstrate that the link would perform satisfactorily under a variety of conditions likely to be encountered in a power system. But first it was necessary, having tested the link, to set the value of the resistance R.

It was mentioned earlier that R was necessary to quench the ferro-resonant response subsequent to a fault. This led to the necessity for a minimum value of  $R = 1.5$  ohms being the value necessary for an inductive load. Resistive and capacitive loads required smaller values of resistance.

It was found that the value of R corresponding to minimum throughput current see fig.3, was approximately 2.0 ohms and this seemed a desirable value giving a small factor of safety over the bare minimum.

In addition it was felt necessary to limit the voltage rise across the capacitor under fault conditions to two or three times the full load value and to about 60% of the system voltage. The value of capacitor voltage increased continuously with R, fig.4.

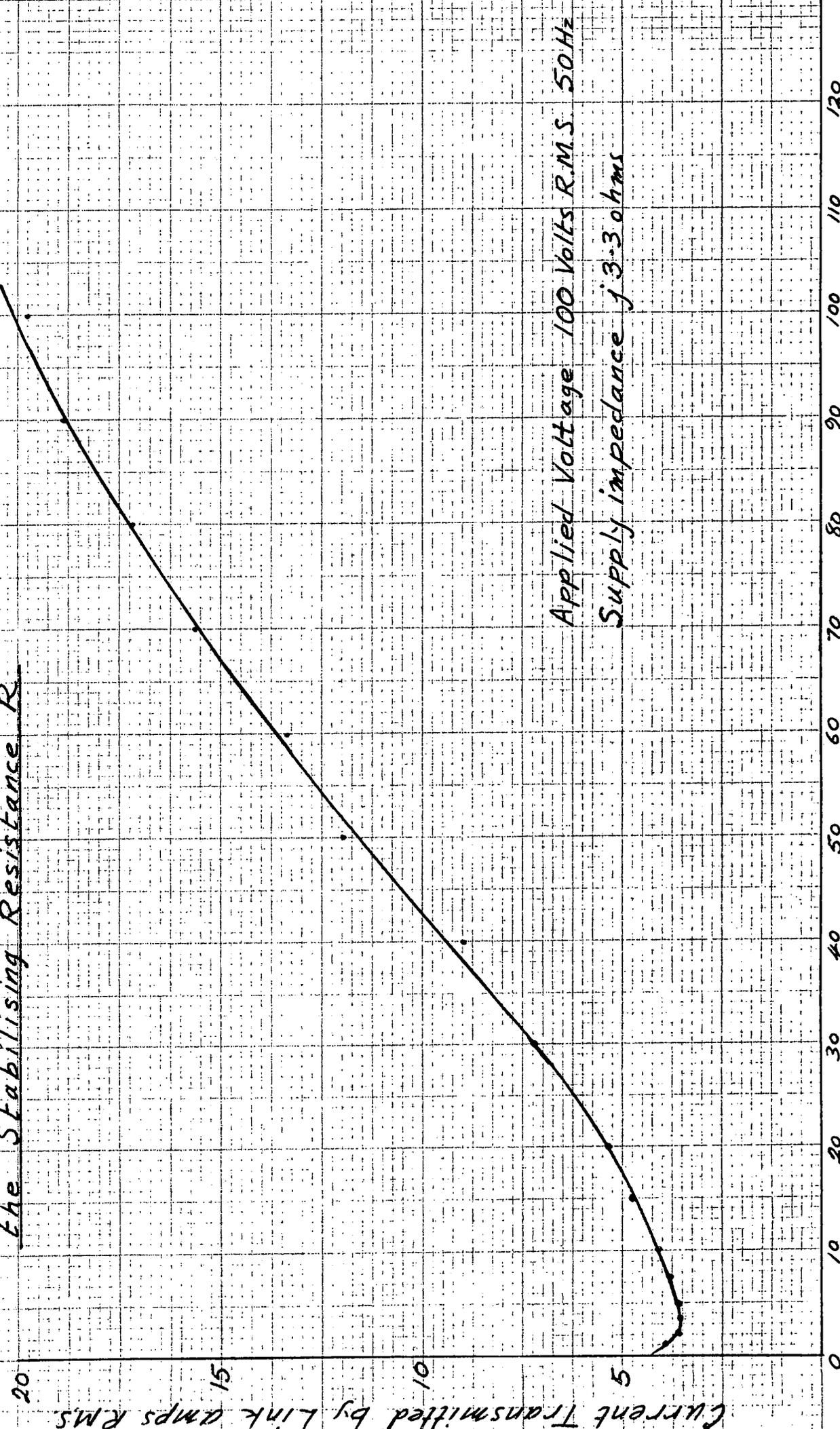
At this point recovery of the circuit was observed following an inductive overload.

If the load on the circuit was increased the circuit eventually limited at a value of load determined by the saturation voltage of the reactor, in this case just over 2 amperes, i.e., twice full load current. Further increase in overload current was limited between 2 amps. and the short circuit limit of 3.65 amps.

When the load was subsequently reduced the saturable reactor-capacitor loop current persisted until the main load current had fallen to a value below the limiting value of 2 amps.

There was thus found to be a current margin between "pick-up" and "drop-off" of the saturable reactor current.

Variation of Transmitted Short Circuit Current with the value of the Stabilising Resistance R



Current Transmitted by Link amps R.M.S.

Applied Voltage 100 Volts R.M.S. 50 Hz  
Supply impedance  $j3.3$  ohms

Stabilising Resistance R Ohms  
Fig 3

Variation of Capacitor Voltage with the value of the  
Stabilising Resistance R

160

140

120

100

80

60

40

20

Capacitor Voltage R.M.S.

0

10

20

30

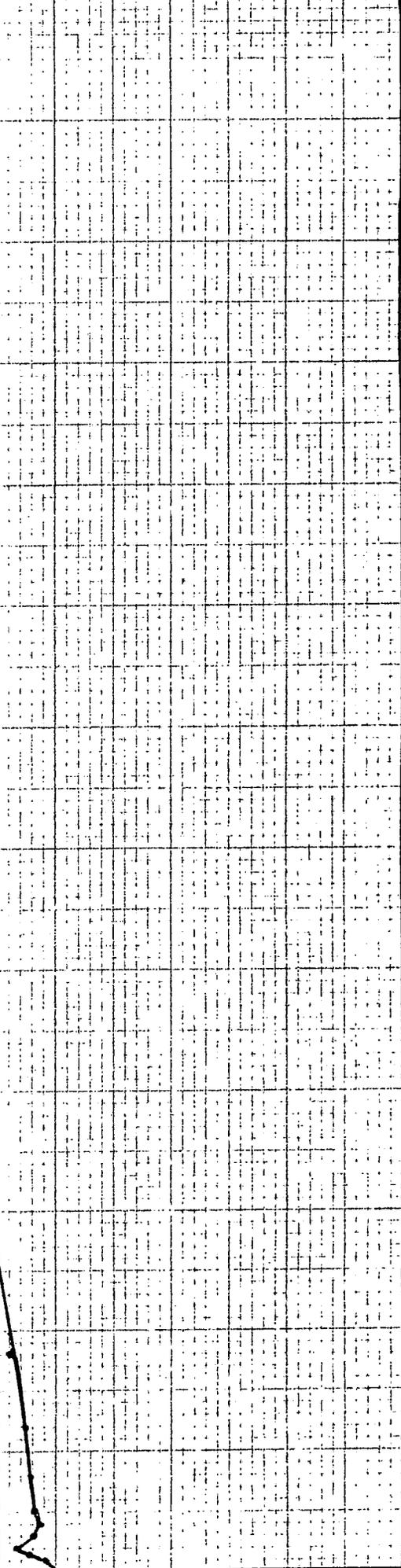
40

50

60

Stabilising Resistance Rohms

Fig 4



It was found that this margin was reduced by increasing R. The drop-off value approaching the pick-up value, and the margin being virtually zero for a value of  $R = 9$  ohms.

Here there was a conflict. It was clear that if the link was to operate effectively it must have an overload capacity which is the drop-off load. To achieve an overload capacity R must be increased, yet increasing R increases the capacitor voltage on short circuit and tends to increase the throughput current.

Some value of R to compromise the two conditions must be arrived at. The value decided upon was 2.6 ohms giving a safe overload capacity of 30% (drop-off at 50% overload). The corresponding capacitor voltage under fault conditions was found to be 64 volts R.M.S. i.e., 64% of system voltage and approximately 3 times full load voltage which was thought to be acceptable.

It was thus seen that the value of R that must be chosen for a link does not depend upon the ferro-resonant condition which it is its prime function to quench, but upon the overload capacity of the link in conflict with the maximum voltage specification for the capacitor. This is so since the value of resistance determined by the overload requirement must, logically, be always greater than the minimum value of R to quench ferro-resonance.

#### 2.4 Auxiliary Resistance 'r'.

About the time that these investigations were being carried out it was becoming clear that something had to be done about the generation of sub-harmonics which were giving

trouble with certain capacitive loads.

Attempts to control and quench this response by adjustment of the resistor R failed, but it was found that very large values of R, above 150 ohms, did show some influence over the sub-harmonics. The use of such a large value of R was clearly out of the question since the circuit limiting effect for short circuits was virtually destroyed.

It was found subsequently that a resistor, 'r', connected to a secondary winding on the saturable reactor (50 turns) was completely effective in destroying the sub-harmonic response of the system, fig.5. In effect this was like having a large value of R in the sub-harmonic mode by reflection. The saturable reactor now behaving as an ordinary transformer. Under fault conditions the saturable reactor core saturated and the reflected resistance dropped considerably thus the limiting effect of the circuit was maintained.

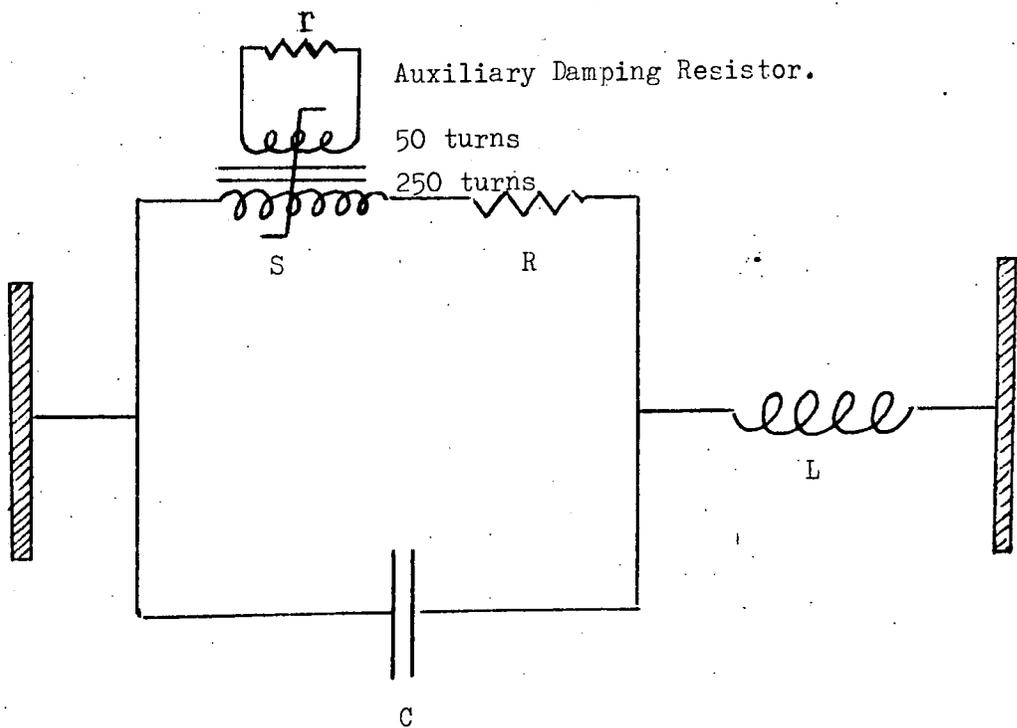
Under normal operation r is reflected into the primary circuit as  $6 \times \left(\frac{250}{50}\right)^2 = 150$  ohms which was found to be sufficient to quench the sub-harmonic response.

Under fault conditions the total resistance in the loop was measured as 6.1 ohms, indicating a reflected resistance of  $6.1 - 2.6 - 0.36 = 3.14$  ohms.

The effect of the resistance 'r' under fault conditions was to increase the fault throughput current slightly from 3.65 amps to 3.7 amps and to increase the capacitor voltage from 64 volts to 65 volts which was thought to be a small sacrifice in performance for such an important stabilising effect.

The discussion of the secondary resistance 'r' should properly be left until the section dealing with sub-harmonics in detail. However, its introduction here is necessary because its beneficial effect was discovered quite early in the investigation, and it was thus used in some of the load test investigations.

As was expected the effect of 'r' was to increase the circuit loss under normal load conditions because it represented a small transformer fed auxiliary load.



Connection of Auxiliary Damping Resistor 'r' to quench the sub-harmonic response.

Fig.5.

Load Flow and Fault Studies.

3.0 Measurement of Link impedance as a function of applied voltage or throughput current.

The link was short circuited and the applied voltage was increased to beyond the limiting value and then returned to zero. No drop-off margin was observed.

The impedance defined as the ratio of R.M.S. voltage to R.M.S. current is shown in fig.2 (a, b).

The justification of using R.M.S. values has been discussed previously and since the distortion of the throughput current waveform is never very great, see recordings Appendix II, the curves of fig.2 should have a useful validity when applied to load flow studies involving resonant links. This is borne out when the load transfer data is considered. The behaviour at least of the infinite busbar case being consistent with the curves of fig.2.

The curves of fig.2 start at approximately 3 ohms, the residual resistance of the resonant link. This minimum resistance was characteristic of the reactors used, but in a real system the series resistance could be exceedingly small.

At limitation the impedance/current curve is seen to double back giving rise to a non-linear region where for a given current, the impedance may have one of two values. This shows that at limitation the impedance does not suddenly become the value of the series linear reactor. In fact the load flow studies indicate that the link impedance initially remains substantially resistive.

The negative, impedance/current characteristic gives rise to the drop in current after limitation in the load flow studies and it is in this region that the drop-off margin occurs in cases where additional series impedance external to the link exists, i.e., in the case of loads.

As the applied voltage is increased the impedance increases reaching a maximum and then decreasing tending to 27 ohms, the saturable reactor/capacitor loop effectively adds 7 ohms to the total series impedance.

### 3.1 Load Flows.

The load flow studies were of two kinds; single fed and double fed systems (interconnected systems). The single end fed systems feeding inductive and resistive loads were fully investigated and found to be satisfactory in all aspects of performance. See Appendix II. With increasing loads the circuits limited at 2.2 amps representing a 120% overload.

Further increase in load up to short circuit caused the throughput current to increase to 3.65 amps.

After limiting the circuit recovered at 150% full load for the inductive case and 190% full load for the resistive case.

As stated before, values of recovery load are determined by the resistor R, and the limiting current and the final fault current are determined jointly by the linear components and by the saturation level of the saturable reactor.

Capacitive loads were also applied with the resistor r connected to suppress sub-harmonics.

Again the circuit limited at 2.2 amps and the drop-off occurred at 2.0 amps.

Increasing the capacitive load indefinitely caused the throughput current to increase beyond the normal limiting value to 9.7 amps with  $C' = 145 \mu F$ .

This must represent the load capacitance resonating with the total inductive reactance of the circuit and source. Beyond  $145 \mu F$  the throughput current dropped to 4.2 amps at  $200 \mu F$  and beyond this towards the short circuit value.

This capacitive case was interesting, but one hardly likely to arise in practice, since it amounted to a load of 4.5 per unit.

In this case the current must have been limited by the total residual series resistance of the circuit.

### 3.2 Fault Performance.

The fault performance of the link was tested under various load conditions.

The responses of the link were recorded and are given in detail in Appendix II. U.V. recordings.

Faults were thrown on an initially loaded link and subsequently cleared.

### 3.3 Inductive Load with Short Circuit.

The initial current inrush transient and asymmetry observed were of a low order and short lived, approximately one cycle.

For the duration of the fault the current was limited normally to the value of 3.7 amps RMS and was seen to have little distortion.

Current in the saturable reactor-capacitor loop showed large complementary spikes and could be explained simply by the switching action of the saturable reactor discharging the capacitor at saturation. The flux wave of the saturable reactor, initially sinusoidal, showed an approximately square form. Recovery of the link upon clearance of the fault was rapid and uneventful except for an odd spike or two of saturable reactor current caused by the collapse of the ferro-resonant state and the redistribution of energy within the recovering circuit.

Recovery was complete in the space of one to two cycles provided the initial load had been below the drop-off overload of the link.

#### 3.4 Resistive Load with Short Circuit.

The effect of fault throwing on the link carrying initially a resistive load was found to be substantially the same as for the inductive case, and the time of recovery was rapid.

#### 3.5 Capacitive Loads with Short Circuit.

During the fault period the circuit behaved as in the first two cases. However, the recovery behaviour was dependent upon three factors.

1) If the circuit load corresponded to a value of capacitance outside the range of sub-harmonic response for the value of applied voltage the recovery was complete after several cycles of jittery response. The length of the irregular response and hence the recovery time could be quite long if the load happened to be close to a range of sub-harmonic responses.

The recovery whilst it lasted showed sub-harmonics of unrecognisable frequencies.

Finally, normal load flow conditions would be restored quite suddenly after the period of irregularity.

2) If the circuit load corresponded to a value of capacitance within the range of sub-harmonic response for the value of applied voltage, the circuit usually, but not always, failed to recover and went into sustained sub-harmonic oscillation after the removal of the fault. About once every ten times the circuit recovered to normal operation, but never recovered once the sub-harmonic response had been established.

The odd cases where the circuit recovered probably correspond to recovery starting at or near a voltage peak or current zero whence the circuit could continue operating on the normal low current characteristic. That is to say, so long as the current had to pass through the sub-harmonic zone then, in this case, a sub-harmonic response must inevitably occur.

3) If resistance  $r$  was connected to the saturable reactor secondary and the tests of 1) and 2) were repeated, in both cases recovery occurred within a few cycles, but within this short time the response was jittery. The time of recovery was, however, not excessive and was considered to be satisfactory. At no time during many switching operations did the circuit fail to recover.

### 3.6 Stability.

Certain transient studies involving switching operations were performed at this stage to test the links stability.

The tests were intended to represent conditions, other than normal load flow, that the link might be faced with in a real system.

- 1) 1 amp inductive load switched on and then off.

Transient conditions were slight and normal load flow was established within approximately two cycles. The switch off performance was uneventful and collapse of the system was rapid involving a single pulse of discharge current in the capacitor saturable reactor loop.

- 2) Energising a dead circuit with a fault. Here limitation of the fault current and the establishment of normal sustained fault conditions was rapid. Assymetry and transient effects being minimal and confined to the first cycle.

In all of the foregoing tests no attempt was made to control the point on wave of the incidence of the fault, yet in all the tests where a variety of initiation angles obtained, the transient and asymmetry in current and voltage waves were seen to be minimal and of very short duration.

### 3.7 Double Fed or Interconnected Systems.

Detailed results are given in Appendix II.

The transfer of load across the link between two bus bars or systems, A and B, where B is an infinite bus bar.

The transference of load from A to B, under the influence of an increased voltage at A, occurred at a high power factor which was to be expected from the point of view of regarding the resonant link as being a purely resistive coupling.

What was unexpected was the very high power factor maintained just after limitation. Although this ultimately deteriorated, the power factor near limitation and drop-off was high indicating that the transfer impedance in these regions was largely resistive. This would go a long way to explaining the tendency to instability when connected to a single synchronous machine.

This would contrast markedly with a switched resonant link where the impedance at limitation would suddenly become highly inductive.

Limitation occurred at a through current of 2.24 amps, an overload of 124%, but immediately upon limitation the current decreased and continued to do so until a minimum of 1.62 amps was reached. Further increase in voltage caused an increase in current and a deterioration of power factor.

This behaviour is entirely consistent with the impedance characteristic of the link and could have been predicted from the impedance/voltage or current characteristics.

Very little measureable drop off margin was observed, the circuit recovered along the loading curve at 2.1 amps at an applied voltage of 107 volts. It is believed that for a truly

infinite bus bar system the drop-off margin would be zero as in the case of the short circuited link.

The transfer of load between two bus bars or systems A and B, where B is not an infinite bus bar.

In this case the impedance behind bus bar B was made arbitrarily equal to  $1.7 + j 5.8 \Omega$  and the load tests were repeated with satisfactory results.

The load transference took place at much lower power factors than those that obtained when end B was an infinite bus bar.

At limitation the power factor suddenly increased as the link impedance predominated and the regulation at the B end increased markedly.

As the voltage at end A was subsequently reduced a drop-off margin was seen to exist as follows:-

At limitation  $V_A = 116$  volts, throughput current  $I = 2.3$  amps.

At drop-off  $V_A = 112$  volts, throughput current  $I = 1.75$  amps.

### 3.8 Parallel Operation of a Resonant Link and a Conventional Supply.

Load sharing with this combination was shown to be feasible, tests being carried out at unity power factor and at low leading and lagging power factors.

An operating chart is shown in fig.6, where the conventional supply is largely inductive having an impedance angle of  $73.7^\circ$  and the resonant link is resistive with an impedance angle of zero. The agreement between this chart and the measured results is good. An indication of this can



be seen by considering the lagging load, point (2) on the chart, fig.6 as follows:-

	<u>Measured</u>	<u>Chart</u>
Link Power P <sub>A</sub>	140 watts export	136 watts export
Link Power Factor	0.67 lag export	0.65 lag export
Conventional supply Power P <sub>B</sub>	60 watts import	63.5 watts import
Conventional supply Power Factor	0.54 lead import	0.52 lead import

power to the load = 140 - 60 = 80 watts. measured.

and losses = link loss + conventional supply loss

$$= 2^2 \times 3.5 \Omega + 1.3^2 \times 1.7 \Omega$$

$$= 14 + 3 = 17 \text{ watts}$$

Nett power to the load = 80 - 17 = 63 watts from results

current in the load = 2.8 amps.

therefore load power =  $2.8^2 \times 7 \Omega = 55 \text{ watts.}$

It is clear, however, that the sharing of the load largely depends upon the link resistance having a comparable value to that of the conventional supply impedance, otherwise the link would tend to take all of the load.

In a real link steps would be taken to minimise the links resistance, so because of this, and the large transfer of power between the link and the conventional supply, it is felt that where this combination may occur in practice some detuning of the link would be desirable to equalise the load sharing.

### 3.9 Effect of Detuning.

These tests were done to record the performance of the link in its detuned state, the reason being that in the case of a link sharing load with a conventional supply some benefit may be

obtained by modifying the links characteristic to influence the load flow.

In the case of an inductive load it can be seen from the results that detuning the link so that it became inductive had the effect of stopping the large power transfer between the two sources; each now operating at a low lagging power factor. Detuning so that the link became capacitive made the power transference larger and hence worse.

It is seen, therefore, that some control through detuning is possible and in the case of a parallel combination of resonant link and conventional supply, desirable, although extensive detuning is not recommended since this would call into question the need for a resonance link in such a case.

### 3.10 Short Circuit Performance.

For a short circuit at the terminals of the parallel combination of resonant link and conventional supply the following results were measured:-

Fault infeed from the conventional supply	$\frac{100}{6\Omega}$ volts	= 16.7 amps = 1670 MVA equivalent.
Fault infeed from the resonant link		= 3.7 amps = 370 MVA equivalent.
total infeed		= 19.6 amps = 1960 MVA equivalent.
alternatively two conventional supplies infeed		= 33.4 amps = 3340 MVA equivalent.

thus there is a total reduction of the fault level by 1,380 MVA equivalent.

Note that the sum of the two infeeds is less than the scalar sum because of phase angle differences. See also the effect of the double resonance link page 72.

### 3.11 Two Resonant Links Operating together and Sharing Load.

For two links having the same parameters, load sharing was equal up to and beyond limitation, both links limited together.

Unstabilised links showed sub-harmonic response with certain capacitive loads, but no other unsatisfactory response was observed.

In the case of links having different parameters and different limiting levels, if the levels were widely different, the links limited in turn as the load increased, but up to limitation the link loadings were similar.

For links with limiting levels within 20% of each other the lower limit became the limit of the whole system both links limiting together. It is believed that the low set link limits first and thus reducing its own share of the load, causes the high set link to limit.

Sub-harmonic Response.

4.0 Overall Link Response.

It was indicated earlier that with certain capacitive loads the circuit could give rise to the generation of sub-harmonics. Whilst it is unlikely that such a load would be encountered in practice extensive cable systems and shunt capacitors could give rise to instability under light load conditions.

The disadvantage of sub-harmonics of current and voltage in a power system can be summarised as follows:-

- 1) Oscillation (hunting) and perhaps instability of rotating machines.
- 2) Interference with and possible mal-operation of earth fault protective systems.
- 3) Hunting and instability of control systems.
- 4) Voltage flicker and dangerous stroboscopic effects from lighting. Overvoltages on E.H.V. systems.
- 5) System resonance with harmonics of the sub-harmonic.

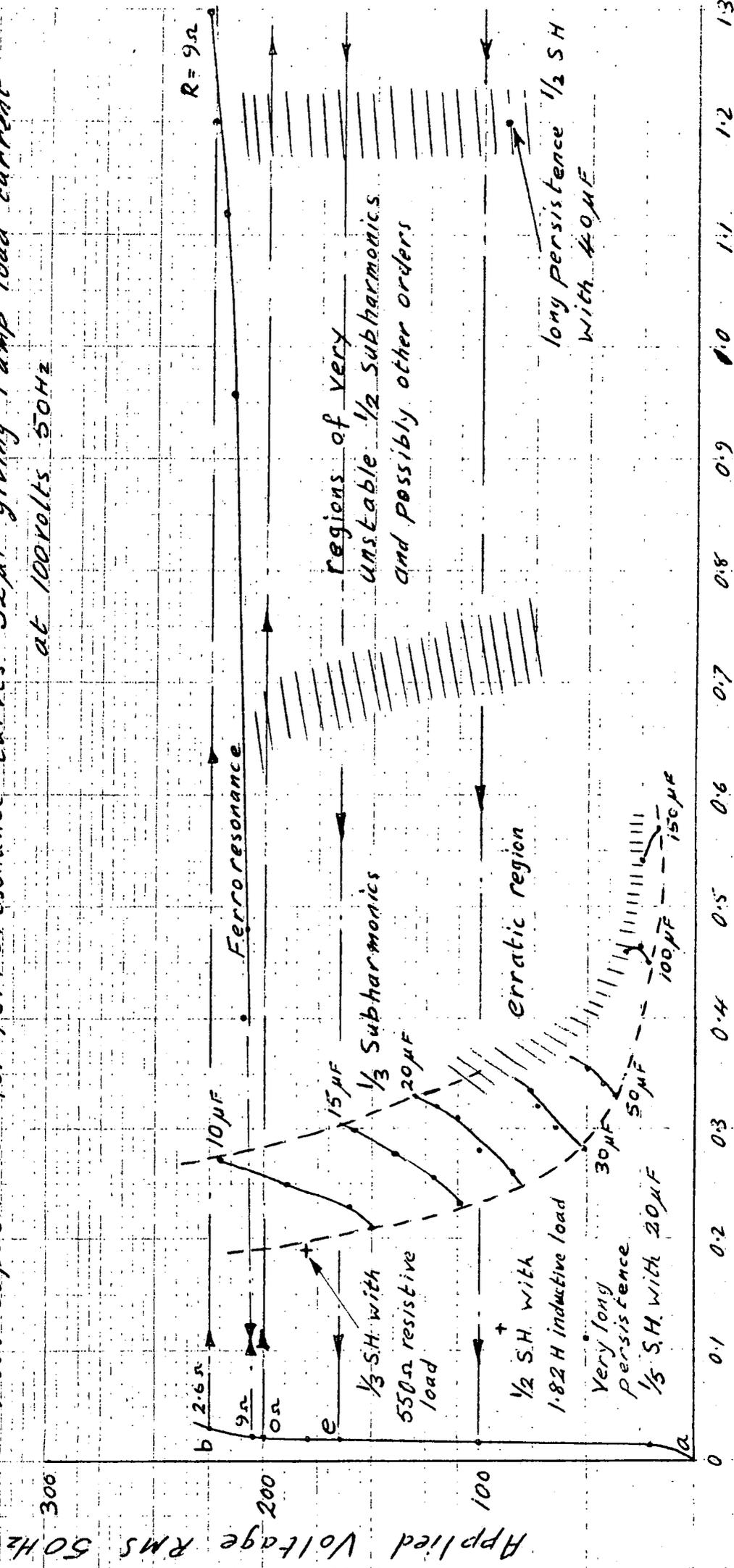
It was discovered early in the investigation that a resistance 'r' connected to a secondary winding on the saturable reactor would quench or prevent the sub-harmonic response. Ultimately, therefore, it would be an advantage to gain some insight into the function of this resistance.

The voltage and current response of the link is shown in fig.7 (1, 2, 3 and 4). It is immediately seen to be complicated and multivalued.

1.

# Range of Subharmonics and Ferroresonance

Load capacitance for ferroresonance curves 32  $\mu\text{F}$  giving 1 amp load current at 100 volts 50Hz



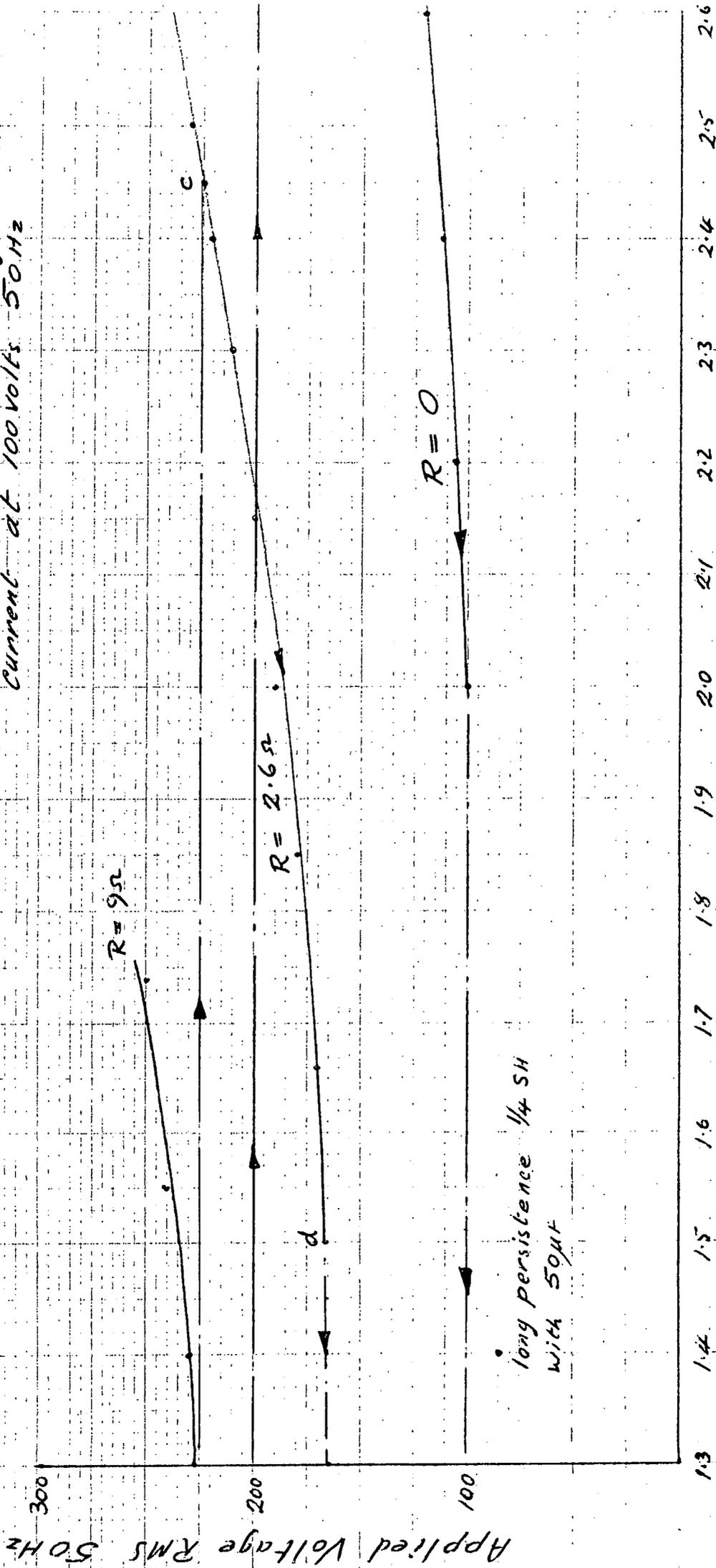
Saturable Non-Linear Reactor R.M.S. Current amps

Fig 7

2.

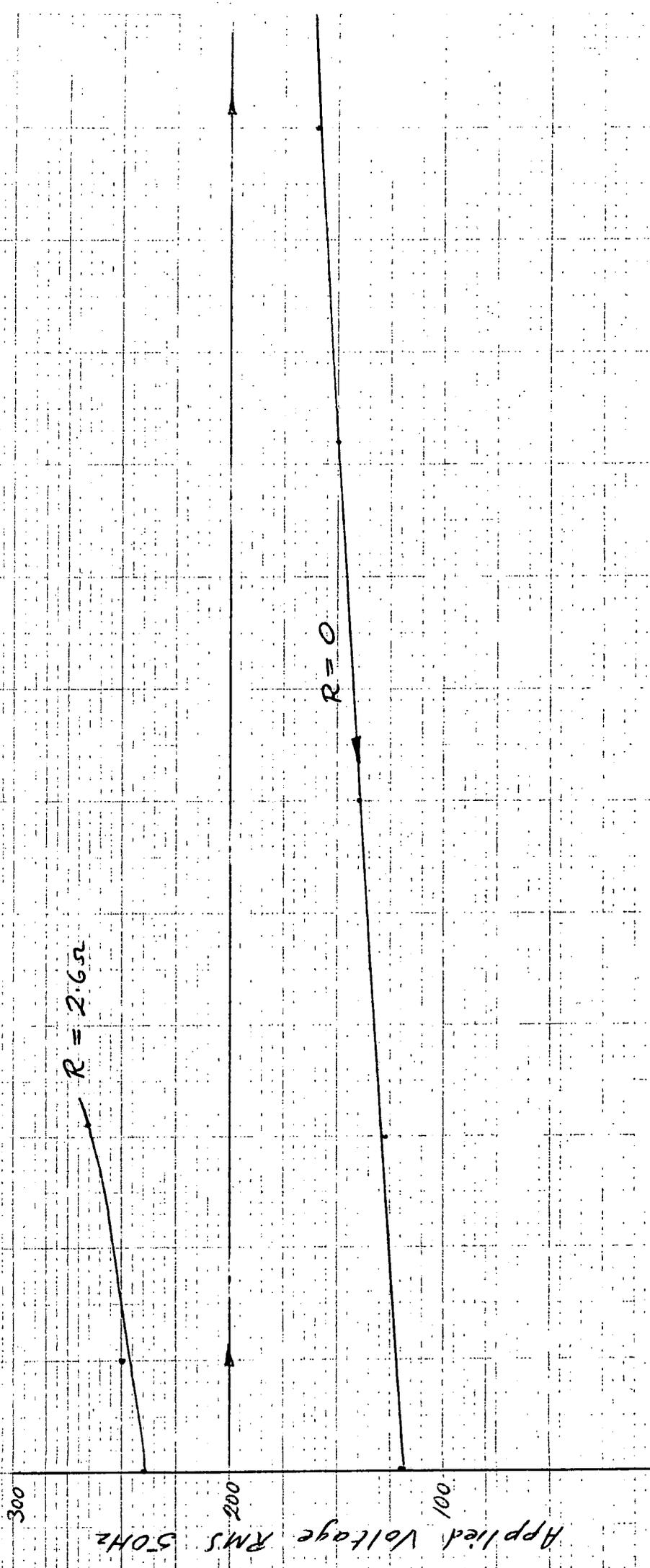
Range of Subharmonics and Ferroresonance

Load capacitance for ferroresonance curves  $32 \mu\text{F}$  giving lamp load current at 100 volts 50 Hz



Saturable Non-Linear Reactor RMS Current amps

3.



26 27 28 29 30 31 32 33 34 35 36 37 38 39

Saturable Non-Linear Reactor RMS Current amps

Fig. 7.

#### 4.1 Ferro-resonant Response.

Referring to fig.7, for any load on the link, if the voltage is increased the saturable reactor current increases very slowly giving rise to the initial steep curve a - b. At b the current suddenly jumps to a high value, c, and the circuit is said to be in a state of ferro-resonance. ref.2. Further increase in voltage causes a continuous and rapid rise in current.

When the voltage is reduced the saturable reactor current falls along the high valued curve but instead of returning to the low value of current at c, the high valued curve is followed to d when the current suddenly falls to e on the low valued magnetisation curve.

This is the "pick-up" and "drop-off" margin already discussed when considering the value of the stabilising resistor R. Fig.7 shows a family of curves for R starting at zero and it can be seen that for values of R = 9ohms and above the margin is reduced to zero.

#### 4.2 Sub-harmonic Response.

Additional to the 50 Hz ferro-resonance response there lies between the upper and lower curves, but not bounded by the "pick-up" and "drop-off" lines, a region in which the circuit response can be sub-harmonic.

Within this region the circuit is capable of oscillation:-

- 1) at a stable sub-harmonic frequency of  $\frac{1}{3}$  fundamental.
- 2) at other sub-harmonic frequencies  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$  and

possibly lower orders, which are unstable and self extinguishing over periods of time ranging from seconds to many minutes, and it is probable that all these sub-harmonics may have stable regions of extremely limited extent requiring a very precise set of circuit parameters and voltage conditions to stabilise them. Such regions if they exist would be very difficult to locate and none were found during the course of this study, except that the  $1/5$  sub-harmonic persisted for so long that it could almost be regarded as a stable mode.

- 3) Regions of erratic response where sub-harmonics of undetermined and possibly changing frequency exist. Operation in this region is characterised by continuously changing waveforms. This response may persist for only a few seconds to several minutes and then like the unstable sub-harmonics of definite period, it suddenly, for no apparent reason, extinguishes. Sometimes extinction occurs after a particularly violent surge of current probably indicating some radical redistribution of energy within the circuit.

Recovery of the circuit is always to the lower magnetisation curve and never to the higher ferro-resonance curve.

However, by adjustment of voltage with a capacitive load the ferro-resonant state can in the process of extinction pass into the sub-harmonic state instead of extinguishing onto the low current curve.

If the voltage is subsequently increased the circuit does not pass from the sub-harmonic state to the ferro-resonant state, but first the sub-harmonic extinguishes and the circuit reverts to the lower curve.

It is clearly seen from fig.7, that for a given voltage at least three stable current conditions can exist and many more unstable current states, which, may nevertheless persist for a long time.

Fig.7 shows a  $1/3$  sub-harmonic region for a variety of load capacitance values. The left hand boundary of this region is characterised by an abrupt cessation of the  $1/3$  sub-harmonic response for a given capacitance as the voltage is reduced and is thus well defined.

The right hand boundary is an uncertain region of jittery response where the  $1/3$  sub-harmonic becomes more and more difficult to discern.

Finally the response in this region degenerates into an unstable self-extinguishing erratic response.

This uncertain boundary of erratic response is relatively broad for large values of load capacitance and low voltages but for lower values of load capacitance and higher voltages the erratic region is narrow and cessation of stable sub-harmonic response becomes abrupt.

The sub-harmonic response in the resonant link is characterised by the existence of large sub-harmonic flux components in the reactor core and excursions into saturation occurring at least once and sometimes twice in any half cycle

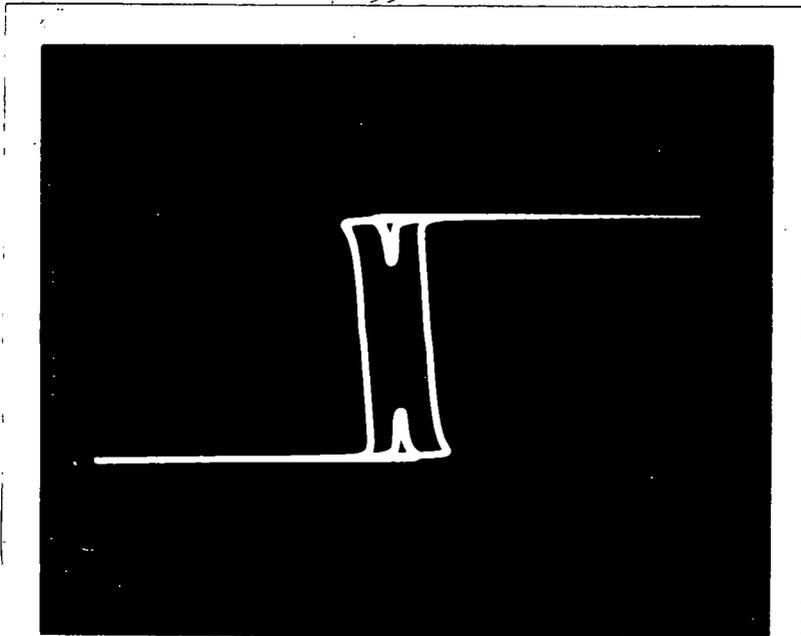
of the flux wave. Accompanying these excursions into saturation are surges of current in the parallel loop that must involve the transference of energy necessary to maintain the oscillations.

Superficially it is tempting to regard the saturable reactor as a sort of switch going on and off as it goes in and out of saturation, discharging or holding the charge on the capacitor. However, the impression that is gained from the results of this study suggest that the simple switch concept is too simple and that the surges of current are in the nature of impulse resonance phenomena occurring as the reactor approaches saturation.

Certainly a simple switching theory could not account for the existence of sub-harmonics in circuits containing stalloy cores where the onset of saturation is a gradual process.

Fig.8 shows a typical hysteresis loop and waveforms of a  $1/3$  sub-harmonic response.

The hysteresis loop is complicated by the existence of re-entrant or minor loops superimposed upon the main loop. The total number of loops giving the order of the sub-harmonic present.



Hysteresis Loop of  $1/3$  sub-harmonic response.

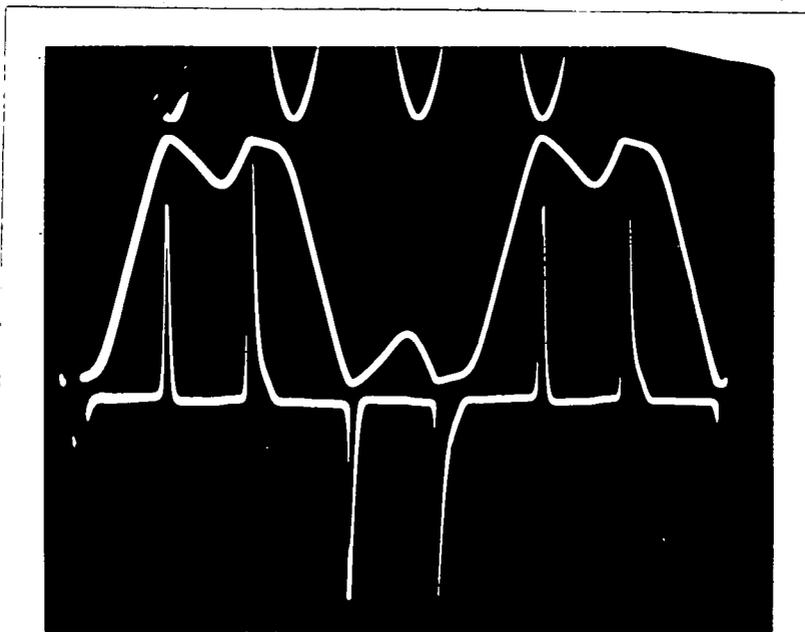
Scales B = 1.00 Wb/m<sup>2</sup>/cm.

V = 105 volts R.M.S.

I<sub>S</sub> = 0.10 A/cm.

C' = 20 μ F

Fig.8a.



Saturable reactor flux and current during  $1/3$  sub-harmonic response.

C' = 20 μ F.

V = 105 volts. R.M.S.

Scales B = 1.0 Wb/m<sup>2</sup>/cm.

I<sub>S</sub> = 1.0 A/cm.

Fig.8b.

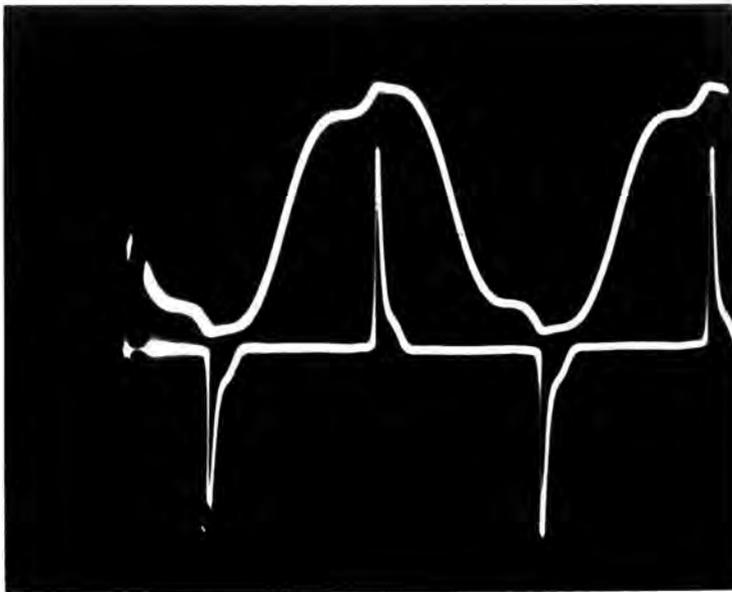


Detail of minor loop of 8a

Scales  $B = 0.5 \text{ Wb/m}^2/\text{cm}.$

$I_S = 0.1 \text{ A/cm}.$

Fig.8c.



Saturable reactor flux and current during  $1/3$  sub-harmonic response.

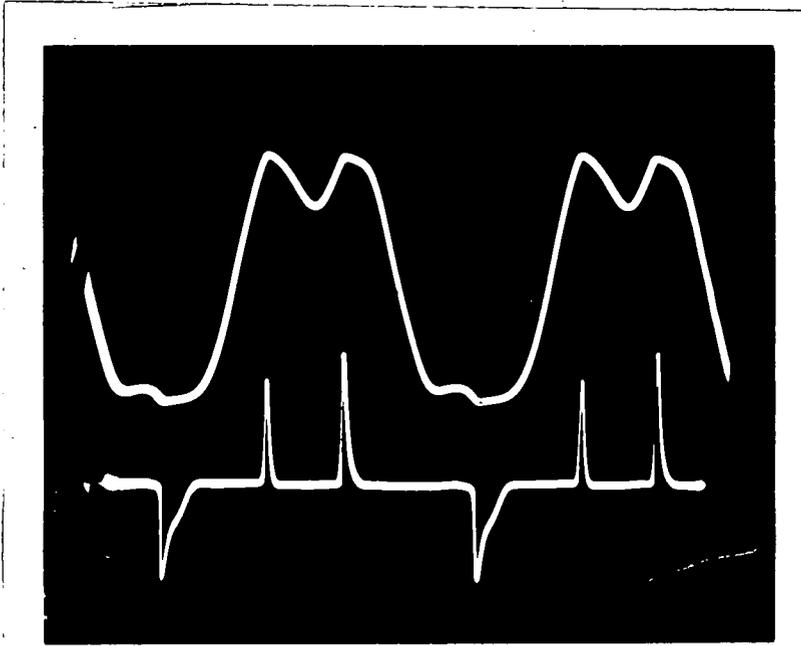
Scales  $B = 1.0 \text{ Wb/m}^2/\text{cm}.$

$V = 100 \text{ volts R.M.S}.$

$I_S = 1.0 \text{ A/cm}.$

$C' = 17 \mu \text{ F}.$

Fig.8d.



$\frac{1}{3}$  Sub-harmonic Response.

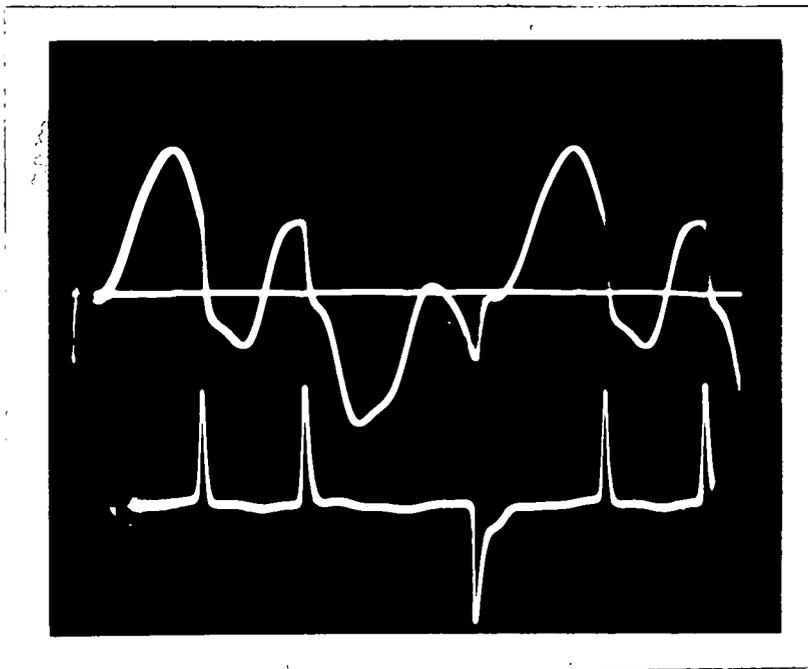
Scales  $B = 1.0 \text{ Wb/m}^2/\text{cm}.$

$V = 100 \text{ volts R.M.S.}$

$I_s = 2.0 \text{ A/cm}.$

$C' = 22 \mu \text{ F}$

Fig. 8e



Capacitor/Saturable Reactor Voltage during  $\frac{1}{3}$  sub-harmonic response.

Scales  $V_c = V_s = 20 \text{ v/cm}.$

$I_s = 2.0 \text{ A/cm}.$

Fig.8f.

#### 4.3 Initiation of Sub-harmonic Response.

As indicated on previous pages with certain capacitive loads a sub-harmonic response results, which requires a switching action or some system disturbance for initiation.

The necessity for some disturbance to start the sub-harmonic response is very significant because without this the circuit behaves normally with sinusoidal response up to the instant of the onset of the high current of ferro-resonance.

As far as can be seen from the literature, the need for a sudden disturbance has been accepted by other investigators as being absolutely necessary (ref: 1 - Travis, and 2 - J.D. McCrumm) and they go on to say that the effect of the disturbance is to produce the correct voltage and current conditions requisite to the initiation of the sub-harmonic response.

If this were entirely correct one might expect some difficulty in arranging for the correct voltage and current conditions to exist for such an arbitrary process as closing a switch, yet if the applied voltage and capacitance have certain values the chances of exciting a sub-harmonic response by simply closing a switch is something over 90%.

The author believes that for the initiation of a sub-harmonic response all that is necessary is for the saturable reactor to have imparted to it an energy that is abnormally high for the applied voltage level, moreover, it will be shown that the energy need not be applied to the circuit as a sudden shock.

Previous workers have always stressed the need for a shock excitation for the establishment of the sub-harmonic response. Usually any switching operation or transient disturbance is sufficient to excite the oscillations provided that the circuit parameters and applied voltage are of the correct order, and yet if the excess energy idea holds true it should be possible to establish the sub-harmonic response simply by slowly imparting energy to the circuit until the resonance phenomena is established.

A test was thus devised to impart energy to the circuit by the application of a d.c. current, to the saturable reactor, the magnitude increased very slowly from zero until the reactors instantaneous inductance reduced as saturation was approached.

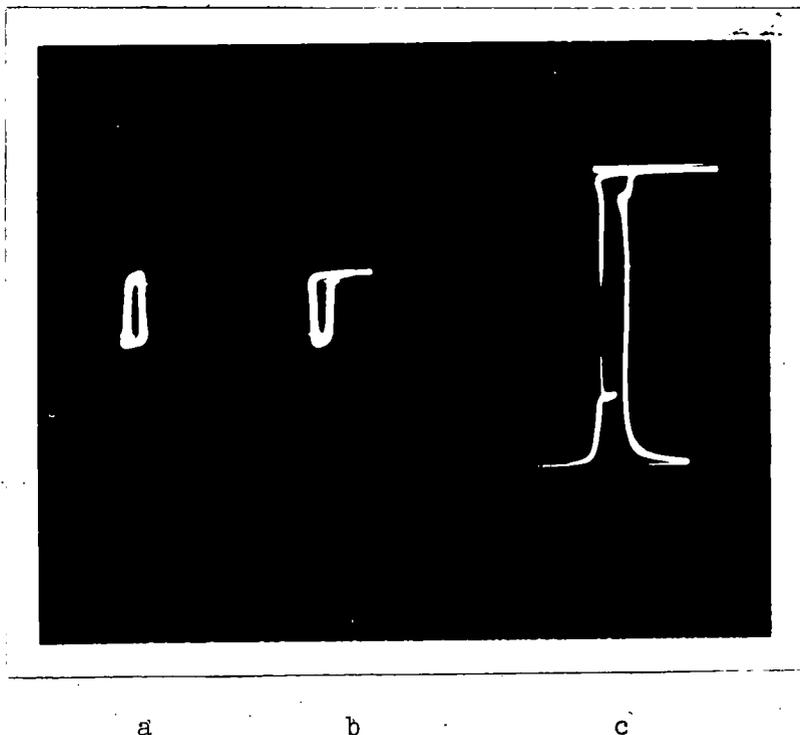
The results of this test are shown in fig.9. It was found that by increasing the d.c. current at the rate of less than an ampere per minute the circuit could be set into a state of sub-harmonic oscillation. The applied d.c. current that achieved this was below that required to effectively saturate the core. Once the sub-harmonic response had been established, the d.c. bias had little effect on the stable sub-harmonic response and could be removed.

In the case where the circuit was set up for unstable self-extinguishing sub-harmonic response the effect of the d.c. bias was to stabilise and sustain the response but as soon as the bias was removed the sub-harmonic response extinguished.

It was conceived that this method might provide a technique for stabilising lower order sub-harmonic responses which were otherwise very fleeting in nature.

Scales B =  $0.715 \text{ Wb/m}^2/\text{cm}$ .

$I_s = 0.3 \text{ A/cm}$ .



Hysteresis Loops showing the excitation of sub-harmonics by the slow application of a d.c. current.

The d.c. current was injected very slowly at the rate of approximately 1 milliamp per second into 10 turns on the saturable reactor.

90 volts was applied to the circuit to keep well below the voltage normally needed to saturate the inductor with a capacitive load of  $20 \mu \text{ F}$ .

- a) No d.c. applied.
- b) 90 milliamp d.c., resonance just beginning.
- c) 99 milliamp d.c.,  $\frac{1}{3}$  sub-harmonic started.

Upon the removal of the d.c. bias the sub-harmonic response continued virtually unaltered.

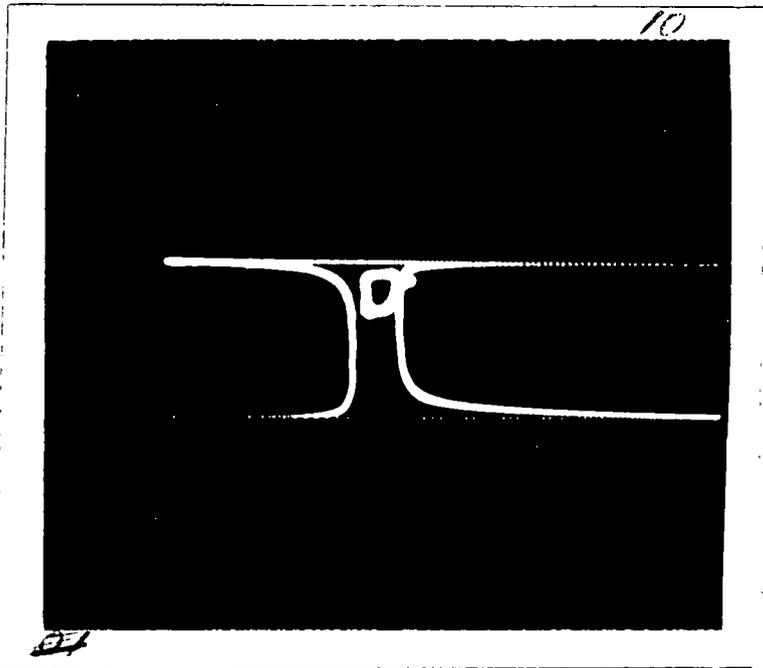
Fig.9.

This was tried and it was found to be sometimes possible to stabilise the higher  $1/2$  sub-harmonic. Fig.10. But the effect on other orders and on the erratic response when there was time to apply the d.c. bias was either to extinguish the sub-harmonics or curiously enough, to establish a  $1/3$  sub-harmonic which extinguished when the d.c. bias was removed.

If the 99 milliamp of d.c. current that was sufficient to establish the stable  $1/3$  sub-harmonic response of fig.9, was further increased the response became erratic, finally extinguishing and at 110 milliamps, ferro-resonance was initiated.

Reduction of the applied d.c. bias to 95 milliamps caused the ferro-resonance to give way to the stable  $1/3$  sub-harmonic.

The complete removal of the d.c. current suddenly, caused the response to revert to either the  $1/3$  sub-harmonic mode or to the low current sinusoidal state.



Hysteresis Loop of an unstable  $1/2$  sub-harmonic stabilised by the application of a d.c. bias of 85 milliamps, on 10 turns.

$$V = 90 \text{ volts.}$$

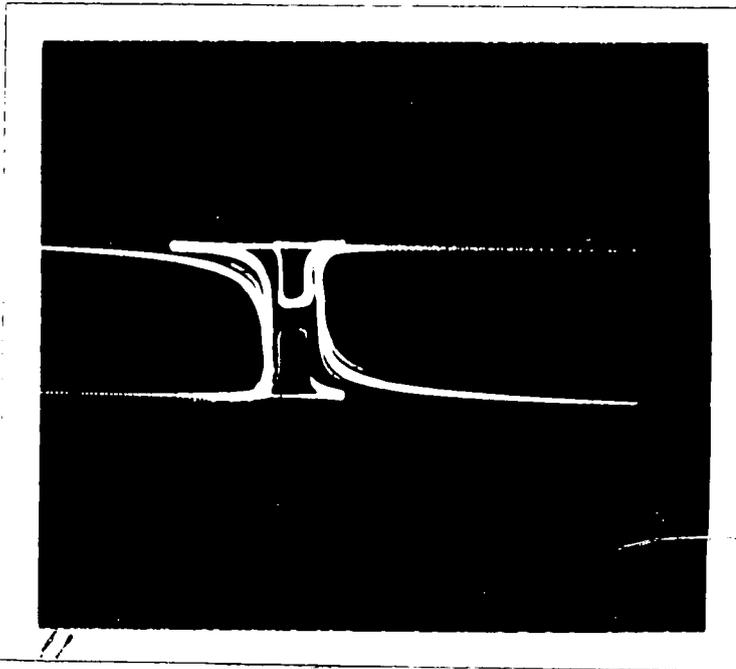
$$C = 160 \mu \text{ F.}$$

$$C' = 40 \mu \text{ F.}$$

Scales  $B = 1.5 \text{ Wb/m}^2/\text{cm.}$

$$I_s = 0.15 \text{ A/cm.}$$

Fig.10.



Hysteresis Loop of long persistence (1 - 2 minutes)  $1/4$  sub-harmonic

Scales  $B = 1.5 \text{ Wb/m}^2/\text{cm}.$

$V = 85 \text{ volts}.$

$C = 160 \mu \text{ F}$

$I_s = 0.15 \text{ A/cm}.$

$C' = 50 \mu \text{ F}.$

Note the four loops and the tendency to jitter.

The response was self extinguishing and although it could be excited several times in succession when the photograph was taken, the next day with the same conditions it could not be excited.

This shows the very fickle nature of the more uncommon sub-harmonics. Perhaps they are difficult to repeat because they depend upon very specific values of resistance and slight changes in resistance may prohibit their excitation. Certainly this is the only parameter that could have changed in the circuit in 24 hours. Reactor core temperature may also be a significant factor.

Fig.11.

The foregoing result is put forward as evidence that the cause of the sub-harmonics is excess energy in the reactor, but it does not explain why the circuit goes into sub-harmonic oscillation.

Travis ref.1 has suggested that the reason for sub-harmonic oscillation lies in the trapping of charge on the capacitor due to the inductor going in and out of saturation.

Whilst this explanation has been widely accepted for many years it is not considered to have sufficient generality to explain all the phenomena associated with the generation of sub-harmonics. In particular, results recorded in this work show that although in some cases charges of a particular polarity can exist on the capacitor for more than half a cycle of the 50 Hz applied voltage so that partial charge trapping could be claimed to exist, in some instances, no evidence of trapped charge was observed and large changes of charge were observed to take place even when the reactor was definitely unsaturated.

The truth behind Travis' theory of trapped charge is, the author believes, the implied idea of excess energy in the system.

It is clear from Fig. 8a, b and other records that if anything is trapped in the circuit it is magnetic flux and hence magnetic energy. Thus it is proposed to take Travis' theory of trapped charge and to extend and broaden it in a theory of excess energy.

#### 4.4 Theory of Excess Energy.

The proposed theory is as follows.

Due to some disturbance or otherwise the circuit has imparted to it an energy which is excessive considering the applied voltage condition. This energy may be manifest as excessive electrostatic charge or excessive magnetic flux or of a combination of the two. Subsequently the circuit endeavours to reject this energy and usually succeeds after several cycles of the 50 Hz frequency. The rejection of this energy can only take place relatively slowly because of the large effective time constant and low mean natural frequency of the circuit. Being a non-linear system both these must be functions of current or voltage but are demonstrably long, Fig.47, Appendix III. Also reference 5 fig. 7 C.F. Spitzer.

It could thus transpire that at certain voltages for a given set of circuit parameters the circuit is forced to run into resonance or saturation again before it has got rid of the excess energy which is thus replenished and the oscillations continue. The order of oscillation will depend upon the mean natural frequency of the circuit being near to a sub-multiple of the supply frequency.

For a given applied voltage if this is close, a stable sub-harmonic will obtain. If there is considerable deviation an erratic possibly self-extinguishing oscillation will occur.

It is significant that no stable sub-frequency is obtained that is not a sub-multiple of the mains frequency,

because the sub-harmonic oscillations are themselves dependent upon resonance phenomena excited by the fundamental frequency.

This theory predicts the result of the application of a d.c. current to the saturable reactor circuit and explains why otherwise a shock excitation is necessary to establish the oscillations.

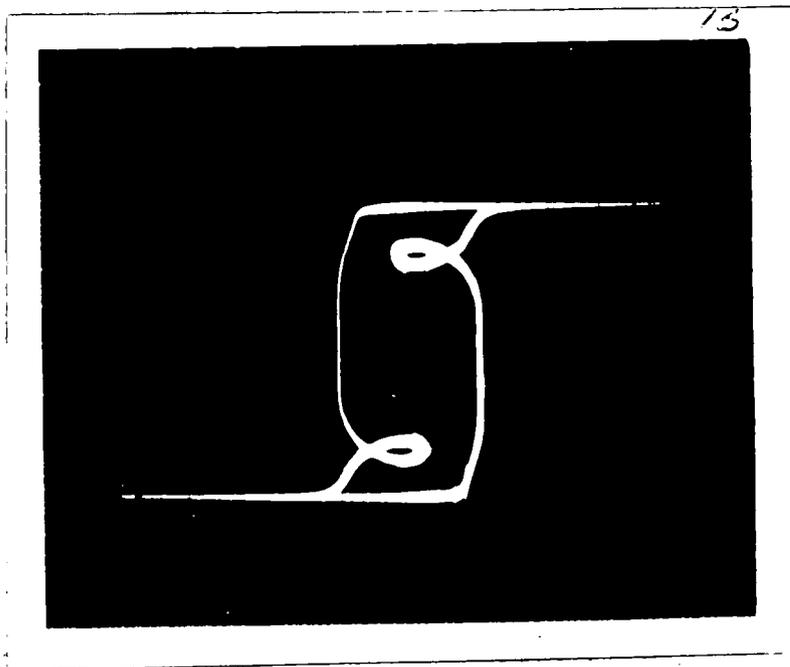
It also predicts that if energy is artificially abstracted from the capacitor saturable reactor loop the sub-harmonic will be damped out or prevented from starting.

Hence it is found that the application of a resistor to the secondary inductor winding quenches the sub-harmonic response as does a resistor applied across the whole parallel combination. The latter, however, is not a good power system arrangement.

#### 4.5 Damping out the Sub-harmonic Response.

The effect of the application of a resistor 'r' across a secondary winding of the saturable reactor is to increase the apparent inductor losses as indicated by the broadening of the hysteresis loop, Fig.12. Subsequently a critical broadening occurs which leads to the extinction of the sub-harmonic response, beyond which no sub-harmonic can be established or maintained. The minimum value of 'r' to achieve this turned out to be  $6 \Omega$  on the 50 turn winding.

That the losses in the saturable reactor profoundly affect the existence of sub-harmonics in the circuit is illustrated by the fact that it is easier to establish and maintain sub-harmonics in a circuit containing a low loss core



Hysteresis Loop with  $r = 31$  ohms on 50 turn winding.

$1/3$  sub-harmonic response just about to extinguish.

Scale  $B = 0.76$  Wb/m<sup>2</sup>/cm.  $V = 100$  volts.

$I_s = 0.11$  A/cm.  $C' = 23 \mu$  F.

Fig.12.

material inductor than in a circuit containing a stalloy core inductor.

Stalloy core inductors do, however, have a sub-harmonic response as indicated by Fig.13 a, b.

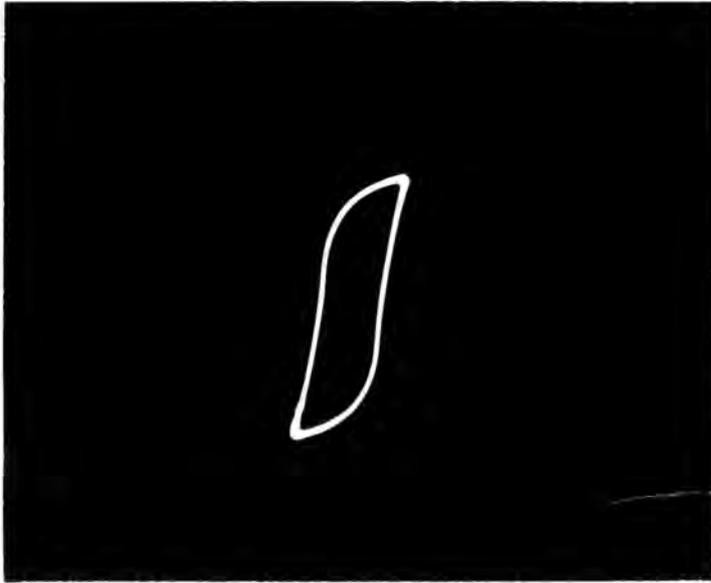
#### 4.6 Sub-harmonics with Stalloy Cores.

By replacing the square loop saturable reactor by one having a stalloy core a resonant link was produced whose operational properties were similar, but inferior to those of a link circuit containing the square loop material.

This link was capable of limiting through fault current, but because of the more gradual onset of saturation it was necessary for successful normal operation, to operate low down on the hysteresis curve. This meant that greater excursions of voltage were necessary under fault conditions in order to saturate the reactor and further since the onset of saturation was not abrupt the limitation of the capacitor voltage was not so good. Interference with the state of resonance was also a gradual process occurring down into the region of normal operation and thus adversely effecting the link's regulation which ideally should be negligible.

With a potential fault current of 30 amps, equivalent to 3,000 MVA, the link limited the throughput current on dead short circuit to 7.4 amperes, more than twice the current limit set by the square loop reactor.

The generation of sub-harmonics in this system was not as easy as in the square loop reactor system and this is thought to be because of the greater loss of the stalloy core.

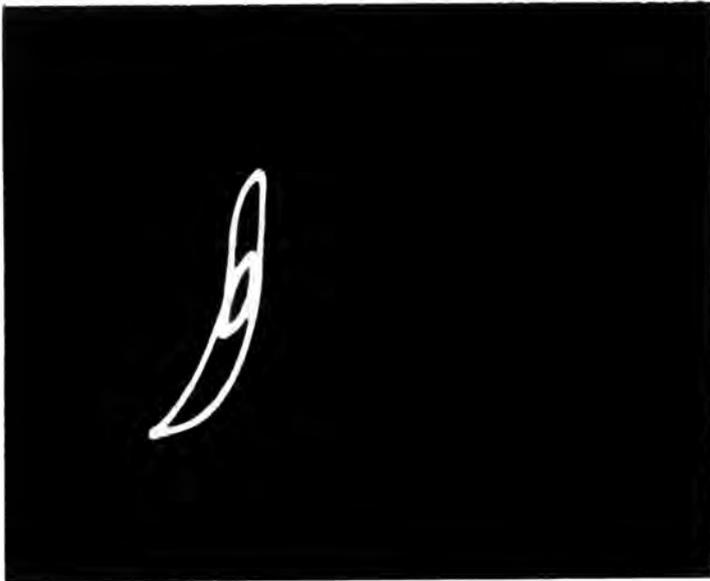


Normal Hysteresis Loop of Stalloy (120 turns C.T.) core.

Scales  $B = 1.0 \text{ Wb/m}^2/\text{cm}.$

$I_S = 0.4 \text{ A/cm}.$

Fig.13a.



$1/2$  sub-harmonic with Stalloy cored reactor (120 turns C.T.)

Scale  $B = 1.0 \text{ Wb/m}^2/\text{cm}.$

$V = 70 \text{ volts}.$

$I_S = 0.8 \text{ A/cm}.$

$C = 160 \mu \text{ F}.$

$C' = 12 \mu \text{ F}.$

Fig.13b.

When sub-harmonics were established their range in terms of load capacitance for a given applied voltage was much narrower, of the order of  $5 \mu\text{F}$ , and the order of stable sub-harmonics was  $1/2$ . Fig. 13b.

#### 4.7 Effect of Remnance in the Reactor Core.

The effect of remnance in the reactor core was not found to have the significance that some theoretical studies would attribute to it. Ref.1.

Certainly it did not determine the form of the resulting sub-harmonics.

Its effect was investigated by magnetising the core by d.c. currents in the dead state and then switching on the supply.

The only effect that the remnance seemed to have was to help the initiation of sub-harmonics if it was large enough. Certainly the incidence of sub-harmonic response with a demagnetised core when the circuit was switched on with a  $20 \mu\text{F}$  load, was less than when the core had a remnant magnetisation.

Such a result was not unexpected in view of the excess energy idea since any remnant flux must represent an amount of stored energy within the system just as an initial charge on the capacitor in the series case has according to some workers, ref.1 helped the initiation of sub-harmonics and also represents stored energy.

The effect of the remnance in relation to the direction of the applied voltage at the time of switch closure was not investigated but is probably of significance.

#### 4.8 Analysis of Waveform Conditions.

Consideration is given to the detail of the circuit waveforms during stable  $1/3$  sub-harmonic oscillation. Fig.14

Starting at the first major flux peak at "a" there is a surge of current in the parallel loop with a positive but rapidly reducing voltage on the capacitor.

Here there is a transfer of energy from the capacitor to the inductor.

Immediately following the surge, the current reverses and the capacitor voltage goes negative as the supply voltage goes negative. This gives rise to a reduction of flux level in the reactor core and a trough in the flux wave is produced. This is an attempt by the system voltage to drive the flux negative; but which does not succeed.

The flux and hence the energy in the field reaches a minimum and begins to rise again as the capacitor voltage and supply voltage go positive.

This reduction of flux level and reversal of current create the minor loop on the hysteresis loop. A second flux peak is reached as the reactor once more saturates and the capacitor is again discharged in a surge of current, which again represents a transfer of energy to the reactor.

This second discharge is significant because it occurs when the supply voltage and capacitor voltage are reaching peak values in the positive direction, but subsequent to the surge the capacitor voltage is reduced to near zero. That is to say the capacitor voltage is zero when it should normally have a peak positive value.

This means that as the driving voltage falls towards the negative direction the capacitor voltage is forced to swing to approximately twice normal voltage in the negative direction because it starts from zero instead of positive peak value.

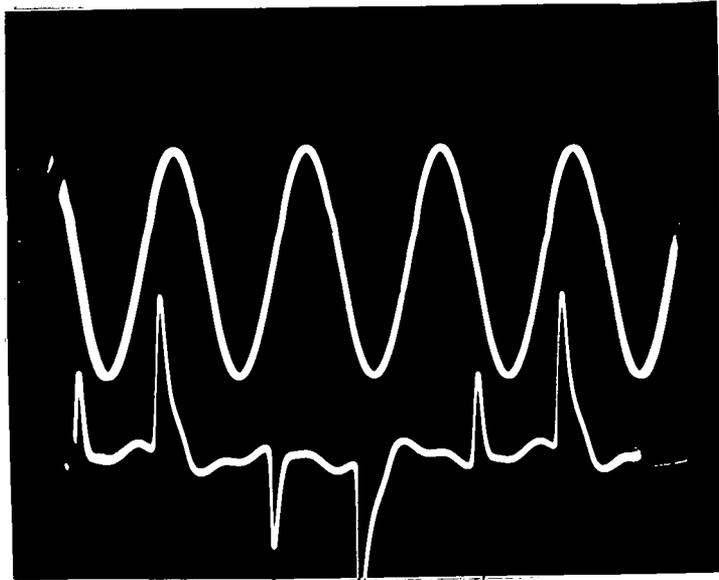
This large voltage excursion on the capacitor is common to all the sub-harmonic responses and probably provides the mechanism of energy transfer that keeps the sub-harmonics going.

The effect of this large voltage excursion on the capacitor is to cause the reactor flux to undergo reversal so that there is a major energy transference during this period.

The reactor flux reaches a peak in the negative direction leading to a current surge as the charge on the capacitor is reduced to zero.

This negative going current pulse is identical to the first positive pulse and all the foregoing events are repeated negatively until the cycle is completed in a period equal to three 50 Hz periods.

If the system voltage is increased the flux troughs and minor loops between the current pulses get deeper until, in this case, at 130 volts, the sub-harmonic extinguishes when the bases of the flux troughs and loops approach zero. This suggests that during the sub-harmonic response the system voltage is not sufficient to drive the flux level to zero, which can only happen when the capacitor voltage achieves a sufficient magnitude as was described above.

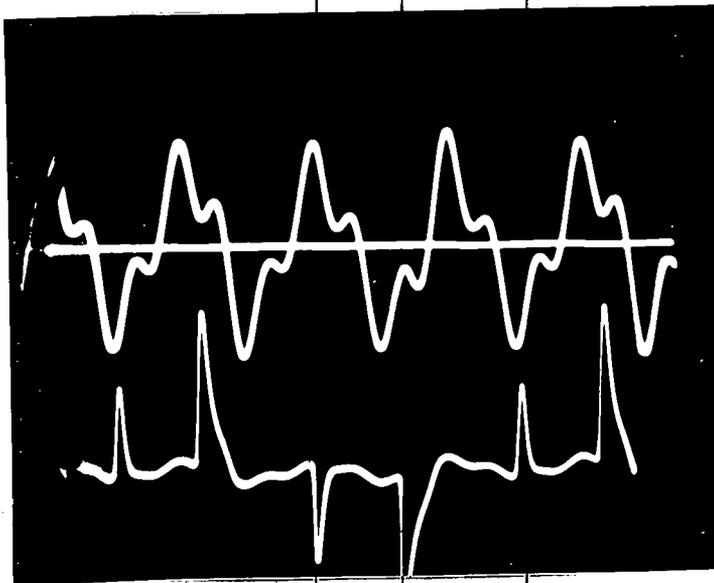


Scales

$$V = 100 \text{ v/cm.}$$

$I_s$

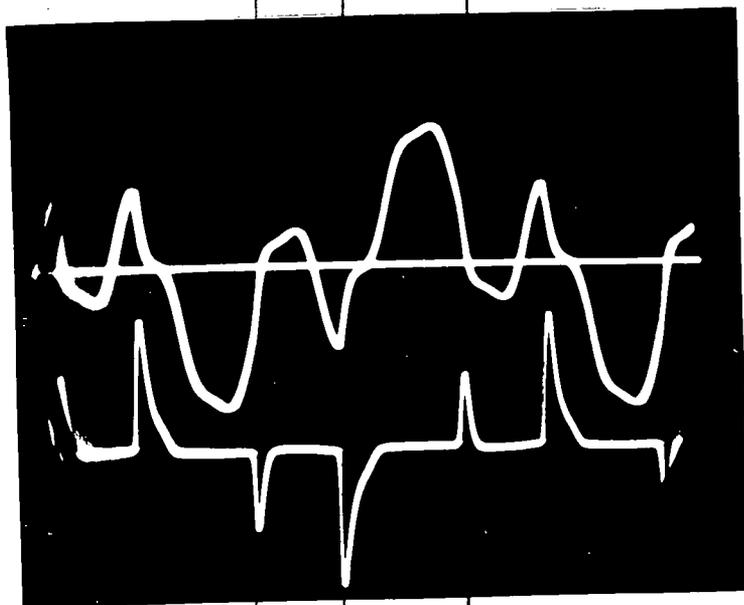
a



$$I = 0.68 \text{ A/cm.}$$

$I_s$

a



$$V_c = 20 \text{ v/cm.}$$

$I_s$

a

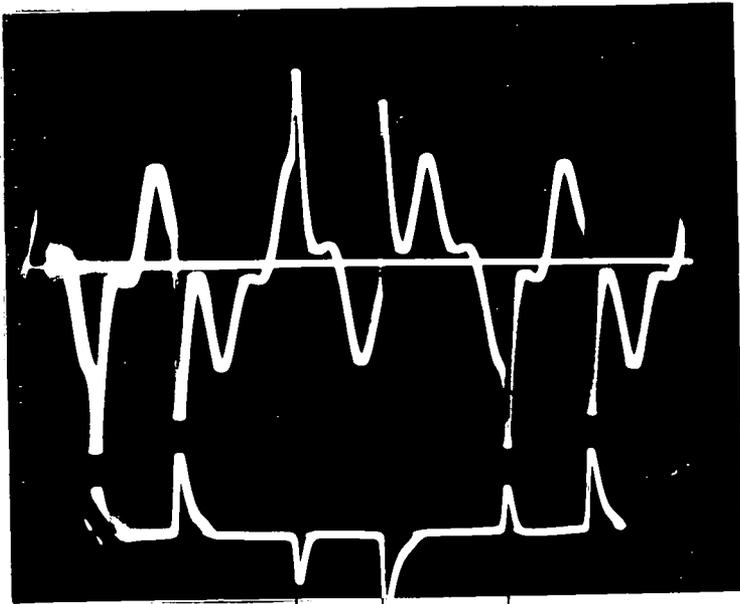
Conditions:  $V = 105 \text{ v}$      $C' = 20 \mu\text{F.}$

Fig.14.

Scales

$$I_C = 0.68 \text{ A/cm.}$$

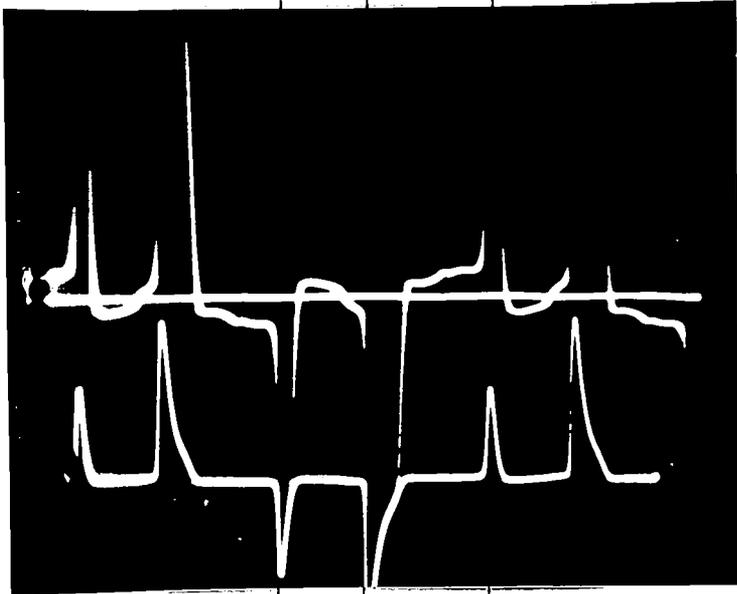
$I_S$



a

$$I_S = 0.34 \text{ A/cm.}$$

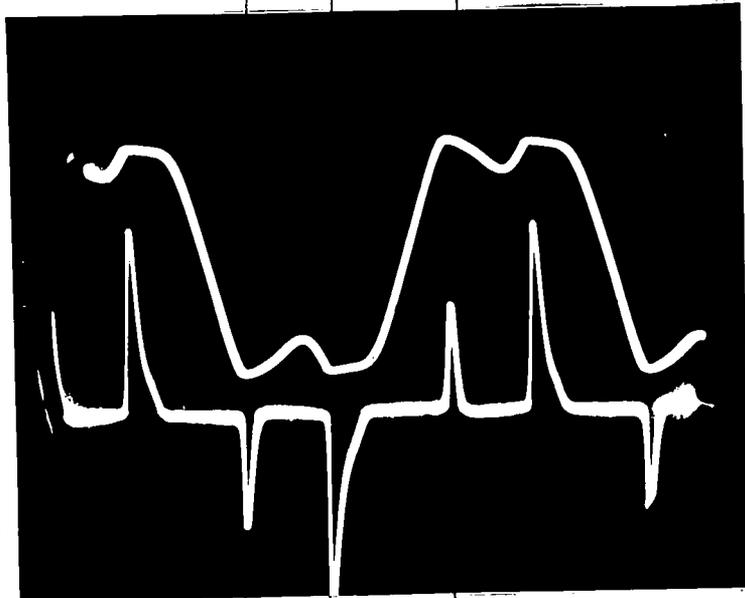
$I_S$



a

$$B = 1.0 \text{ Wb/m}^2/\text{cm.}$$

$$I_S = 2 \text{ A/cm.}$$



a

Conditions:  $V = 105 \text{ v}$      $C' = 20 \mu \text{ F.}$

Fig.14.

In Travis' analysis, Ref.1, it is supposed that the cause of the sub-harmonic response is trapped charge on the capacitor. This may be so for some series circuits but in this case if anything is trapped it is magnetic energy, although some degree of charge storage does occur in that, the charge on the capacitor persists for a period of up to 75% of the 50 Hz period. But as stated this arises not so much because the charge is trapped but because the capacitor suffers a peak to peak voltage excursion in one direction following its discharge. The persistence of magnetic flux in any one direction is 150% of the 50 Hz period. Travis also says that the mechanism of the trapping of charge is the reactor going out of saturation and limiting the transfer of charge, yet it is clear from the results of this work that major transfers of energy occur when the reactor is unsaturated.

It is for these reasons that it is felt that Travis' theory lacks generality, certainly it does not hold for the case of a stalloy core where the onset of saturation is gradual.

The theory of excess energy is, therefore, put forward as a possible explanation of the sub-harmonic phenomena and this theory would be applicable to any circuit containing any core material. It embraces Travis' theory where stored charge is involved and explains why lossy circuits discourage the sub-harmonic response.

It also provides a possible link between the sub-harmonic response and the ferro-resonant response which some workers seem to think are unrelated, ref. 2, J.D. McCrumm.

4.9 Relationship between the Sub-harmonic and Ferro-resonance Phenomenae.

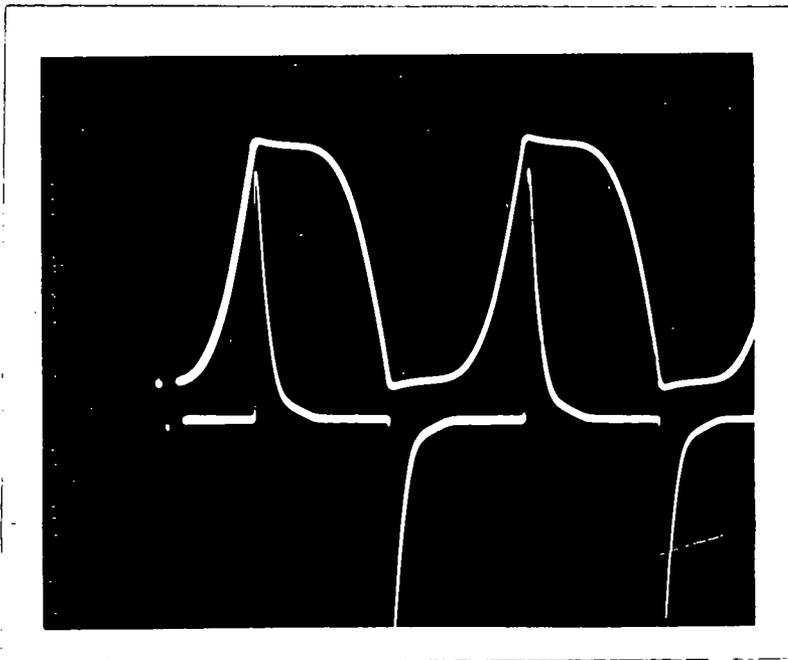
The theory of excess energy suggests that sub-harmonics occur when the circuit receives an energy input that is abnormally high for the applied voltage condition prevailing and which is of sufficient magnitude to take the saturable reactor to saturation; to a region where impulse resonance phenomena can occur between the capacitance and falling inductance of the saturable reactor.

If the applied voltage is of sufficient magnitude that it can, without the additional injection of energy, drive the inductor to near saturation on each half cycle then a ferro-resonant condition will result of fundamental frequency.

This is believed to be the relationship between the two phenomena, that they are both essentially impulse resonance phenomena driven by the fundamental applied frequency and dependent largely upon the magnitude of the applied voltage.

Furthermore, if the circuit is operating stably with the saturable reactor on the lower current curve, but within the overlap margin previously discussed, a shock or slowly applied d.c. current will excite the ferro-resonance condition which will persist after the d.c. bias is removed.

Fig.15 shows the waveforms of two ferro-resonance conditions. The flux and current waves have a fundamental frequency of 50 Hz and those of 15a are typical of the single resonance link limiting fault current.



Saturable Reactor Flux and current waveforms of ferro-resonance phenomena during fault current limitation.

Scales B = 1.0 Wb/m<sup>2</sup>/cm.

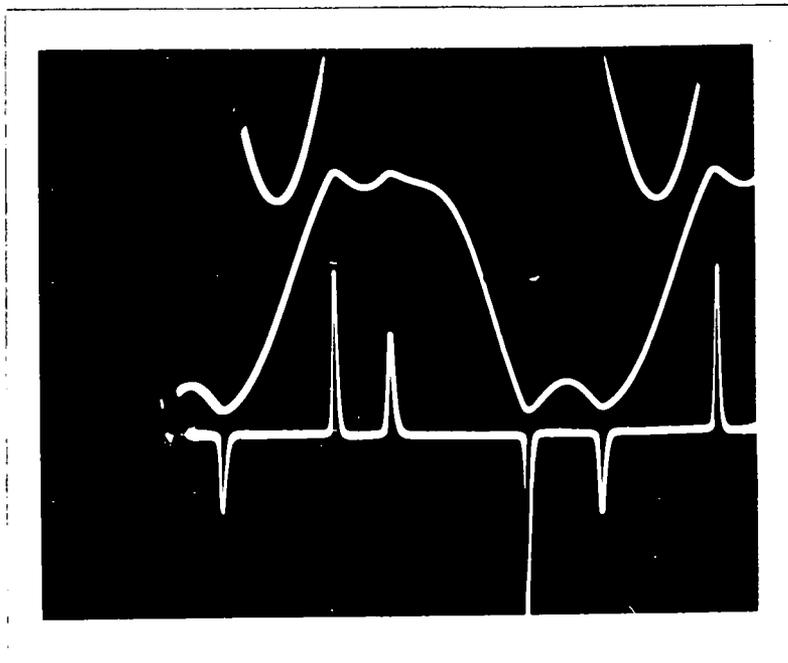
Conditions V = 100 volts RMS.

I<sub>S</sub> = 4 A/cm.

C = 160 μ F.

Link short circuited.

Fig.15a.



Saturable Reactor flux and current waveforms of ferro-resonance phenomena during fault limitation, but with gross detuning of the link.

Scales B = 1.0 Wb/m<sup>2</sup>/cm.

Conditions V = 100 volts.

I<sub>S</sub> = 4 A/cm.

C = 80 μ F.

Fig.15b.

The wave of 15b is for the case where there is gross detuning of the link ( $C = 80 \mu F$ ) and is produced to show the complexity that the ferro-resonance response can achieve, and to demonstrate the similarity between these waveforms and those of the sub-harmonic response. Yet the waveforms of 15b are of fundamental frequency and are produced here to support the belief that the sub-harmonic and ferro-resonance phenomena are related by a common excitation process.

#### 4.10 Possibility of the Sub-harmonic Response with other than Capacitive Loads.

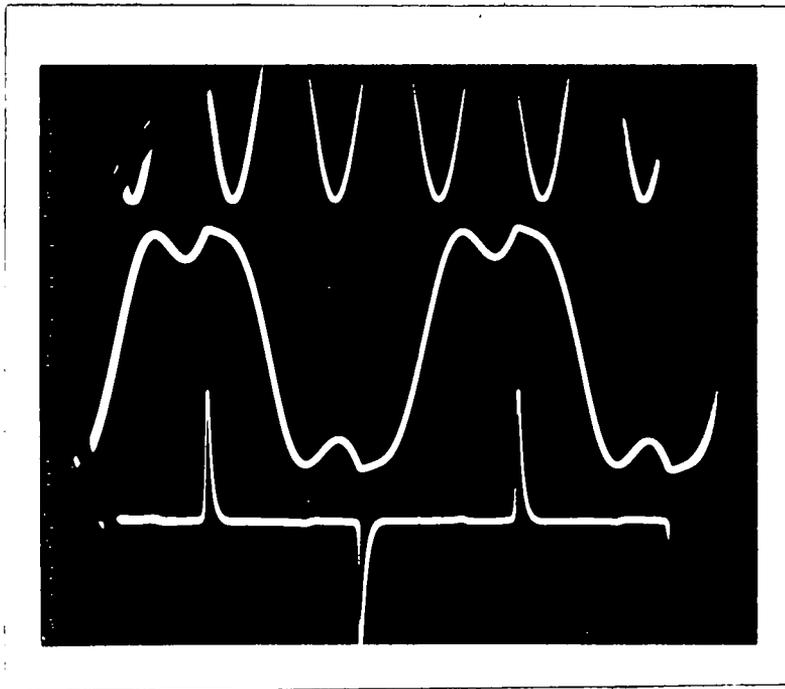
In the course of the sub-harmonic investigation it soon became clear that the linear series reactor was of no consequence in the generation process of these low frequencies and could in fact be shorted out with little effect.

At first it was accepted that in order to excite a sub-harmonic response the load impedance had to be capacitive and yet as experience of the circuit was gained it became clear that the source of the sub-harmonic and ferro-resonant response was the parallel loop, although the load capacitance did determine the waveform and range of the sub-harmonics.

It was conceived that it might be possible to excite a sub-harmonic response with other high impedance loads and such was found to be the case if a d.c. bias or a sufficiently large shock was applied to the saturable reactor and capacitor loop. Thereafter the sub-harmonics were stable and self supporting of orders  $1/2$  and  $1/3$ , Fig.16 & Appendix III, Fig.54.

The load impedances for this response had to be high of the order of several hundred ohms resistive or inductive. This points to the probability that the load capacitance that allows a sub-harmonic response, acts simply to allow sufficient loose coupling of the parallel loop to exist so that any excess energy imparted to the loop cannot be easily dissipated to the external circuit.

This is consistent with the observation that any strong coupling of the 50 Hz supply or supplies to the link, as in the case of parallel operation with a conventional source, eliminates the sub-harmonic response.



$\frac{1}{3}$  Sub-harmonic with a resistive load of 550 ohms with an applied voltage of 180 volts. R.M.S.

Scales  $B = 1.0 \text{ Wb/m}^2/\text{cm}.$

$I_s = 1.0 \text{ A/cm}.$

Fig.16.

Analysis of Resonant Link Circuit.

5.0 In the analysis of circuits containing magnetic material it is often useful to represent the  $B/H, \psi/I$  curve as a single valued function by ignoring hysteresis.

The elimination of the hysteresis means that a loss is eliminated from the system and this should assist the generation of harmonics.

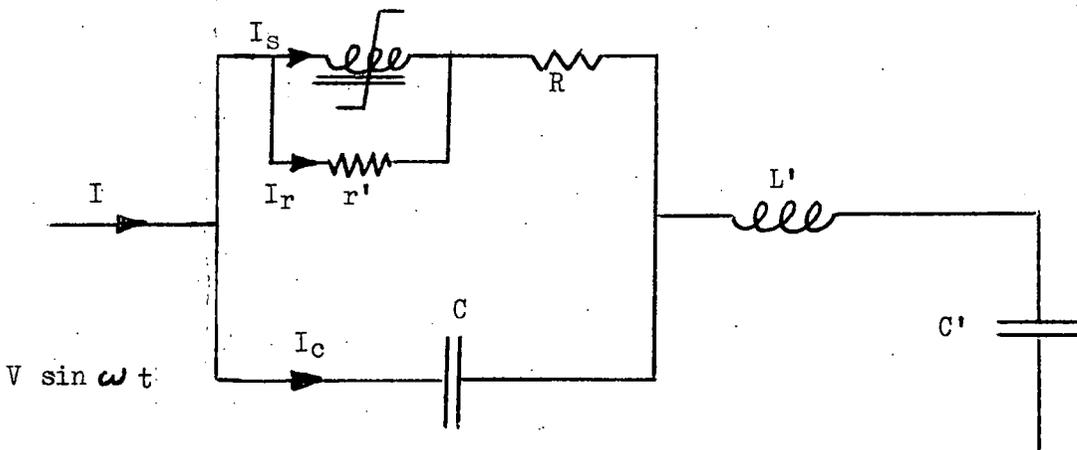
In any formal analysis the representation of the  $B/H$  curve presents a problem. Some workers have represented the curve by two straight lines, ref.1., which gives a good approximation for square loop material but eliminates the knee region and is a crude model for stalloy type materials.

Other workers have advocated the use of polynomial series or part series containing only odd terms ref. 8, 9., which can be shown to give good representation of stalloy materials and systems containing added reactance, but are not very accurate for square loop materials.

Two methods of representation have been investigated.

5.1 Polynomial Method.

Considering the circuit:-



where  $r'$  represents the reflected damping resistor  $r$ .

Neglecting the voltage drop across  $L'$  compared to the voltage drop across  $C'$ .

$$I_s = f(\psi) \dots \dots \dots (1)$$

$$V_c = \frac{d\psi}{dt} + R(I_s + I_r) \dots \dots \dots (2)$$

differentiating:-

$$\frac{dV_c}{dt} = \frac{d^2\psi}{dt^2} + R \frac{dI_s}{dt} + R \frac{dI_r}{dt} \dots \dots \dots (3)$$

$$V \sin \omega t \doteq \int \frac{I_c}{C} dt + \int \frac{I}{C'} dt \dots \dots \dots (4)$$

differentiating:-

$$\begin{aligned} V \omega \cos \omega t &= \frac{I_c}{C} + \frac{I}{C'} \\ &= \frac{I_c}{C} + \frac{I_c + I_s + I_r}{C'} \end{aligned}$$

$$\therefore \frac{I_c}{C} = \frac{dV_c}{dt} = \frac{C'}{C+C'} \left( V \omega \cos \omega t - \frac{I_s}{C'} - \frac{I_r}{C'} \right) \dots \dots \dots (5)$$

and  $I_r = \frac{1}{r'} \frac{d\psi}{dt} \doteq \frac{1}{r'} V_c$  if R is small

thus equating 2 and 5.

$$\frac{V \omega C'}{C+C'} \cos \omega t = \frac{d^2\psi}{dt^2} + R \frac{df(\psi)}{dt} + \frac{R}{r'} \frac{d^2\psi}{dt^2} + \frac{f(\psi)}{C+C'} + \frac{1}{r'(C+C')} \frac{d\psi}{dt}$$

which is if  $r' \gg R$

$$\frac{V \omega C'}{C+C'} \cos \omega t \doteq \frac{d^2\psi}{dt^2} + R \frac{df(\psi)}{dt} + \frac{1}{r'(C+C')} \frac{d\psi}{dt} + \frac{f(\psi)}{C+C'} \dots (6)$$

and following the method of Cunningham ref.8 for the series circuit

and using a part polynomial representation:-

$$I_s = f(\psi) = A\psi + B\psi^3 \dots \dots \dots (7)$$

and substituting in (6)

$$\frac{V \omega C'}{C+C'} \cos \omega t = \frac{d^2\psi}{dt^2} + \left( RA + 3RB\psi^2 + \frac{1}{r'(C+C')} \right) \frac{d\psi}{dt} + \frac{A\psi}{C+C'} + \frac{B\psi^3}{C+C'} \dots \dots \dots (8)$$

which is a form of Duffings equation with a non-linear damping term.

The use of higher order polynomial terms results in higher order forms of Duffings equation.

The emergence of Duffings equation and higher order forms of Duffings equation from the analysis directly indicates the possibility of a sub-harmonic response see Cunningham, Non-linear Analysis, ref.8.

Several methods of solution of non-linear differential equations are applicable. Perturbation techniques, see ref.8, are rather long and involved and are not conveniently suitable if one wishes to try the effect of varying the circuit parameters.

Two obvious methods are analogue and digital techniques. The former was at first attempted. However, the large range of parameters involved made scaling very difficult, at least on the machine available at the time.

A digital solution was thus felt to be the most feasible. The object of the analysis was to see if:-

- 1) Given the right circuit conditions a steady sub-harmonic response could be generated.
- 2) With such a response to see whether it could be damped out by adjustment of 'r' in the region of values corresponding to the reflected value of r the secondary damping resistor in the model.

The Duffing equation was programmed using a Runge-Kutta procedure and the computed results in real time are shown in fig.17a.

The results show that after an initial transient period the response settles down to a regular  $\frac{1}{3}$  sub-harmonic. This response was found to extinguish when r' was made equal to 100 ohms, the response then being sinusoidal. This value of r' compares to the value of 150 ohms measured on the model.

The foregoing method provides a way of estimating the value of the reflected damping resistance  $r'$  necessary to extinguish the sub-harmonic response, as an alternative to simple trial and error.

However, the analytical representation of the B/H curve for high permeability materials is still rather crude and subject to considerable error. This being so better methods of representation and analysis are currently under investigation as an extension of the present work.

The new approach attempts to represent the B/H,  $\psi/I$ , curve as a series of points, defining small straight lines, fed into a computer and the system equations then being solved by a step by step process as follows.

5.2 Step by Step Integration Method.

Taking equations (1), (2) and (5) modified and integrating for small steps of time  $\Delta t$  beginning at time  $t = 0$  when  $I_s = 0$  and  $V_c$  and  $\psi$  having initial values specified.

The analytical procedure is defined as follows:-

$$SI = I_s = f(\psi) \text{ a set of points for SI and } \psi \dots \dots \dots (9)$$

$$\frac{d\psi}{dt} = V_c - RI_s$$

$$\Delta\psi = \frac{d\psi}{dt} \Delta t = (V_c - RI_s) \Delta t$$

$$\begin{aligned} PSI = \psi &= \psi + \Delta\psi \\ &= \psi + (V_c - R.SI) \Delta t \dots \dots \dots (10) \end{aligned}$$

and using equation (5),

$$\Delta V_c = \frac{dV_c}{dt} \Delta t = \frac{C'}{C+C'} \left( V\omega \cos \omega t - \frac{I_s}{C'} - \frac{I_r}{C'} \right) \Delta t$$

and  $I_r \doteq \frac{V_c}{r'}$  for small values of R

$$V_c = V_c + \frac{C'}{C+C'} \left( V\omega \cos \omega t - \frac{SI}{C'} - \frac{V_c}{r'C'} \right) \Delta t \dots (11)$$

$$t = t + \Delta t \dots (12)$$

The foregoing equations are solved in the order 11, 10, 9, 12, starting with initial values at  $t = 0$  of  $SI = 0$ ,  $\psi = \psi_0$  and  $V_c = V_{c0}$ , and using a prescribed small value for  $\Delta t$ .

Equation(9) acts as a control producing a new value for SI using the value of PSI ( $\psi$ ) from equation (10)

New values of  $V_c$  and SI are then substituted back in (11) and the process repeats.

The results of using a simple three point i.e., two slope approximation to the  $\psi / I_s$  curve and an integration step time  $\Delta t$  of 0.0001 second are shown in fig. 17b for which the following data was also used:-

at  $t = 0$ ,  $V_{c0} = 0$       PSI ( $\psi_0$ ) = 0.195 Wb turns, SI = 0,

DELTA =  $\Delta t = 0.0001$  sec.      V = 141.4 volts,       $\omega = 314.16$  rad/sec.

C = 160  $\mu$ F,      C' = 20  $\mu$ F,      R = 2.6 ohms,      r' = 1,400 ohms

Break point PSIM = 0.195 Wb turns.

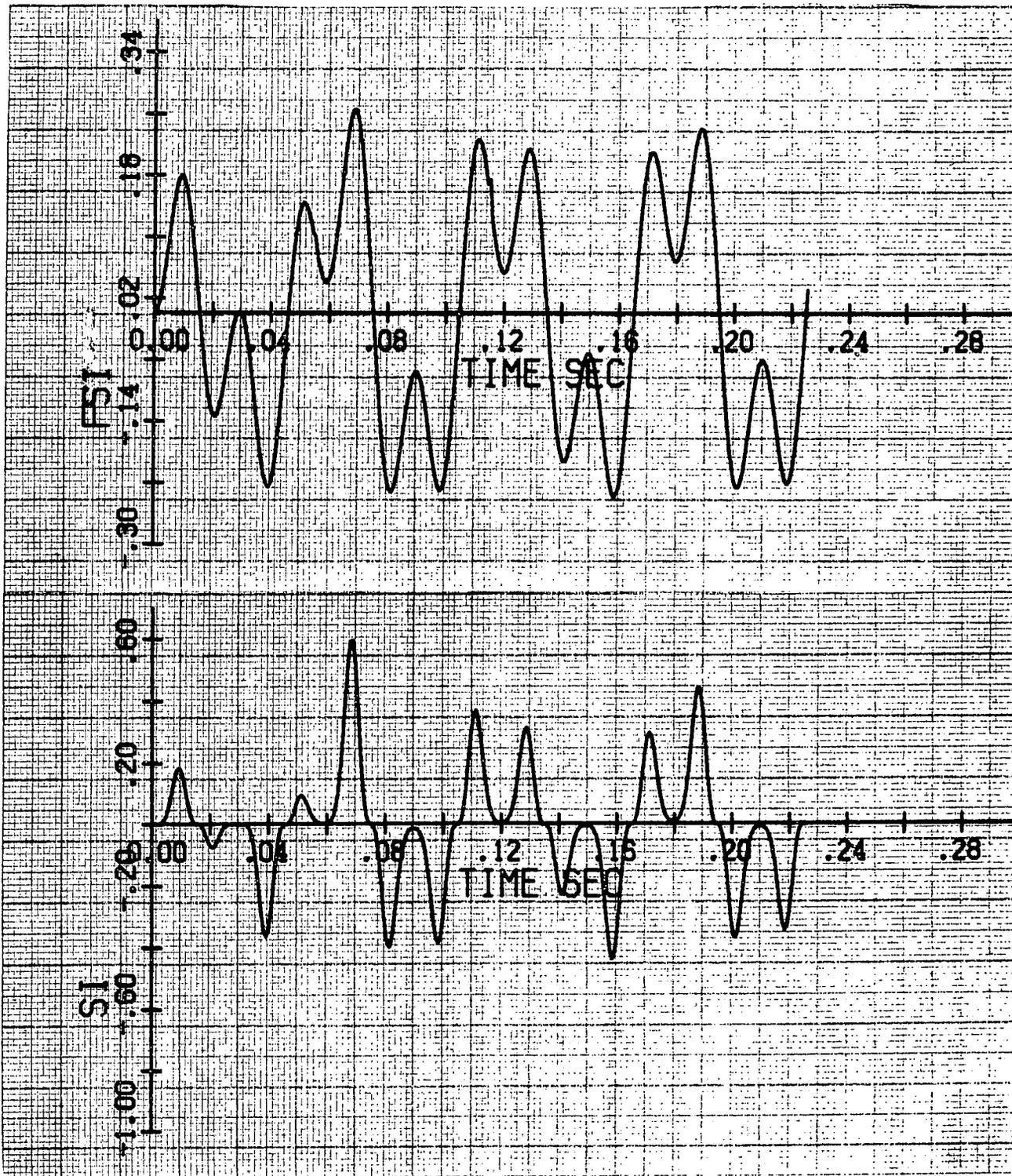
Slopes defining  $\psi / I$  curve,

up to the break point SLP1 = 0.05 amps/Wb turn.

after the break point SLP2 = 250 amps/Wb turn.

The emergence of  $1/3$  sub-harmonics from the digital computer studies with only fundamental excitation present and the similarity in the shape and detail between the computed waveforms and the actual waveforms shown in Appendix III, adds credence to the analytical procedures adopted.

However, a complete mathematical model of this non-linear system must represent the B/H curve in accurate detail and ultimately the full hysteresis loop must be represented.



Computer drawn curves for digital solution of Duffings equation, equation 8, showing  $\frac{1}{3}$  sub-harmonic response after the initial transient period.

Conditions:

$$V = 141.4 \text{ volts}$$

$$C' = 20 \mu\text{F}$$

$$\omega = 314.16 \text{ rad/sec}$$

$$R = 2.6 \text{ ohms}$$

$$C = 160 \mu\text{F}$$

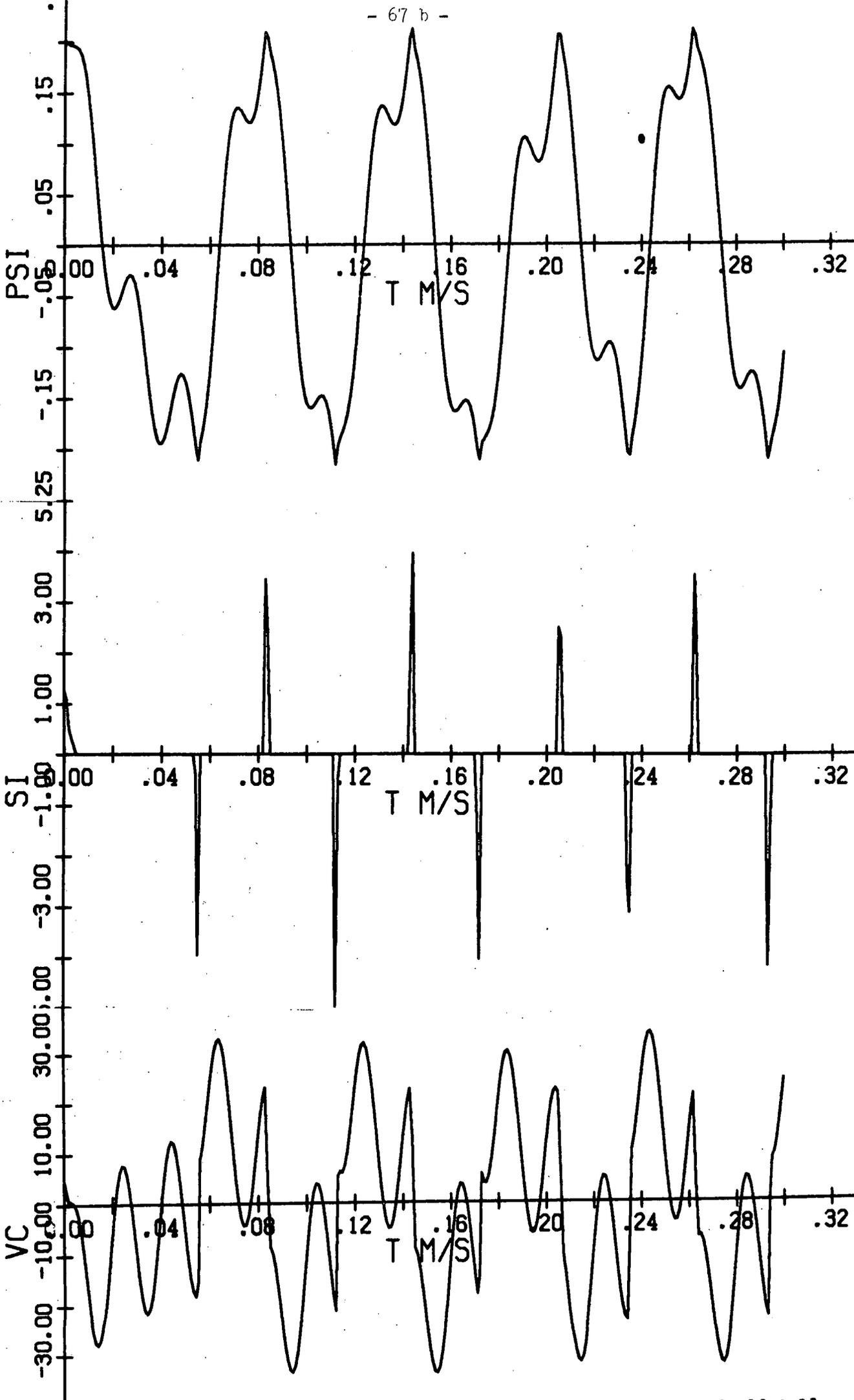
$$r' = 1,000 \text{ ohms.}$$

From a least squares approximation to the  $\psi/I_s$  curve

$$A = 0.12$$

$$B = 54$$

Fig. 17a.



Computer drawn curves for step by step solution of equations 9, 10 & 11.

Fig.17b.

The Double Resonance Link.

6.0 The single resonance link discussed so far relies upon the series linear reactor as the main limiting element to the flow of fault current, when the state of resonance is destroyed.

A more enhanced limitation would be achieved if the circuit could be arranged to switch automatically from the series resonant condition to the high impedance parallel resonant condition when a fault is being fed by the link.

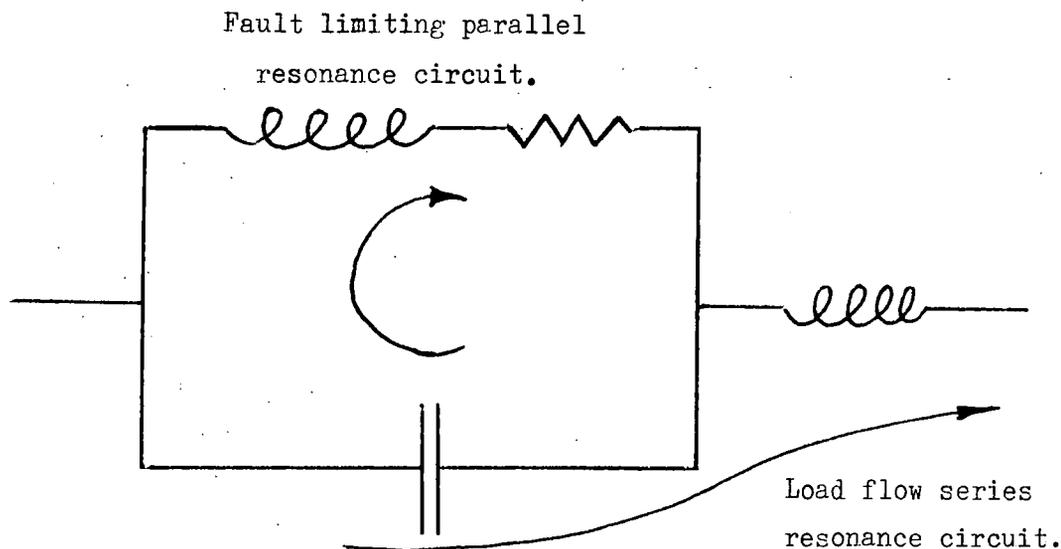


Fig.18.

In the circuit shown the linear reactor and capacitor are in series resonance for load flow conditions, but the value of the capacitance is chosen so that it resonates with the saturated inductance of the saturable reactor under fault conditions. With such an arrangement the current through-put could theoretically be very small, indeed smaller than the load current.

### 6.1 Practical Considerations.

Such a link was set up but it was initially found that the value of capacitance required to resonate with the saturated non-linear reactor was over 350  $\mu$  F and this was considered to be too large for practical reasons and in any case with the resistance necessary in the loop the state of resonance was poor and the current limitation was just over 3 amps, little better than that of the single resonance link. The resistance in the loop, the value of R, had a limiting lower value of 3 ohms necessary to quench the ferro-resonance current which otherwise persisted after a fault was cleared.

An auxiliary linear ballast reactor was tried in the loop to increase the reactance and this was found to be very successful.

Its value was set by making the capacitance 160  $\mu$  F as previously, and applying such a voltage as to saturate the non-linear reactor and then tuning the parallel loop for minimum through current by means of the auxiliary reactor. The operational circuit as set up to study this system is shown in fig.19., shown overleaf.

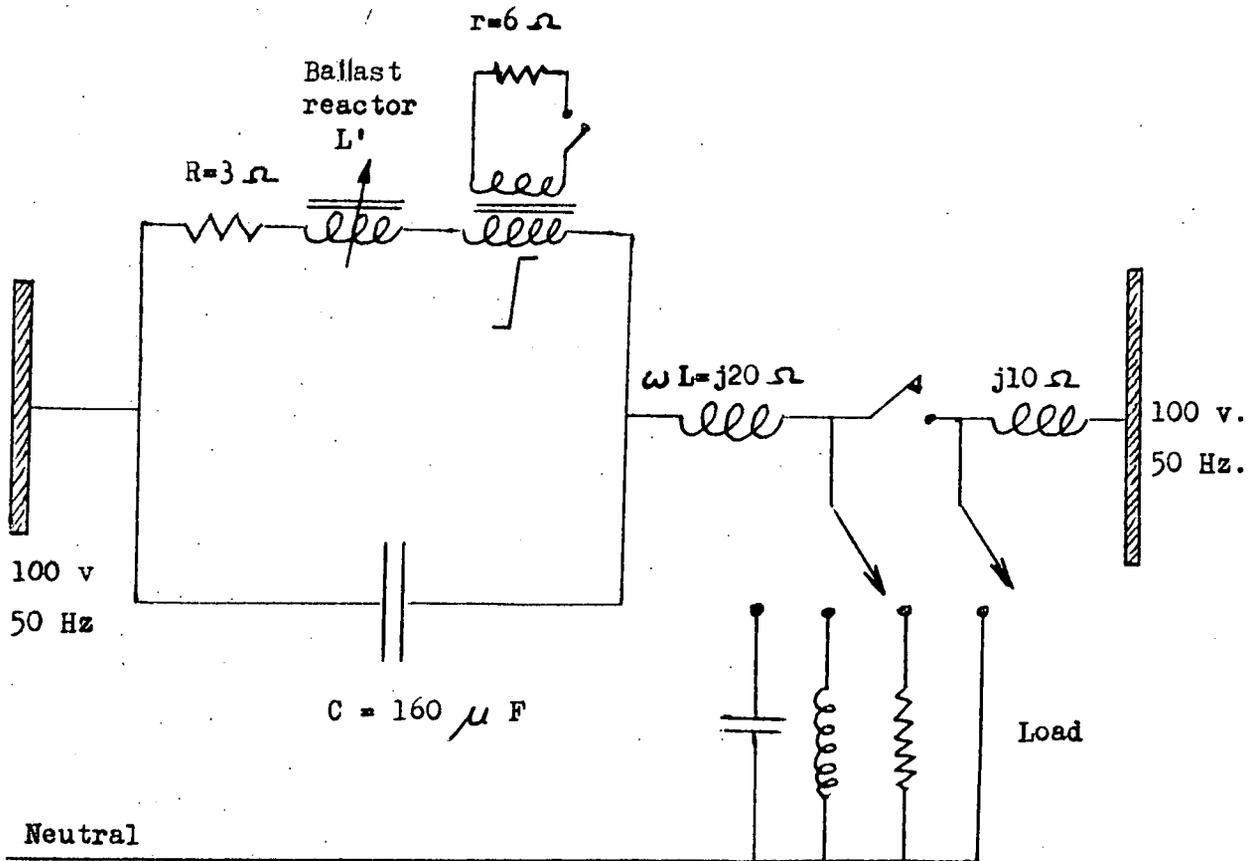


Fig.19.

Again the circuit was set up to take a normal load current of 1 ampere representing 100 MVA at 132 kV.

### 6.2 Load Flow.

The link was used to supply resistive, inductive and capacitive loads and to transfer load between two busbars.

In all cases it behaved like the single resonance link and the regulation was negligible. Indeed for the reactive loads slight detuning could produce negative regulation which might be useful for maintaining voltage levels. With some capacitive loads sub-harmonic response occurred.

With an inductive load the throughput current could be

increased up to 2.5 amps and then the link limited to 1.35 amps. As the load was reduced the current increased very slightly to 1.4 amps before recovery took place, the load dropping to 1.3 amps.

Furthermore, a load of 1.3 amps could just be switched onto the circuit without its limiting, so in this case the circuit had a firm 30% overload capacity. Unlike the single resonance link the circuit, as indicated, had very little drop-off margin.

### 6.3 Short Circuit Performance.

The improvement over the single resonance link was considerable and indicated an impedance to fault current of 74 ohms. On short circuit the current limited to only 1.35 amps, with a potential fault current of 34 amps. This fault throughput current is dependent upon the value of the damping resistance R necessary to quench the ferro-resonance current in the saturable reactor loop following a disturbance.

The necessity to have a value of R of 3 ohms in the parallel resonance loop tended to hinder its performance. Indeed with R reduced to zero the current on short circuit could be made to fall to 0.28 amps, which is about a quarter of the load current and indicates an impedance of 430 ohms.

However, that such a dramatic fault current limitation cannot be utilised because of the need for finite loop resistance turned out to be hardly a disadvantage.

A bus bar was established, fed by a 100 volt source through an inductive impedance of 10 ohms  $\angle 80^\circ$ , giving a fault

current of 10 amps at a power factor of approximately 0.2 lag.

The resonant link was attached to this bar from another 100 volt source to determine the actual fault contribution to a bus bar when other sources were also connected.

When a fault was established on the busbar although the current throughput of the link was 1.3 amps it was discovered that the actual fault level of the busbar had dropped to 9.8 amps. So that far from contributing, the link was now reducing the fault level of the bar to which it was attached.

In fact it was found that, with the very small fault infeed of 0.28 amps with  $R = \text{zero}$ , the fault level reduction was negligible so that there was some advantage in having a link fault current of the same order as the load current.

It turned out that with  $R$  increased to 6 ohms the link throughput was 2 amps and the combined busbar fault level was 9.5 amps, thereafter as  $R$  was increased and the link throughput increased, the fault level increased to and beyond 10 amps.

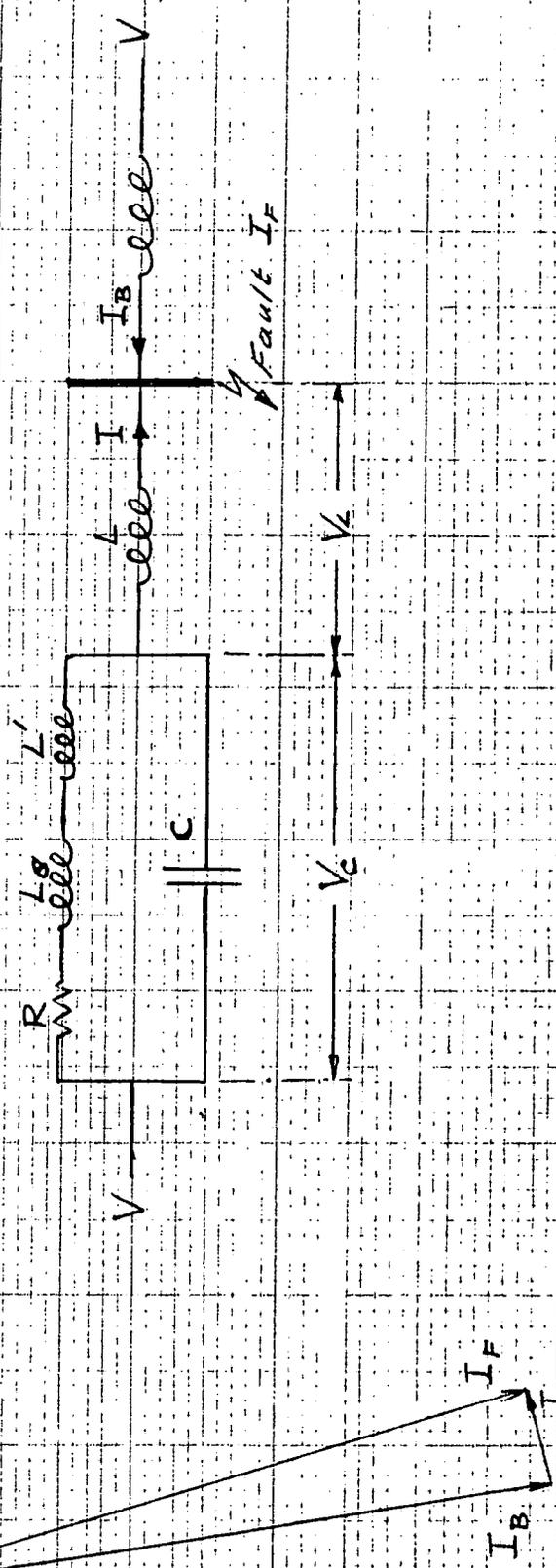
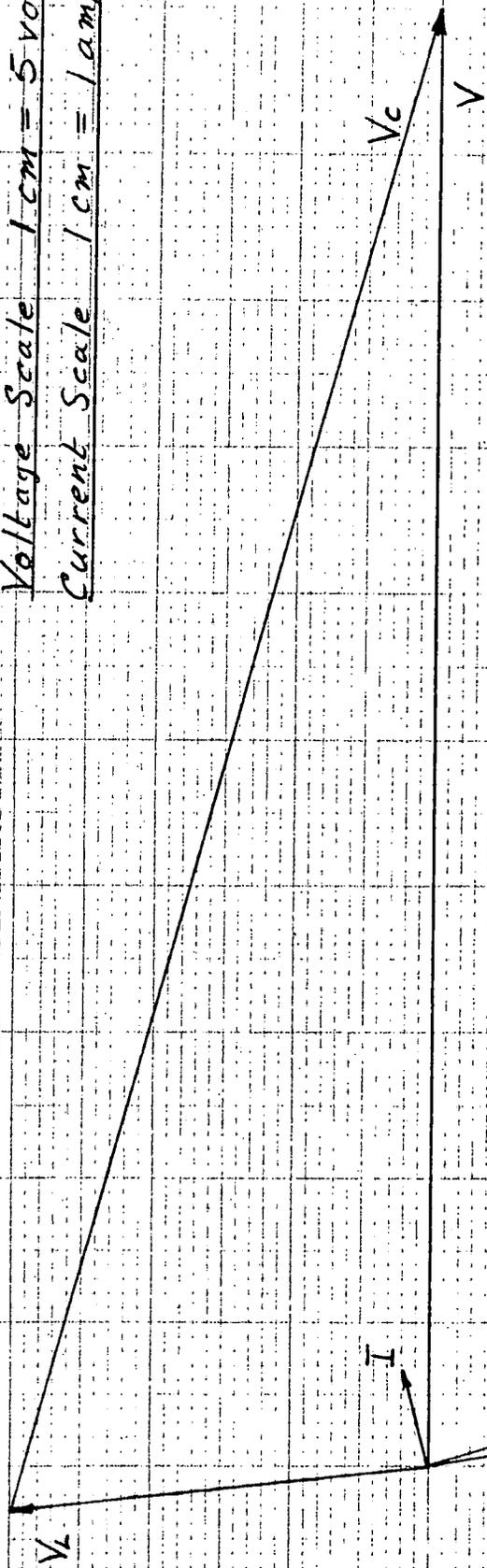
The reduction in fault level was an unexpected result since it had hitherto been considered that the limited double resonant link might be like a high resistance, but instead it was behaving like a high impedance capacitive infeed.

The reason for this behaviour can be seen from the construction of a phasor diagram for the limiting link Fig.20, using measured values of current and voltage and assuming that these are sinusoidal. An assumption which is not true, but is sufficiently accurate to demonstrate why an overall fault level reduction is achieved.

# Phasor Diagram of Double Resonance Link feeding a Bus Bar Fault

Voltage Scale 1 cm = 5 volts

Current Scale 1 cm = 1 amp



It is at once seen that the voltage across the main capacitor and hence the parallel loop is high, 107 volts, and this directly gives rise to a leading current through the link.

This current, although its leading component is small, is sufficient to cause an overall reduction in the total fault current as fig.20, shows.

From a practical standpoint such a reduction in fault level would be a considerable advantage in places where the fault level is already approaching the rating of the switchgear.

However, the use of such a link in series with a synchronous machine is not indicated since its high resistive impedance with a capacitive component would inevitably lead to instability.

#### 6.4 Link Impedance.

The link impedance as a function of voltage or current is shown in figs. 21 and 22.

What is striking about these curves is the remarkable similarity in shape with those of the single resonance link fig.2, although the mechanism of fault current limitation is apparently different in each case.

The word apparent is used since in reality the phenomenon of ferro-resonance takes place in both links when they limit. In effect what is happening in the double resonance link is that this resonance process is optimised to give the maximum impedance and hence the minimum current.

Again it can be seen that the link was not operated optimally to give maximum limiting impedance, but to achieve this would have meant altering the value of the capacitance and this was not considered to be worth while for a mere 5 ohms increase.

Double Resonance Link Impedance on Short Circuit as a function of Voltage

Link Resistance  $R = 3 \Omega$

$r = 6 \Omega$

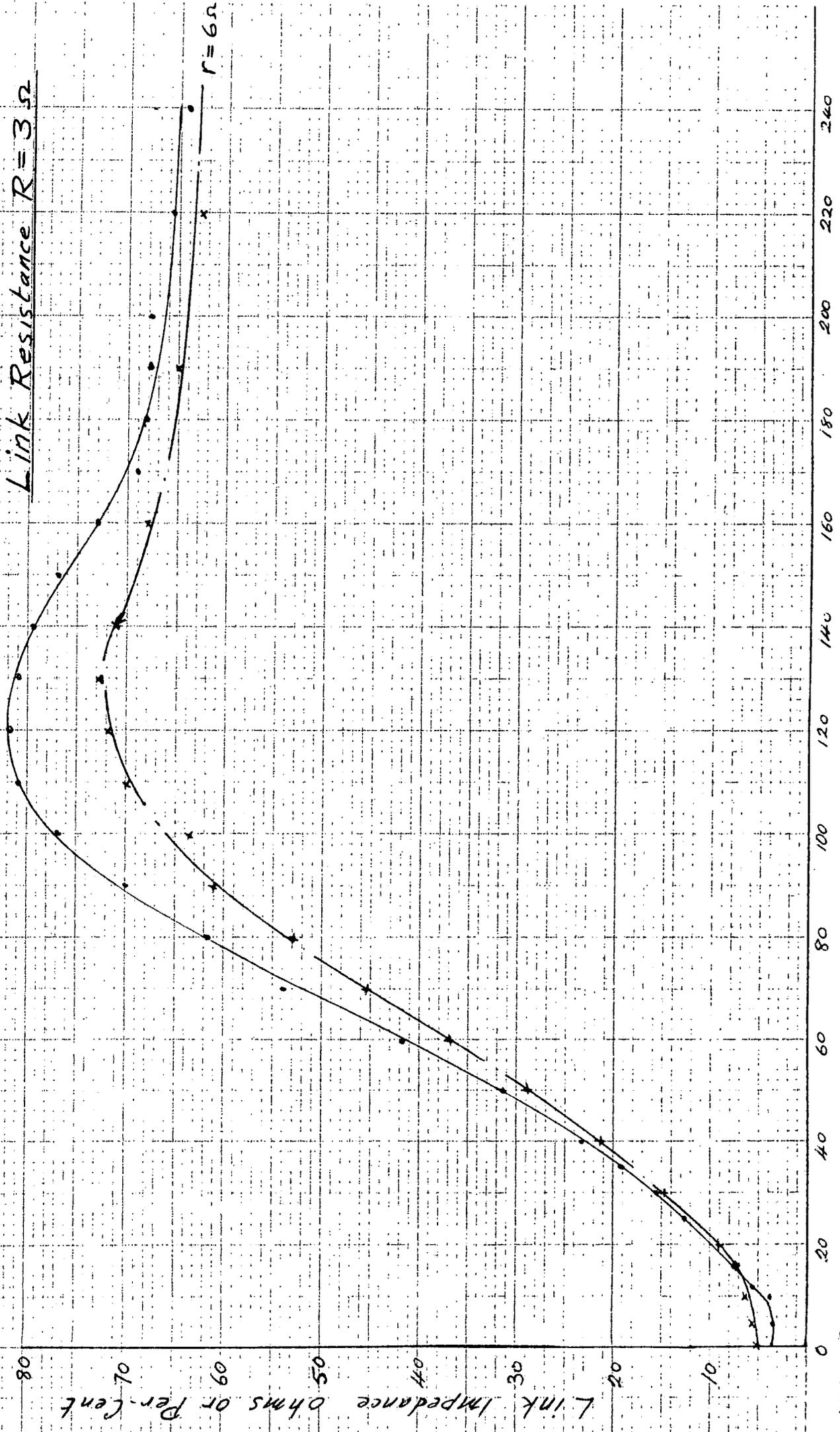


Fig 21

Double Resonance Link Impedance on Short Circuit as a function of Current  
Link Resistance  $R = 3 \Omega$

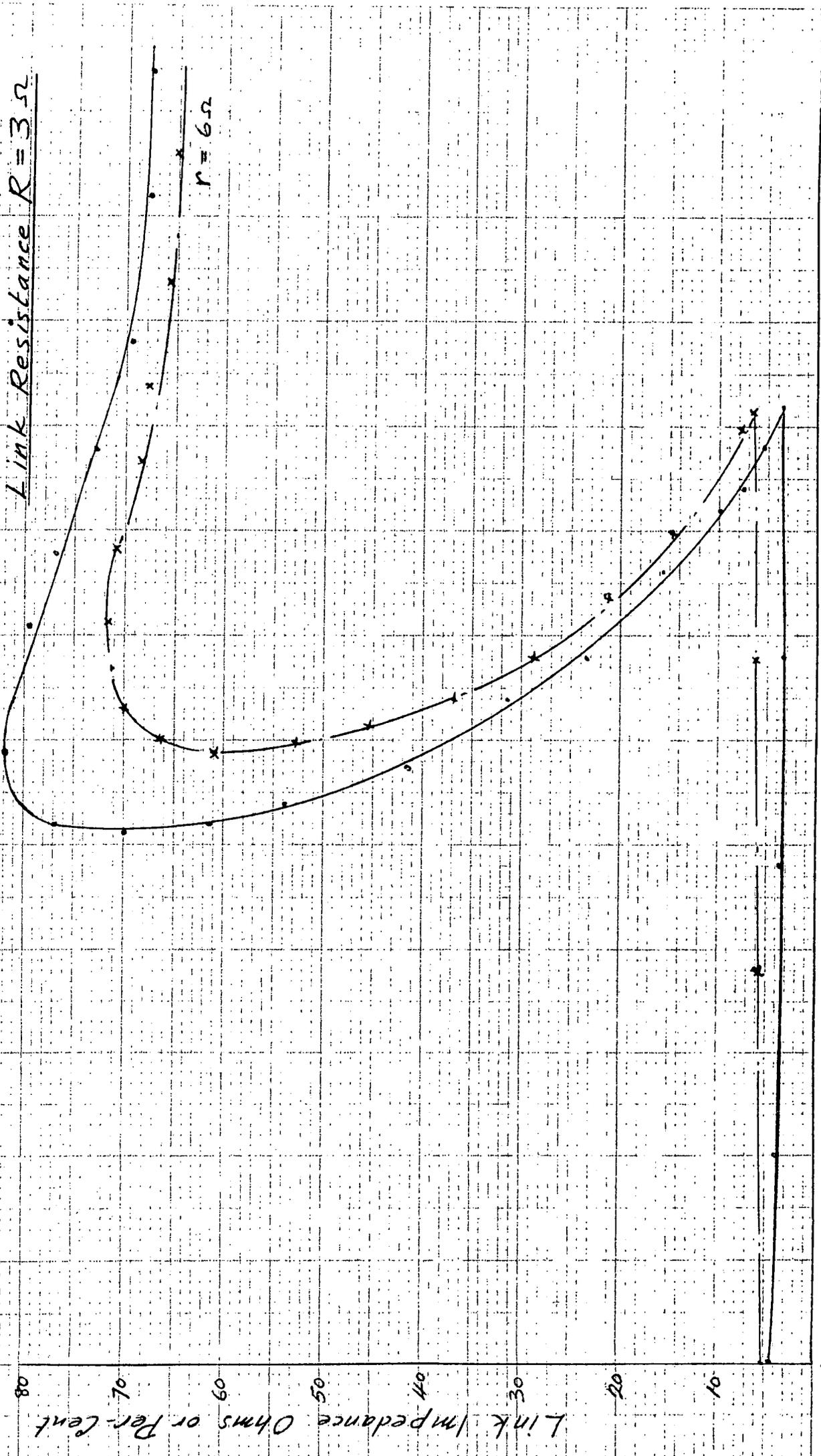


Fig 22 Current Through Link Amps RMS.

The setting of the auxiliary ballast reactor for minimum transmitted current is different for different applied voltages although the range of variation is not large.

#### 6.5 Ferro-resonance and Sub-harmonics.

One significant difference between the ferro-resonance and sub-harmonic currents of the double resonance link compared to those of the single resonance link was the relatively long duration of the current peaks and their rounded nature.

This was found to be a direct result of the auxiliary ballast reactor and was considered to be beneficial since the absence of very large spiky currents in the parallel loop must lead to reduced stressing of the capacitor and inductor.

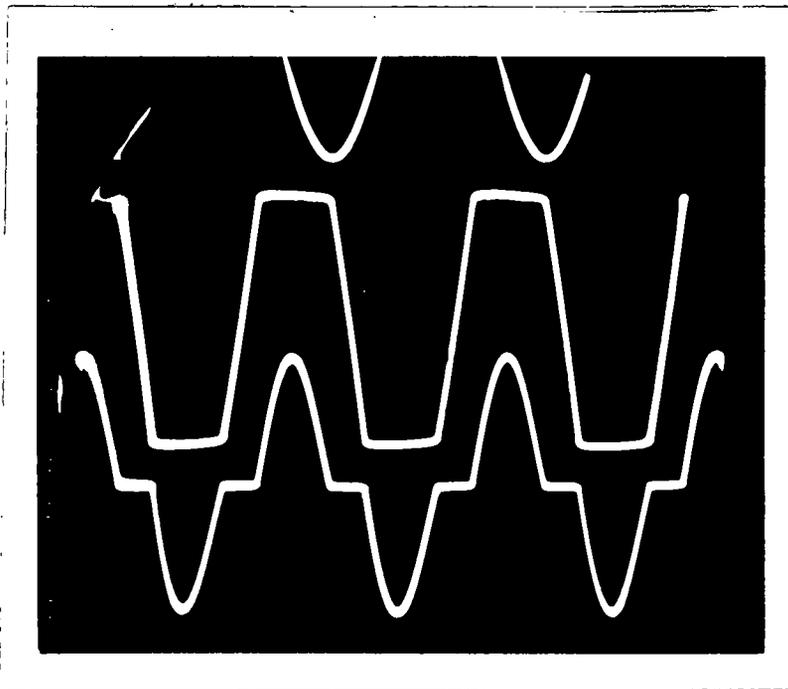
What is striking is the similarity of features of the ferro-resonance currents and sub-harmonic currents providing further evidence that the two phenomena are related, figs. 23 and 24.

The establishment of sub-harmonics in the circuit was rather more difficult than in the single resonance circuit, requiring a greater shock excitation although a slowly increasing d.c. bias current applied to the saturable reactor was just as effective.

The need for an increased shock to establish the sub-harmonic state was thought to be due to the difficulty of imparting sufficient energy to the parallel resonant loop.

The sub-harmonics once generated were always of  $\frac{1}{3}$  order and were limited in range of capacitance and voltage, more so than in the single resonance circuit.

Double Resonant Link.



Saturable reactor flux and current during fault current limitation.

Scales  $B = 1.0 \text{ Wb/m}^2/\text{cm}.$

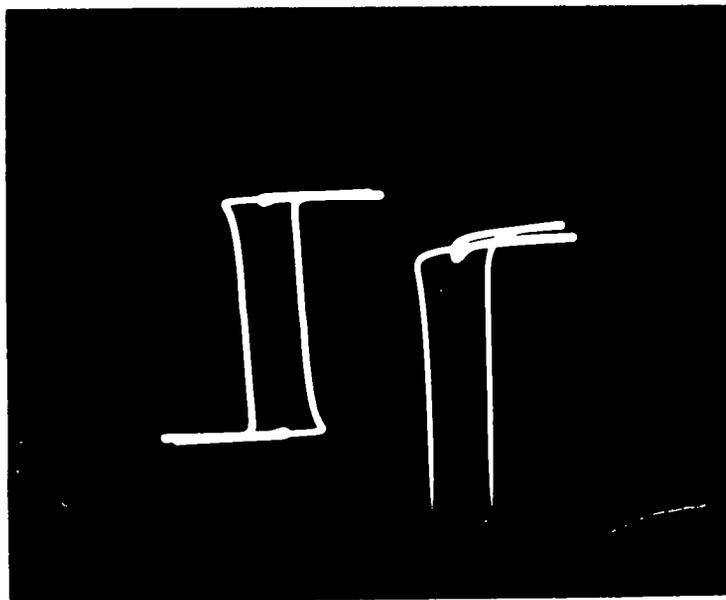
$I_s = 4 \text{ A/cm}.$

Conditions.

$V = 100 \text{ volts}.$

$C = 160 \mu \text{ F}.$

Fig.23.



Hysteresis Loop for sub-harmonic response showing details of minor loop.

Scales B = 100 Wb/m<sup>2</sup>/cm.

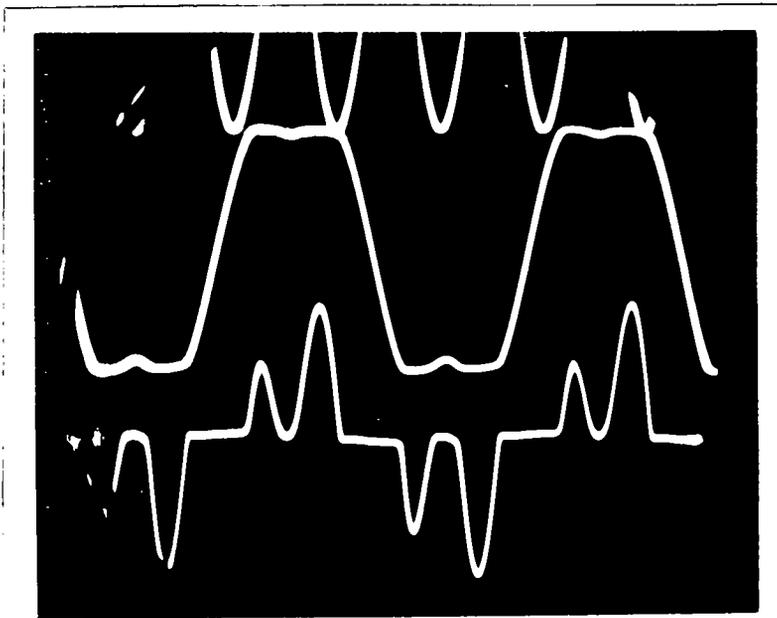
Minor Loop B = 0.5 Wb/m<sup>2</sup>/cm.

I<sub>S</sub> = 0.1 A/cm.

I<sub>S</sub> = 0.1 A/cm.

Note current impulses are truncated.

Fig.24a.



Saturable reactor flux and current during <sup>1</sup>/<sub>3</sub> sub-harmonic response.

Scales B = 1.0 Wb/m<sup>2</sup>/cm.

Conditions V = 100 volts.

I<sub>S</sub> = 1.0 A/cm.

C' = 17 μ F.

C = 160 μ F.

Fig.24b.

There was some evidence to suggest that this poorer sub-harmonic response was due to the existence in the parallel loop of the linear ballast reactor since a circuit was set up without this, and although of little practical use showed a much richer sub-harmonic response with spiky currents.

Again the sub-harmonics were easily quenched or prevented from arising by the use of a 6 ohm auxiliary resistor supplied from a 50 turn secondary winding on the saturable reactor.

During the sub-harmonic oscillation the reactor flux was predominately sub-harmonic, and practically trapezoidal in shape.

#### 6.6 Summary.

Where isolated machines are not involved the performance of the double resonance link is undoubtedly superior to the single resonance circuit. In particular the much greater fault transfer impedance and the actual reduction of fault levels at busbars to which it is connected are of considerable practical importance.

### Harmonic Analysis.

7.0 Harmonic analysis was carried out on the various current and voltage waves of the resonant link, in order to determine the frequency spectra within the system, the results of which are given in appendix IV.

The flux wave of the saturable reactor during sub-harmonic oscillation was found to be remarkably pure, being mostly  $1/3$  sub-harmonic. Indeed the total harmonic content, if one can regard the mains frequency as a third harmonic of the sub-harmonic, was just over 10%, in one case, virtually the whole flux wave was made up of 50 Hz and  $16^{2/3}$  Hz frequencies, other harmonics being of negligible proportion.

The peaky current wave of the saturable reactor was found to be rich in harmonics of both the fundamental and sub-harmonic frequencies, containing a large component at  $83^{1/3}$  Hz this being the fifth harmonic of the sub-harmonic. The spectra tailed off above 500 Hz and at the lower end cut off at the sub-harmonic frequency of  $16^{2/3}$ , there being nothing below this frequency.

As expected the ferro-resonant current and flux waves contained only fundamental and odd harmonic frequencies of 50 Hz. Generally speaking the waves of current and flux for the double resonant link had a smaller harmonic content than those of the single resonance system and the range of the spectrum was more limited, a result that might be expected from the more rounded waveforms of current obtaining in the double resonant circuit.

Although the relative phase displacements of the harmonics are unknown it is relatively easy to assume these for the simple spectra and to see how the waveforms of flux and in some cases the current and capacitor voltage are built up.

It is clear from the results that the bulk of the harmonics occur in the currents circulating in the parallel capacitor-saturable reactor loop. Moreover it can be seen that for the sub-harmonic and its harmonics the capacitor and reactor components are fairly evenly matched, indicating that the loop is the origin of the sub-harmonic and its harmonics. Dissimilarity between the components of current in the loop occurs only in the fundamental and harmonics of 50 Hz. The larger component always occurring in the main capacitor and it is thus indicated that these harmonics are associated more with the supply circuit than with the saturable reactor-capacitor loop. Certainly the main flow paths of these two types of harmonics are different.

This similarity in the harmonic components within the loop is also evident in the analysis of the ferro-resonant condition although in this case no sub-harmonic is present. This would seem to further indicate a basic similarity between the sub-harmonic and ferro-resonant responses and points to a resonance origin.

Protection of Resonant Link.

8.0 The problem of protecting a resonant link against internal faults is not difficult if one discounts the use of distance protection.

To substantiate this three modern unit systems were connected to a resonant link and tests were made to determine the performance under a variety of conditions likely to be encountered in practice. In particular internal faults were thrown in the positions indicated in fig.25, for both single end and double end fed faults.

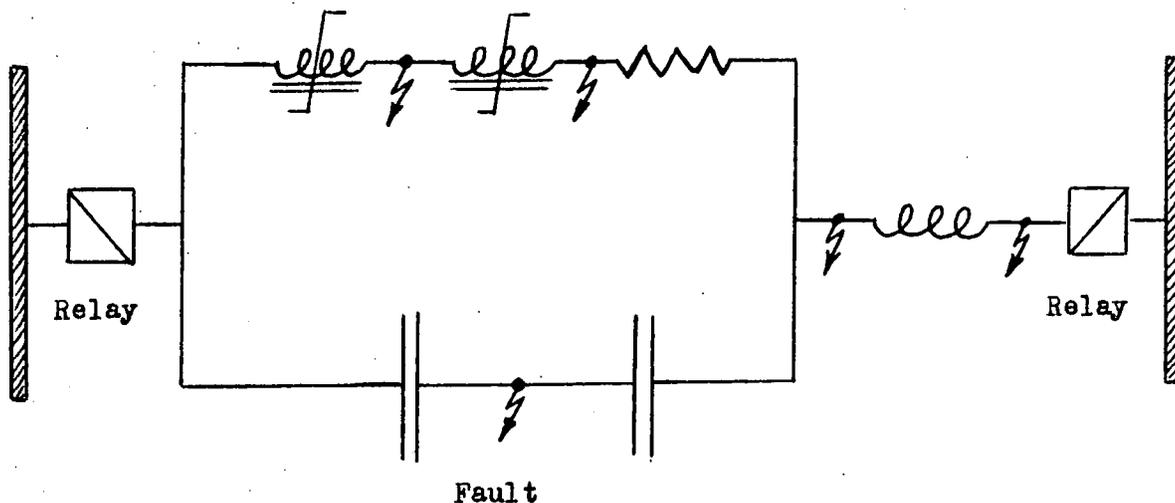


Fig.25.

In all cases the protective systems operated satisfactorily and in the case of the single end fed faults both ends cleared so that no need for intertripping was indicated.

8.1 Solkor R - Reyrolles 410A16315 Pamphlet 1328.

The protection was stable on through faults and under transient conditions when the system was switched onto an external fault from the dead condition.

For internal faults the fault settings obtained were close to those indicated in the manufacturers pamphlet.

<u>Single end faults 1 amp relays</u>	<u>Measured fault setting.</u>	<u>Manuf. fault setting figure.</u>	<u>Remote end fault setting.</u>
Red phase-earth.	0.25	0.25	0.55
Red-yellow phase.	1.29	1.25	
Red-blue phase.	0.64	0.62	
3 phase.	0.70	0.72	

8.2 Translay - A.E.I. HOZ pamphlet 22161C.

Again the protection was stable on through faults and under transient conditions when the system was switched onto a fault from the dead condition.

Internal fault settings are indicated:

<u>Single end faults 1 amp relays.</u>	<u>Measured faults setting.</u>	<u>Manuf. fault setting figure.</u>	<u>Remote end fault setting.</u>
Red phase-earth.	0.22	0.22	0.65
Red-yellow phase.	0.93	0.90	
Red-blue phase.	0.47	0.45	
3 phase.	0.51	0.52	

8.3 Merz-Price Balanced Current.

A.E.I. with 1 : 5 C.T's, Protection stable under all external conditions. Internal fault settings are indicated:

<u>Single end faults 5 amps relays.</u>	<u>Measured fault setting.</u>	<u>Manuf. fault setting figure.</u>
Red phase-earth.	0.31 (1.55)	0.30 (1.50)

No maloperation occurred in any of the protective systems when sub-harmonics were generated by the link, although when these were excited they appeared in the pilot voltages and currents. As expected none of the systems saw series faults within the link and these, where they may occur, must be catered for separately.

#### 8.4 Earth Fault Protection (Back-up).

Back-up protection could be provided by timed earth fault relays in the normal way.

However, sub-harmonics can flow in the neutral circuit or be summed by current transformers connected in the phase conductors.

Thus where sub-harmonics may occur and maloperate the earth fault relays, these relays could be made insensitive by either increasing the fault setting to override the sub-harmonic level or by providing relays sensitive to 50 Hz only.

The former solution would be preferable to special relays since the sub-harmonic level is never more than 0.3 per unit in the phase currents, although in the case of the  $\frac{1}{3}$  sub-harmonic, these can sum to:-

$$\begin{aligned} I_N &= I \sin \frac{\omega t}{3} + I \sin \left( \frac{\omega t}{3} + 40^\circ \right) + I \sin \left( \frac{\omega t}{3} + 80^\circ \right) \\ &= 2.53 I \sin \frac{\omega t}{3} \end{aligned}$$

giving a minimum fault setting of 0.759, which is high for an earth fault relay.

Of course where a secondary damping resistor "r" is connected no such modification to the normal earth fault protection would be necessary.

8.5 Open Circuit or Short Circuit of Capacitors.

In medium voltage systems, say up to 20 kV, the capacitor units within the capacitor bank could be protected against internal short circuits by fuses, however, it could be possible for several units to be shorted out or become open circuited and some method of detection applicable to all systems is necessary.

A proposed system is as follows and necessitates the splitting of the capacitor bank into two parallel balanced sections fig.26.

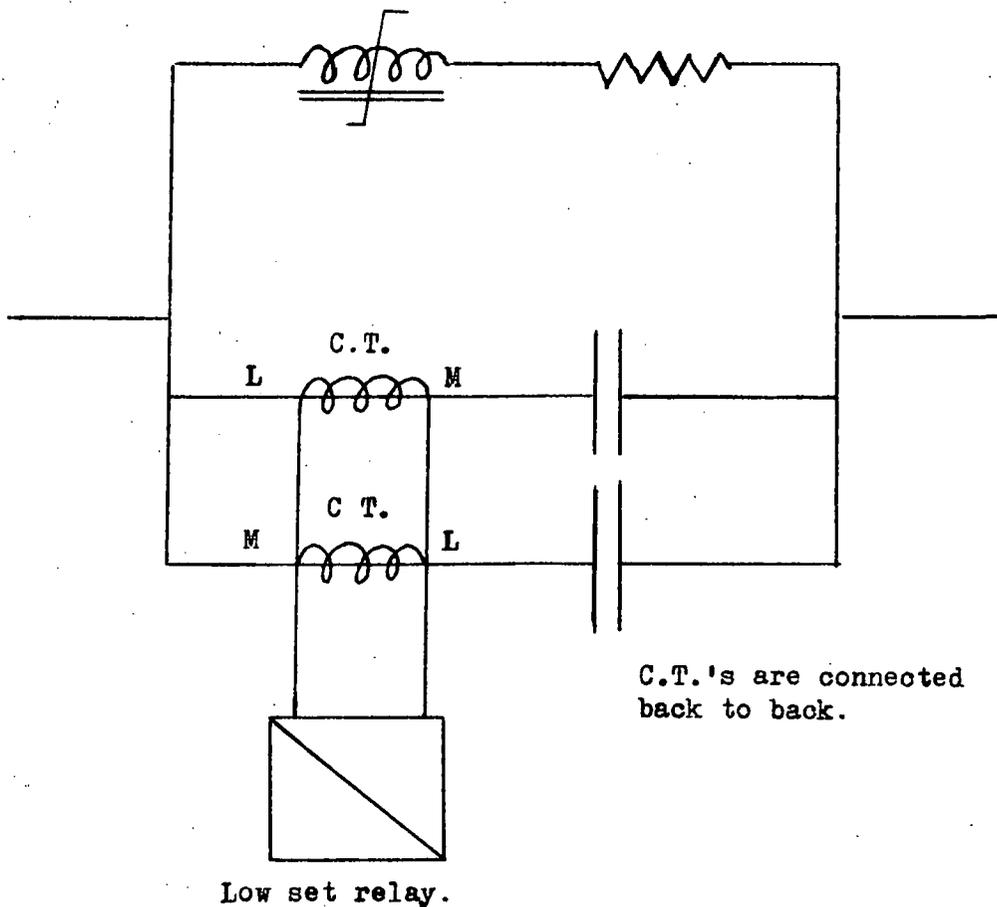


Fig.26.

such a system would detect any imbalance in the capacitor branches.

Finally some means of detecting capacitor over voltage would be necessary and this could be achieved by the use of a voltage transformer connected across the parallel loop and backed up by a simple spark gap.

8.6

A complete protective system is shown in fig.27.

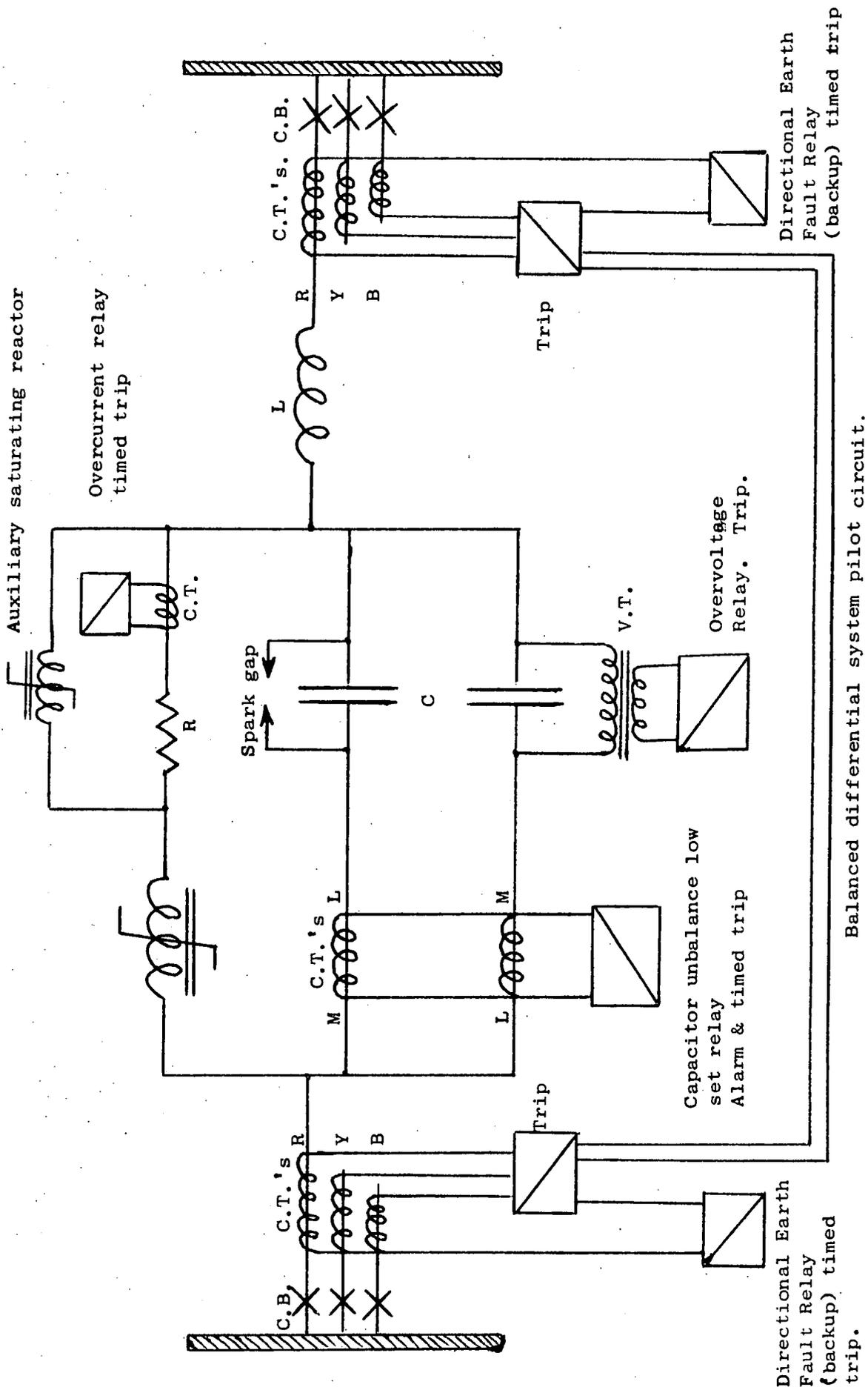


Fig.27.

### 8.7 Distance Protection.

In the case where the linear series reactor is an overhead line it would be difficult to cover the whole scheme by distance protection because of the large change in impedance that the circuit is capable of producing when limiting fault current. However, it is rather difficult to see where the need for a resonant link incorporating a long line would arise since with long lines there is usually no short circuit problem and from a stability aspect a resonance link, with its high short-circuit impedance, would be undesirable where the interchange of synchronising power over the long distance transmission system may be important. However, the line itself could be covered by a distance scheme leaving the parallel part of the link to be protected by another system. Back up by distance relays in zones behind the link would again be difficult because the link during limitation would cause these relays to under reach.

For shorter lines less than 100 miles and for links between busbar sections any differential system (carrier protection for lines above 30 miles) would be satisfactory as indicated in this investigation.

Review of Previous Work.

9.0           The peculiarities of the non-linear series resonant circuit have been studied since as early as 1907. However, the first major contribution to the understanding of the sub-harmonic phenomenon was undoubtedly the paper published in 1938 by Travis & Weygant, ref.1, entitled Sub-harmonics in Circuits Containing Iron-Cored Reactors. This paper has been regarded as a standard reference text by many subsequent workers and by industry.

          The paper studies the response of a series resonant circuit containing a saturable reactor, supplied from a sinusoidal source.

          The most important result to emerge from Travis' work was the theory that the sub-harmonics in these circuits are generated because of trapped charge on the capacitor.

          The theory explained in detail in the paper states that when the inductor saturates a large current flows imparting a charge to the capacitor, subsequently when the inductor becomes unsaturated some of the charge remains trapped on the capacitor and can only escape when next the inductor goes into saturation in the opposite sense, which with a limited applied voltage may be several cycles after the initial trapping.

          Thus the existence of sub-harmonics is attributed to this charge trapping and Travis goes on to analyse the series circuit with the aid of a differential analyser the solutions of which confirm the existence of trapped charge. Fig.3, ref.1.

There is some doubt about the complete generality of this theory because whilst charge does persist on the capacitor when the saturable reactor is unsaturated in the series-parallel circuit, it does not seem to be trapped since it changes continuously, rising and falling to zero.

Perhaps the word stored would convey a better idea of the situation than the word trapped.

When a series circuit was connected up similar to Travis', the expected trapped charge was not recorded.

Fig.28, a and b.

What was recorded was the existence of substantial sub-harmonic flux in the inductor core indicating a storage of magnetic energy over 1 cycle of the 50 Hz frequency.

This remained a puzzle for some time and it was decided to look at the series circuit case in more detail.

At first no trapped charge was observed, with capacitors of a few  $\mu$  F giving rise to sub-harmonics of  $1/2$  supply frequency. But the sub-harmonic flux was again observed. At higher values of capacitance another stable region of sub-harmonics was found and here, indeed, was evidence of charge on the capacitor persisting for several loops of applied frequency. Here then was Travis' trapped charge, but if this was the cause of the sub-harmonics how were the other cases to be explained, and in any case sub-harmonic flux was again observed in the reactor at the same time, Fig.29 a and b.

#### 9.1 Is Charge Trapped.

The idea that the charge that persists on the capacitor, if it persists, is trapped, arises largely out

of Travis' assumption that the inductance and hence the effective impedance of the saturable reactor is infinite at times when it is unsaturated. Thus it is stated that charge imparted to the capacitor when the inductor is saturated, cannot leak away when the inductor is unsaturated.

Undoubtedly, the weakness of this idea is that whilst it might be arguable that the unsaturated reactance of square loop material tends to infinity, this is certainly not the case for stalloy cores and yet sub-harmonics can be generated in circuits containing stalloy cores.

The author also questions the validity of representing the B/H curve by two straight lines, one vertical and the other having low finite slope to represent the saturated condition, Fig.2 in Travis' Paper, Ref.1.

This technique is sometimes used with success to represent the behaviour of C.T. cores in protective systems, but here the question is simply, is the core saturated or not.

And again this representation in the case of stalloy cores is crude.

There is abundant evidence to show that the sub-harmonic phenomena is closely associated with changes taking place in the knee regions of the B/H curve and any valid analysis must take into account the finite curvature of these regions. To represent the B/H curve by two straight lines is to eliminate this vital region. Certainly all the re-entrant loops of the B/H curve, characteristic of the sub-harmonic response occur near this region.

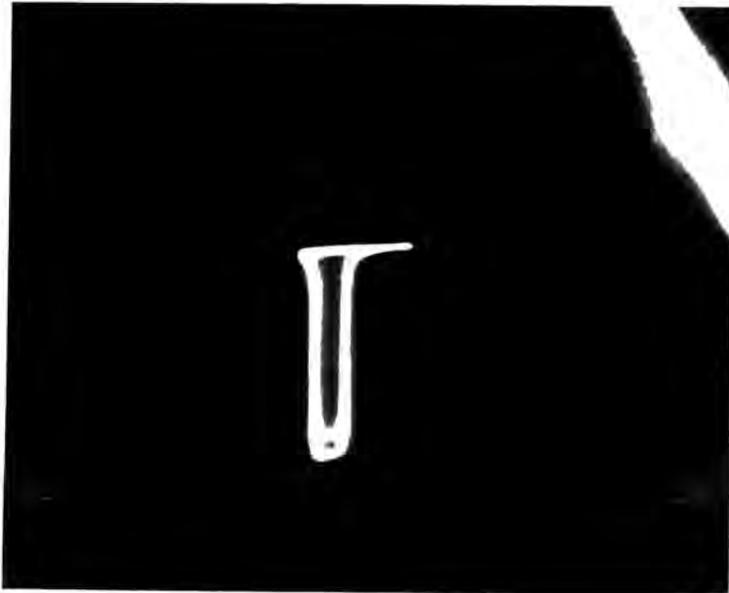
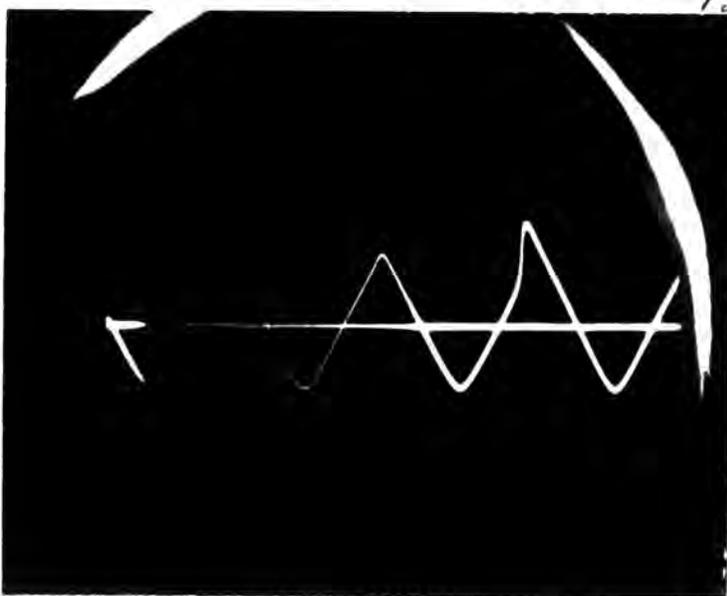


Fig.28a.



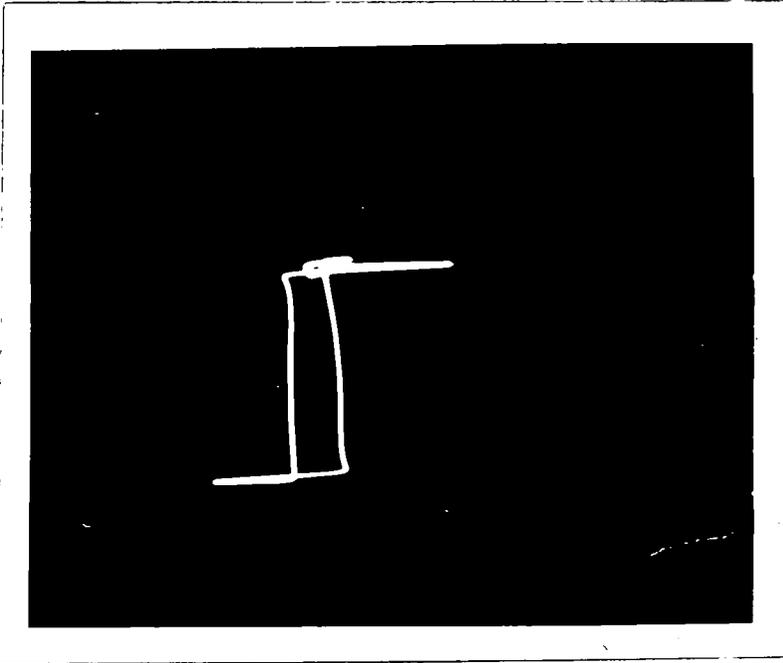
Hysteresis Loop and waveform of charge on the capacitor for a series circuit exhibiting  $1/2$  sub-harmonic response.

Note that there is no trapped charge but that the hysteresis loop is double and indicates a  $1/2$  sub-harmonic flux.

$$V = 25 \text{ volts.}$$

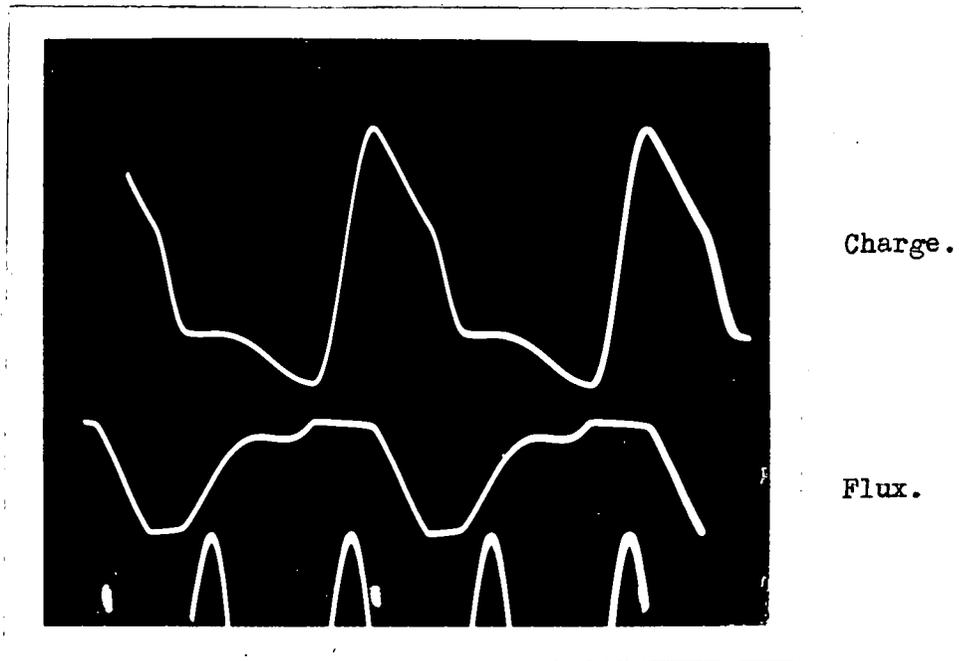
$$C = 3 \mu \text{ F.}$$

Fig28b.



Saturable reactor hysteresis loop for the series circuit with  $1/2$  sub-harmonic response.

Fig. 29a.



Waves of capacitor charge and saturable reactor flux for the series circuit with  $1/2$  sub-harmonic response. Both charge and flux show  $1/2$  sub-harmonic. This is the trapped charge of Travis' paper.

Conditions  $V = 22$  volts RMS.  $C = 87 \mu$  F.

Fig. 29b.

Figure 3

(A)—Differential-analyzer solution of the equation

$$\psi(t) = \int \left[ 0.7 \sin t - 0.05i - 0.197 \frac{di}{dt} - 0.214 \int i dt \right] dt$$

(B)—Approximate check of analyzer solution by step-by-step calculation

(C)—Approximate steady-state solution calculated from recursion relations

Scales:

Flux linkage is shown in per unit  
Time is shown in radians of the impressed sinusoid

To obtain current in per unit multiply scale shown by 0.542

To obtain capacitor voltage in per unit multiply scale shown by 0.27

Maximum value of impressed voltage is 0.7 per unit

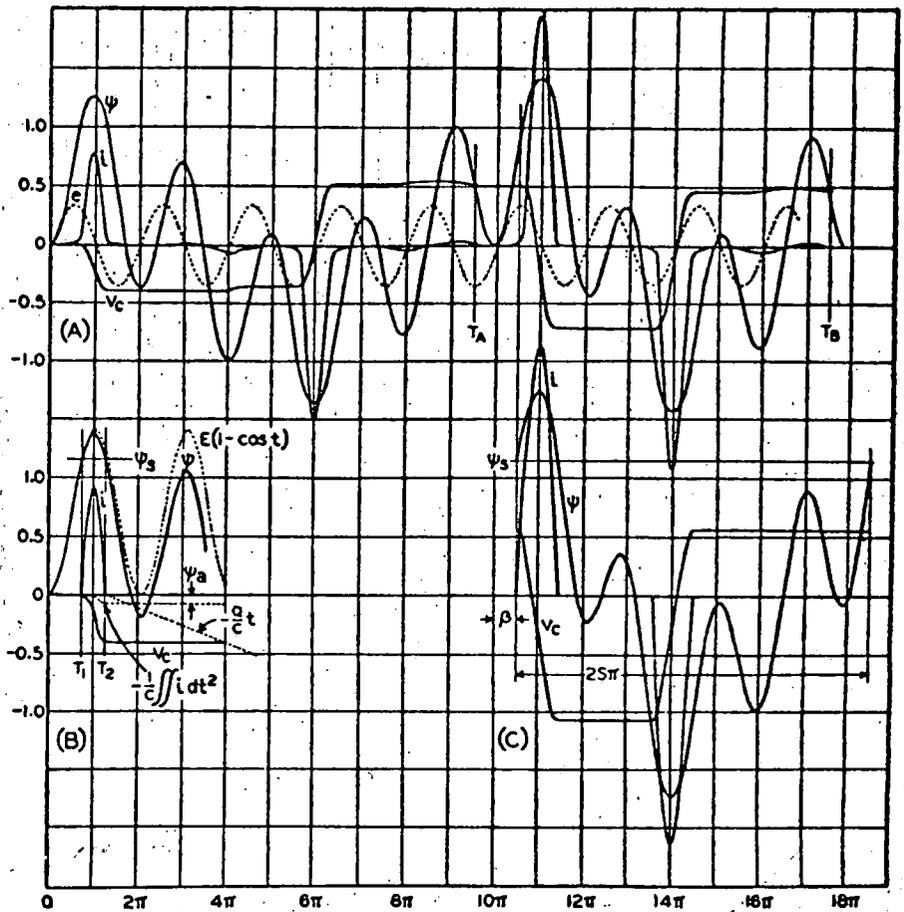


Figure 3 of Travis & Weygandts paper, Sub-harmonics in Circuits Containing Iron-Cored Reactors, Ref.1.

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Furthermore, the idea that the charge cannot transfer during periods when the reactor core is unsaturated is certainly not borne out by the results obtained in this present work, see Fig.14.

Fig.14 shows that there is considerable transfer of energy within the circuit at times when the reactor is unsaturated and indeed during these periods the charge on the capacitor can change sign.

The impression that comes from a study of the results of this work is not so much that there is anything trapped, but that there is a slow transference of energy going on most of the time.

Finally, although Travis produces analytical curves to demonstrate trapped charge no wave forms of charge for actual systems are given. However, it is interesting that Travis' waveforms of Fig.3, ref.1, reproduced here for convenience, have a remarkable duality with the waveforms recorded for the series-parallel circuit Fig.55. Appendix.III. Travis' waveforms of trapped charge  $V_c$ , and flux  $\psi$  are similar respectively to the waveforms of flux and charge recorded for the series-parallel circuit.

It is such considerations as these backed up by the recorded results that have led to the broader notion that the sub-harmonics are the result of excess energy within the system, originally imparted to the system by some disturbance and cyclically replenished in the case of sub-harmonic response and dying away in the majority of cases.

Much of the appendix of Travis' paper is devoted to methods of solution of his equations using step by step procedures and taking into account specified initial conditions.

His analysis yield useful descriptions of the different types of sub-harmonics observed, but suffers from a difficulty that besets any attempt to analyse this type of circuit.

In order to start any analysis of these circuits one must define a B/H curve or loop and thereafter ones analytic results are constrained by this curve. The trouble is that nature knows no such constraints and often does things outside the scope of our restricted analysis, i.e., the response including minor loops of complicated and changing shape. This criticism can be levelled at all analyses carried out on this type of non-linear circuit and the author's own analysis, which is far less rigorous and comprehensive than some of the analyses applied to the series circuit, falls down in this way.

Thus results of such analyses must always be suspect in certain detail and this certainly applies to Travis' B/H curve excursion of Fig.5 entitled Sub-harmonics of the First Kind.

Such a single excursion is never observed in nature from either the series or parallel circuits, since it implies the existence of only the sub-harmonic frequency in the flux and in fact the fundamental frequency is always present giving rise to the minor loops, as are shown in Travis' excursion, Fig.7, ref.1.

Hence the total number of loops gives the order of sub-harmonics being generated. The minor loops are caused by the fundamental frequency flux wave being superimposed on a predominant sub-harmonic flux wave.

Travis' single excursion of Fig.5, ref.1 arises because of the neglect of resistance in his analysis and is physically unrealistic.

Both E. Weber and McCrumm have criticized Travis' analysis on the grounds of his neglect of resistance or dissipation and McCrumm in his paper has shown the importance of resistance by practical demonstration.

The role of resistance or dissipation in these circuits is at once simple and complicated. It is simple by its damping action as it appears, albeit in non-linear form, only in the damping term of the Duffing equation and in practice it has a positive effect in quenching the sub-harmonics. But also it is capable of changing the range and form of the sub-harmonic waveforms and the reasons for this are obscure.

Travis & Weygant also conclude from their analysis that initial conditions play an important part in determining the form of the sub-harmonic response. It has been the Authors' experience, over many months of experimental study, that initial conditions play little or no part in determining the final response of the circuit although inevitably, as in the linear circuit, the initial conditions will determine the transient response. To give an instance, if the series-parallel circuit was set up with 100 volts R.M.S. applied voltage and a load capacitance of 20 ~~μ~~ F, the response was consistent and repeatable irrespective of how the sub-harmonics were started or of the instantaneous value of the applied voltage when the disturbances took place.

Dr. Ernest Weber, in the discussion that follows the paper, attacks Travis' analysis on the grounds that resistance has been ignored which results in the conclusion that switching

angle determines the mode of sub-harmonic response. Weber writes, "the conclusion of the authors that there can exist entirely different characteristic responses to an applied a.c. voltage for different switching angles is not justified. In fact, experience shows that the final steady state as seen in oscillograms is typically the same for various switching angles, though not necessarily of exactly the same form", this is the experience of the present author.

The value of Travis & Weygants paper lies undoubtedly in the idea of trapped charge as the cause of sub-harmonics which can be developed and generalised into the idea of excess energy as the cause of sub-harmonics.

The weakness in Travis' thesis is that too much reliance is placed upon the results of differential analyser solutions of equations of only limited validity. Had the results been supported by actual results obtained from such a non-linear circuit, some of the invalidities would have been exposed.

9.2 McCrumm's paper is essentially concerned with the experimental investigation of sub-harmonics in the series circuit.

In a rather pleasing way he shows in some detail the complicated influence of resistance in the circuit and uses families of constant capacitance curves for given resistance, plotted on applied voltage/current graphs to indicate the zones of sub-harmonic response. Having demonstrated the complicated effect of resistance in this way he rightly concludes when referring to other papers that the neglect of resistance in the analysis renders the results questionable.

The author agrees with McCrumm that the initial conditions of capacitor charge, switching angle, etc., have no effect on the final form of the sub-harmonic response, but disagrees with the statement that the sub-harmonic phenomenon and ferro-resonance are unrelated for reasons already given on pages 56 - 58.

Furthermore, the statement that a shock excitation is always required to initiate the sub-harmonic response has been shown to be untrue pages 39, 40. In addition to this McCrumm states that one cannot produce a sub-harmonic response simply by applying a high voltage magnitude to the circuit and then reducing the voltage to the right value for sub-harmonic response. This may well be true for a series circuit, but for the series-parallel circuit, used by the author, it was certainly possible to go from the high state of ferro-resonance into a sub-harmonic mode as the applied voltage was reduced. See fig.48. Appendix III.

It was, however, not possible to go from a sub-harmonic mode to the high current ferro-resonant mode by increasing the voltage. Always the sub-harmonics extinguished before the high ferro-resonant mode was reached. This behaviour is entirely consistent with the response diagram fig.7. For a falling voltage the operating point may pass through a sub-harmonic zone as the ferro-resonance is extinguished but for a rising voltage the operating point passes out of the sub-harmonic zone before the high state of ferro-resonance is reached.

9.3 Charles F. Spitzer's paper, ref.5, is interesting because it is one of the few papers that shows a sub-harmonic voltage for the saturable reactor and also the re-entrant loops on the hysteresis loop and corroborates McCrumm's evidence that the effect of

resistance in the circuit is complicated and goes beyond the simple effect of damping. Evidence is given to show that a minimum resistance is required below which sub-harmonic responses cannot be sustained.

Ignoring the fact that, strictly, superposition cannot be applied to the non-linear series circuit, Spitzer gives a convincing construction for the hysteresis loop showing re-entrant loops, as being simply the result of the superposition of fundamental on predominantly sub-harmonic flux waves, fig.10 ref.5.

The validity of Spitzer's construction is borne out by the results of the harmonic analysis of this present work which shows that in the cases considered the flux waves were composed predominantly of a sub-harmonic and a fundamental component, all other harmonics being of low order.

9.4. The most recent American papers on sub-harmonic oscillations in power systems ref.9 and 10, are still concerned with the series non-linear circuit, the non-linear inductor in these cases being a power transformer. Paper 9 is mainly concerned with an analytical approach to the problem primarily in an effort to establish a suitable computational technique and to determine the existence zones of sub-harmonic response in the voltage-current plane. In this, the paper gives a good degree of mathematical substantiation to McCrumm's experimental results ref.2.

The authors represent the power transformer magnetising characteristic by the expression,  $i = C_1 \psi + C_n \psi^n$ ,

where  $n$  takes the values 3, 5 or 7, similar to the representation adopted in this present work. The first linear term is then eliminated by combining it with the series capacitance and hence the representation becomes,  $i = C_n \psi^n$ .

This elimination of the linear term would seem to be difficult to justify since several frequencies are involved in the problem and in any case such an elimination could not be made in the case of the parallel circuit.

The main objection that can be made against the analysis is that having established the necessary circuit equations the authors assume trial solutions containing fundamental and  $1/3$  sub-harmonic terms. Whilst such procedures are often adopted in analysis it is more impressive to see sub-harmonic solutions emerge by direct computation, because otherwise some a priori knowledge of the solution is necessary and one may not necessarily determine other modes such as  $1/5$  sub-harmonic, which may, nevertheless, appear in a real system.

The authors claim good agreement between theoretical and experimental results. But the experimental results are obtained from a transient analyser model and not from a real system, so that it is not a really good check. G.W. Swift of Manitoba University writes in the discussions, "it should be emphasised that the experimental results of the paper are really analogue computer results and verify the mathematics but not the mathematical model".

The papers main contribution is analytical but unlike Travis' much earlier contribution, does not come near to giving a physical reason for the generation of sub-harmonics.

Paper 10, from an engineering standpoint, is more useful since it investigates sub-harmonics in a 3 phase system. Particular attention is paid to the sub-harmonic current in the neutral circuit and one important result to emerge is that the neutral current can take three different modes for given sub-harmonic phase current waveforms, depending upon the sub-harmonic current phase displacement which can be  $40^\circ$ ,  $80^\circ$  or  $160^\circ$ . The  $40^\circ$  displacement gives the greatest sub-harmonic neutral current and was the displacement assumed in the protection section of this present work page 82.

The authors verify that all modes of three phase oscillation can in the case of solidly earthed systems be represented by per phase systems, a result that is rightly claimed to be useful for the power systems analyst.

Certain operating states which obtain when neutral resistance is present and which are termed asymmetric or degenerate modes are investigated. These are:-

- 1) Sub-harmonics in two phases and ferro-resonance in the third phase.
- 2) Sub-harmonics in two phases and a low magnetising current in the third phase.
- 3) As case one, but with the ferro-resonant state continuously cycling round all three phases.

This last case is very interesting and gives rise to an apparent beating as the phase current amplitudes slowly change.

These three responses occur near to the upper and lower voltage limits of sub-harmonic response and show a complexity that cannot be accurately represented by a per phase circuit for the resistance earthed case.

#### 9.5 Series-Parallel Circuit. The Resonant Link.

A search through the technical literature has yielded surprisingly little on the series-parallel resonant circuit as a power system link. This is all the more surprising because a prototype system is in commission at the St. Helens works of Pilkington Brothers.

This sparsity of papers on the resonant link and lack of any paper to compare with the standard of the American papers on the series circuit, has been discussed by letter with M.N. John the author of ref.13, who agrees that the subject is poorly documented and suggests that "commercial secrecy concerning the development of resonant links" is responsible.

A few important contributions have been found. The first, "Tuned Interconnectors " by M.W. Bonell and E. Friedlander deals with a resonant link in which the capacitor is protected by a short circuiting spark gap and circuit breaker. See fig.1c and ref.12.

This arrangement has the advantage of being entirely linear and no sub-harmonic response arises. However, the system is not as automatic as that employing a saturable reactor and auxiliary supplies for the circuit breaker must be available. Also for E.H.V. systems, this system will probably be more expensive than the saturable reactor system, because of the high cost of circuit breakers for grid voltages.

A serious disadvantage that emerges concerning the switched link is that limited overloads may overstress the capacitor without operation of the spark gap or circuit breaker. Furthermore, there would seem to be some difficulty in getting the circuit to recover properly after operation because the opening of the circuit breaker would give rise to current surges if the voltage vectors on either side of the link were different and or, widely out of phase. Such current surges could again operate the spark gap and reclose the circuit breaker.

In order to overcome these difficulties the authors propose the splitting of the capacitor bank with isolators and switches so that the link may be brought back into circuit untuned as the system recovers.

Such complexities must further complicate the control and protection systems necessary and increase the cost of the scheme.

9.6 In a paper entitled "An Automatic Resonance Link", ref.13, M.N. John looks at the practical aspects of incorporating a link or links in strategical positions in power systems. Unfortunately, John does not deal with the problem of sub-harmonics, but his paper is based upon the practical applications of links acting between sections of power systems in which situations sub-harmonics are unlikely to occur. That is to say, the links are intended to bridge two systems for security purposes, and to block fault current infeed from one system to another.

In such a situation the only time that there would be a likelihood of sub-harmonic response would be at light load, low plant conditions when there might be a predominance of leading VARs



on the system. However, under these conditions the link would not be required and would probably not be in circuit.

9.7 C.I.G.R.E. paper 301 by B. Kalkner, ref.11, describes an experimental German system operating at 10 kV and limiting a prospective fault current of 31,000 amps to a mere 190 amps when the actual full load current is 140 amps.

Kalkner does not say whether his circuit is of the single or double resonance variety since such severe limitation could be achieved with either system depending upon the parameters chosen.

No indication of the values of reactance or capacitance are given in the paper and his circuit and text would suggest that the single resonance circuit is involved.

If this is the case such extreme limitation implies high values of capacitive and inductive reactance with considerable voltages being developed across the equipment under normal load conditions.

The total short circuit impedance of the German link is  $\frac{10,000}{190} = 53$  ohms, and assuming that the actual series reactor impedance is of the order of 50 ohms, this would imply a voltage of 7 kV, that is 70% system voltage, developed across the series capacitor and reactor during normal operation.

On the other hand some of Kalkner's waveforms look remarkably similar to those obtained for the double resonance circuit, particularly his waveforms of loop current. Also the waveform of the transmitted current during fault conditions is practically in phase with the line voltage but shows a slight

leading component as observed in this present study of the double resonant link. It therefore seems fairly certain that the double resonance link is involved here although no mention of this is made. Kalkner goes on to show that the resonant link can be useful in limiting the surges of an out of step machine and indicates that automatic resynchronisation is possible if the limit of the link is set well above the normal synchronising power level. It may well be the case that this paper covers the results obtained with both types of link otherwise it is remarkable that Kalkner does not highlight the stability problem that undoubtedly exists when a double resonance circuit is used in conjunction with a synchronous machine. It is rather disappointing that no reference was made in the text to the generation of sub-harmonics, since it would have been interesting to have had some data concerning this mode of behaviour from a large high voltage system.

9.8           The last three references 14, 15 and 16 are mainly concerned with the practical aspects of the location of resonance links at strategical positions in power systems to limit fault current and to buffer the system against voltage disturbances created by fluctuating loads.

A new innovation that emerges from these papers is the shunting of the damping resistor  $R$  by an auxiliary saturating reactor.

This reactor, like the main non-linear saturating reactor, is quiescent under load flow conditions but under fault conditions it saturates after the main reactor and shorts out the damping resistor.

The need for this from a practical stand point arises because under fault limiting conditions considerable currents flow in the parallel loop and this means that the dissipation of R must be large. By automatically shorting out R during fault limiting the need for a physically large and costly resistor is overcome.

After the fault is cleared the auxiliary reactor is set to recover first so that the resistor R suddenly becomes available to damp out the ferro-resonance loop current and the link recovers.

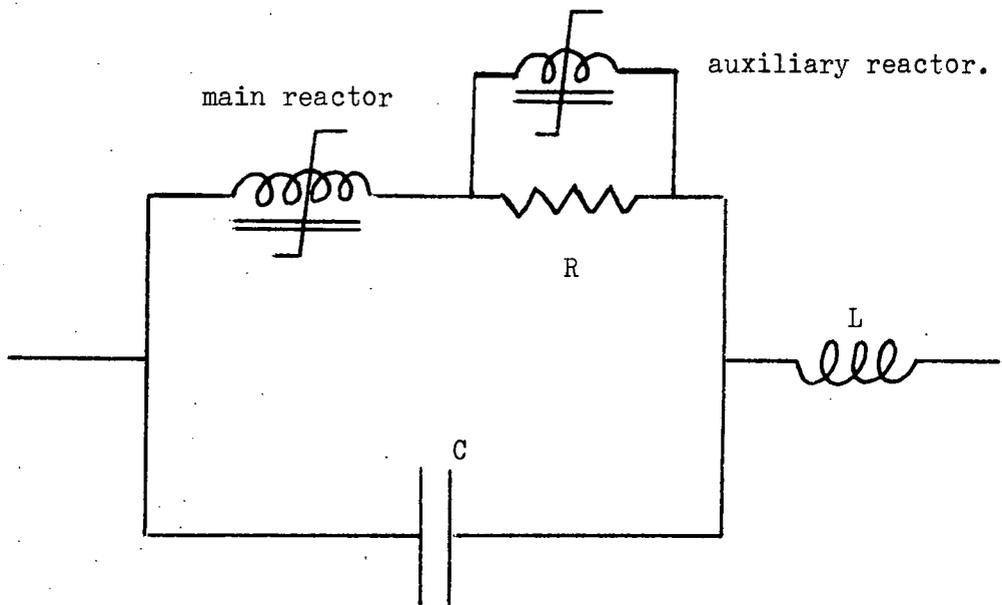


Fig. 30.

Such a system was tested in the laboratory to assess its effectiveness.

A saturable reactor with a similar core to the main reactor, but with a 10 turn winding, was connected across the 3.0 ohms damping resistor R.

The reactor had no noticeable effect on the generation of sub-harmonics and its effect on the throughput current of the link was a very slight reduction; 3.7 amps to 3.6 amps.

Its effect on the short circuit current in the damping resistor R was considerable, the current being reduced from 4.1 amps RMS to 0.8 amps RMS, which is a reduction in the dissipation of R by a factor of 26, so its usefulness was amply demonstrated.

All of the papers reviewed concerning the resonant link failed to deal with the sub-harmonic problem, except for a brief mention in reference 16.

The point seems to be that although the sub-harmonic response is rather involved it can fairly easily be eliminated so that there is a concentration in these papers on the practical aspects and economics of the resonant link.

There is very little theoretical work in any of the papers on resonant links, very few waveforms are given and no attempt at analysis is made.

Summary and Conclusions.

10.0           The object of this work has been to show that the non-linear resonant link has a satisfactory performance as a fault limiting device in a power system and that certain problems associated with it can be satisfactorily overcome without the need for expensive elaboration.

It is difficult to make a clear economic assessment of the link in relation to alternative modes of system operation for a given degree of security and power transference.

This is because every case of possible application is different and must be treated individually. To effect a valid economic comparison in a particular case as to whether to instal a link or to, say, up rate or replace circuit breakers for ones with higher rating, one would need to know the cost of circuit breakers, and the numbers involved, and the possible future development of the system. The amount of regulation that could be tolerated for the use of simple reactors as a cheap alternative, or the possible reduction of the fault levels by a certain amount of system sectioning at the expense of some security.

M.N. John, ref.13, makes a simple economic study of a simple four transformer 11 kV system and arrives at a figure of £2.4/kVA of firm load which he shows to be cheaper than network reinforcement to achieve the same result. However, for systems at high voltages one would expect a figure substantially higher than this to obtain. Indeed, the volume of the main capacitor

bank, given by John as 550 cu.ft./MVA, would require careful consideration.

Certainly the resonant link will not find wide application in power systems, but it is believed that, like the d.c. link, there are cases in large A.C. systems where its unique properties would be beneficial. See fig. 31.

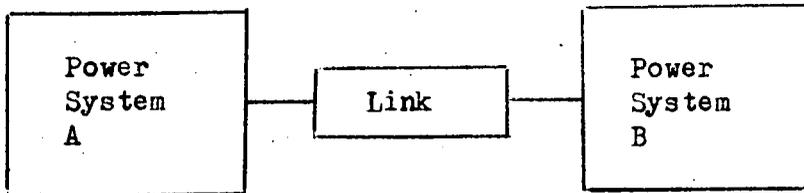
Cases (a) and (b) concern the linking of systems for the transfer of power, but which require a severe limitation on the fault contribution from either system to avoid the necessity for large scale changing or uprating of switchgear and or sectioning of the network within either system to reduce fault duty in the case of a direct coupling.

Case (c) might involve in addition to the short circuit problem, a regulation problem with a direct connection. The two links (for security) limit the fault contribution from either end, but for load flow virtually connect the two bars solidly thus limiting regulation.

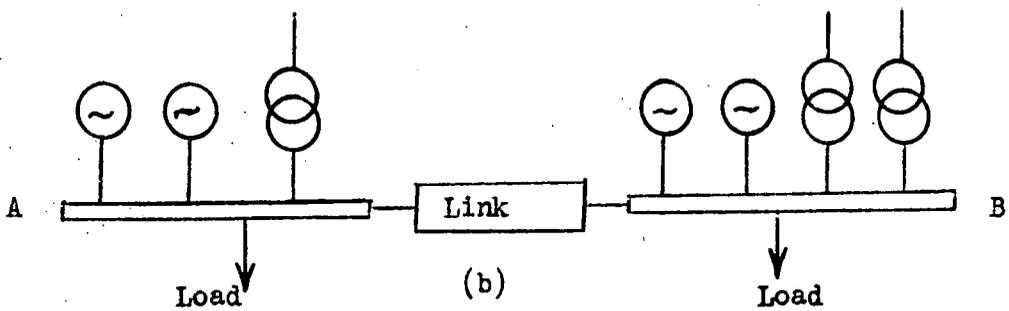
In the foregoing cases consideration would have to be given to machine stability and this factor would have to be taken into account when deciding the limiting levels of the links.

Case (d) involves a load fed from a system and a number of reasons here might make a link desirable between load and supply.

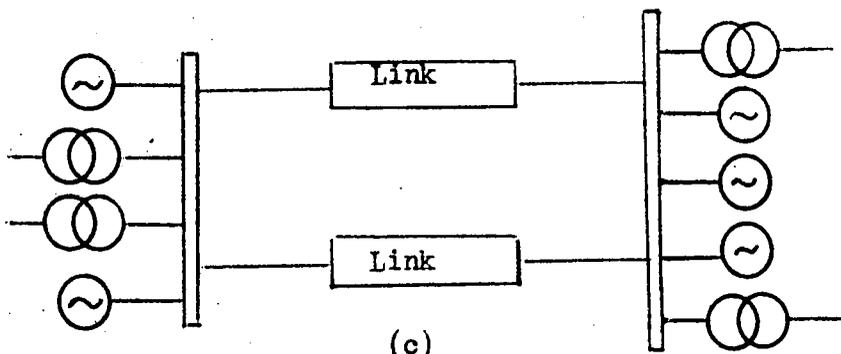
- 1) to limit the fault level at the load busbar.
- 2) to improve or eliminate regulation at the load busbar.
- 3) to limit fault back feed from a synchronous load to the supply system.



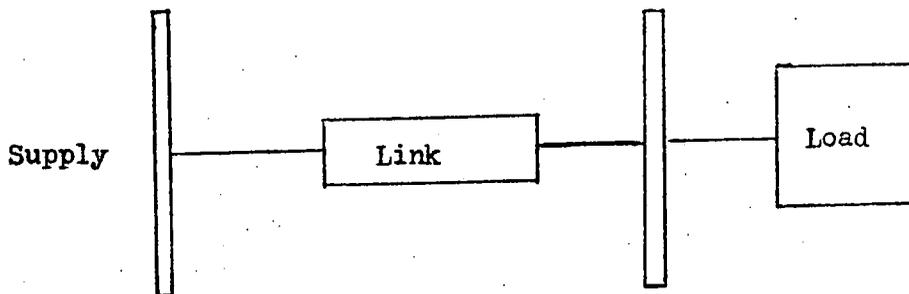
(a)



(b)



(c)



(d)

Fig.31.

- 4) to reduce the flow of harmonics, from high power rectifiers into the supply system.
- 5) the load may be fluctuating (arc furnaces etc.) and a resonant link would buffer the supply system against violent load surges that might interfere with system voltages.

In some of the applications of case (d), an auxiliary damping resistor would be required to prevent the excitation of sub-harmonics, particularly in the case of item 4).

10.1 The conclusions derived from this investigation may be summarised as follows:-

Load Flow and Fault Studies.

- 1) The resonant link can be designed to operate satisfactorily as a load transferring element in a power system, with negligible regulation, which nevertheless severely limits fault currents to a few times full load value.
- 2) The resonant link will operate satisfactorily in parallel with another resonant link and in its detuned state with a conventional supply for load sharing.  
  
The load flow and sharing can to a limited extent, be controlled by variable detuning.
- 3) The link is stable under a variety of transient conditions. Its limitation of fault current is rapid, in the first half cycle, and its recovery after a fault is rapid and complete within a few cycles at most.
- 4) The links overload capacity is good and is adjustable depending upon the value of the series resistance R.

- 5) The dissipation and physical size of the damping resistor R may be considerably reduced by the use of an auxiliary saturating reactor shunting R.
- 6) The link can only be used in conjunction with an isolated synchronous machine if its limiting current is well above the maximum synchronising surges the machine is capable of producing, otherwise the machine will not resynchronise following pole slipping.
- 7) Protection of the link against internal faults is a straightforward matter and no difficulties were encountered in tests using modern differential unit systems.
- 8) Sub-harmonics will flow in the neutral circuit of a three phase system and for the common  $\frac{1}{3}$  sub-harmonic in a solidly earthed system may achieve a maximum value of 2.53 times the phase RMS value.

#### 10.2 Sub-harmonics and Ferro-resonance.

- 1) It is suggested that sub-harmonics are generated when an excess energy is imparted to the system and cyclically replenished by impulse resonance surges driven by the 50 Hz supply.

This is the proposed theory of excess energy put forward as a generalisation of Travis' theory of "trapped charge".

- 2) The sub-harmonic and ferro-resonant phenomena are basically similar. Both are excited by the impulse resonance surges driven by the fundamental frequency.

Sub-harmonics will occur if an excess energy is imparted to the circuit when the driving voltage is below that necessary to maintain ferro-resonance.

Ferro-resonance occurs when the driving voltage is above a certain minimum level, sufficient to cause the reactor to approach saturation in each half cycle of the 50 Hz frequency.

- 3) It is possible to go from the ferro-resonance state to the sub-harmonic state as the applied voltage is reduced or as a fault current is thrown off, but it is not possible to go from the stable sub-harmonic state to the ferro-resonant state with increasing applied voltage.
- 4) It is believed that stable sub-harmonics are produced when the mean natural frequency of the circuit falls near a sub-multiple of the supply frequency. Otherwise an erratic extinguishing sub-harmonic is produced.
- 5) Sub-harmonic zones are restricted to definite ranges of applied voltage, capacitance and resistance.
- 6) The effect of resistance in the circuit is more complex than simple damping although this is its main effect. For any circuit there is a range of resistance above which it is impossible to generate sub-harmonics and below which it is impossible to maintain stable sub-harmonics, the response being erratic.
- 7) Taking into account both the sub-harmonic and ferro-resonance phenomena there may be 2 or 3 stable current states for a given applied voltage, and several unstable states which nevertheless may persist for several minutes.

- 8) Contrary to the assertion of many workers a shock excitation is not necessary to start a sub-harmonic response. All that is necessary is that a certain value of energy be imparted to the system, which energy may be applied rapidly as a shock or very slowly.

Indeed sub-harmonics may be initiated by the slow application of a d.c. current, rising at a rate of less than an amp per minute, to a winding on the saturable reactor.

After the sub-harmonics have been established, if it is a stable response, it becomes independent of the applied d.c. current, which may be removed.

- 9) Such an applied d.c. bias may stabilise an unstable sub-harmonic, convert an erratic response to a stable sub-harmonic response or extinguish it and may initiate a ferro-resonant response which also becomes independent of the d.c. bias once it is established.
- 10) The application of a heavy d.c. bias with a low applied voltage results in a ferro-resonant response in an unsymmetrical mode. The polarity of the bias determining the asymmetry.
- 11) Initial conditions are unimportant in determining the form of a stable sub-harmonic response. The initial conditions are only significant for the transient period as in the case of linear systems.
- 12) Remnance in the reactor core assists the initiation of sub-harmonics, but has no noticeable effect upon the steady

state sub-harmonic response although it must influence the transient period.

- 13) Hysteresis loops of the saturable reactor with sub-harmonic response show re-entrant or minor loops, the total number of loops giving the order of sub-harmonic present.
- 14) The  $\frac{1}{3}$  sub-harmonic is by far the most common sub-harmonic response of the resonant link.  $\frac{1}{2}$  and  $\frac{1}{4}$  and  $\frac{1}{5}$  sub-harmonic responses are encountered but these are unstable although they may persist for several minutes. The  $\frac{1}{2}$  sub-harmonic may be stabilised by the application of a d.c. bias and the  $\frac{1}{5}$  sub-harmonic persists for so long that it could almost be regarded as a stable mode.
- 15) It is possible to generate sub-harmonics with resistive and inductive loads of high impedance.
- 16) Auxiliary damping resistors "r" connected to a secondary winding on the saturable reactor or across the parallel loop are effective in quenching the sub-harmonic response or in preventing their excitation.

### 10.3 Double Resonance Links.

- 1) Broadly these links exhibit a similar behaviour to that of the single resonance link.

However, the double resonance link is superior to the single resonance link since its fault throughput current may be only a fraction greater than full load current.

- 2) The phase angle of the fault current of a double resonance link has a leading component and therefore leads to a reduction of the fault level of a source busbar to which it is connected with other inductive supplies.

- 3) Its overall sub-harmonic and ferro-resonant response is better since the range of sub-harmonic response is narrower and the current surges involved are not spiky like those of the single resonant link, thus leading to a reduction in the stressing of components.
- 4) The double resonance link is not suitable for use with isolated or loosely coupled synchronous machines since its ability to transmit synchronising power is very poor.

10.4 Finally the open ended nature of this project must be stressed.

During the work that has led to the presentation of this thesis many issues have arisen and have not been fully investigated.

For instance the whole problem of sub-harmonics and ferro-resonance needs further investigation particularly with regard to non-linear reactors with different core materials particularly stalloy and composite cores.

With the increasing use of series and shunt capacitor systems often associated with reactors and transformers as static compensating devices the general problem of sub-harmonic and ferro-resonant response will continue to demand attention and suitable remedies can only be expected to emerge from studies into the physical nature of the generation processes involved, ref.10 discussion, 14, 15, 16.

Improved analysis of this non-linear system needs further attention particularly with regard to the accurate representation of the B/H curve and hysteresis loop and this is being actively pursued as an extension to the present work.

The subtle effects of dissipation in the circuit is something that needs further attention since it is suspected of having some influence upon the generation of lower order sub-harmonics of fleeting existence.

There were certain aspects of behaviour of this circuit which although of insignificant effect, arrested ones attention, but which were too fleeting and uncertain as to be capable of documentation. For instance it remains a puzzle why for a few days it seemed relatively easy to excite a  $1/4$  sub-harmonic which lasted for seconds or minutes and even the  $1/7$  sub-harmonic was seen long enough to establish its frequency. But then for days or even weeks try as one may these sub-harmonics refused to appear with the same undisturbed apparatus and identical voltage conditions.

Does the temperature of the saturable reactor core play a part ?

The theory of excess energy needs to be tested further since in its elementary form it might be too simple to explain all the oddities of behaviour that this circuit is capable of producing.

Probably the most profitable and fruitful field of investigation lies in the operational behaviour of the resonance link in conjunction with a synchronous machine.

Preliminary investigations indicate that there is a stability problem involved which is very acute in the case of the double resonance link.

It is felt that the complexity of this problem will be considerable because part of the response of such a system will involve a synchronous machine operating asynchronously (pole slipping)

against a synchronous supply through a non-linear coupling.

Also the effect of sub-harmonics, generated by the link, being supplied to rotating machinery (particularly small induction motors) needs to be looked at.

On the other hand sub-harmonics have mostly been generated by the use of capacitive loads, yet it has been observed that a type of erratic sub-harmonic response can be produced by any load rapidly switched on and off. Thus it may well be the case that fluctuating loads could give rise to this adverse response.

Again the jittering and slow oscillations can be damped by the use of a secondary damping resistor supplied from the saturable reactor and this is important because the link should be useful as a buffer between a fluctuating load (arc furnaces etc.) and a supply system to prevent the propagation of voltage disturbances into the system.

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Appendix I.

Design of the Saturable Reactors.

The saturable reactor used in this investigation is shown in fig.32., and its characteristic magnetisation curve and hysteresis loop are shown in fig. 33 and 34 for 50 Hz excitation.

Core material; H.C.R. square loop alloy type 12C, see TELCON magnetic alloys publication TP 25 - 666.

Dimensions of Toroidal Cores.

Inside diameter	82.5 mm
Outside diameter	127 mm
Core depth	25.3 mm
Cross sectional area of core	564 mm <sup>2</sup>
Stacking factor	0.88
Nett cross sectional area of core	496 mm <sup>2</sup>
Mean magnetic length of core	329 mm
Core tape thickness	0.0158 mm (0.004")

Volts per turn RMS =  $4.44 \hat{B} \cdot A \cdot f = 0.12$  volts/turn.

where  $\hat{B} = 1.1$  Wb/m<sup>2</sup>       $f = 50$  Hz       $A = 4.96 \times 10^{-4}$  m<sup>2</sup>

Thus 250 turns give a nominal 30 volts.

Saturation occurs at  $\hat{B} = 1.54$  Wb/m<sup>2</sup> i.e., at 44 volts RMS

(sinusoidal) on the 250 turn winding.

Windings.

Main windings 16 S.W.G. 5A continuous rating, current density

$$J = 2000 \text{ amps/inch}^2$$

Winding Regime

100 turns	}	Normal operating combination to give 250 turns - 30 volt coil. D.C. resistance = 0.36 ohms.
100 turns		
30 turns		
20 turns		
20 turns		
10 turns		

Auxiliary windings 20 S.W.G.

50 turns

50 turns

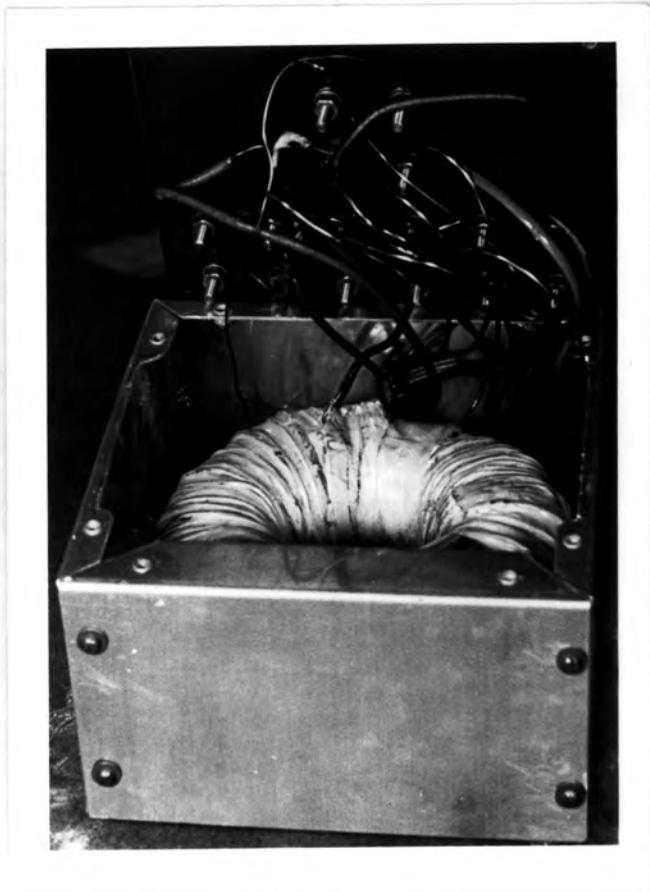
The reactors were designed with a large number of separate windings to allow for flexibility and to save the possibility of having to add further windings later.

This proved to be very convenient as the project progressed.

The 50 turn auxiliary windings were placed respectively next to the core and on the outside of the reactor and were used to measure flux waveforms and to supply the auxiliary damping resistor  $r$  when required.

Summary of magnetic properties taken from manufacturers data manual.

Maximum relative permeability	$10^5$
Saturation	$1.54 \text{ Wb/m}^2$
Remnance maximum	$1.5 \text{ Wb/m}^2$
Hysteresis loss at $B = 1.5 \text{ Wb/m}^2$	$65 \text{ Joules/m}^3/\text{cycle}.$



The Saturable Reactor.

Fig. 32.

Magnetisation Curve for the H.C.R. 12c Alloy Core

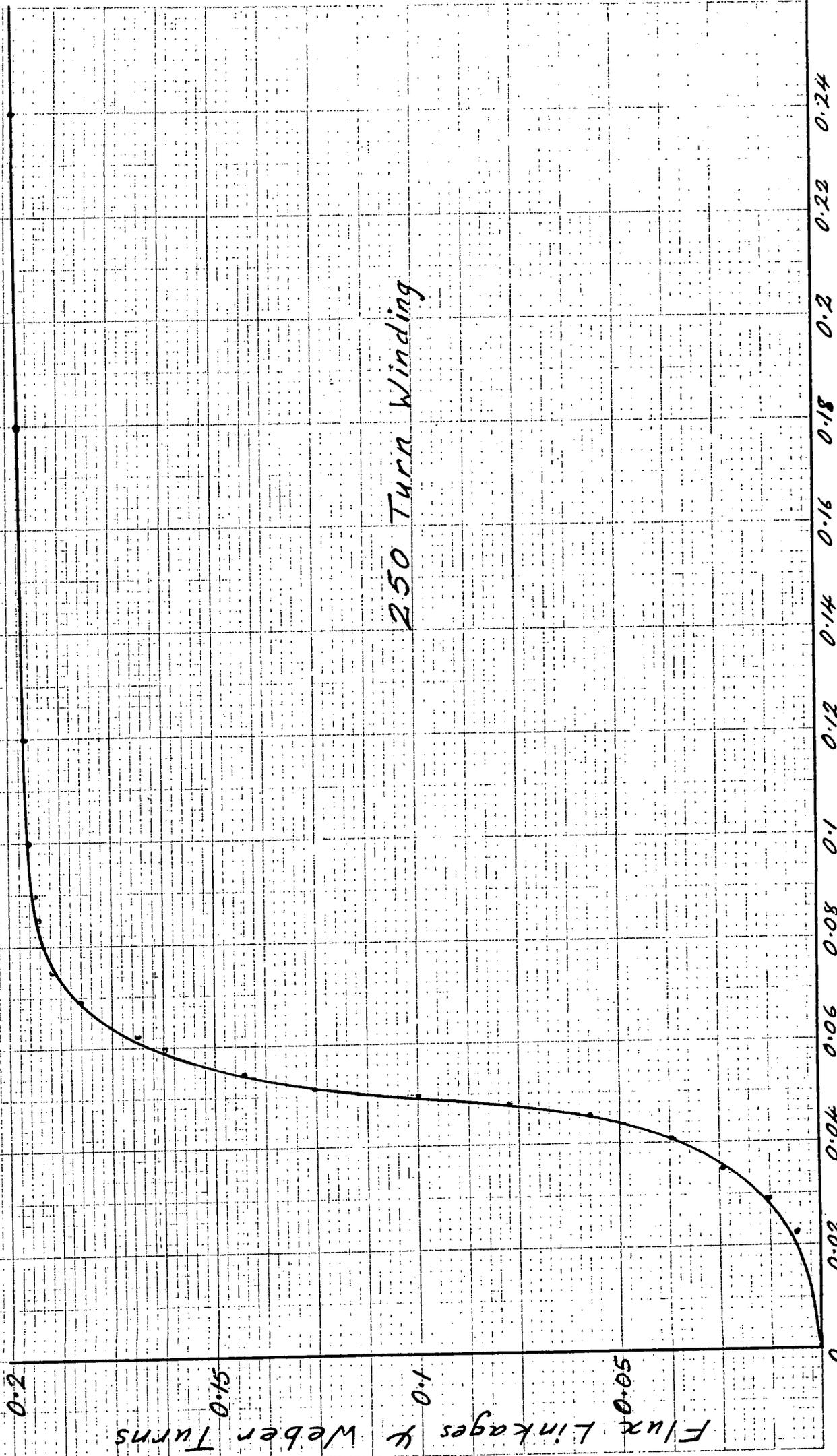
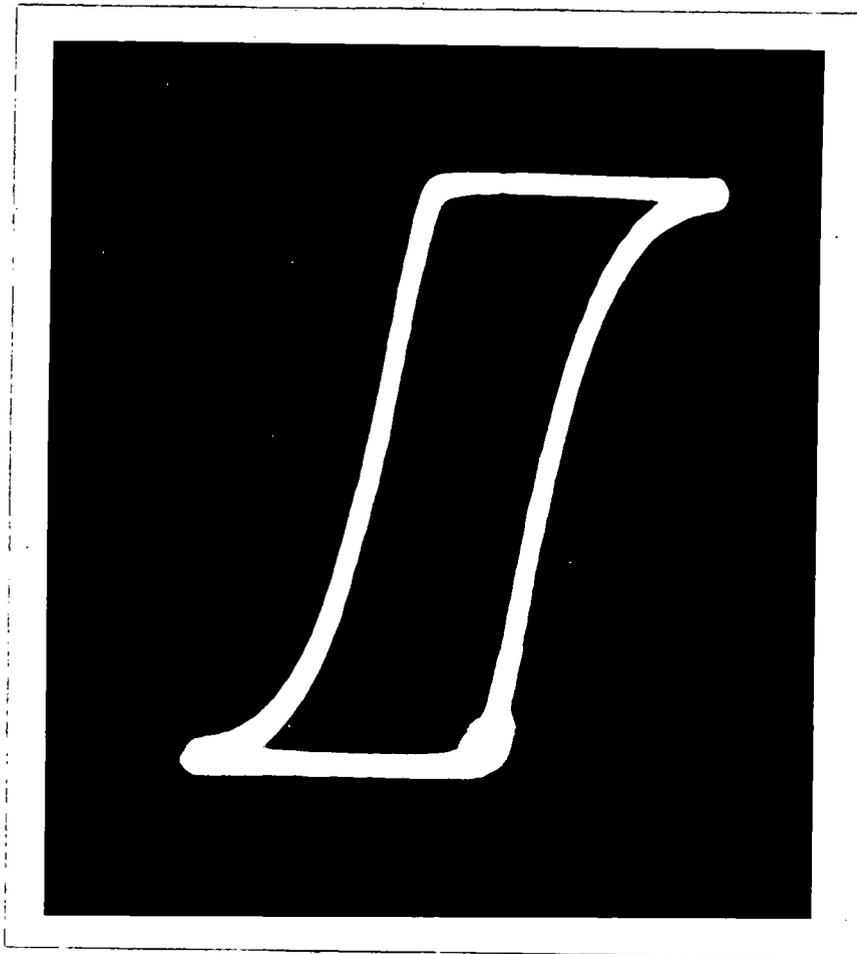


Fig 33 Saturable Reactor Current AMPS.

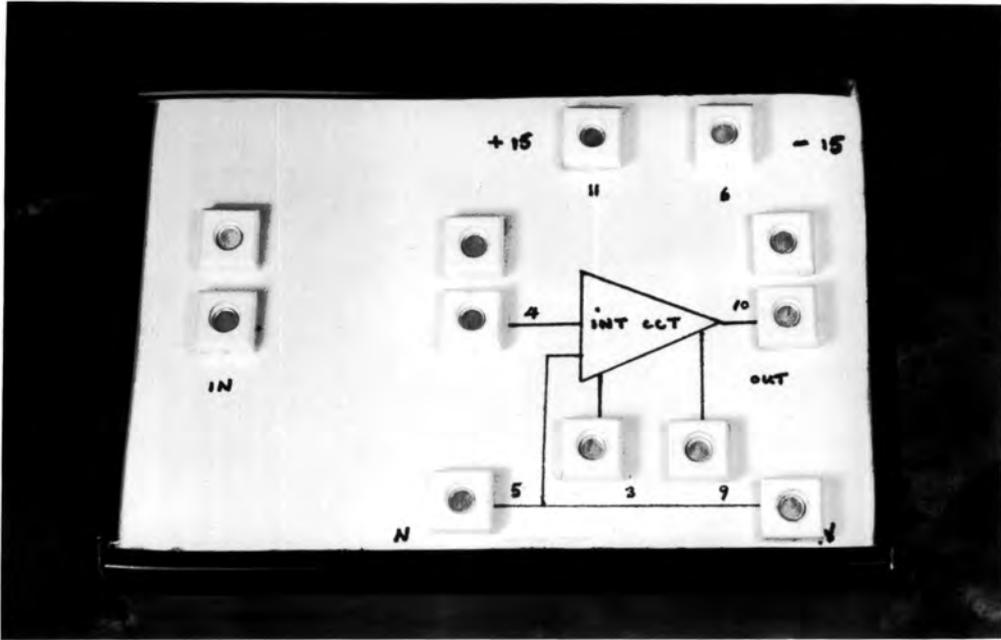


Normal Hysteresis Loop of H.C.R. Alloy core with 50 Hz  
sinusoidal excitation on 250 turn winding.

Scales  $B = 0.414 \text{ Wb/m}^2/\text{cm}$ .

$I_s = 0.05 \text{ A/cm}$ .

Fig.34.



This shows the high performance integrated circuit amplifier No.5741A used for the integration process in the production of the flux waves and hysteresis loops.

With the resistive and capacitive components of 300 K ohms and 1.0  $\mu$  F respectively, the integrator had good linearity over the frequency range indicated in the harmonic analysis. Linearity was tested using a variable frequency square wave generator working down to a frequency of 10 Hz.

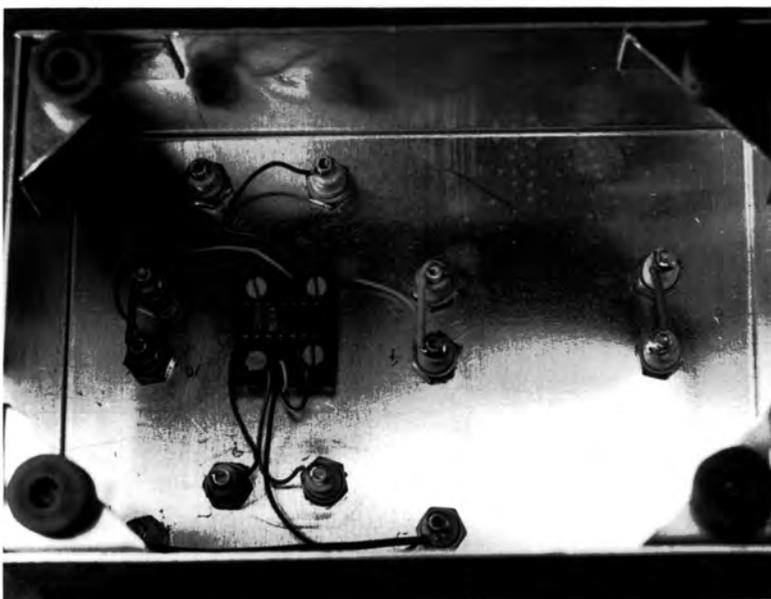
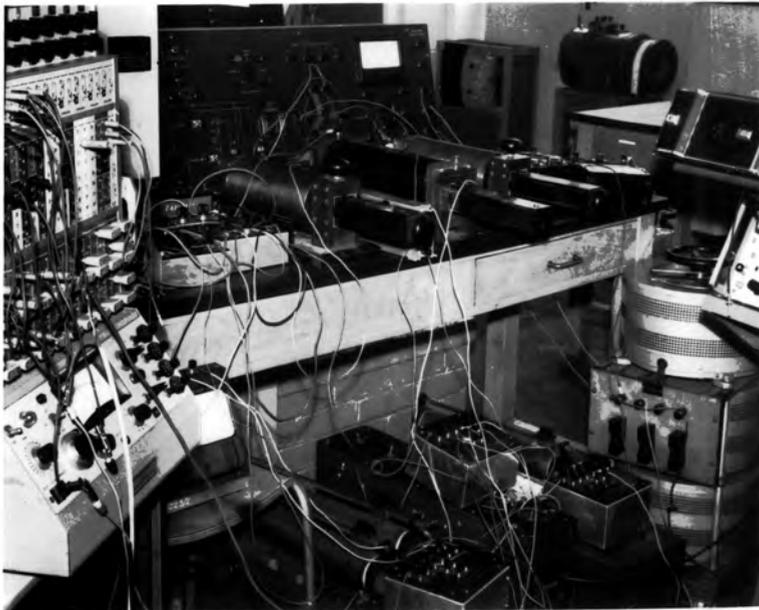


Fig.35.



Single phase - Single/Double Resonance - Resonant Link.



3 phase - Resonant Link.

Fig.36.

Appendix II.

Load Flow Data.

The following are load flows taken with a single resonance link operating in conjunction with conventional supplies.

This data forms the basis of the discussion of section 3.00.

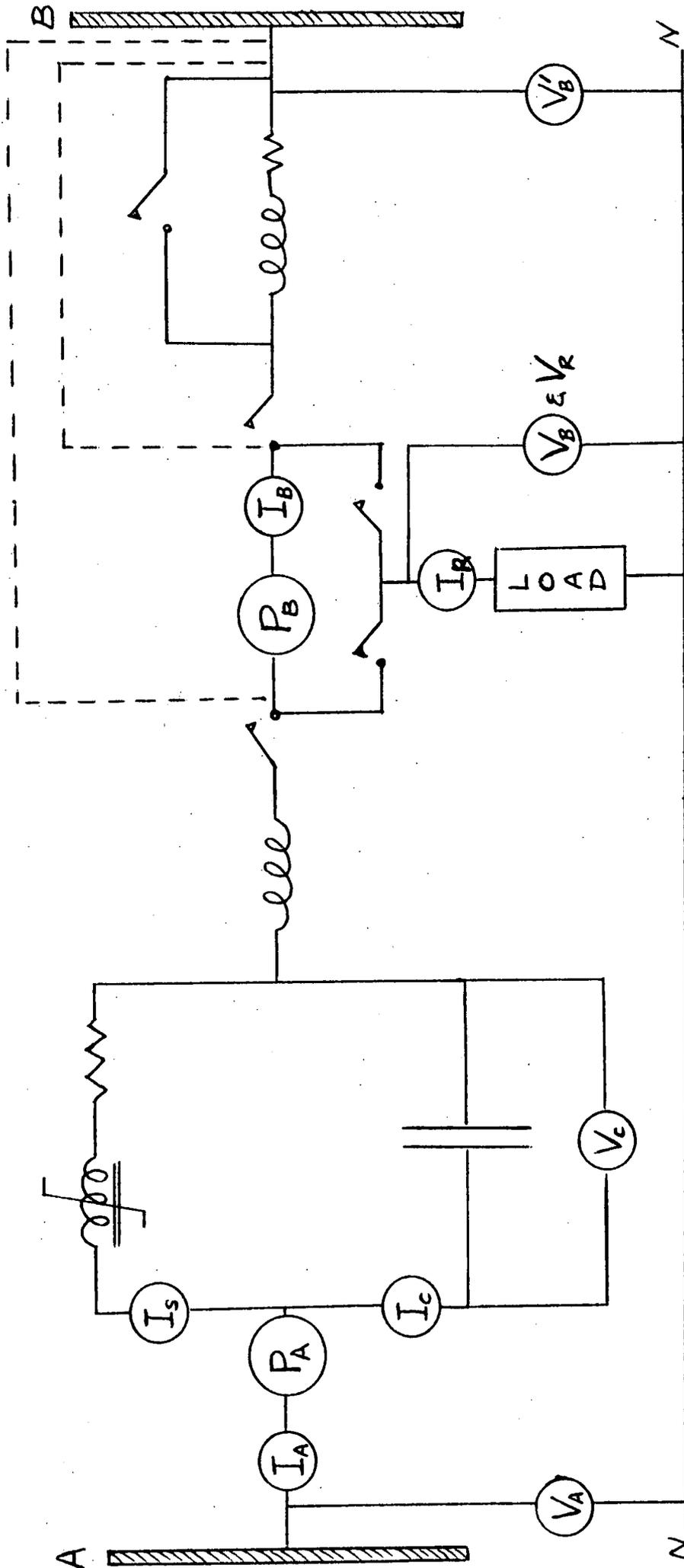
The driving voltages in the load flows have often been taken to levels far higher than would occur on a real power system having regard to the nominal system voltage. This was done to get a broader picture of the behaviour of the circuit particularly beyond limitation.

Fig.37 shows the metering arrangements for the load flows and fault studies.

Where non-sinusoidal currents or voltages could be encountered moving iron or dynamometer instruments were used to determine the RMS values.

The waveforms are reproduced from U.V. recordings and show a variety of conditions of load flow and fault limitation discussed in section 3.00.

Metering may be connected directly to busbar B.



Metering Arrangement for Load Flows

Fig. 37.

Transfer of Load from A - B.

B not an infinite busbar       $R_b = 1.7 \text{ ohms}$      $X_b = 5.8 \text{ ohms.}$

<u>V<sub>a</sub></u> <u>volts</u>	<u>I<sub>a</sub></u> <u>amps</u>	<u>P<sub>a</sub></u> <u>watts</u>	<u>PF<sub>a</sub></u>	<u>V<sub>b</sub></u> <u>volts</u>	<u>P<sub>b</sub></u> <u>watts</u>	<u>PF<sub>b</sub></u>
100	-	0	-	100	-	-
102	0.4	31	.76 lg.exp.	102	28	.7 lg.imp.
103	0.65	54	.8 "	102.5	49	.74 "
105	0.91	76	.79 "	104	71	.75 "
107	1.17	100	.8 "	106	92	.75 "
111	1.70	147	.79 "	110	136	.73 "
116	2.3	204	.77 "	115	190	.72 "
120	1.7	196	.96 "	102	170	.98 " limiting
122	1.65	192	.95 "	102	160	.97 "
126	1.58	190	.95 "	102	154	.96 "
136	1.57	206	.97 "	103	153	.95 "
145	1.65	234	.98 "	104	164	.95 "
116	1.75	200	.98 lg.exp	101	170	.95 "
115	1.85	212	UPF	101	184	.95 "
112	1.75	150	.76 lg.exp.	110	140	.73 " drop-off

Transfer of Load from A - B.

B an infinite busbar.

<u>V<sub>a</sub></u> <u>volts</u>	<u>I<sub>a</sub></u> <u>amps</u>	<u>P<sub>a</sub></u> <u>watts</u>	<u>PF<sub>a</sub></u>	<u>V<sub>b</sub></u> <u>volts</u>	<u>P<sub>b</sub></u> <u>watts</u>	<u>PF<sub>b</sub></u>
100.5	0.34	34	UPF	100	33	UPF
101	0.74	73	"	100	70	"
102	0.95	97	"	100	93	"
103	1.05	107	"	100	104	"
103.5	1.4	144	"	100	138	"
104	1.5	153	"	100	147	"
104.5	1.64	170	"	100	160	"
105	1.85	190	"	100	180	"
106	2.1	220	"	100	210	"
106.5	2.24	235	"	100	220	"
107	2.05	216	"	100	198	" limiting
111	1.93	212	"	100	193	.99 lg.imp.
126	1.62	196	.96 lg.ex.	100	166	.99 "
130	1.76	206	.9 "	100	159	.90 "
140	1.9	226	.85 "	100	162	.85 "

Transfer of Load from A - B.

B. not an infinite busbar  $R_b = 1.7$  ohms  $X_b = 5.8$  ohms.

6 ohm Auxiliary Stabilising Resistor "r" connected.

<u>Va</u> <u>volts</u>	<u>Ia</u> <u>amps</u>	<u>Pa</u> <u>watts</u>	<u>PFa</u>	<u>Vb</u> <u>volts</u>	<u>Pb</u> <u>watts</u>	<u>PFb</u>	
105	.32	27	.8	104	22	.85	
			lg.exp.			lg.imp.	
109	.74	67	.83 "	106	64	.85 "	
114	1.18	108	.8 "	108	100	.8 "	
118	1.65	154	.8 "	111	140	.78 "	
123	2.05	200	.8 "	114	174	.75 "	
128	2.5	250	.79 "	116	214	.74 "	
130	1.93	244	.97 "	108	198	.98 "	limiting

B an infinite busbar

6 ohm Secondary Stabilising Resistor "r" connected.

<u>Va</u> <u>volts</u>	<u>Ia</u> <u>amps</u>	<u>Pa</u> <u>watts</u>	<u>PFa</u>	<u>Vb</u> <u>volts</u>	<u>Pb</u> <u>watts</u>	<u>PFb</u>	
101.5	.26	26	-	100	25	-	
103	.59	60	UPF	100	59	UPF	
105	.89	94	"	100	90	"	
107	1.3	138	"	100	128	"	
109	1.7	184	"	100	167	"	
110	2.03	220	"	100	201	"	
111	2.43	276	"	100	240	"	
212	2.25	260	"	100	234	"	limiting

Effect of Detuning on the Transfer of Load between A - B.

B not an infinite busbar  $R_b = 1.7$  ohms  $X_b = 5.8$  ohms.

<u>Va</u> volts	<u>Ia</u> amps	<u>Pa</u> watts	<u>PFa</u>	<u>Vb</u> volts	<u>Pb</u> watts	<u>PFb</u>	<u>C</u> <del><math>\mu</math></del> <u>F</u>	<u>Link</u> <u>Impedance.</u>
108	1.3	137	.98 lg.exp.	104	130	UPF	120	R C
108	1.23	115	.89 "	106	110	.85 lg.imp.	140	R C
108	1.18	101	.8 "	106	96	.82 "	160	Resistive
108	.85	62	.65 "	106	58	.64 "	190	R L
108	.76	52	.63 "	106	48	.6 "	210	R L

B an infinite busbar

<u>Va</u> volts	<u>Ia</u> amps	<u>Pa</u> watts	<u>PFa</u>	<u>Vb</u> volts	<u>Pb</u> watts	<u>PFb</u>	<u>C</u> <del><math>\mu</math></del> <u>F</u>	<u>Link</u> <u>Impedance.</u>
97	1.05	102	.93 lg.exp.	100	95	.96 lg.imp	120	R C
98	0.68	76	.9 "	100	62	.97 "	140	R C
103	.65	67	UPF	100	62	UPF	160	Resistive
103	.6	51	.8 lg.exp	100	48	.8 Lg.imp	180	R L
103	.53	37	.73 "	100	34	.65 "	200	R L

Unity Power Factor Load Supplied from a Resonant Link and Conventional Supply Operating together.

A - A resonant link with series resistance of 3 ohms.

B - Normal supply with  $R_b = 1.7$  ohms  $X_b = 5.8$  ohms.

In all cases  $V_a$  and the driving voltage behind B is equal to 100 volts.

<u>I<sub>R</sub></u> <u>amps</u>	<u>I<sub>a</sub></u> <u>amps</u>	<u>P<sub>a</sub></u> <u>watts</u>	<u>PF<sub>a</sub></u>	<u>Load</u> <u>volts</u>	<u>I<sub>b</sub></u> <u>amps</u>	<u>P<sub>b</sub></u> <u>watts</u>	<u>PF<sub>b</sub></u>
.5	.4	35	.91 ld.exp.	98	.26	19	.75 lg.exp.
1.0	.72	64	.89 "	97	.52	28	.75 "
1.5	1.05	94	.9 "	95	.76	57	.76 "
2.0	1.4	124	.89 "	95	1.0	72	.75 "
(1) - - - 2.5	1.82	156	.85 "	94	1.3	93	.76 " - - - (1)
3.3	1.7	40	.25 "	81	3.8	280	.9 " limiting

As above but with 6 ohms secondary stabilising resistor r in circuit.

<u>I<sub>R</sub></u> <u>amps</u>	<u>I<sub>a</sub></u> <u>amps</u>	<u>P<sub>a</sub></u> <u>watts</u>	<u>PF<sub>a</sub></u>	<u>Load</u> <u>volts</u>	<u>I<sub>b</sub></u> <u>amps</u>	<u>P<sub>b</sub></u> <u>watts</u>	<u>PF<sub>b</sub></u>
.5	.3	25	.84 ld.exp.	99	.25	23	.92 lg.exp.
1.0	.57	50	.87 "	97	.47	42	.9 "
2.0	1.2	104	.87 "	94.5	1.3	91	.87 "
3.0	1.85	60	.35 "	88	1.9	162	.89 " limiting

Note with a throughput of 104 watts in the resonant link the loss in the 6 ohm resistor amounted to 2.7 watts measured.

Effect of Detuning.

<u>I<sub>R</sub></u> <u>amps</u>	<u>I<sub>a</sub></u> <u>amps</u>	<u>P<sub>a</sub></u> <u>watts</u>	<u>PF<sub>a</sub></u>	<u>Load</u> <u>volts</u>	<u>I<sub>b</sub></u> <u>amps</u>	<u>P<sub>b</sub></u> <u>watts</u>	<u>PF<sub>b</sub></u>	<u>C</u> <u>μ</u> <u>F</u>
1.5	1.24	48	.5 ld.exp.	90	1.55	94	.67 lg.exp.	140
1.5	1.05	94	.9 "	95	.76	57	.76 "	160
1.5	.8	74	.95 "	96	.82	84	.97 "	210

Lagging Load (approximately .2 PF) supplied from a resonant link and conventional supply operating together.

A - A resonant link with series resistance of 3 ohms.

B - Normal supply with  $R_b = 1.7$  ohms  $X_b = 5.8$  ohms.

In all cases  $V_a$  and the driving voltage behind B is equal to 100 volts.

<u>I<sub>R</sub></u> <u>amps</u>	<u>I<sub>a</sub></u> <u>amps</u>	<u>P<sub>a</sub></u> <u>watts</u>	<u>PF<sub>a</sub></u>	<u>Load</u> <u>volts</u>	<u>I<sub>b</sub></u> <u>amps</u>	<u>P<sub>b</sub></u> <u>watts</u>	<u>PF<sub>b</sub></u>	
.5	.44	22	-	100	.2	14	.7	ld.imp.
1.0	.84	43	.5	100	.38	26	.69 "	lg.exp.
1.5	1.25	78	.6 "	100	.7	52	.75 "	
2.0	1.67	116	.66 "	97	.94	61	.67 "	
(2) - - - 2.5	2.0	140	.67 "	95	1.3	60	.54 "	- - - (2)
3.0	1.8	86	.96	86	3.0	100	.4 "	limiting
			ld.exp.					

Effect of Detuning.

<u>I<sub>R</sub></u> <u>amps</u>	<u>I<sub>a</sub></u> <u>amps</u>	<u>P<sub>a</sub></u> <u>watts</u>	<u>PF<sub>a</sub></u>	<u>Load</u> <u>volts</u>	<u>I<sub>b</sub></u> <u>amps</u>	<u>P<sub>b</sub></u> <u>watts</u>	<u>PF<sub>b</sub></u>	<u>C</u> <del><u>μ</u></del> <u>F</u>
1.0	1.25	120	.98	96	1.35	100	175	120
			ld.exp.				ld.imp.	
1.0	1.1	87	.77 "	100	.78	76	.93 "	140
1.0	.84	43	.5	100	.38	26	.7 "	140
			lg.exp.					
1.0	.78	30	.35 "	100	.32	14	.45 "	180
1.0	.69	21	.3 "	99	.33	5	.2 "	210

Leading Load (zero power factor) supplied from a resonant link and conventional supply operating together. 6 ohm stabilising resistor connected.

A - A resonant link with series resistance of 3 ohms.

B - Normal supply with  $R_b = 1.7$  ohms  $X_b = 5.8$  ohms.

In all cases  $V_a$  and the driving voltage behind B is equal to 100 volts.

<u>I<sub>R</sub></u> <u>amps</u>	<u>I<sub>a</sub></u> <u>amps</u>	<u>P<sub>a</sub></u> <u>watts</u>	<u>PF<sub>a</sub></u>	<u>Load</u> <u>volts</u>	<u>I<sub>b</sub></u> <u>amps</u>	<u>P<sub>b</sub></u> <u>watts</u>	<u>PF<sub>b</sub></u>
.5	.34	20	.6 ld.imp.	101.5	.22	15	.7 ld.exp.
1.0	.7	36	.5 lg.imp.	101.5	.6	39	.65 "
1.5	1.06	53	.5 "	102	.85	57	.67 "
2.0	1.36	64	.47 "	102	1.1	72	.65 "
(3) - - - 2.5	1.67	79	.47 "	103	1.45	90	.64 " - - - (3)
3.0	2.25	98	.43 "	106	1.75	118	.65 "
3.3	1.8	170	UPF	119	3.8	196	.5 " limiting

Effect of Detuning.

<u>I<sub>R</sub></u> <u>amps</u>	<u>I<sub>a</sub></u> <u>amps</u>	<u>P<sub>a</sub></u> <u>watts</u>	<u>PF<sub>a</sub></u>	<u>Load</u> <u>volts</u>	<u>I<sub>b</sub></u> <u>amps</u>	<u>P<sub>b</sub></u> <u>watts</u>	<u>PF<sub>b</sub></u>	<u>C</u> <u>μF</u>
1.0	.98	92	.97 ld.exp.	104	1.4	95	.67 ld.exp.	110
1.0	.85	63	.74 "	102.5	.8	65	.8 "	140
1.0	.7	36	.5 lg.imp.	101.5	.6	39	.65 "	160
1.0	.63	25	.4 "	102	.62	27	.42 "	180
1.0	.54	14	.26 "	102	.6	18	.25 "	210

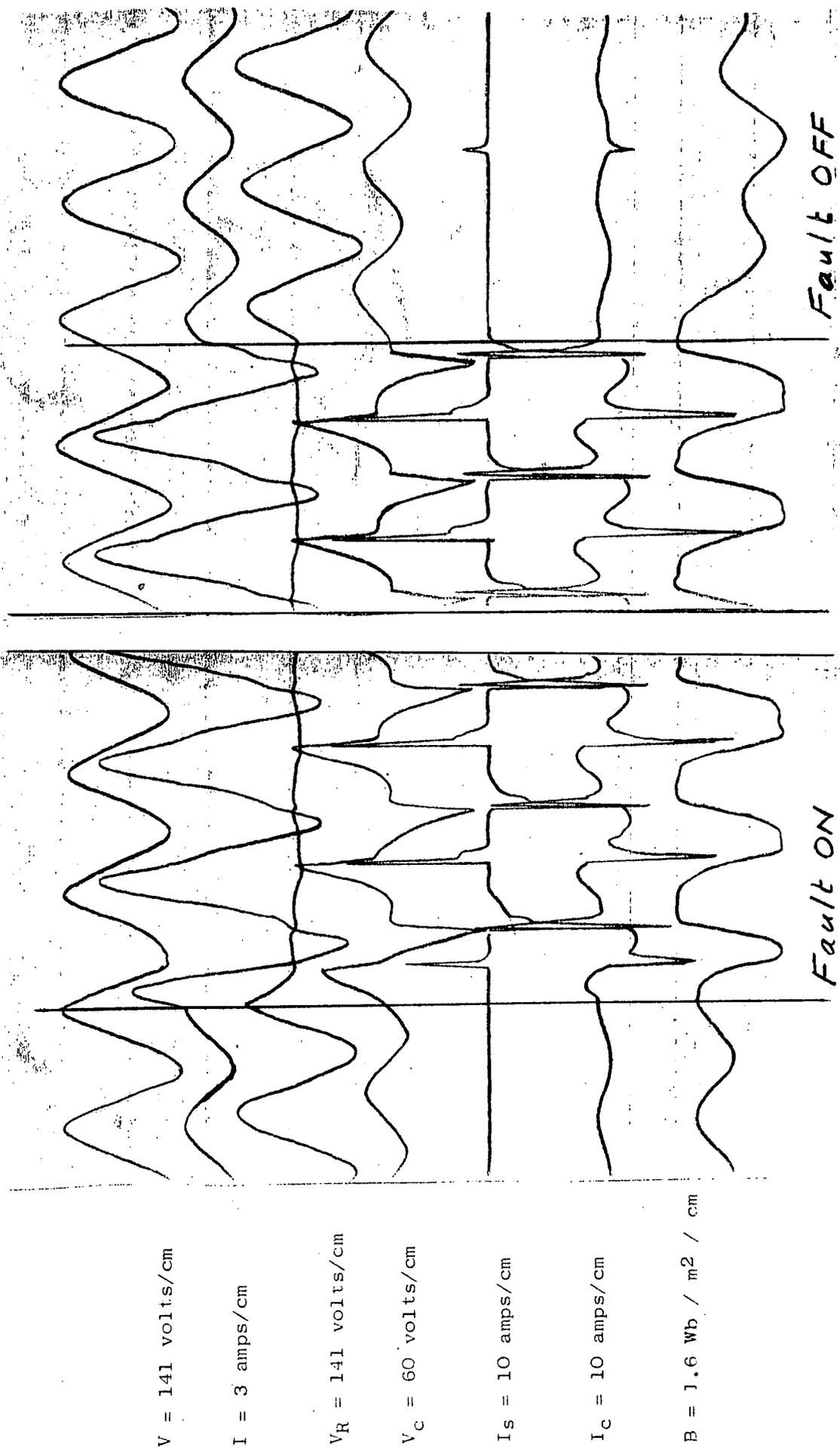
Without the stabilising resistor r connected the capacitive load of between 15 and 25  $\mu$  F caused the generation of  $1/3$  sub-harmonics when single end fed.

Closing end B extinguished the generation of the sub-harmonics.

Two Resonant Links operating together transferring load between A and B.

The links were identical with C equal to 160  $\mu$  F and with the same limiting level.

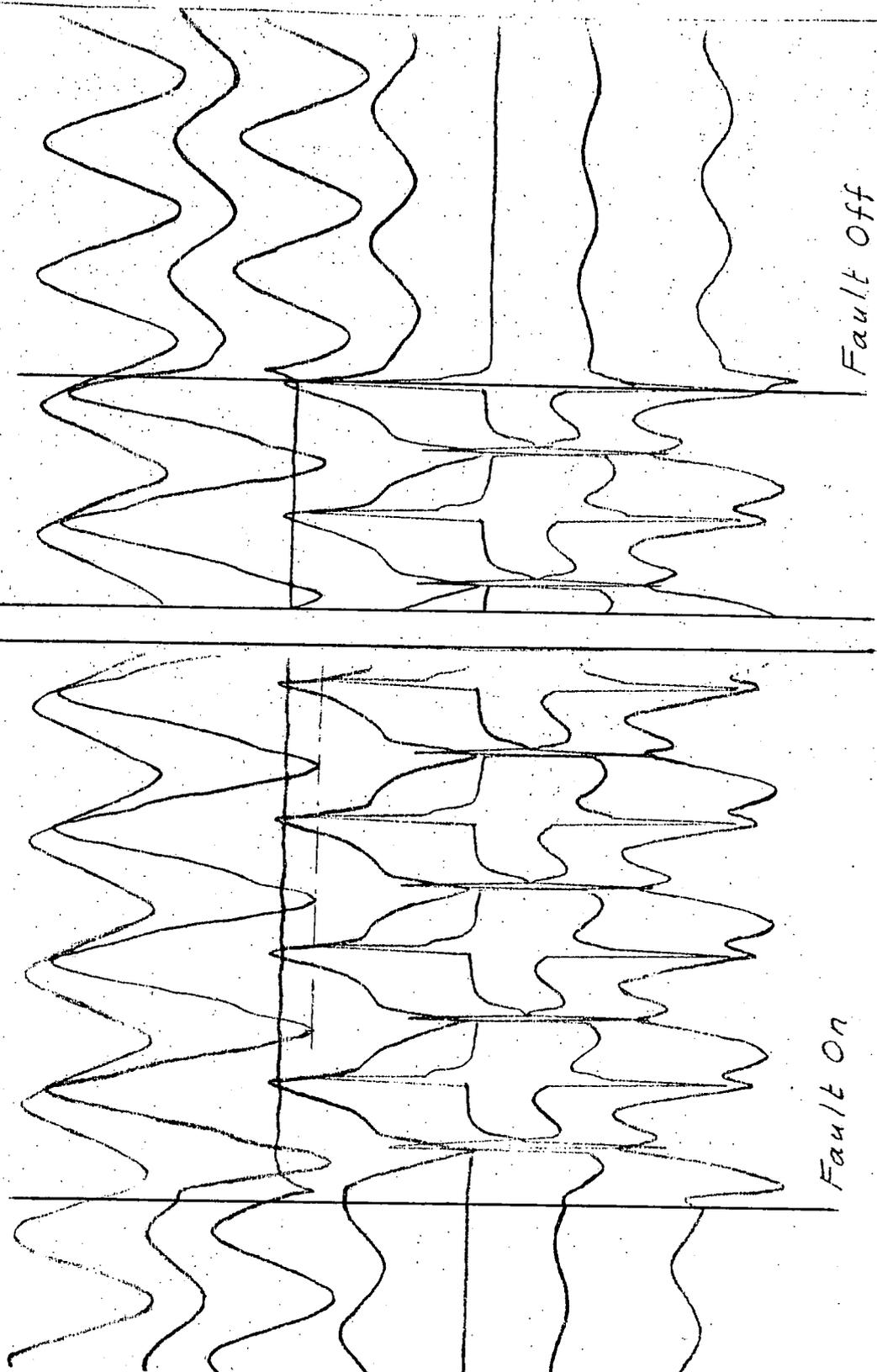
<u>Va</u> <u>volts</u>	<u>Ia</u> <u>amps</u>	<u>Pa</u> <u>watts</u>	<u>PFa</u>	<u>Vb</u> <u>volts</u>	<u>Pb</u> <u>watts</u>	<u>PFb</u>	
102	.32	32	UPF	101	31	UPF	
104	.65	68	"	102	67	"	
106	1.1	118	"	103.5	114	"	
108	1.3	148	"	104.5	135	"	
110	1.7	185	"	105	175	"	
112	2.1	239	"	106.5	222	"	
114	2.0	225	"	107	212	"	limiting
116	1.9	220	"	108	204	"	
120	1.8	214	"	111	196	"	
130	1.6	205	"	116	183	"	
112	2.2	240	"	106	226	"	drop-off



1 amp Resistive Load Condition with Development and Clearance of Short Circuit.

Note that B is reversed to overcome leakage problem in U.V. recorder.

Fig. 38.



$V = 141 \text{ volts/cm}$

$I = 3 \text{ amps/cm}$

$V_R = 141 \text{ volts/cm}$

$V_C = 60 \text{ volts/cm}$

$I_S = 10 \text{ amps/cm}$

$I_C = 10 \text{ amps/cm}$

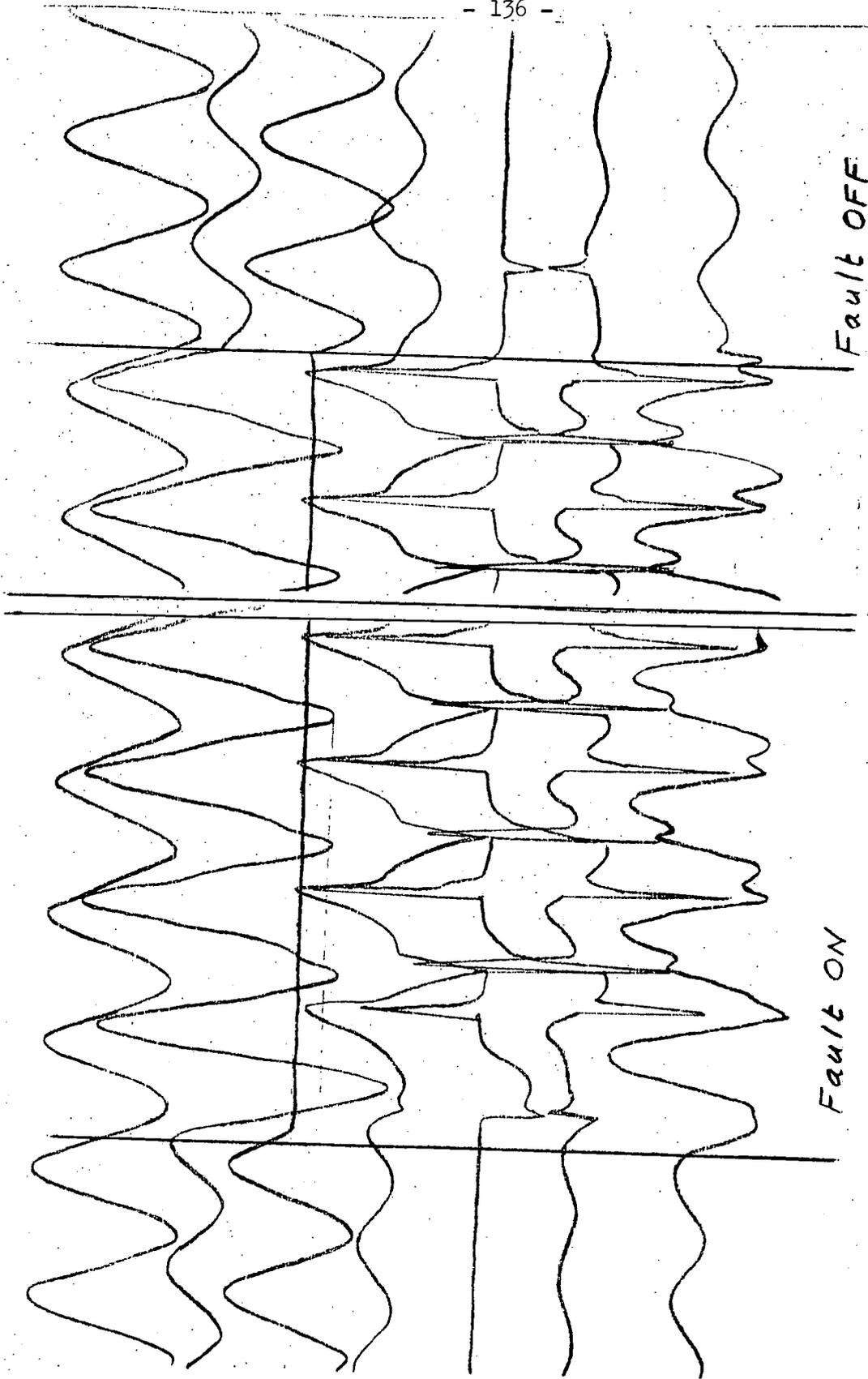
$V_L = 141 \text{ volts/cm}$

*Fault On*

*Fault Off*

Fig. 39.

1 amp Resistive Load Condition with Development and Clearance of Short Circuit.  
Note that  $V_L$  is reversed to overcome leakage problem in U.V. recorder.



$V = 141$  volts/cm

$I = 3$  amps/cm

$V_R = 141$  volts/cm

$V_C = 60$  volts/cm

$I_S = 10$  amps/cm

$I_C = 10$  amps/cm

$V_L = 141$  volts/cm

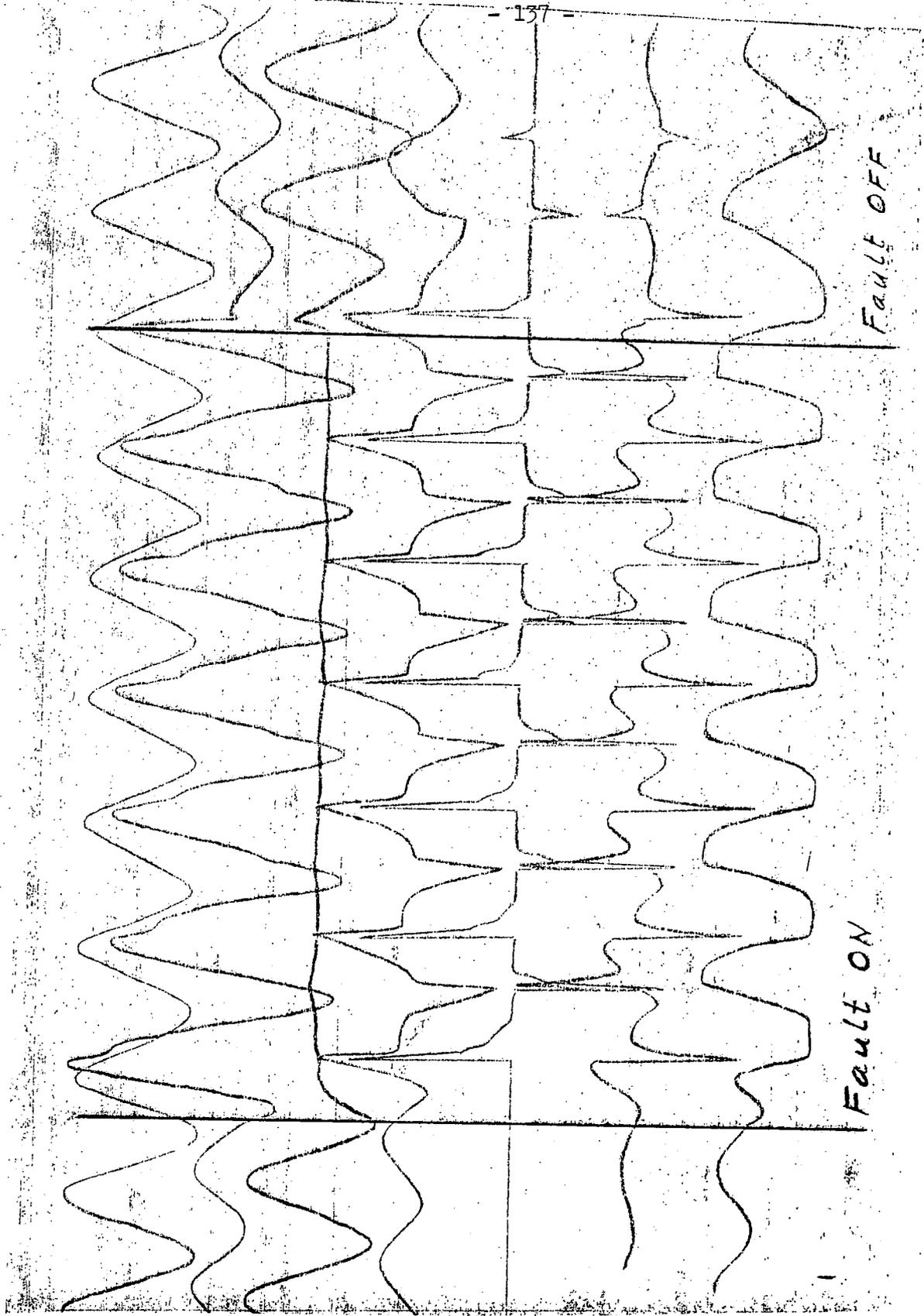
*Fault ON*

*Fault OFF*

1 amp Inductive Load Condition with Development and Clearance of Short Circuit.

Note that  $V_L$  is reversed to overcome leakage problem in U.V. recorder.

Fig.40.



$V = 141 \text{ volts/cm}$

$I = 3 \text{ amps/cm}$

$V_R = 141 \text{ volts/cm}$

$V_C = 60 \text{ volts/cm}$

$I_S = 10 \text{ amps/cm}$

$I_C = 10 \text{ amps/cm}$

$B = 1.6 \text{ Wb / m}^2 \text{ / cm}$

1 amp Inductive Load Condition with Development and Clearance of Short Circuit.

Note that B is reversed to overcome leakage problem in U.V. recorder.

Fig.41.

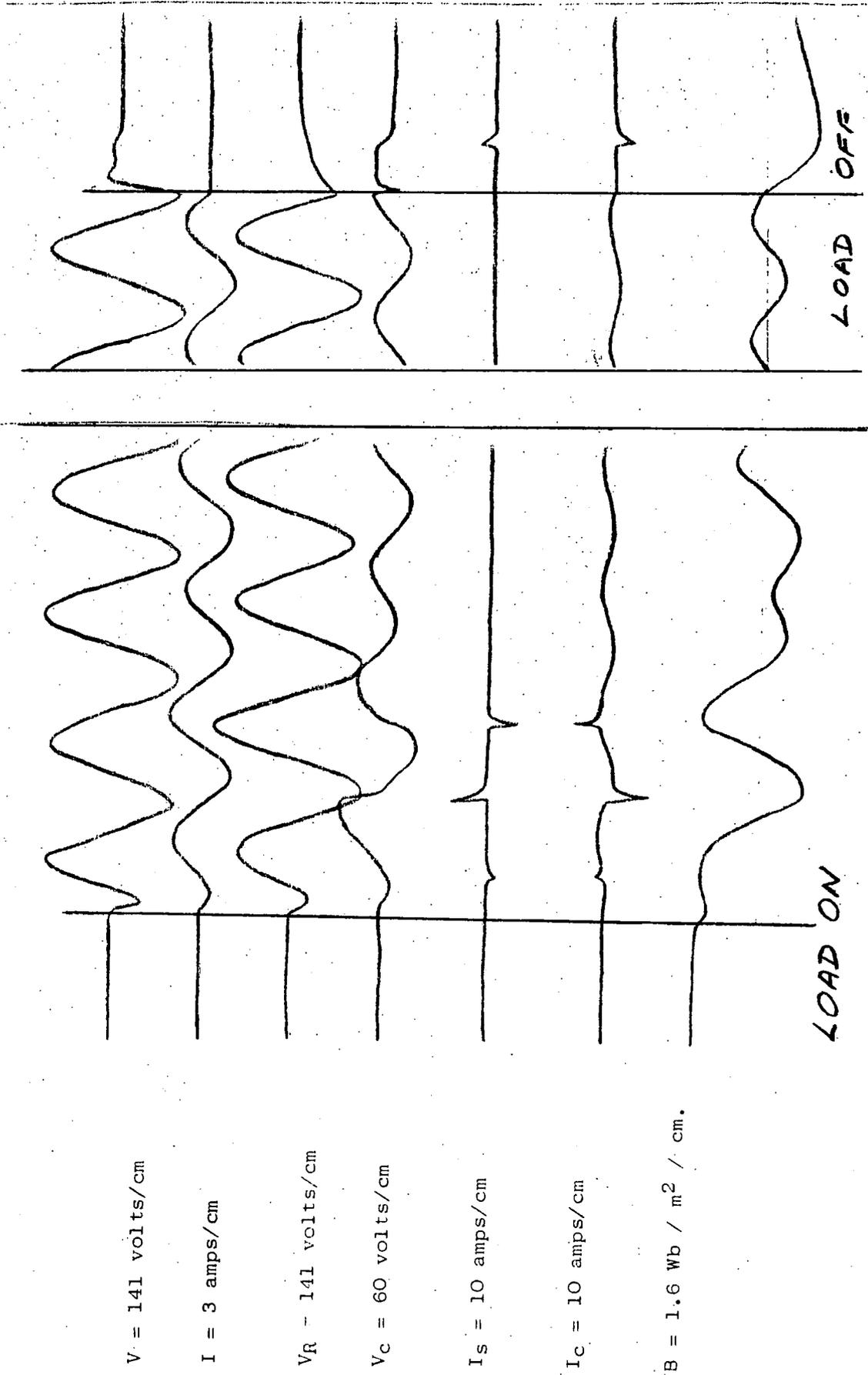
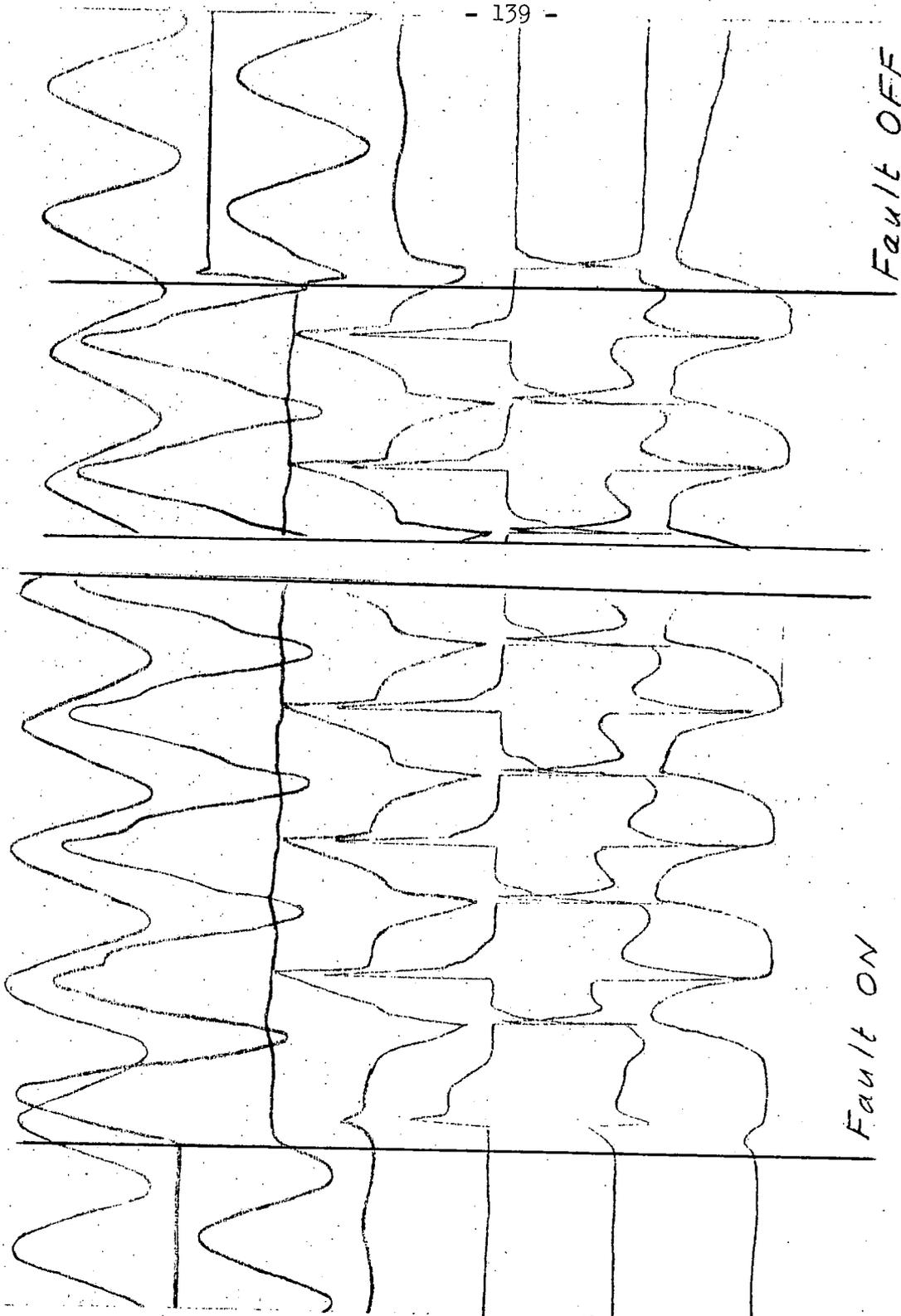


Fig.42.

Switch on and off of 1 amp Inductive Load.

Note that B is reversed to overcome leakage problem in U.V. recorder and that the different zero levels are caused by the operation of amplifier and do not indicate remanence.



V = 141 volts/cm

I = 3 amps/cm

VR = 141 volts/cm

Vc = 60 volts/cm

Is = 10 amps/cm

Ic = 10 amps/cm

B = 1.6 Wb / m<sup>2</sup> / cm.

Fault ON

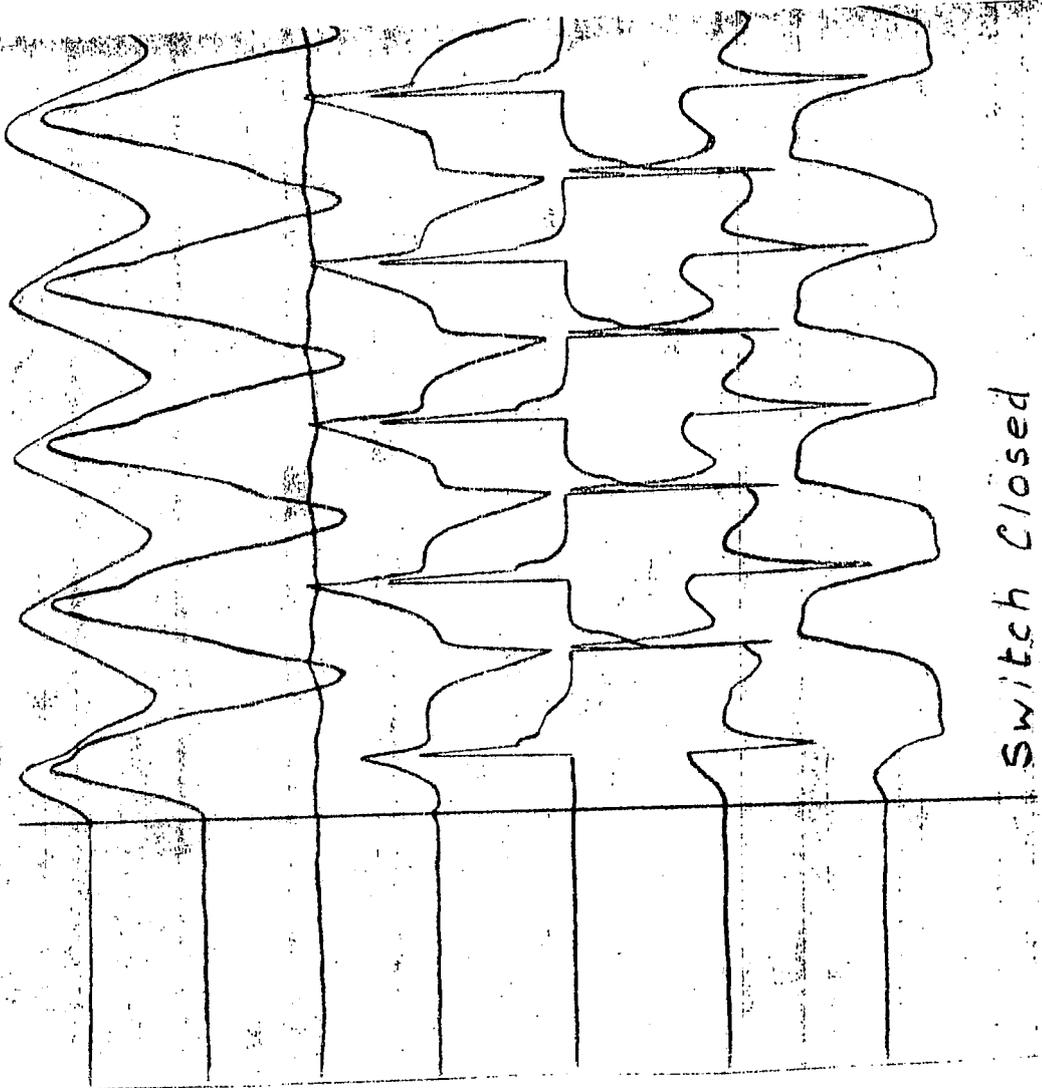
Fault OFF

Short Circuit from No-load Condition.

Note that B is reversed to overcome leakage problem in U.V. recorder.

Fig.43.

$V = 141 \text{ volts/cm}$   
 $I = 3 \text{ amps/cm}$   
 $V_R = 141 \text{ volts/cm}$   
 $V_C = 60 \text{ volts/cm}$   
 $I_S = 10 \text{ amps/cm}$   
 $I_C = 10 \text{ amps/cm}$   
 $B = 1.6 \text{ Wb / m}^2 / \text{cm}$

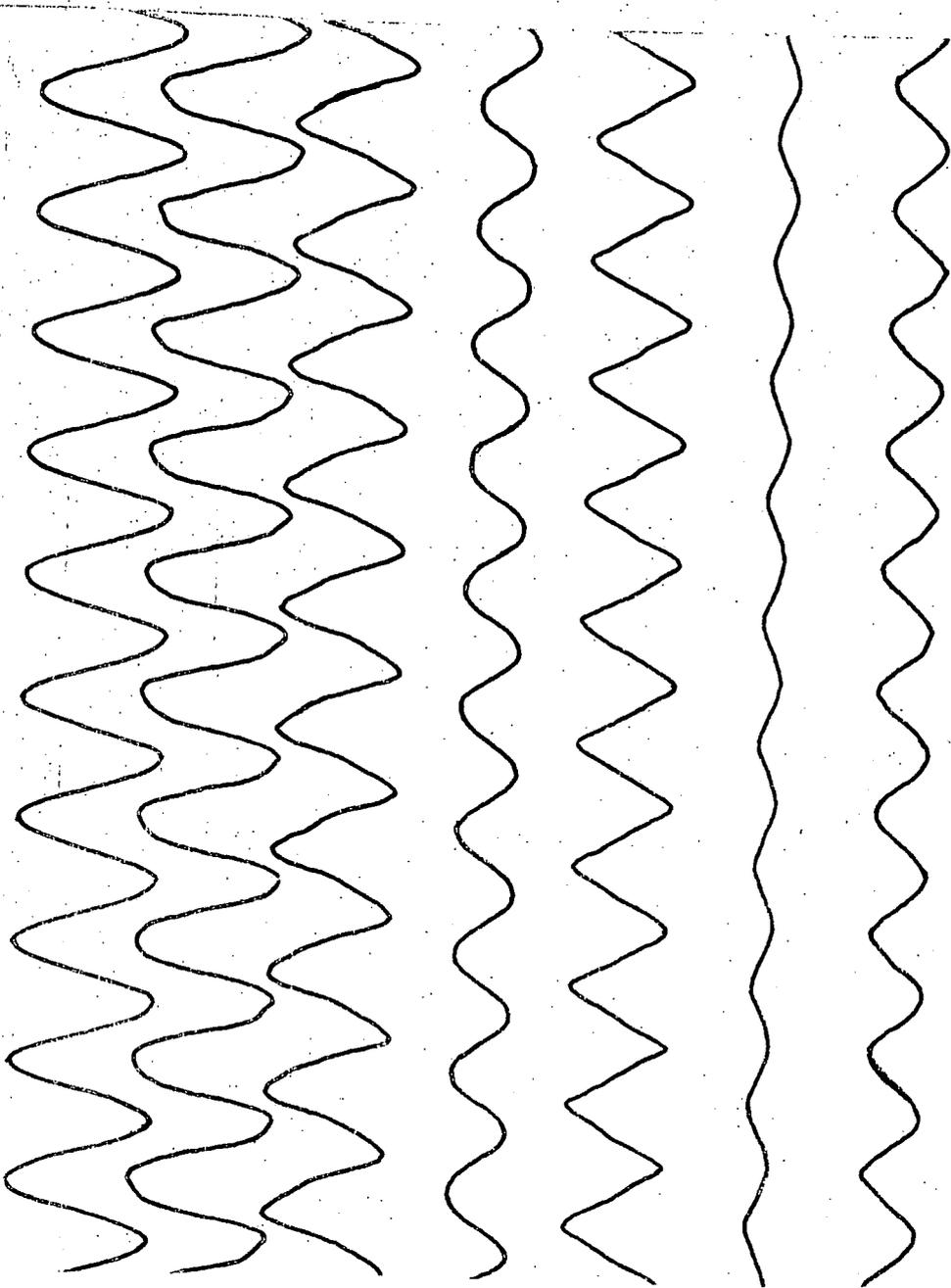


Switch Closed

Energising of Dead Circuit with a Fault.

Note that B is reversed to overcome leakage problem in U.V. recorder.

Fig.44.



$V = 141 \text{ volts/cm}$

$V_R = 141 \text{ volts/cm}$

$I = 1 \text{ amp/cm}$

$V_C = 30 \text{ volts/cm}$

$I_C = 1 \text{ amp/cm}$

$I_S = 0.5 \text{ amp/cm}$

$B = 1.2 \text{ Wb / m}^2 / \text{cm}$

Normal 50 Hz Response with Capacitive Load -

$V = 100 \text{ volts}$

$C' = 20 \mu\text{F}$

Resistor r

connected

Fig. 45.

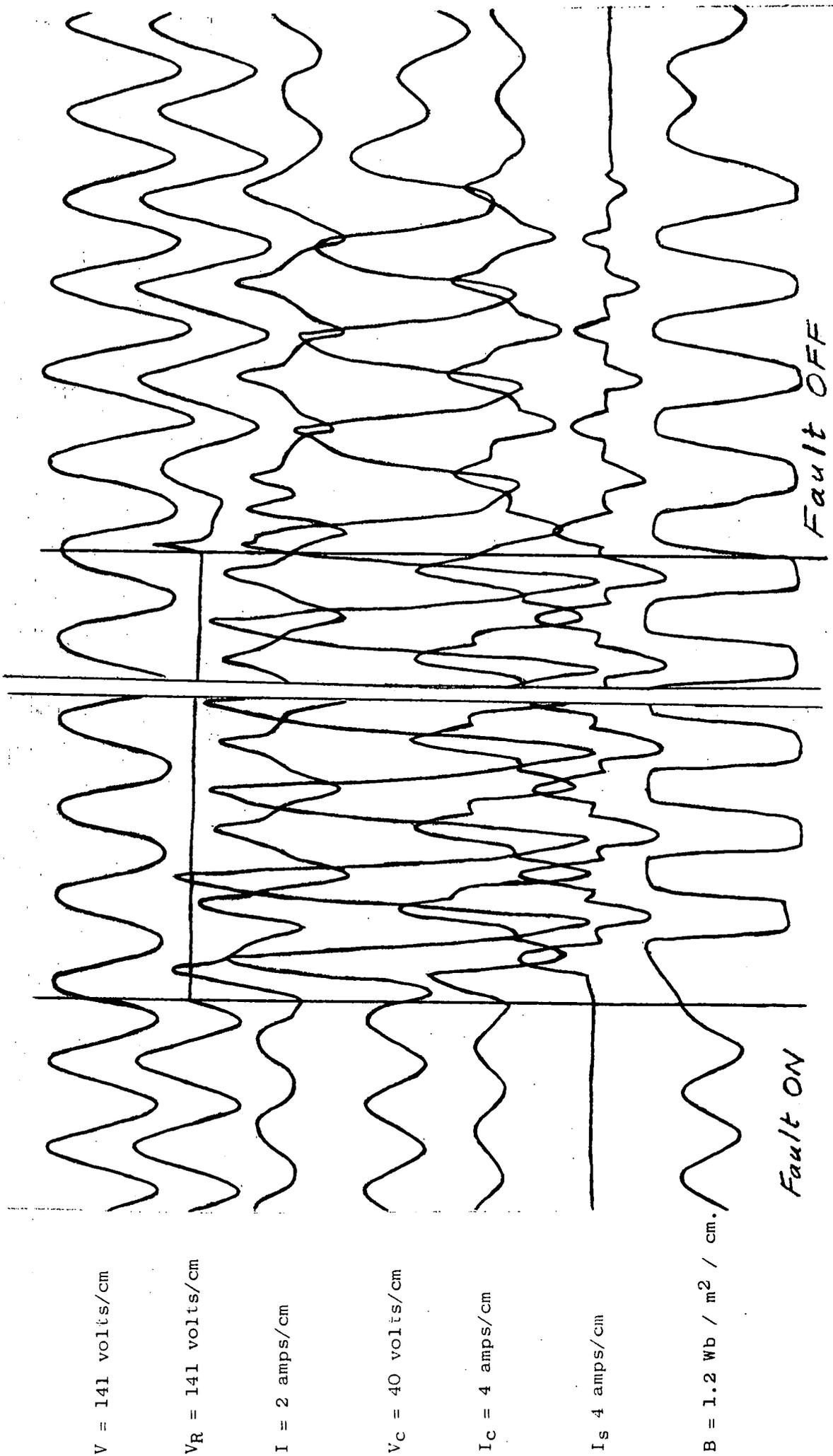


Fig.46.

1 amp inductive load with Development and Clearance of Short Circuit for the Double Resonance Link.

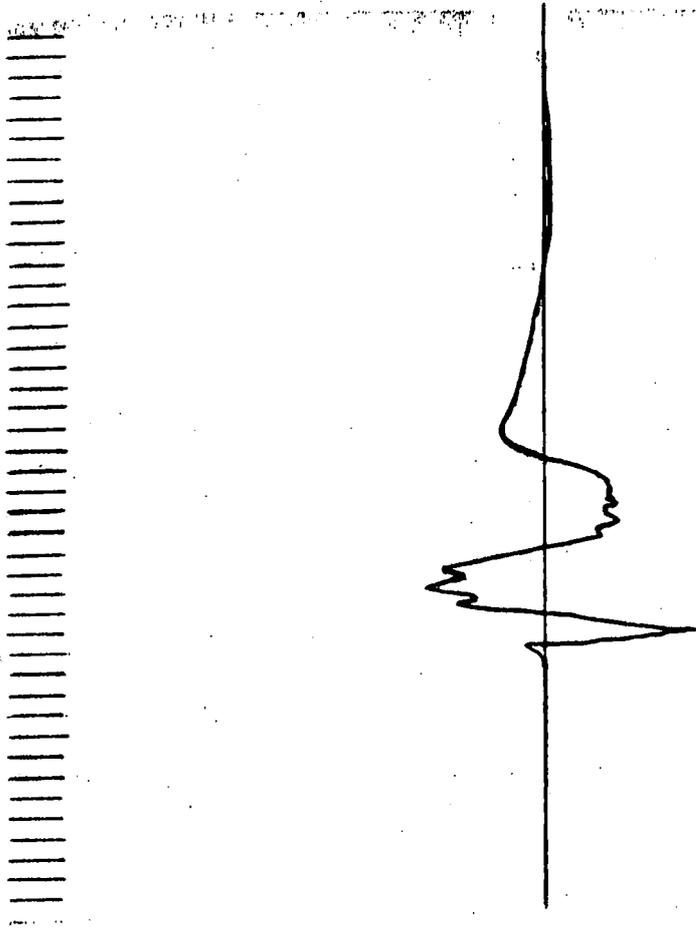
Appendix III

Sub-harmonic Response of the Resonant Link.

All time markers on recordings are at 0.01 sec. intervals.

S.R.L. means single resonance link.

D.R.L. means double resonance link.



Oscillatory decay of  $V_s$  following discharge of parallel capacitor.

Note the low but varying natural frequency.

Time spacings at 0.01 second.

Fig.47.

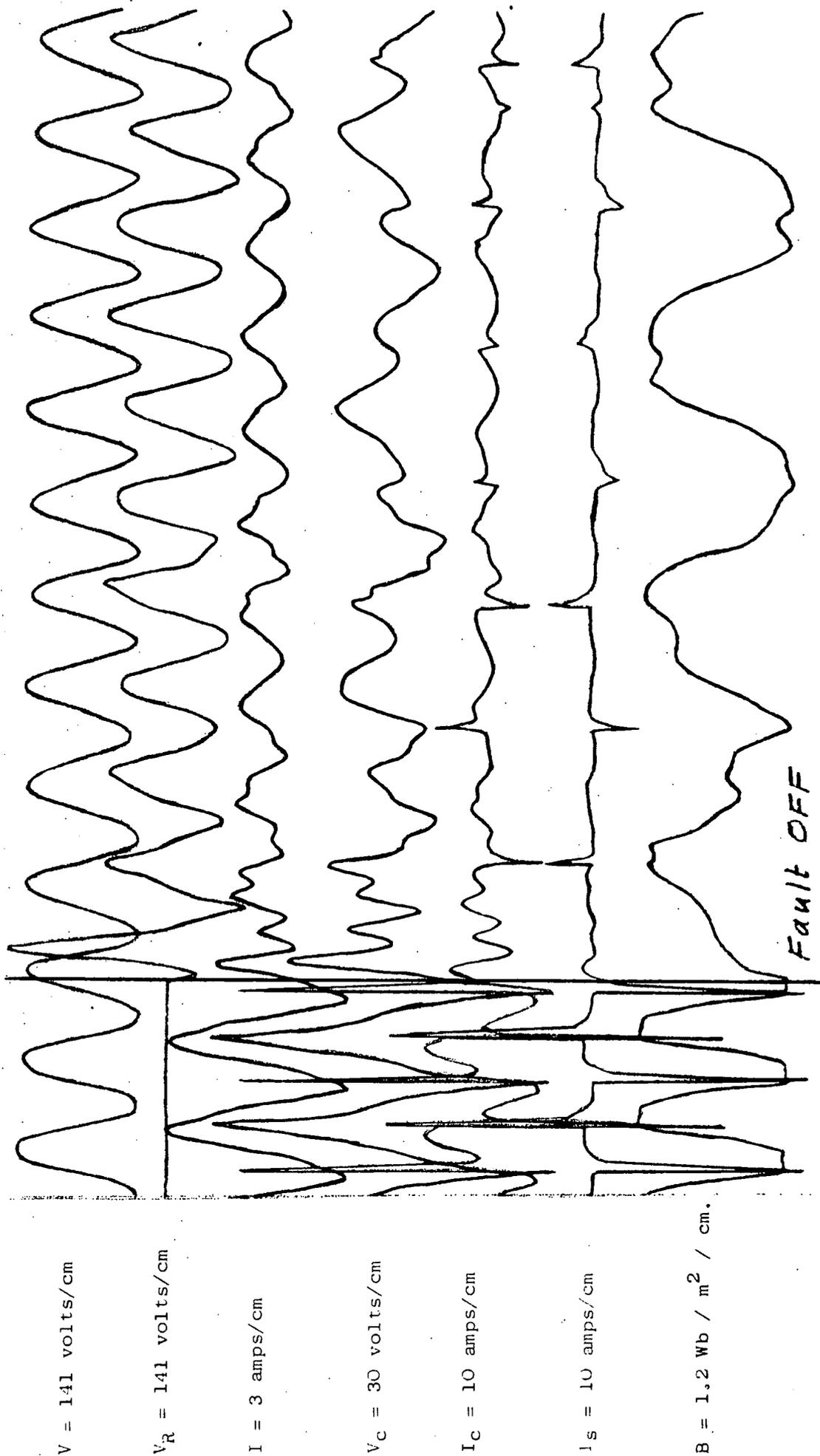
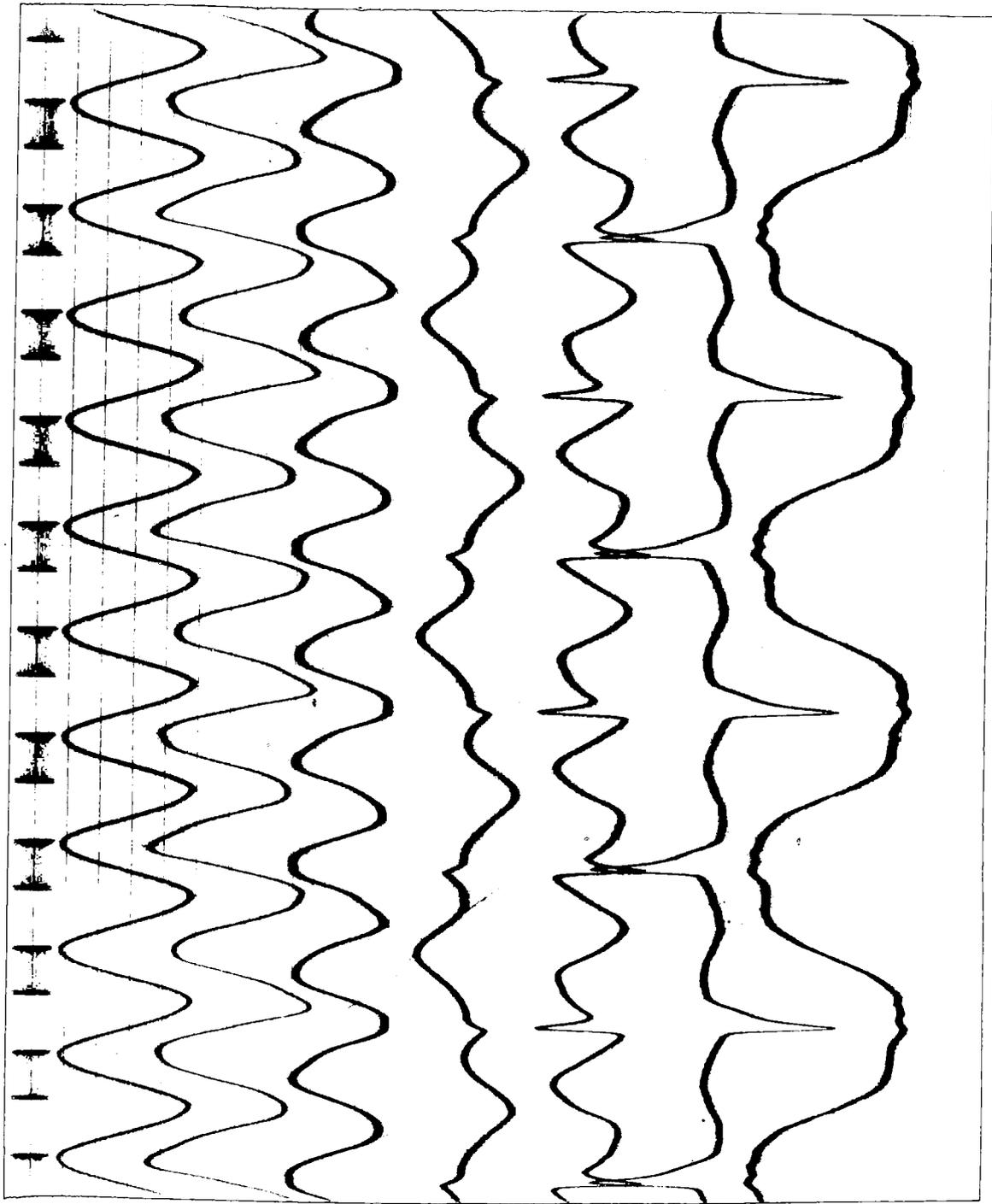


Fig.48.

Collapse of Ferro-resonant state to Sub-harmonic Response -  $V = 105 \text{ volts}$   $C' = 20 \mu \text{ F.}$



$V = 141$  volts/cm

$V_R = 141$  volts/cm

$I = 1$  amp/cm

$V_c = 30$  volts/cm

$I_c = 1$  amp/cm

$I_s = 0.5$  amp/cm

$B = 1.2$  Wb / m<sup>2</sup> / cm.

Fig.49

Sub-harmonic Response (S.R.L.) -  $V = 100$  volts  $C' = 18 \mu F$

$V = 141$  volts/cm

$V_R = 141$  volts/cm

$I = 1$  amp/cm

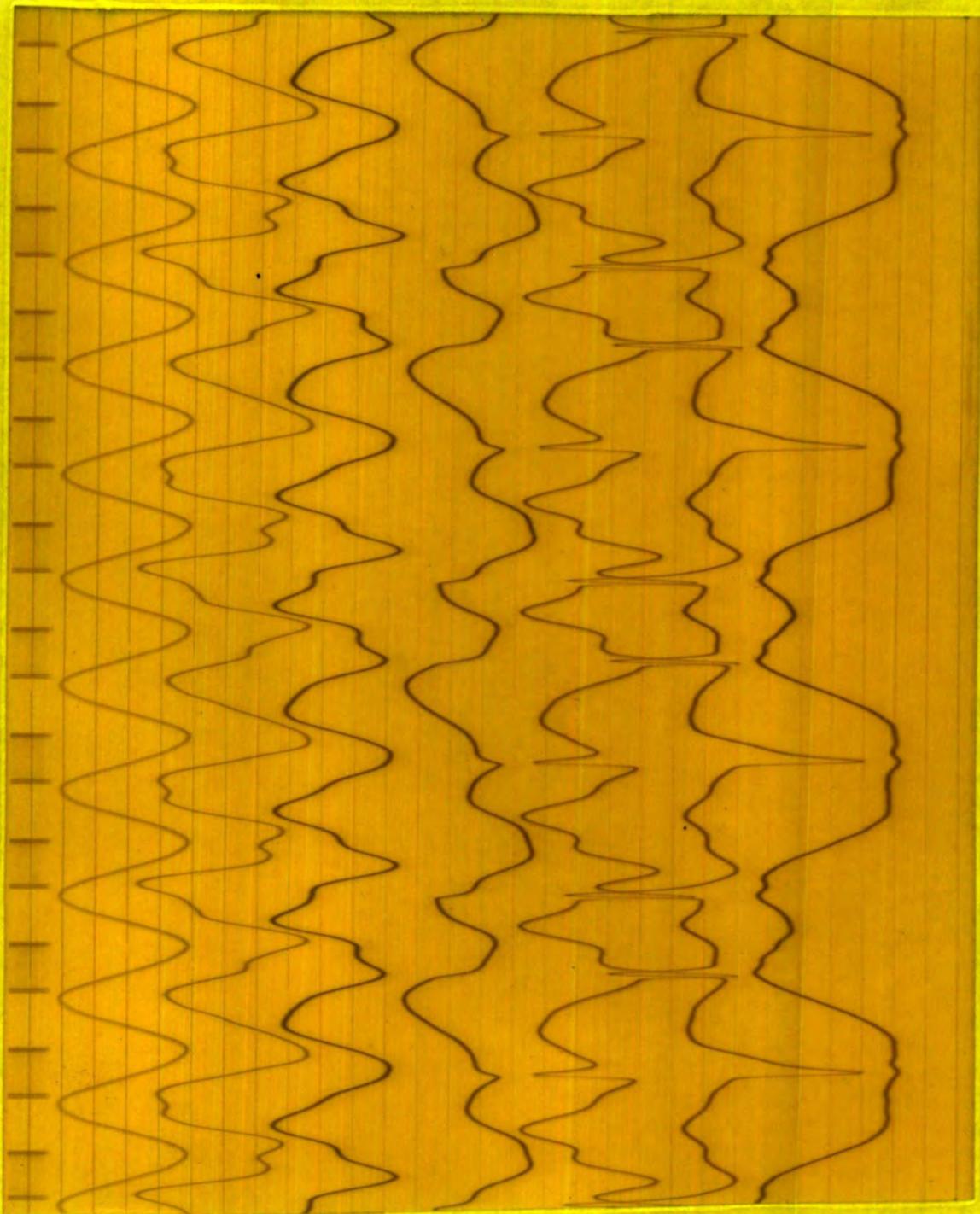
$V_c = 30$  volts/cm

$I_c = 1$  amp/cm

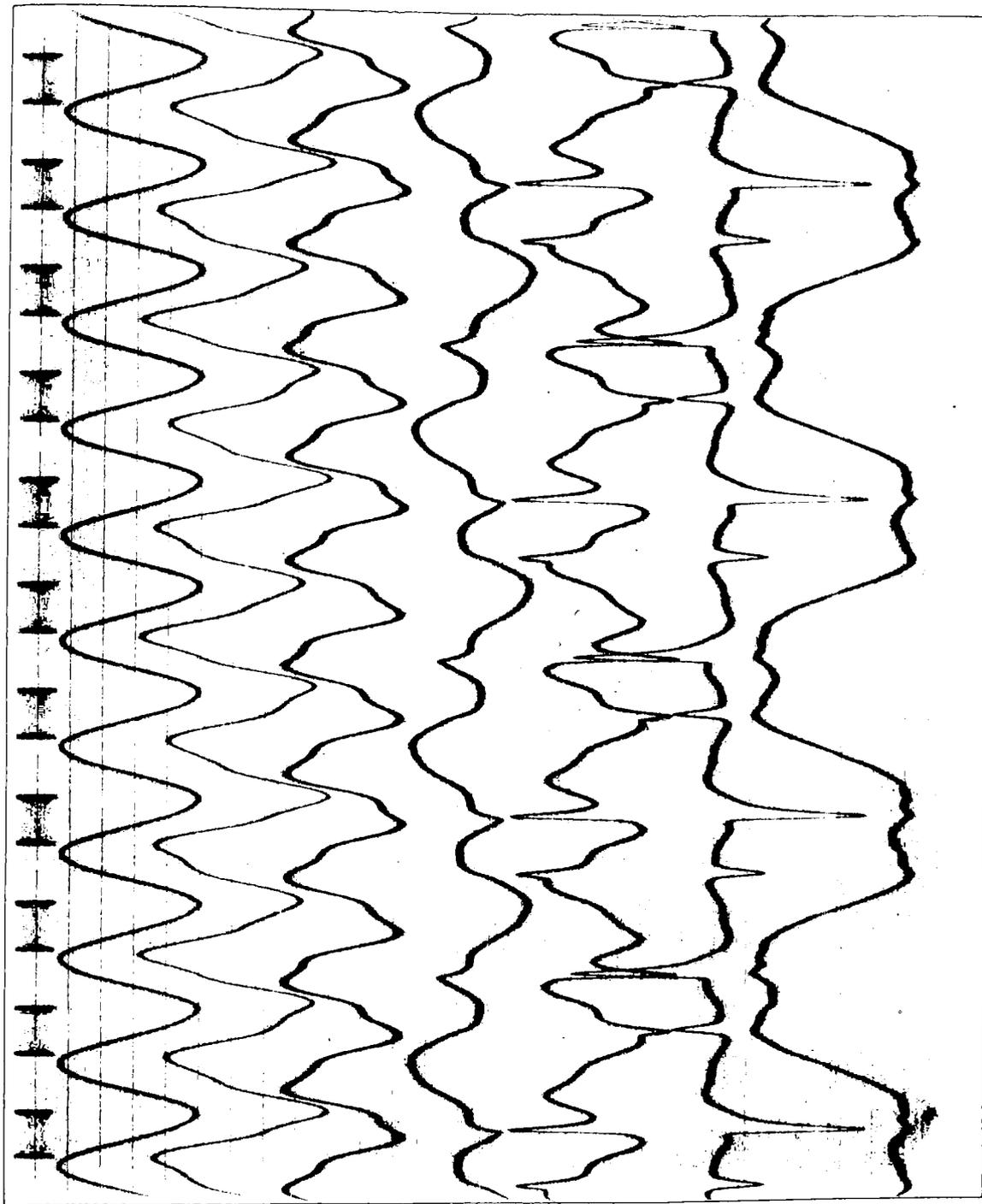
$I_s = 0.5$  amp/cm

$B = 1.2$  Wb/ m<sup>2</sup> / cm.

Fig.50.



‡ Sub-harmonic Response S.R.L.)      -       $V = 100$  volts       $C' = 22 \mu F$



$V = 141$  volts/cm.

$V_R = 141$  volts/cm.

$I = 1$  amp/cm

$V_C = 30$  volts/cm

$I_C = 1$  amp/cm

$I_S = 0.5$  amp/cm.

$B = 1.2$  Wb / m<sup>2</sup> /cm.

Fig.51.

‡ Sub-harmonic Response Single Resonance Link (S.R.L.) -  $V = 105$  volts  $C' = 20 \mu F$

$V = 141$  volts/cm

$V_R = 141$  volts/cm

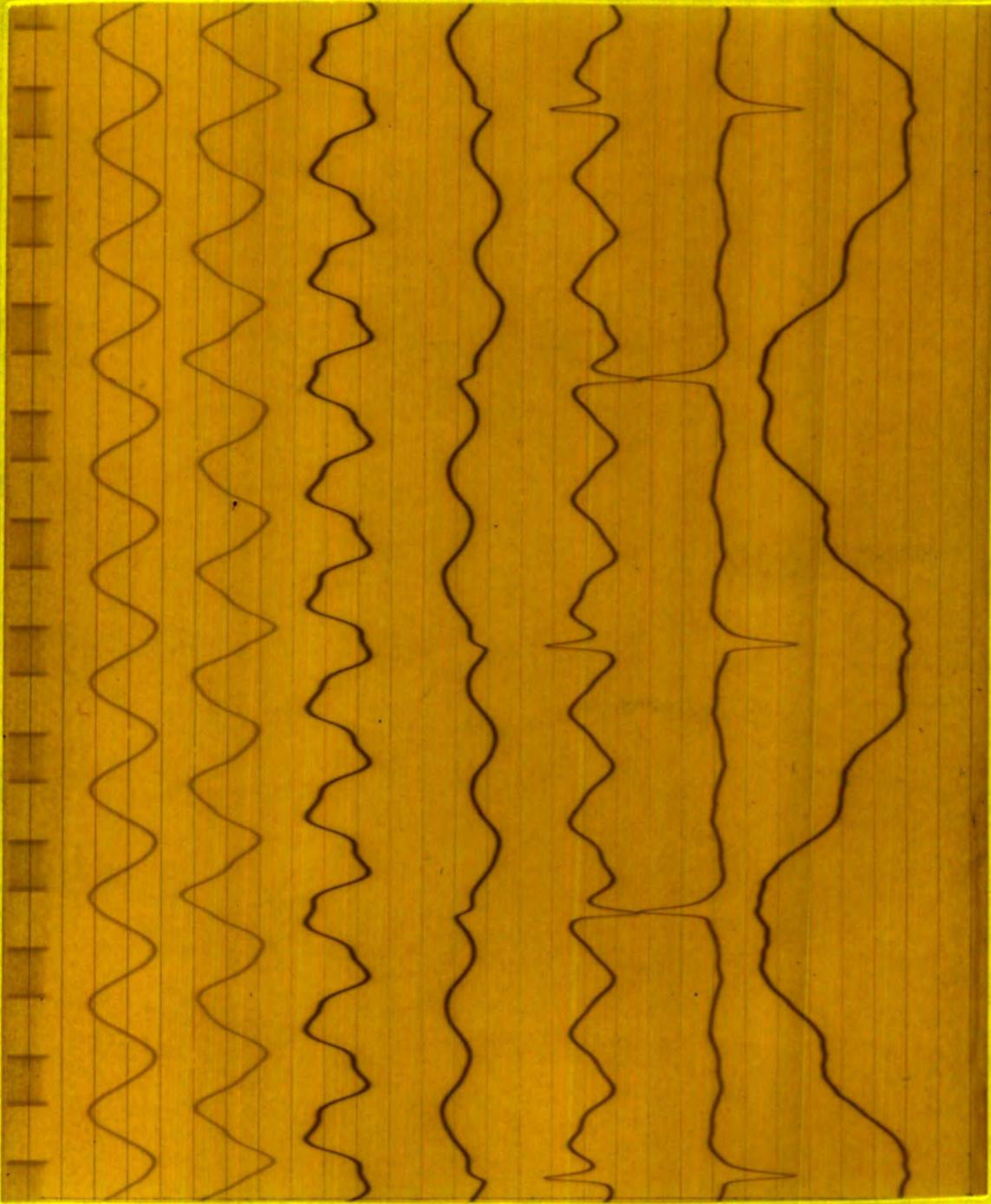
$I = 1$  amp/cm

$V_c = 30$  volts/cm

$I_c = 1$  amp/cm

$I_s = 0.5$  amp/cm

$B = 1.2$  Wb / m<sup>2</sup> / cm.



$1/5$  Sub-harmonic Response (S.R.L.) -  $V = 50$  volts  $C' = 20 \mu F$

Fig.52.

$V = 141$  volts/cm

$V_R = 141$  volts/cm

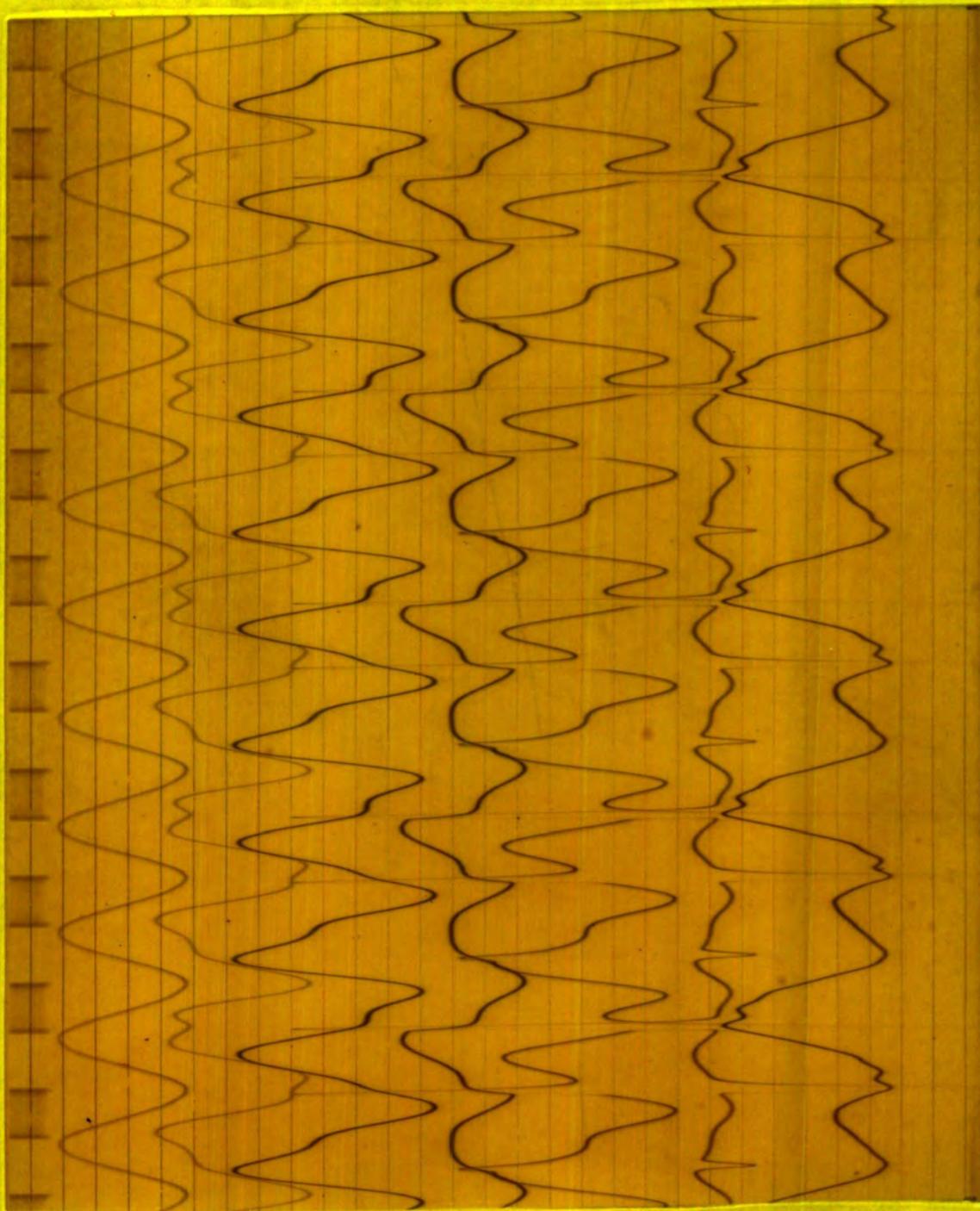
$I = 1$  amp/cm

$V_C = 30$  volts/cm

$I_C = 1$  amp/cm

$I_S = 0.5$  amp/cm

$B = 1.2$  Wb / m<sup>2</sup> / cm



$\frac{1}{2}$  Sub-harmonic Response (S.R.L.)  $V = 100$  volts.  $C' = 40 \mu F$

Fig.53.

$V = 141$  volts/cm.

$V_R = 141$  volts/cm

$I = 0.5$  amps/cm

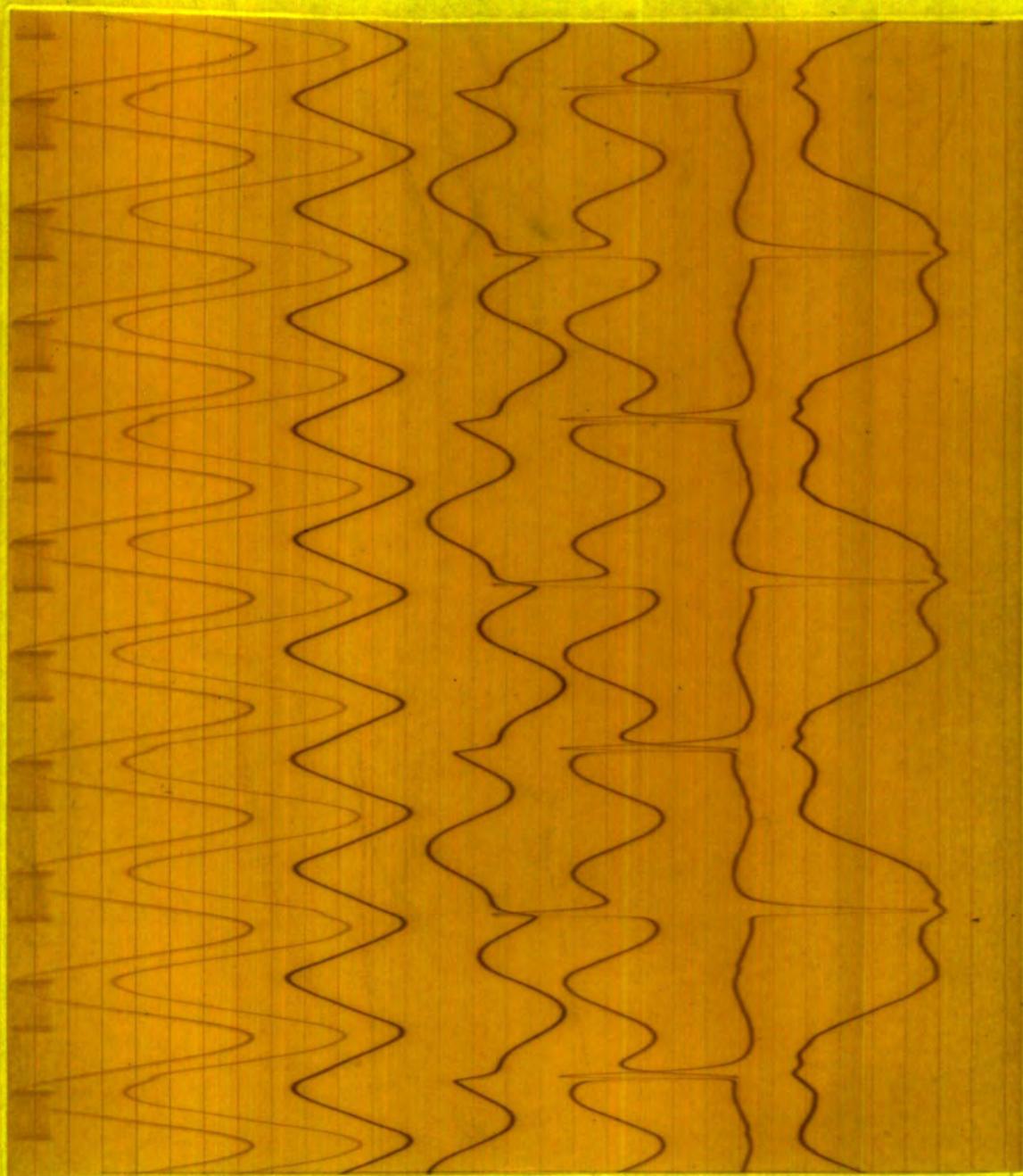
$V_C = 30$  volts/cm

$I_C = 0.5$  amps/cm

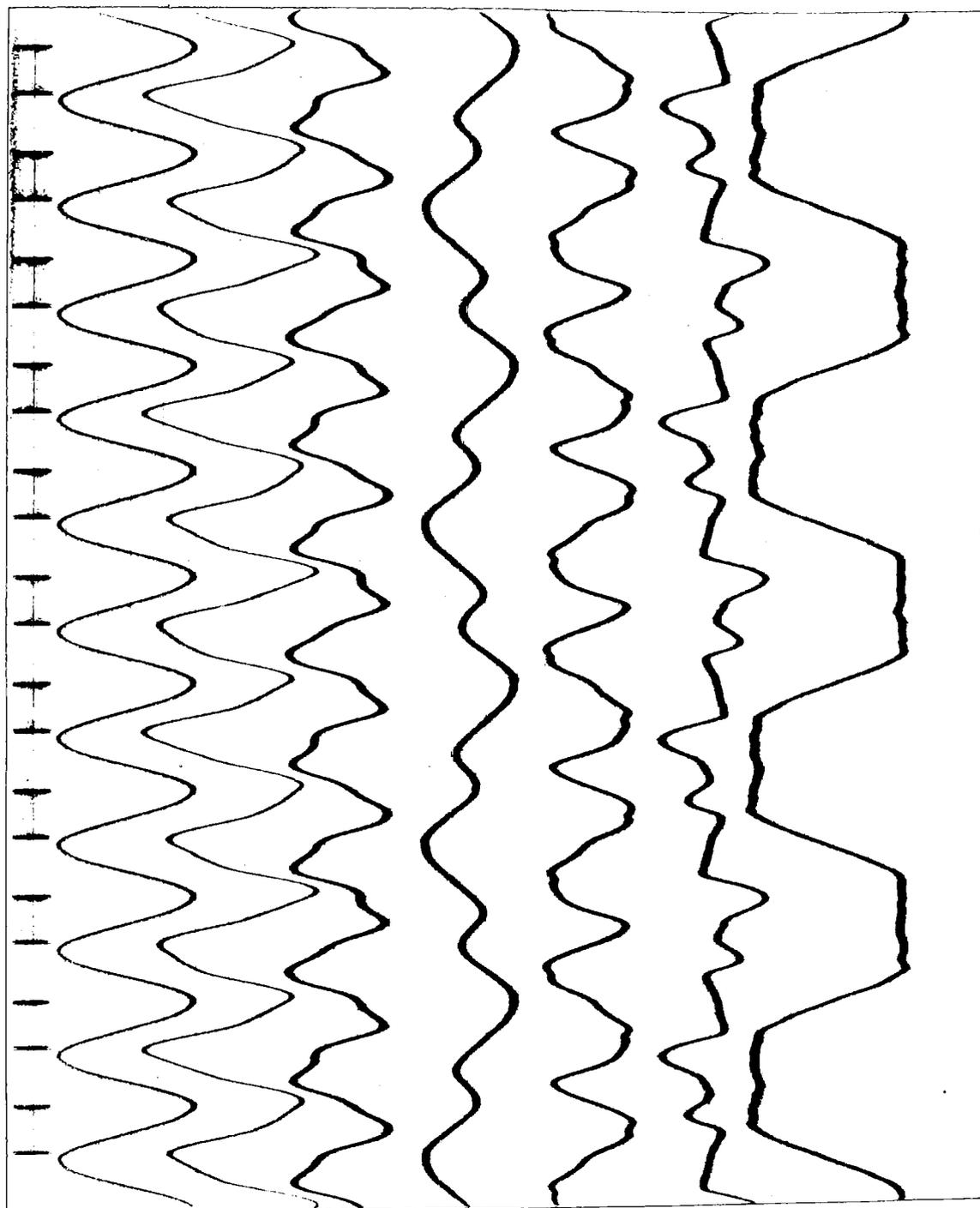
$I_s = 0.25$  amps/cm

$B = 1.2$  Wb/m<sup>2</sup>/cm.

Fig.54.



Sub-harmonic Response (S.R.L.) -  $V = 180$  volts     Resistive Load - 550 ohms.



$V = 141$  volts/cm

$V_R = 141$  volts/cm

$I = 1$  amp/cm

$V_c = 30$  volts/cm

$I_c = 1$  amp/cm

$I_s = 0.5$  amp/cm

$B = 1.2$  Wb/ m<sup>2</sup> / cm.

Fig. 55.

$\frac{1}{3}$  Sub-harmonic Response Double Resonance Link (D.R.L.) -  $V = 105$  volts  $C' = 17 \mu F$

Appendix IV - Harmonic Analysis.

Asymmetrical Sub-harmonic Waves ref. Figs. 8e, f.

Conditions  $V = 100v.$   $C = 160 \mu F.$   $C' = 22 \mu F.$

Hz	(- - - Per Unit Magnitude - - - - -)						Order ( of Harmonics )	
	I	I <sub>s</sub>	I <sub>c</sub>	V <sub>c</sub>	V <sub>R</sub>	B	50 Hz.	16 $\frac{2}{3}$ Hz.
16 $\frac{2}{3}$	.3	.5	.45	.71	.16	1.0	$\frac{1}{3}$	1
33 $\frac{1}{3}$	.1	.28	.28	.35	.035	.24	-	2
50	1.0	.41	.95	1.0	1.0	.25	1	3
66 $\frac{2}{3}$	-	.28	.27	.1	-	-	-	4
83 $\frac{1}{3}$	.22	.425	.346	.1	.02	-	-	5
100	-	.05	.05	-	-	-	2	6
116 $\frac{2}{3}$	.1	.2	.173	.02	-	-	-	7
113 $\frac{1}{3}$	-	.06	.06	-	-	-	-	8
150	.412	.28	.51	.17	.14	.03	3	9
166 $\frac{2}{3}$	-	.02	.02	-	-	-	-	10
183 $\frac{1}{3}$	.17	.35	.39	.005	-	-	-	11
200	-	.01	.01	-	-	-	4	12
216 $\frac{2}{3}$	-	.07	.07	-	-	-	-	13
233 $\frac{1}{3}$	-	-	-	-	-	-	-	14
250	.1	.1	.1	-	-	.01	5	15
266 $\frac{2}{3}$	-	-	-	-	-	-	-	16
283 $\frac{1}{3}$	.1	.2	.2	-	-	-	-	17
300	-	-	-	-	-	-	6	18
316 $\frac{2}{3}$	-	.14	.14	-	-	-	-	19
333 $\frac{1}{3}$	-	-	-	-	-	-	-	20
350	.02	.05	.05	-	-	-	7	21
366 $\frac{2}{3}$	-	-	-	-	-	-	-	22
383 $\frac{1}{3}$	-	-	-	-	-	-	-	-
400	-	-	-	-	-	-	8	24
416 $\frac{2}{3}$	-	.02	.02	-	-	-	-	25
433 $\frac{1}{3}$	-	-	-	-	-	-	-	26
450	-	.01	.01	-	-	-	9	27
466 $\frac{2}{3}$	-	-	-	-	-	-	-	28
483 $\frac{1}{3}$	-	.005	.005	-	-	-	-	29

Symmetrical Sub-Harmonic Waves, ref. Fig. 8d.

Conditions  $V = 100$  v.  $C = 160 \mu F.$   $C' = 17 \mu F.$

Hz	Per Unit Magnitude.				Order of Harmonics.	
	I	I <sub>s</sub>	I <sub>c</sub>	B	50 Hz	16 $\frac{2}{3}$ Hz.
16 $\frac{2}{3}$	.17	.35	.33	1.0	$\frac{1}{3}$	1
50	1.0	.175	.94	.07	1	3
83 $\frac{1}{3}$	.07	.17	.14	.02	-	5
116 $\frac{2}{3}$	.05	.19	.19	.005	-	7
150	.09	.1	.1	.01	3	9
250	.005	.002	.002	-	5	15

Symmetrical Very Long Persistence  $\frac{1}{5}$  sub - harmonic ref. Appendix III.

Conditions  $V = 50$  V.  $C = 160 \mu F.$   $C' = 20 \mu F.$

Hz	Per Unit Magnitude.		Order of Harmonics.	
	I <sub>s</sub>	B	50Hz	10 Hz.
10	1.0	1.0	$\frac{1}{5}$	1
30	.81	.14	-	3
50	.74	.265	1	5
70	.79	.1	-	7
90	.66	.004	-	9
110	.53	-	-	11
130	.37	-	-	13
150	.24	-	-	15
170	.12	-	-	17
190	.06	-	-	19
210	.05	-	-	21

Double Resonance Link.

Symmetrical Sub-harmonic Waves ref. Fig.24b.

Conditions V = 100 v.

C = 160  $\mu$  F.

C' = 17  $\mu$  F.

Hz	( - - - Per Unit Magnitude - - - )					( Order of Harmonics )	
	I	I <sub>s</sub>	I <sub>c</sub>	V <sub>c</sub>	B	50 Hz	16 $\frac{2}{3}$ Hz.
16 $\frac{2}{3}$	.2	.515	.495	.79	1.0	$\frac{1}{3}$	1
50	1.0	.375	.85	1.0	.24	1	3
83 $\frac{1}{3}$	.1	.3	.3	.1	.05	-	5
116 $\frac{2}{3}$	.02	.08	.08	.02	-	-	7
150	.17	.12	.14	.03	-	3	9
183 $\frac{1}{3}$	-	.005	.005	-	-	-	11
216 $\frac{2}{3}$	-	-	-	-	-	-	13
250	.02	.004	.014	-	-	5	15

Ferro-resonance.

Single resonance link ref. fig. 15a.

Conditions V = 100 v. Link short circuited.

Hz	( Per Unit Magnitude. )				Order of Harmonics. 50 Hz.
	I	I <sub>s</sub>	I <sub>c</sub>	B	
50	.84	1.0	.945	1.0	1
150	.12	.81	.85	.2	3
250	.07	.89	.89	.1	5
350	.02	.65	.65	.07	7
450	-	.49	.48	-	9
550	-	.35	.37	-	11
650	-	.3	.31	-	13
750	-	.28	.29	-	15
850	-	.24	.24	-	17
950	-	.2	.22	-	19

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Double Resonance Link ref. Fig. 23

Conditions V = 100 v. Link short circuited.

Hz.	( Per Unit Magnitude. )				Order of Harmonics. 50 Hz.
	I	I <sub>s</sub>	I <sub>c</sub>	B	
50	.425	.965	1.0	1.0	1
150	.17	.28	.32	.24	3
250	.1	.17	.17	.1	5
350	-	.1	.1	.01	7

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