Some problems in gear metrology

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SOME PROBLEMS IN GEAR METROLOGY

by

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SUMMARY

Some problems in gear metrology.

The inspection of each individual tooth element of a spur gear wheel can be a tedious and expensive process. Two methods of composite gear testing are available, namely, the single flank rolling gear test and the dual flank rolling gear test. The first method is well documented but the dual flank system tends to have been neglected, few papers being available on the subject.

As a preliminary stage to the investigation into the dual flank testing technique two methods of pitch error measurement using ball ended contact probes are examined. These are:

(i) the direct measurement of cumulative pitch measurement using a single contact probe and dividing arrangement and

(ii) the span gauging of adjacent teeth, commercial measuring equipment being used in both cases.

The two methods are compared theoretically in terms of the maximum cumulative pitch error of a gear, and their probable accuracy of measurement compared with the allowable pitch tolerances quoted in the relevant British Standard Specifications.

A theory for the dual flank test is developed relating the variation in centre distance of a pair of gears in close mesh to the errors in tooth pitch and involute profile. The

validity of the theory is tested by experimental measurements on actual gears, and a means of determining the permissible variation in the dual flank test error trace from the allowable tolerances on tooth pitch error and tooth profile error suggested. The identification of the type of elemental error from the pattern of the dual flank trace is also examined together with the limitations of the test.

ACKNOWLEDGEMENTS

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CONTENTS

CHAPTER 1.

Introduction

1.1. The primary requirement of a gear train is to transmit uniform rotary motion from one shaft to another under the operating conditions for which they were designed. Any departure from the design dimensions will affect the fulfilment of this aim, but since perfection in manufacture is not possible some departure from the design dimensions must be tolerated.

The function of the gears will determine the magnitude of tolerance permitted on the gear tooth elements. Gears for accurate indexing operating under light loads (e.g. servomechanisms, machine tool positional "read out" mechanisms) require that the involute profile, tooth spacing and alignment be as near perfect as is possible.

For high speed and highly loaded gears the tooth profile is modified from a true involute form to allow for deformation of the tooth under load. The tips of the teeth are relieved to prevent the teeth gouging into the gear tooth flanks of the mating gear when coming into contact. Errors in manufacture will also be reflected in the noise of operation of the gears.

Gear tooth elements can be inspected individually but it will not always be easy to predict how they will combine in practice under the operating conditions. It would seem preferable to use some form of composite meshing test even though the exact working conditions can not in many cases be simulated. Two such tests are employed (i) dual flank and (ii) single flank meshing tests.

1.2. Chapter 2 of this thesis reviews the existing techniques for both elemental and composite testing of straight tooth spur wheels.

Of the two methods of composite testing used, the single flank test has been thoroughly investigated by Munro (5) (6), but the dual flank technique, although earlier in concept, tends to have been neglected. No published work appears to be available which relates the dual flank error to errors in tooth pitch and tooth profile. This investigation is an attempt to rectify the situation by considering the dual flank mesh testing of straight spur gears.

Initially, in order to provide information required in the later stages of the work, two well established methods of pitch measurement using ball ended contact probes are examined. These are ; (i) the direct measurement of cumulative pitch error using a single contact probe and indexing arrangement and (ii) the determination of adjacent tooth pitch error from which the cumulative pitch error can be obtained. The two methods are compared theoretically in terms of the cumulative pitch error of an eccentrically mounted gear and their probable accuracy of measurement compared with the allowable pitch tolerances quoted in the relevant British Standard Specifications. The theory for the single contact method is given in ref. (4) for the α tooth flanks, but the work does not indicate the small difference required in the analysis when considering the β tooth flanks. The theory for the span gauging technique was developed for this thesis and the differing methods of building up the cumulative pitch error from the individual adjacent pitch errors for the α and β flanks which is detailed in chapter 4 is not emphasised in any of the standard texts or publications.

In chapter 5 a theory for the dual flank meshing test is evolved which relates the variation in centre distance of a pair of gears in tight mesh to the errors in tooth pitch and tooth profile. The authenticity of this theory is tested experimentally and a means of obtaining the allowable variation in the dual flank error trace from the allowable tolerances for tooth pitch error and tooth profile error suggested. The possible diagnosis of the form of gear tooth elemental error from the pattern of the dual flank trace is also examined together with the limitations of the test.

CHAPTER 2.

Gear Measurement.

2.1. Timms produced the following list of items which require checking for the complete inspection of a gear wheel. Although the list deals specifically with the examination of a spur gear the principles involved apply equally to other forms of gear wheel (1).

Gear Blank

(i) Bore diameter

(ii) Outside diameter

(iii) Face width

(iv) Concentricity of outside diameter relative to bore

(v) Axial float of each end face

Positional Measurements

(i) Flank to flank pitch (adjacent pitch)

(ii) Spacing of each set of tooth flanks around a circle concentric with the axis of rotation (cumulative)

(iii) Variations in tooth thickness

(iv) Eccentricity of teeth relative to axis of rotation.

Tooth Profile

(i) Tooth form errors

(ii) Symmetry of form of the two flanks

(iii) Tooth thickness

(iv) Tooth depth

Tooth Alignment

(i) Alignment of each tooth flank parallel to the axis

Meshing of Gears

(i) Centre distance

(ii) Backlash at nominal centre distance

(iii) Uniformity of motion measured relative to a mating gear or an appropriate master gear

2.2. Accuracy of Gear Blank

The items quoted in this section of the above list form datum faces for mounting the blank on the gear cutting machine. Thus their accuracy can directly influence the final accuracy of the finished product. They may also be used as locating faces for portable inspection equipment e.g. pitch and profile measuring instruments.

2.3. Pitch Measurement

Two methods are in common use; (i) direct measurement of cumulative pitch error relative to a pre-selected datum tooth flank, see Fig. 2 - 3a and (ii) span gauging of adjacent pitch error relative to a pre-selected datum span, Fig. 2 - 3b. The data obtained by the second method may be used to obtain the cumulative pitch error.

It is important in both cases that each successive measurement is made at the same radial distance from the centre of the gear, usually at the point at which the tooth flank and reference circle (formally known as the pitch circle) intersect. By adopting this position the effect of errors in tooth profile will have a minimal influence on the measurement of pitch error.

A single point stylus may be used, as shown in the diagrams, but this will involve taking a number of sets of readings across the face of wide gears in order to check for variations in pitch error. An alternative arrangement is to use a pneumatic gauging head with a number of measuring jets spaced across the face width of the gear, thereby giving an indication of the mean pitch error over the whole gear face in one measurement. (2)

As a preliminary to measurement by either method the gear wheel must be cleaned and the teeth numbered for identification purposes, the terms α and β being used to distinguish between the two flanks of a particular tooth, see Figs. 2 - 3a and 2 - 3b.

(i) Direct measurement of pitch error

For medium sized gears up to say 700 mm (2 ft) diameter (although weight will also be a factor to consider) the essential equipment required is an accurate indexing arrangement and a measuring stylus.

Indexing mechanism

It is preferable to use a horizontal rotary table as this allows the gear wheel to be clamped in position on the table without the use of a supporting arbor. The gear bore can then be used for centralising the gear with the axis of rotation of the rotary table, this setting is important since an eccentricity in the positioning of the gear will produce a sinusoidal pitch error. A wheel which is cut integral with its shaft can best be supported on a rotary table by means of a piece of tubing having parallel end faces.

If a table which rotates in the vertical plane is used, an accurate supporting arbor which fits the gear bore will normally be required together with a second support or centre to support the overhanging weight. To facilitate the "setting up" of the gears for measurement they are sometimes provided with a reference or setting ring, which is an external diameter machined concentric with the gear bore, or bearing surfaces.

The accuracy required of the indexing arrangement is a function of the size of the gear to be measured, since it directly effects the accuracy of the pitch measurement. It should be noted that an error of 1 second of arc represents an error of 0.00005 mm at a radius of 10 mm. Thus for very small gears a precision worm and wheel drive table with an accuracy of 10 seconds of arc may be adequate. Generally an optical dividing head or rotary table is better; such tables are available reading to 1 second of arc, although the overall cumulative error may be greater, perhaps 5 seconds, but corrections can be made for this error if a calibration chart is available for the instrument used. Each indexing for pitch measurement should be such as to position the gear to the nearest 1 second of arc.

Measuring stylus

This must be rigidly mounted on some form of withdrawal mechanism in order that the stylus (or probe) clears the gear when indexing, but returns to the same position for each measurement. The measuring head may be a precision mechanical type or electrical transducer giving repeatable readings to 0.001 mm and having a measuring range adequate for the pitch variation of the gear. Care must be taken to ensure that the correct sign is allotted to the pitch error, excess metal represents a positive error for the β flanks and a negative error for the α flanks.

The measurement of each individual tooth of a gear having a large number of teeth can be a long tedious process subject to human error. For this reason automatic indexing and measuring machines have been developed which give a graphical display of the results. If an accurate rotary table is not available pitch measurements can be made on a limited scale using an autocollimator and precision polygon.

Figs. 2 - 3c and 2 - 3d show the cumulative pitch error curves for a 36T 7DP 20° A.O.P. spur gear generated with a 0.050 mm eccentricity mounting error on a gear shaping machine.

(ii) Span gauging of pitch error

This method compares the adjacent pitch errors of pairs of teeth, although greater spans are used when inspecting gears having large numbers of teeth. From the results of a span gauging a cumulative pitch error curve can be constructed. The method used to obtain such a curve involves the addition of the individual span errors, thus any inaccuracies in the individual measurements are also added. Span gauging of pitch error is therefore potentially more inaccurate than the direct measurement method.

For medium sized gears the measuring instrument used is often manufactured as an accessory to be used on either a rolling gear testing machine or an involute testing machine. Fig. 2 - 3b shows a diagramatic arrangement of the measuring system. The three probes are attached to a single head which may be fitted to one of the above machines. No indexing arrangement is required for the gear being inspected, points P for successive teeth being located in position by the fixed datum probe and the spring loaded finger. The measuring stylus indicates variations in adjacent pitch, the accuracy required of the reading head being repeatable readings to 0.001 mm.

The sliding carriage of the gear testing machine provides the means of withdrawing the measuring head clear of the gear teeth so allowing indexing of the wheel, and the returning of the measuring head to its original position for the next measurement. Since the angular positioning of the gear wheel is dependent upon the fixed datum probe and spring loaded finger, the wheel must be able to rotate about its axis very freely. It is best if the gear being inspected is mounted between centres, the centres used must be concentric with the gear bore (in which case an accurate arbor will be required) or bearing surfaces of the gear.

Very large gears, having many teeth, for example ships reduction gears are checked for pitch error by span gauging. But in this case because of the size and weight of the gears involved the measuring instrument must be taken to the gear cutting machine. This procedure has certain advantages, firstly the hobbing machine table is useful for revolving the gear during the measurement and also the gear wheel is in a position which makes any rectification, if required, easy to carry out. When inspecting such gears for pitch error the greatest possible span is measured and the cumulative pitch error built up from these spans. Individual tooth spans can then be measured if required, this method reduces the effect of the inaccuracies in the measurements on the accuracy of the cumulative pitch error curve.

As in the case of direct measurement of pitch error, automatic span gauging machines are available, so reducing the possibility of human reading errors (2) & (4).

2.4. Variations in Tooth Thickness

A graph showing variations in tooth thickness can be obtained from the cumulative pitch error curves for the α and β tooth flanks, the same tooth must have been used as datum for the two pitch error curves. The tooth thickness variation curve so obtained will show the variation relative to the same datum tooth. The magnitudes of the tooth thickness variations are obtained from:-

$$(\Delta p_{\beta 1} - \Delta p_{\alpha 1}),\quad (\Delta p_{\beta 2} - \Delta p_{\alpha 2}),\quad (\Delta p_{\beta 3} - \Delta p_{\alpha 3})\quad \text{etc}$$

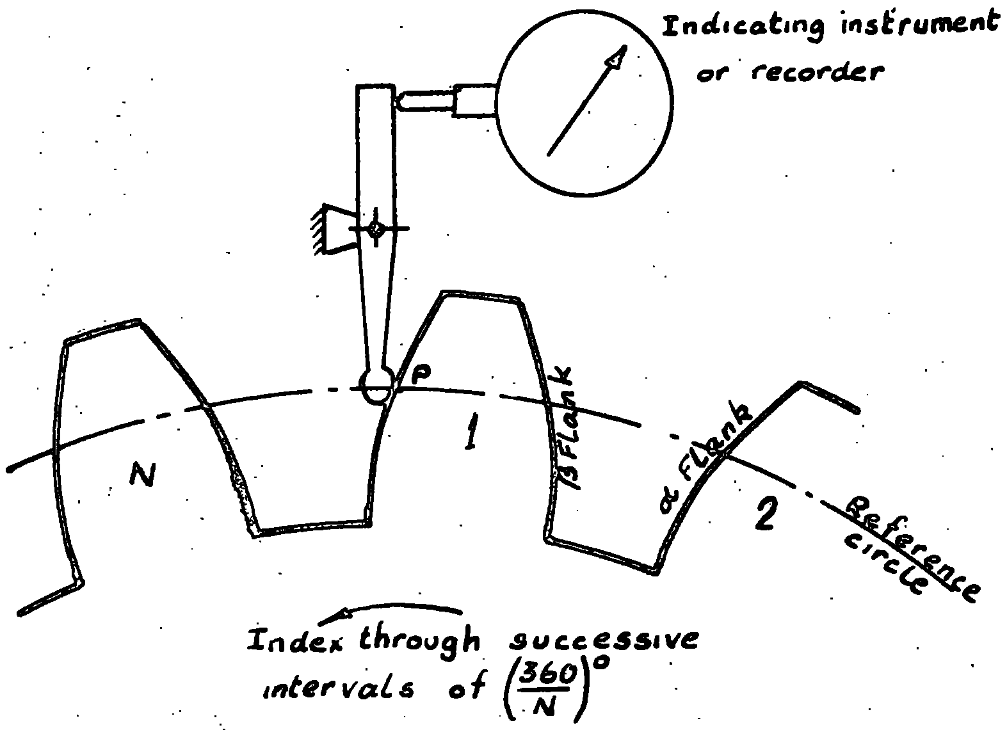


Fig 2.3a. DIRECT MEASUREMENT OF CUMULATIVE PITCH ERROR

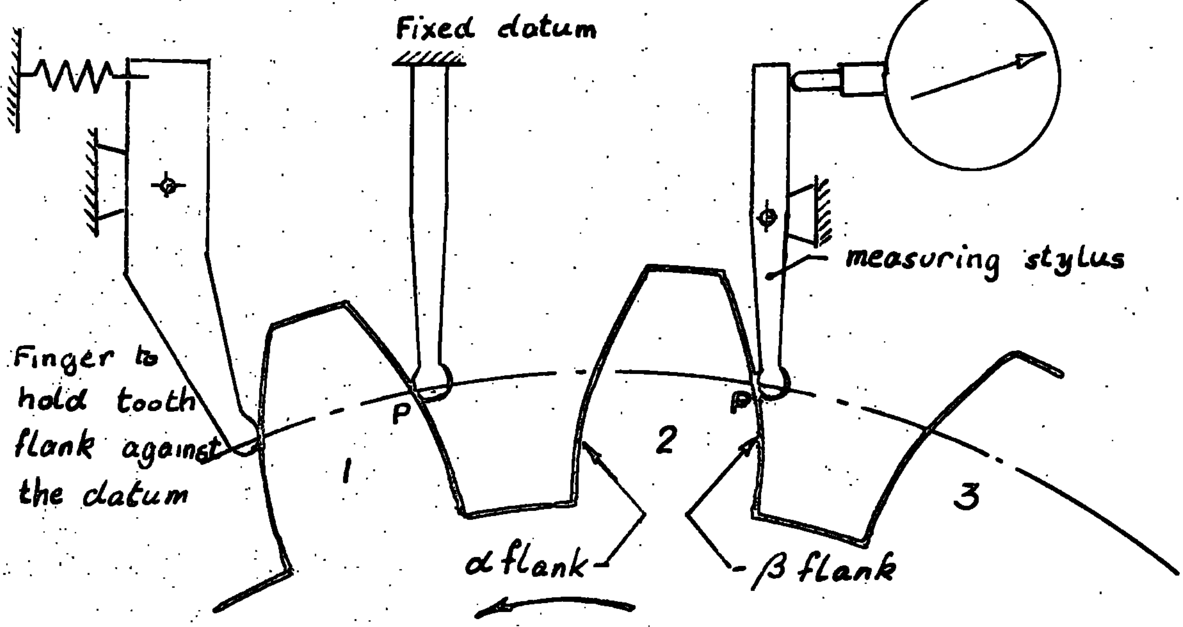


Fig 2.3b STEP BY STEP OR SPAN MEASUREMENT OF ADJACENT PITCH ERROR

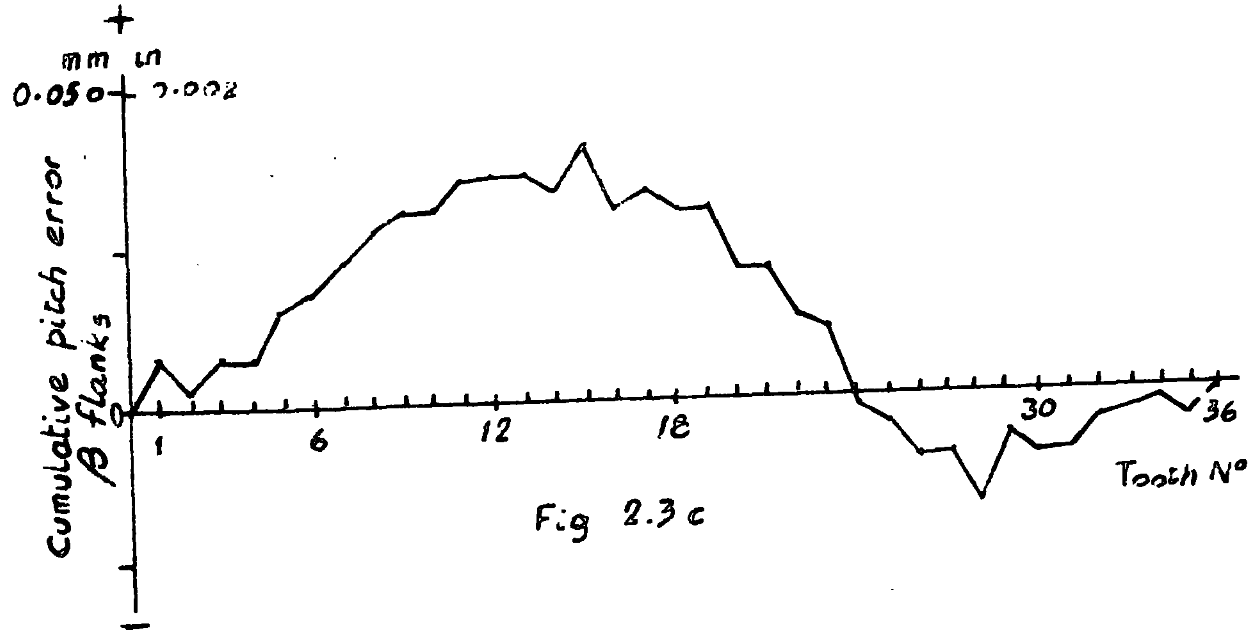


Fig 2.3c

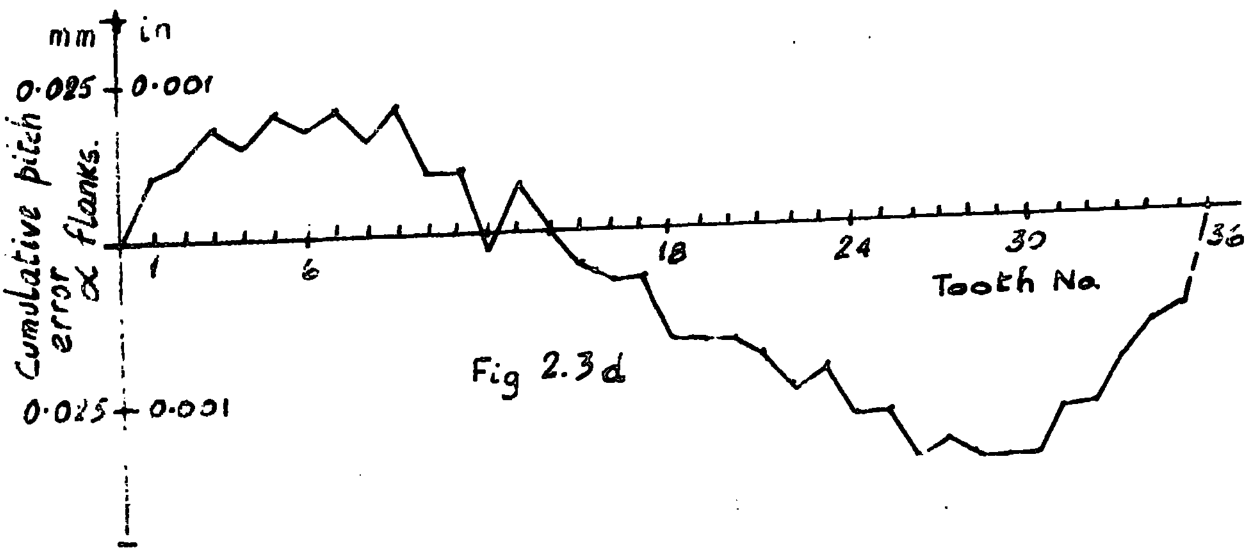


Fig 2.3d

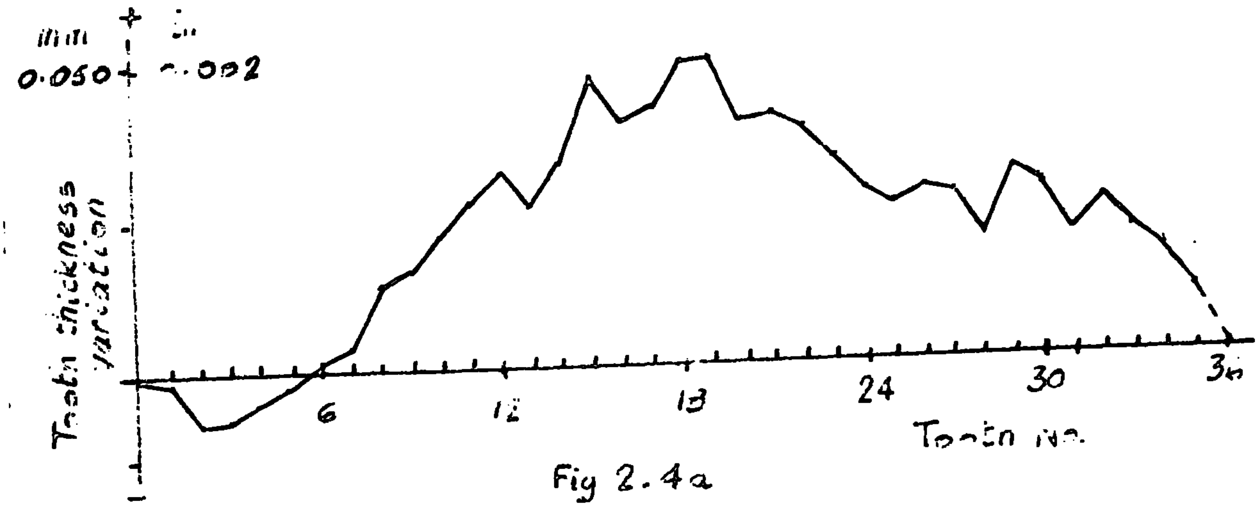


Fig 2.4a

where Δp = cumulative pitch error

β and α refer to the respective tooth flanks and the numbers identify the tooth numbers. (4)

Such a curve is shown by Fig. 2 - 4a for the 36T 7DP 20° A.O.P. gear cut with a 0.050 mm eccentricity.

One indication of the eccentricity of the teeth relative to the axis of rotation of the gear is shown by the general sinusoidal shape of the pitch error curves. But it should be noted that the eccentricity 0.050 mm is quite large. In normal gear cutting practice the sinusoidal effect may not be so pronounced.

2.5. Involute Profile

This particular gear tooth element is subject to a small manufacturing tolerance. It is possible to modify a machine tool operating system in order to minimise variations in pitch error (3) and (4), but it is much more difficult to control profile errors which are due to inaccuracies in the cutting tool.

(i) Very small gears:

These can be examined for accuracy of profile and tooth pitch by means of an optical projector and master profile.

(ii) Medium sized gears:

Theoretically using the properties of the involute curve, such gears can be examined using only an accurate dividing head and height gauge. Unfortunately when the method is applied in practice it may not indicate the true profile error, nor does it give a visual record of the results.

It is therefore more usual to use special purpose measuring equipment, the operating principle of which is that of a base disc rolling without slip along a straight edge. Fig. 2 - 5a. Two types of machine are available; base disc machines and linkage machines.

(a) A base disc machine requires a base disc having a diameter to suit the particular gear to be measured, base disc diameter equals reference circle diameter of the gear multiplied by the cosine of the pressure angle. The accuracy of the diameter of the disc and its concentricity are of prime importance since they directly effect the accuracy of the measurements made. The layout of the machine is such that the base disc is mounted on a "live" centre, and the gear being inspected is located between the "live" centre and a dead centre. The straight edge is pressed up against the base disc, a power drive is provided for the straight edge so giving a rotation to the base disc and hence the gear under inspection. The power drive is also synchronized with a chart recorder, which displays deviations of the measuring stylus against an axis proportional to the rotation of the gear.

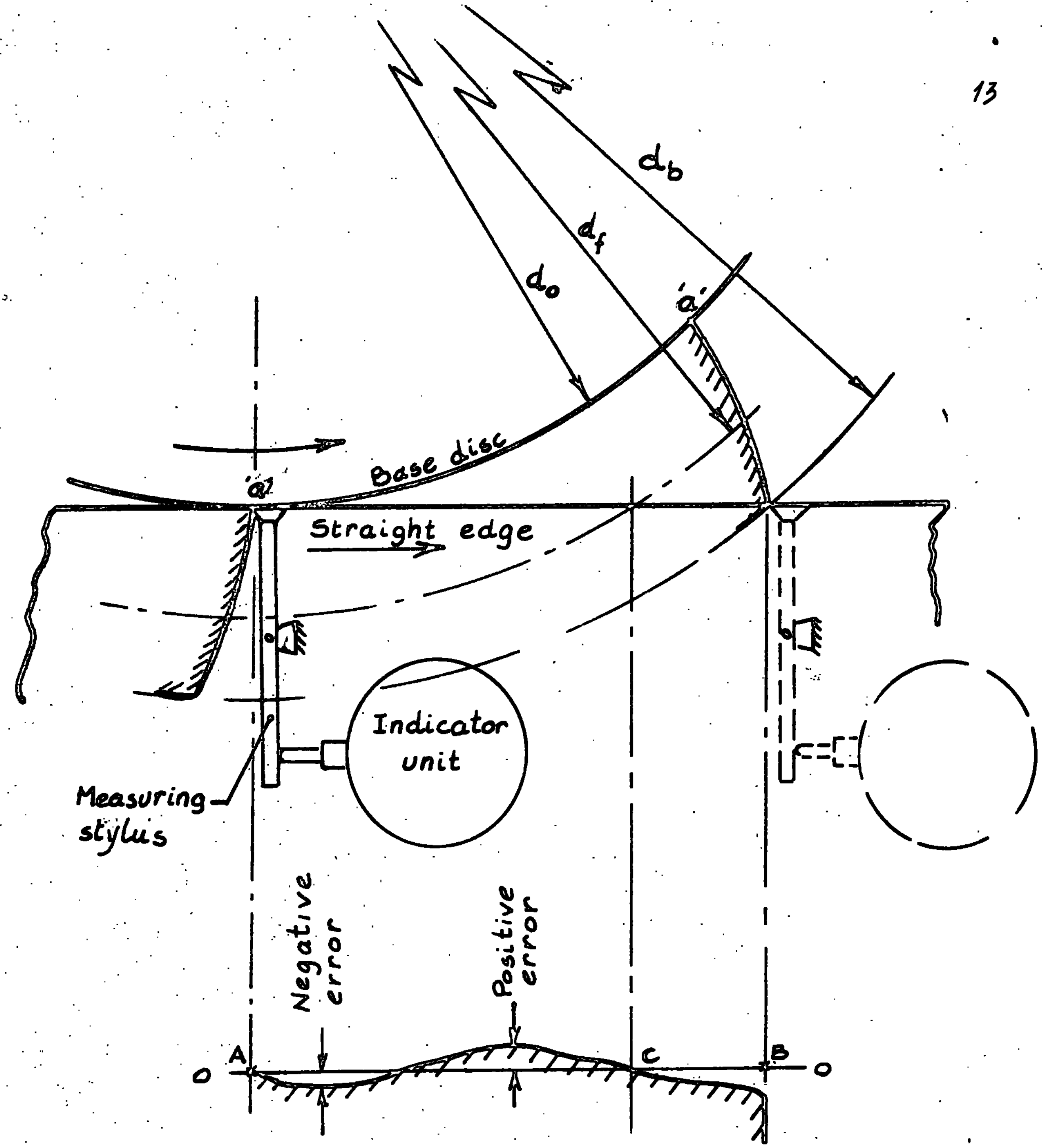
(b) Linkage machines, do not require individual base discs, but are of a more complex design and hence more expensive. The machine has a master base disc or involute cam, coupled to a linkage system which allows the base radius of the generated curve to be varied to suit the gear under inspection. The result of which is the production of the same relative movements between gear and measuring stylus as for a base disc machine.

Very large gears.

Again as for pitch measurement the measuring equipment has to be portable and taken to the gear to be measured.

The graph shown by Fig. 2.- 5a is representative of the style in which the results are presented by a base disc type involute testing machine. The horizontal axis is proportional to the rotation of the gear being inspected, the vertical axis indicating the profile error measured normal to the true involute. A straight line "0 - 0" representing a true involute since the stylus describes an involute relative to the point "a" on the base circle.

In practice the two magnifications will be different, a horizontal magnification up to 5 to 1, the vertical (error scale) magnification up to 2 000 to 1. When errors are required from the graph at a specific point the distances AB, AC and BC given on Fig. 2 - 5a must be multiplied by the horizontal magnification.



$$AB = \sqrt{\left(\frac{d_b}{2}\right)^2 - \left(\frac{d_o}{2}\right)^2}$$

$$AC = \sqrt{\left(\frac{d_f}{2}\right)^2 - \left(\frac{d_o}{2}\right)^2}$$

$$BC = AB - AC$$

Fig 2.5a Principle of involute testing machine.

2.6. Tooth Thickness

The measurement of tooth thickness is useful in that it can be carried out on the gear wheel during the machining operation, thus giving the machine operator an indication of when the correct size has been obtained. Figs. 2 - 6a and 2 - 6b show the involute geometry necessary to calculate the tooth thickness at any radii of a spur gear.

Methods of measurement of tooth thickness:

(i) Constant chord method, Fig. 2 - 6c in this case the thickness dimension is independent of the number of teeth in the gear for particular values of arc tooth thickness at reference circle and pressure angle.

(ii) Chordal thickness, corrected addendum method, Fig. 2 - 6d, this is the more commonly used method, since the tooth thickness is measured at the reference circle and will not therefore be effected by any profile error of the gear tooth.

Both methods (i) and (ii) require that the height and thickness be measured simultaneously and are also dependent on the accuracy of the blank diameter of the gear wheel. Correction can be made to the height reading if the blank diameter is known. The instruments used to carry out such measurements are:

(a) Gear tooth vernier caliper, which is a vernier caliper designed so as to make two readings at right angles. It is therefore subject to the setting difficulties of a vernier caliper and dependent in use on the "feel" of the operator. Probable accuracy of measurement 0.05 mm under the best conditions.

(b) Precision tooth thickness indicator which is similar to the tooth caliper, but with micrometer setting facilities and dial indicator "read out" of tooth thickness. The height setting can be made to 0.01 mm and the dial gauge graduations indicate to 0.005 mm.

(iii) Base tangent method, Fig. 2 - 6c, this is potentially more accurate than methods (i) or (ii) since only one measurement is employed, but it will be influenced by errors in gear tooth pitch. Measuring equipment which may be used; (a) Vernier caliper, (b) micrometer having special anvils

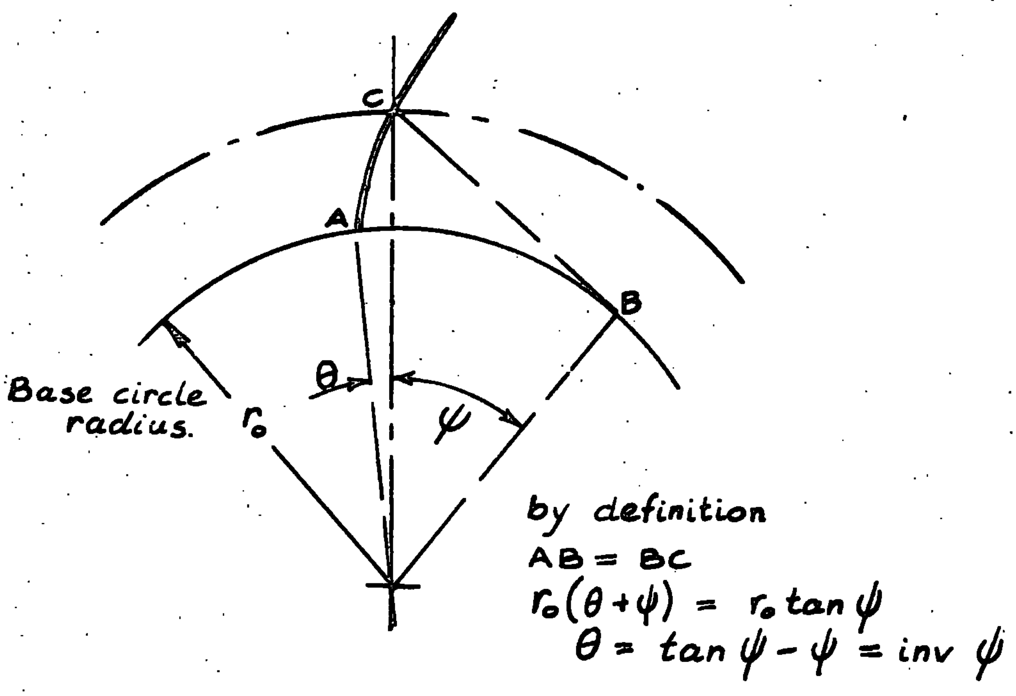
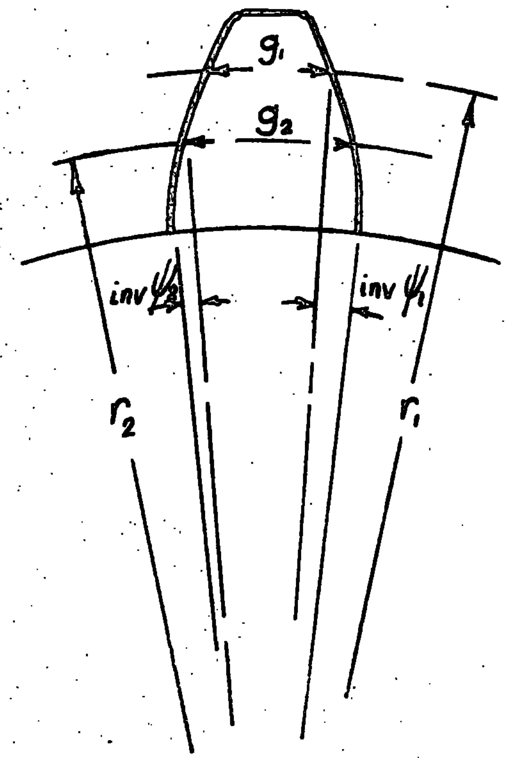


Fig 2.6a Involute function. [inv.]

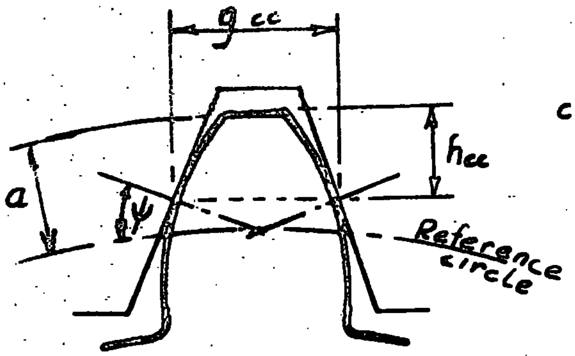


$$r_o = r_1 \cos\psi_1 = r_2 \cos\psi_2$$

$$\frac{g_1}{r_1} + 2\,\text{inv}\,\psi_1 = \frac{g_2}{r_2} + 2\,\text{inv}\,\psi_2$$

$$\therefore\ g_2 = 2r_1\left[\frac{g_1}{2r_1} + \text{inv}\,\psi_1 - \text{inv}\,\psi_2\right]$$

Fig 2.6b
Determination of arc tooth thickness at any radius.

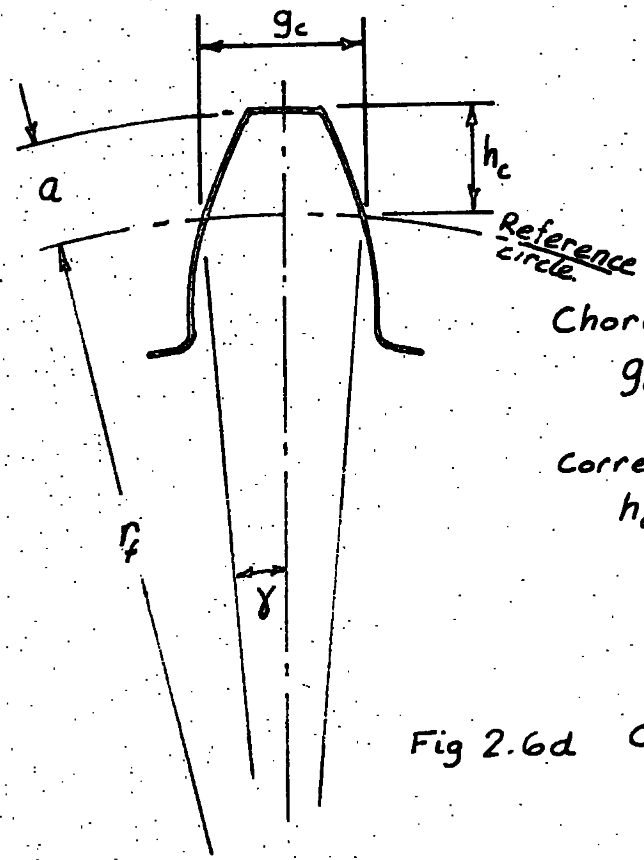


constant chord

$$g_{cc} = g_f \cos^2\psi$$

$$h_{cc} = a - \frac{g_f}{2}\cos\psi \sin\psi$$

Fig 2.6c Constant chord.



$$\gamma = \frac{g_f}{2 r_f}$$

Chordal thickness

$$g_c = 2 r_f \sin\gamma$$

Corrected addendum

$$h_c = a + r_f(1 - \cos\gamma)$$

Fig 2.6d Chordal thickness and corrected addendum.

and (c) precision tooth span indicator having dial gauge "read out."

(iv) Measurement over rollers in tooth spaces, Fig. 2 - 6f, this method really measures variation in tooth spaces rather than tooth thickness. For a gear having an even number of teeth the rollers are placed in diametrically opposite tooth spaces and the dimension over the rollers is given by:

$$M = 2r_2 + W$$

In the case of a gear wheel having an odd number of teeth the rollers are placed in the nearest diametrically opposed tooth spaces in which case:

$$M = (2r_2 \cos \frac{90^\circ}{T}) + W$$

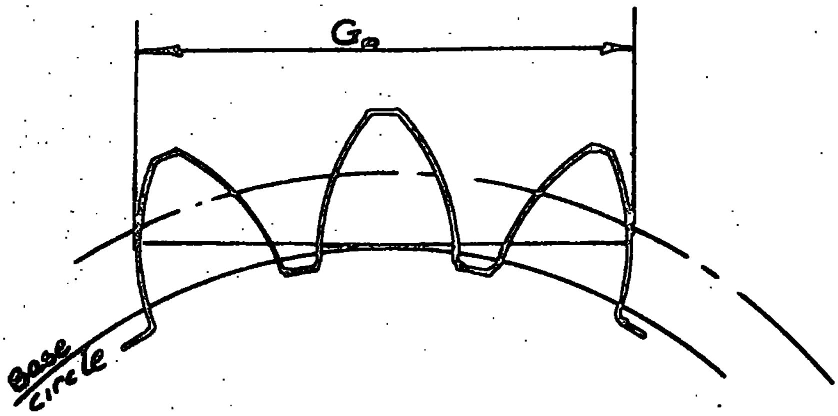
2.7. Tooth Depth

This dimension can be determined by making measurements of the outside and root diameters of the gear wheel, but care must be taken to ensure that the correct result is obtained. Difficulties arise owing to the fillet radius in the root of the teeth and for gears having an odd number of teeth. In practice it can be assumed that the tooth depth is satisfactory if the tooth thickness is correct (section 2 - 6) and the cutting tool used to manufacture the gear had the correct proportions.

From an operational point of view it is the length of active tooth profile which is important and this dimension can be checked using the involute profile record (section 2 - 5).

2.8. Tooth Alignment

The accuracy of tooth alignment relative to the axis of the gear is important to ensure correct load distribution across the face of the gear. It may be measured on a point to point basis using standard metrology equipment, although special purpose machines giving a charted record of the results are more usual.



g_o = arc tooth thickness at base circle

p_o = base pitch (p_o = circular pitch at reference circle x cosine of the pressure angle)

n = number of teeth spanned

$$G_o = (n-1)p_o + g_o$$

For base tangent to intersect tooth flank at reference circle

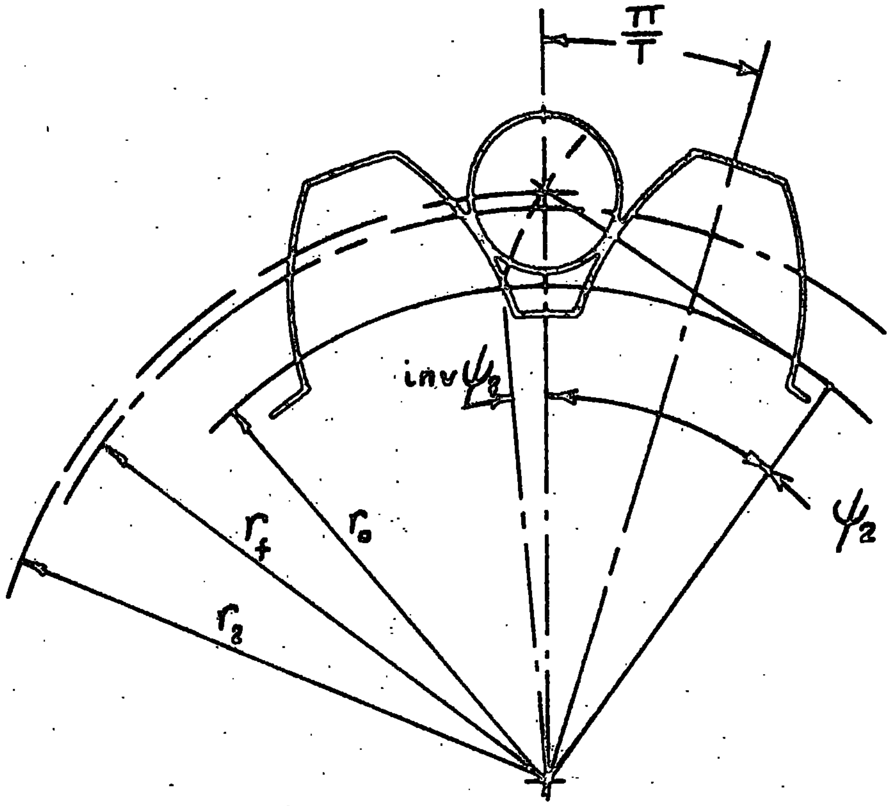
p = circular pitch of teeth at reference circle.

$$(n-1)\,p + g_f = 2 . r_f . \psi^\circ . \frac{2\pi}{360^\circ}$$

$$n = \frac{\psi^\circ}{180^\circ} . T - \frac{g_f}{p} + 1$$

where T = number of teeth in gear

Fig 2.6e BASE TANGENT MEASUREMENT.



T = number of teeth in gear.
ω = diameter of roller.

$$\frac{\pi}{T} + \text{inv}\,\psi_2 = \frac{g_f}{2r_f} + \text{inv}\,\psi_f + \frac{\omega}{2r_o}$$

$$\text{inv}\,\psi_2 = \frac{g_f}{2r_f} + \text{inv}\,\psi_f + \frac{\omega}{2r_o} - \frac{\pi}{T}$$

and $r_2 = r_o \sec\psi_2$

Diameter of roller to contact flank at reference circle [ω_f]

$$\omega_f = 2r_o\left[\tan(\psi_f + \theta) - \tan\psi_f\right]$$

where $\theta = \frac{g_s}{2r_f}$ g_s = arc width of tooth space at reference circle.

in this case $r_2 = r_o \sec(\psi_f + \theta)$

Fig 2.6f Roller in tooth space.

2.9. Meshing Tests

The measurement of errors in individual gear tooth elements can be a long tedious process especially if it is required to predict their combined effect on the resulting performance of the gear wheel in its working situation.

To help alleviate this problem two forms of composite testing are used, (i) dual flanking meshing test and (ii) single flank meshing test. (4), (5), (6) & (7)

(i) Dual flank meshing test

For this method the gear wheel under inspection is held tight in mesh with a master gear thus giving contact on both the α and β flanks of the teeth, the gears are then rotated and the variation in centre distance measured. The obvious disadvantage of this system is that such a meshing condition will rarely occur in practice, but the method requires relatively simple and inexpensive equipment. This test is the subject of a detailed investigation in a later chapter.

(ii) Single flank meshing test

This method approaches the operating conditions of a gear pair, in that the gear being inspected and the master gear are run at the correct centre distance, thus only one set of tooth flanks will be in contact. Errors in the gear tooth elements produce variations in the velocity ratio of the gear train and it is these variations in terms of angular displacement that are measured. The apparatus is thus more complex and costly than the dual flank mesh testing machine.

Theory for this test has been developed by Munro (5)(6) and will be briefly described as it is used to derive a theory for the dual flank meshing test in a later chapter.

Fig. 2 - 9a shows the principle of operation of a single flank testing machine (using radial diffraction gratings) originally designed at the National Engineering Laboratory and now available commercially. The error trace output of the chart recorder indicates the error in angular displacement of one gear relative to the other.

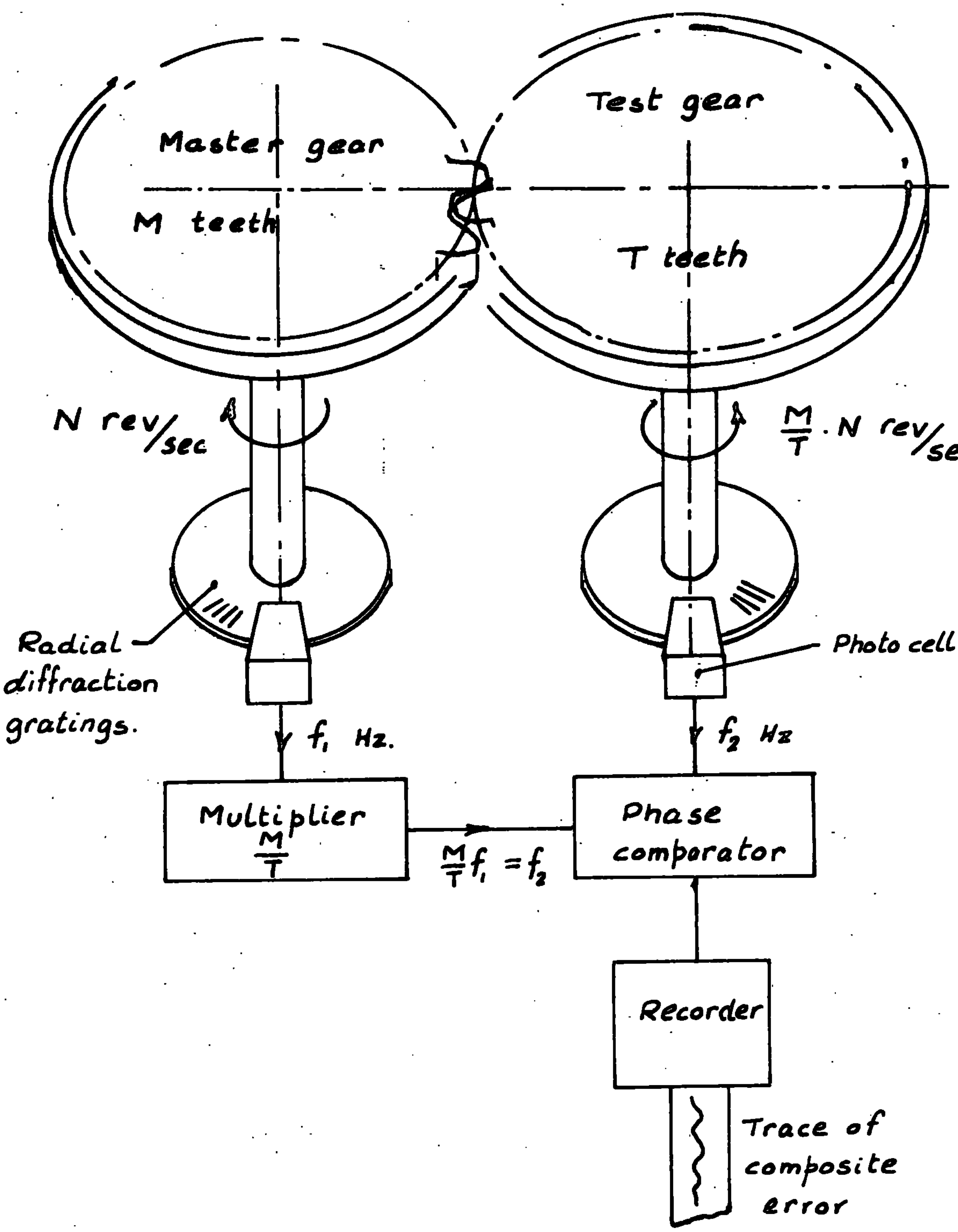


Fig 2.9a PRINCIPLE OF SINGLE FLANK MESHING TEST.

2.10. Single Flank Transmission Error Theory.

Initially it will be assumed that a perfect master gear is used to inspect a production spur gear, in which case all the transmission error will be due to errors in the production gear. For the position indicated by Fig. 2.10a the combined elemental error of the gear under test at the point of contact between the two gears (point P) will determine the angular position of the master gear. Thus as the gear pair rotate the transmission error will vary in direct proportion to the combined elemental error variation of the gear under inspection. It is therefore possible to obtain a graph of transmission error to a base of gear rotation.

(i) Assuming the gear under test to have no pitch error, in which case the involute profile error of the teeth will determine the transmission error and the graph will take the form shown in Fig. 2.10b. The curve R, P, T, indicates the profile error (measured normal to the profile) of tooth 1 of the gear under test. Point R_1 indicating the initial point of contact at the root of the tooth 1 and T_1 the final point of contact at the tip. As the gears revolve each tooth of the gear under test will come into contact with the master gear and in order that the rotation shall be continuous there must be some degree of overlap between successive tooth contacts. The transmission error curve being built up from successive profile error curves of the gear under inspection spaced at intervals equal to the base pitch of the gears. Fig. 2.10b.

(ii) The effect of a tooth pitch error will be to raise or lower the profile error curve in the direction of the transmission error by an amount proportional to the pitch error. Fig. 2.10c.

The diagrams show the transmission error for the α tooth flanks, reversal of the direction of gear rotation will give an error curve for the β flanks.

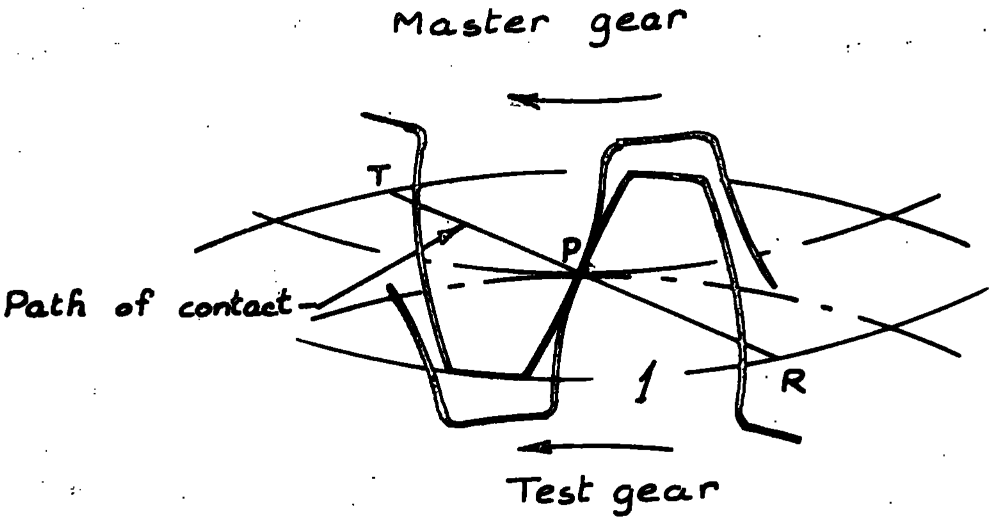


Fig 2.10a

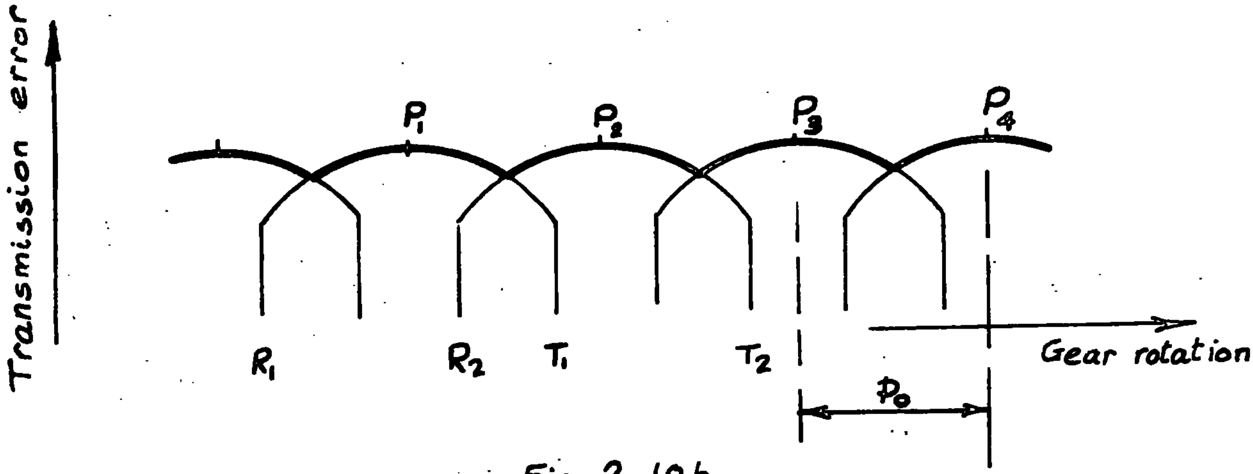


Fig 2.10b

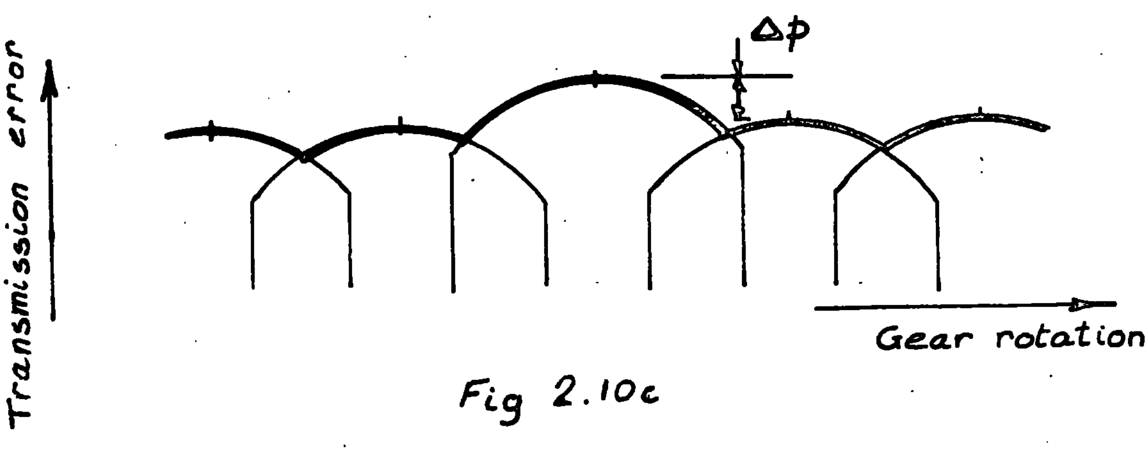


Fig 2.10c

The above description assumed a perfect master gear. As this is not possible in practice the transmission error actually indicated by a single flank meshing test will be that due to the algebraic sum of the errors of the master and test gears.

2.11. Other Single Flank Testing Machines

(i) Friction disc machines

This type of machine requires two metal discs the diameters of which are equal to the pitch circle diameters of the two meshing gears. The disc having a diameter equal to the pitch circle diameter of the master gear is firmly fixed to the shaft carrying the master gear. The other disc which has a diameter equal to the pitch circle diameter of the test gear, is mounted on the shaft supporting the test gear but is free to rotate independently of the shaft.

When the two gears are in mesh at the correct centre distance the two pitch circle discs will be pressed against each other. On rotation of the master gear together with its disc, the disc on the test gear shaft will rotate at the correct speed consistent with the required gear ratio. The variation in angular position of the test gear relative to its pitch circle disc is measured and magnified, so indicating the composite gear tooth error of the gears in single flank contact.

The above arrangement is suitable for spur and helical gears. Refinements in the design using a belt drive between two pulleys the diameters of which are in the same ratio as the pitch circle diameters enables friction disc machines to inspect spur, helical, bevel and worm gears. (8)

(ii) Seismic or inertia machines

Single flank testing machines using angular oscillation "pick ups" have been developed in Germany. Accelerations and decelerations of the rotating gears under test are detected relative to a steadily rotating mass, these fluctuations in speed being due to the composite errors of the gears in single flank contact. Such instruments can cope with all error frequencies provided they are above 1 Hz, this limit being set by the natural frequency of the instrument. The gears being tested must therefore be rotated at speeds greater than sixty revolutions per minute, there is also an upper speed limit of six hundred revolutions per minute in order to prevent damage to the instrument. (4) & (9)

(iii) Temac system

Originally developed in Czechoslovakia for measuring hobbing machine table motion errors, but now available in the form of single flank gear inspection machines. The system involves the measurement of phase errors between electrical signals obtained from rotary sensors. Chart recordings are obtained which indicate errors of tooth spacing and profile. (4) & (9)

2.12. Difficult Gears

The development of single flank testing machines and portable measuring heads based on the same operating principles has simplified the inspection of gears which can not be economically inspected on an elemental basis, e.g. bevel, spiral bevel, worm and wormwheel gears. Previously these types of gears were often checked by simple meshing tests after "blueing" one set of teeth with toolmakers marking blue and observing the tooth contact pattern. Tests on large turbine reduction gears are also carried out in this manner using specially designed rigs.

For spiral bevel and hypoid gears inspection machines have been designed (using vibration "pick ups") which relate gear testing to actual vehicle tests. Such machines employ a measurement system that automatically identifies gear sets as acceptable or reject on a basis of running quality.

In spite of the advantages of single flank mesh testing, measurement on an elemental basis cannot be neglected altogether. The individual measurement of gear elements may be the only method of identifying a particular error and locating its cause.

CHAPTER 3.

Measurement of tooth pitch - theory

3.1. In practice the actual numerical value of the circular pitch of the gear teeth at the pitch circle of the gear is not determined. The measurements taken are such that they give the tooth positional error relative to equal angular spacing of the gear.

3.2. Tooth pitch and eccentricity

The two factors cannot be separated; a gear devoid of any pitch error if mounted eccentrically will display a cumulative pitch error.

3.3. Direct measurement of pitch error - theory

(see also section 2.3.)

Consider a master gear assumed to be devoid of any pitch error mounted with an eccentricity equal to "e"

(i) For the α tooth flanks the cumulative pitch error will be given by:

$$D\,E = e \cdot \sec\psi \cdot \sin(\theta + \psi) \qquad 3.3a$$

ref. (4)

where θ = the angle turned through from the position of maximum negative eccentricity. "e" when used in equation 3.3a must be made numerically positive in order to conform to the sign convention for cumulative pitch error. Fig. 3.3a gives the derivation of equation 3.3a. Fig 3.3c illustrates the equation graphically. This equation is also quoted in reference (4).

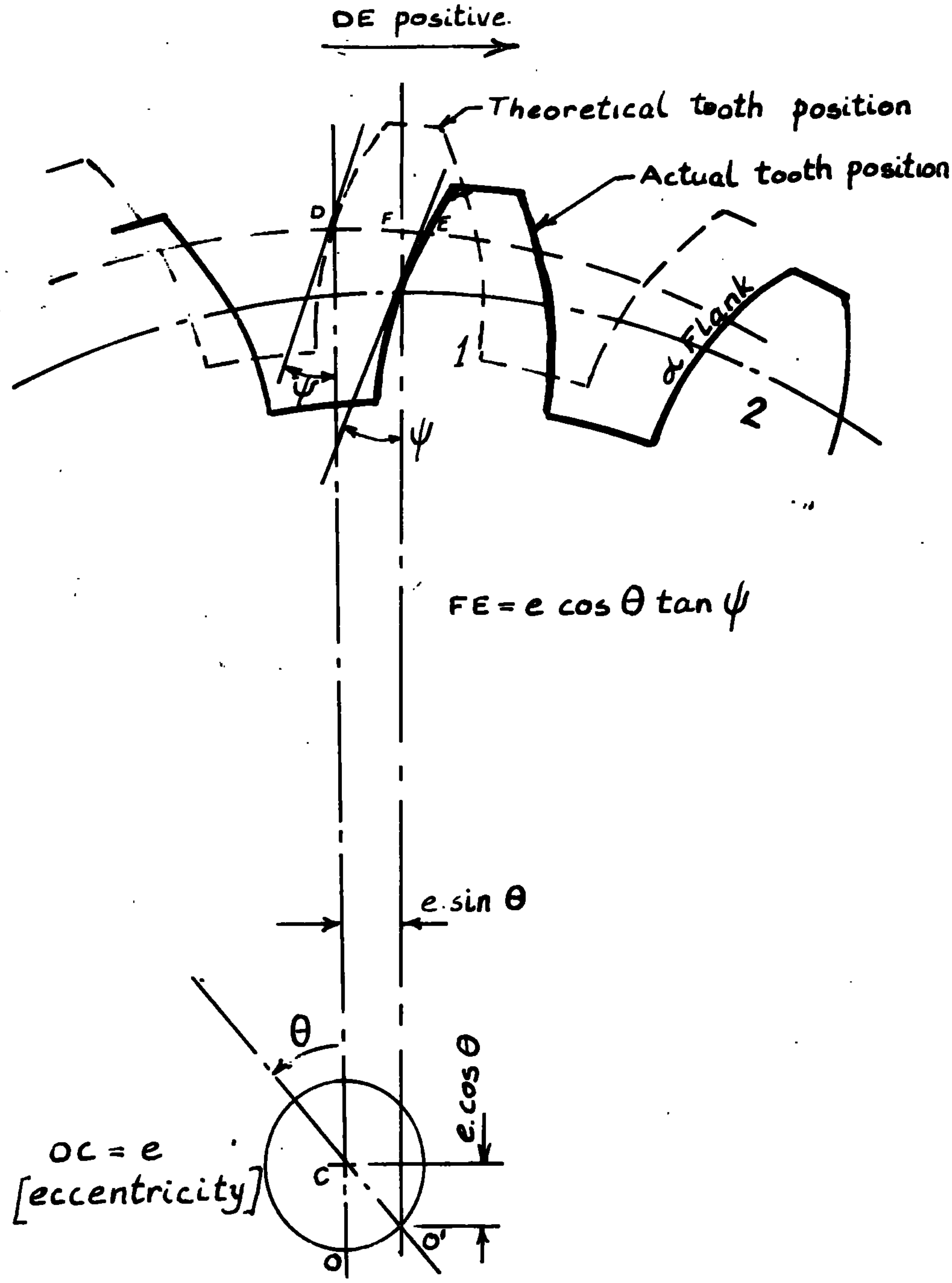
(ii) For the β tooth flanks the cumulative pitch error will be given by:

$$D\,E = e.\sec\psi \ \sin(\theta - \psi) \qquad 3.3b$$

in order to conform to the above sign convention for pitch error, "e" is made numerically positive in this case.

Fig. 3.3b gives the derivation of equation 3.3b

Fig. 3.3d illustrates the equation graphically.



C = centre of rotation of gear.
O' = true centre of the gear.
D = position of measuring stylus set relative to axis of rotation though 'C'.

$$DE = DF + FE = e\sin\theta + e\cos\theta.\tan\psi$$

$$DE = e.\sec\psi.\sin(\theta+\psi)$$

Fig 3.3a DIRECT MEASUREMENT OF PITCH ERROR α FLANK.

3.4. Span gauging of tooth pitch error - theory

(see also section 2.3.)

Consider a master gear devoid of any pitch error mounted with an eccentricity equal to "e". For the case in which the measuring and locating styli contact only the β tooth flanks the method of measurement employed is as follows; see Fig. 3.4a.

(1) The gear is rotated about the centre of rotation 'C' through an angle θ measured from the position of maximum negative eccentricity. This results in the gear tooth pitch point taking up the position indicated by 'E'.

(2) Because of the position of the fixed datum stylus (Fig. 2.3b), an additional rotation through the angle δ is given to the gear about centre C, i.e. through the arc DE measured at the reference circle. The angle δ may be numerically positive or negative relative to the angle θ.

DE = absolute cumulate pitch error of the tooth in contact with the datum stylus (tooth N).

(3) The actual span measurement is made along the line DX.

(4) The actual reference circle of the eccentrically mounted gear passes through the points PG,PG being a chord across the true circular pitch of the gear.

The adjacent pitch error

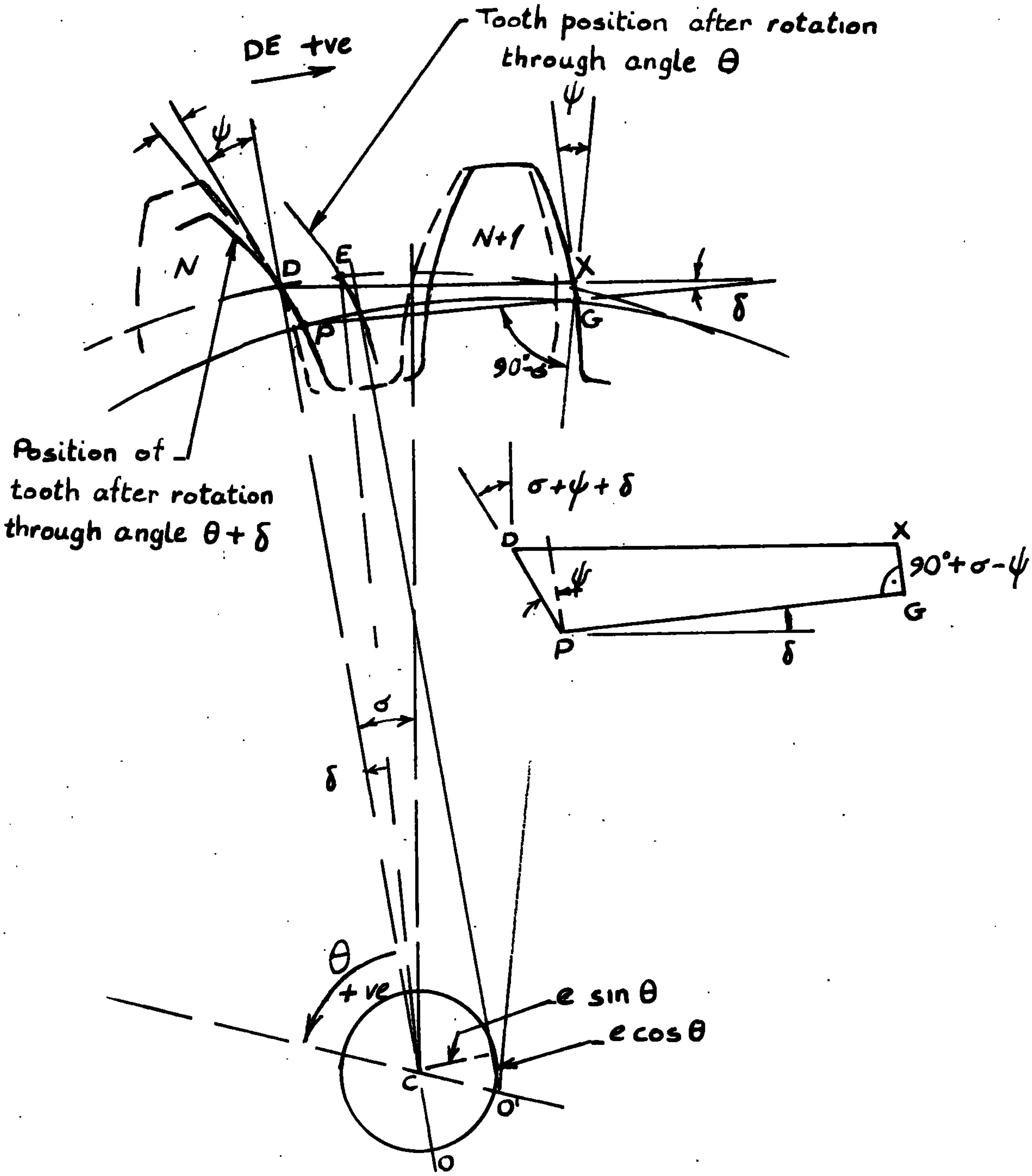
$$\Delta p_a = DX - PG$$

$$= \frac{PG.\,\delta.\tan\gamma + e.K\cos}{1 - \delta.\tan\gamma} \qquad \text{———} \qquad 3.4a$$

It is important to use the correct sign convention for δ when calculating Δp_a from equation 3.4a. The sign of the value of DE will give the sign of δ. The derivation of equation 3.4a is given on the pages immediately following Fig. 3.4a.

3.5. A cumulative pitch error curve can be obtained from the algebraic summation of the adjacent pitch errors. It should be noted that the cumulative pitch error graph is made up of a number of definite points each representing a particular tooth flank. These points are usually joined together by straight lines in order to guide the eye from one point to the next, see Fig. 3.6a.

Because of the design of the single point contact span gauging instrument the gear must be turned over to measure the adjacent pitch error of the α tooth flanks.



D = Fixed datum relative to centre of rotation C

O' = true centre of gear

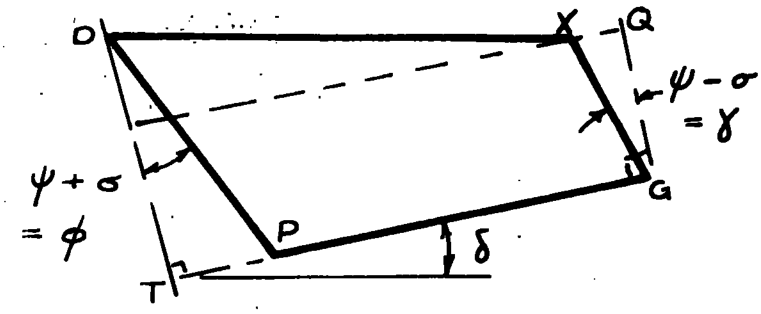
θ = nominal angle turned through from position of maximum negative eccentricity

$$\sigma = \frac{180^\circ}{\text{Number of teeth in gear}}$$

SPAN GAUGING OF ADJACENT PITCH ERROR

Fig. 3.4a

Diagram from
Fig 3.4a



DX = chordal adjacent pitch measured

PG = true chordal adjacent pitch

$DX \cos\delta = PG + TP - XQ$ and $PG = 2R_f \sin\sigma$

$TP = DP \sin\phi = e\cos\theta \sec\psi \sin\phi$

$XQ = QG \tan\gamma$ where $QG = DT - DX\sin\delta$

$= DP\cos\phi - DX\sin\delta$

$$DX\cos\delta = PG + e.\cos\theta\sec\psi\sin\phi - [e\cos\theta.\sec\psi\cos\phi - DX\sin\delta]\tan\gamma$$

$$= PG + e\cos\theta\sec\psi[\sin\phi - \cos\phi\tan\gamma] + DX\sin\delta\tan\gamma$$

$$\therefore DX = \frac{PG + e\cos\theta\sec\psi[\sin\phi - \cos\phi\tan\gamma]}{[\cos\delta - \sin\delta\tan\gamma]}$$

Adjacent pitch error $\Delta p_a = DX - PG$

$$\Delta p_a = \frac{PG + e\cos\theta.K}{[\cos\delta - \sin\delta\tan\gamma]} - PG$$

where $K = \sec\psi[\sin\phi - \cos\phi\tan\gamma]$
which is a constant for a particular gear

$$\delta = \frac{e\sec\psi\sin(\theta - \psi)}{R_f} = \frac{\text{cumulative pitch error}}{R_f}$$

δ will in most practical cases be small for a 48T 12DP (module) 20° AOP gear wheel set 0·0596mm (0·0025 in) eccentric

$$\delta_{max} = 0\cdot00130 \text{ radians}$$

$$\therefore \delta \simeq \sin\delta$$

$$\cos\delta = (1 - \sin^2\delta)^{\frac{1}{2}} \simeq 1\cdot000$$

$$\therefore \quad \Delta p_a = \frac{PG + e\cos\theta . K}{[1 - \delta\tan\gamma]} - PG$$

$$\Delta p_a = \frac{PG.\,\delta.\tan\gamma + e.K.\cos\theta}{1 - \delta\tan\gamma} \qquad 3.4a$$

3.6. Figs. 3.6a and 3.6b show the comparison in magnitudes of the cumulative pitch errors for gears having 12 teeth and 48 teeth, the values having been calculated using equations 3.3b (direct measurement theory) and 3.4a (span gauging theory).

Fig. 3.6c indicates how the ratio, Cumulative pitch error by direct measurement theory to cumulative pitch error by span gauging theory varies with the number of teeth in the gear. This variation is probably explained by considering the assumptions made in the derivation of equations 3.3b and 3.4a;

(i) the teeth are assumed to be straight sided and inclined at the pressure angle in the region of the reference circle, for this reason the values of "e" used must be numerically small. This assumption is applicable to both equations.

(ii) in the derivation of equation 3.4a the circular pitch is assumed to be approximately equal to the chord length at the reference circle. But as the difference in two chord lengths is used to obtain the adjacent pitch error, the resulting error owing to this assumption will be small unless the adjacent pitch error is very large coupled with a small number of teeth in the gear.

(iii) relative to the tooth flanks the pitch error is not measured in the same direction for equations 3.3b and 3.4a. But as the number of teeth in the gear increases this difference in direction decreases.

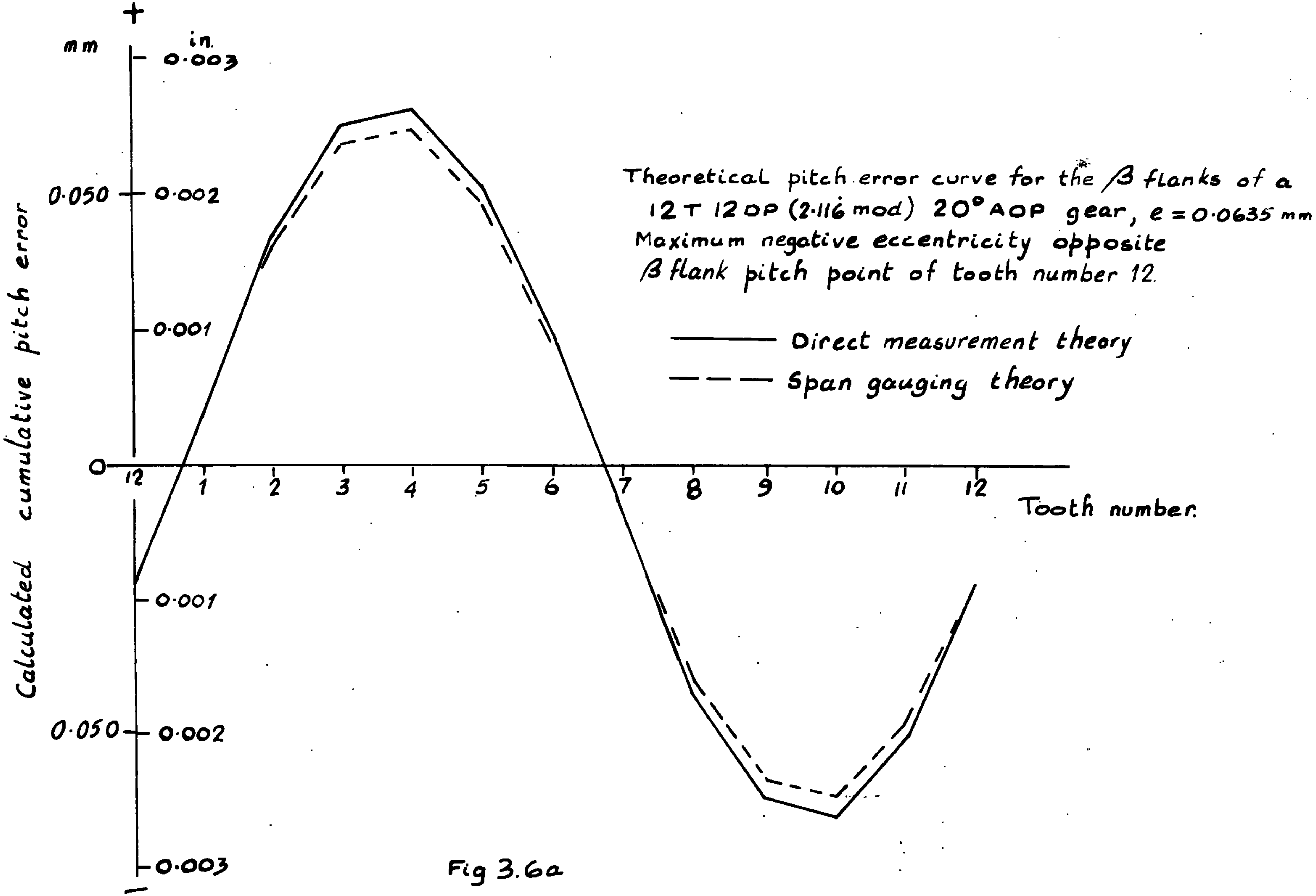

Theoretical pitch error curve for the β flanks of a 12T 12DP (2·116 mod) 20° AOP gear, e = 0·0635 mm
Maximum negative eccentricity opposite β flank pitch point of tooth number 12.
Direct measurement theory
Span gauging theory
Calculated cumulative pitch error
mm
in.
+
0·003
0·050
0·002
0·001
0
0·001
0·050
0·002
0·003
−
12
1
2
3
4
5
6
7
8
9
10
11
12
Tooth number

Fig 3.6a

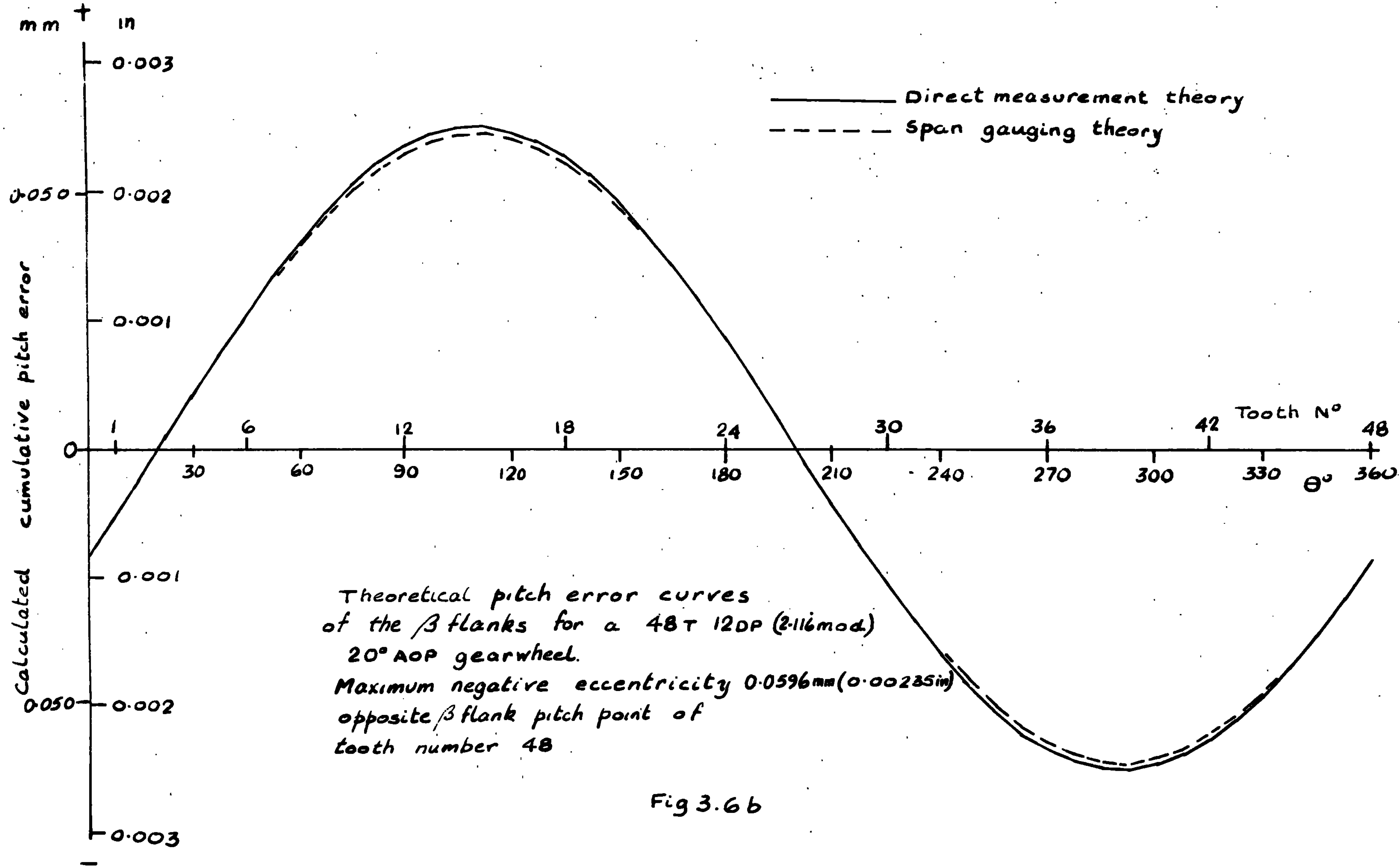
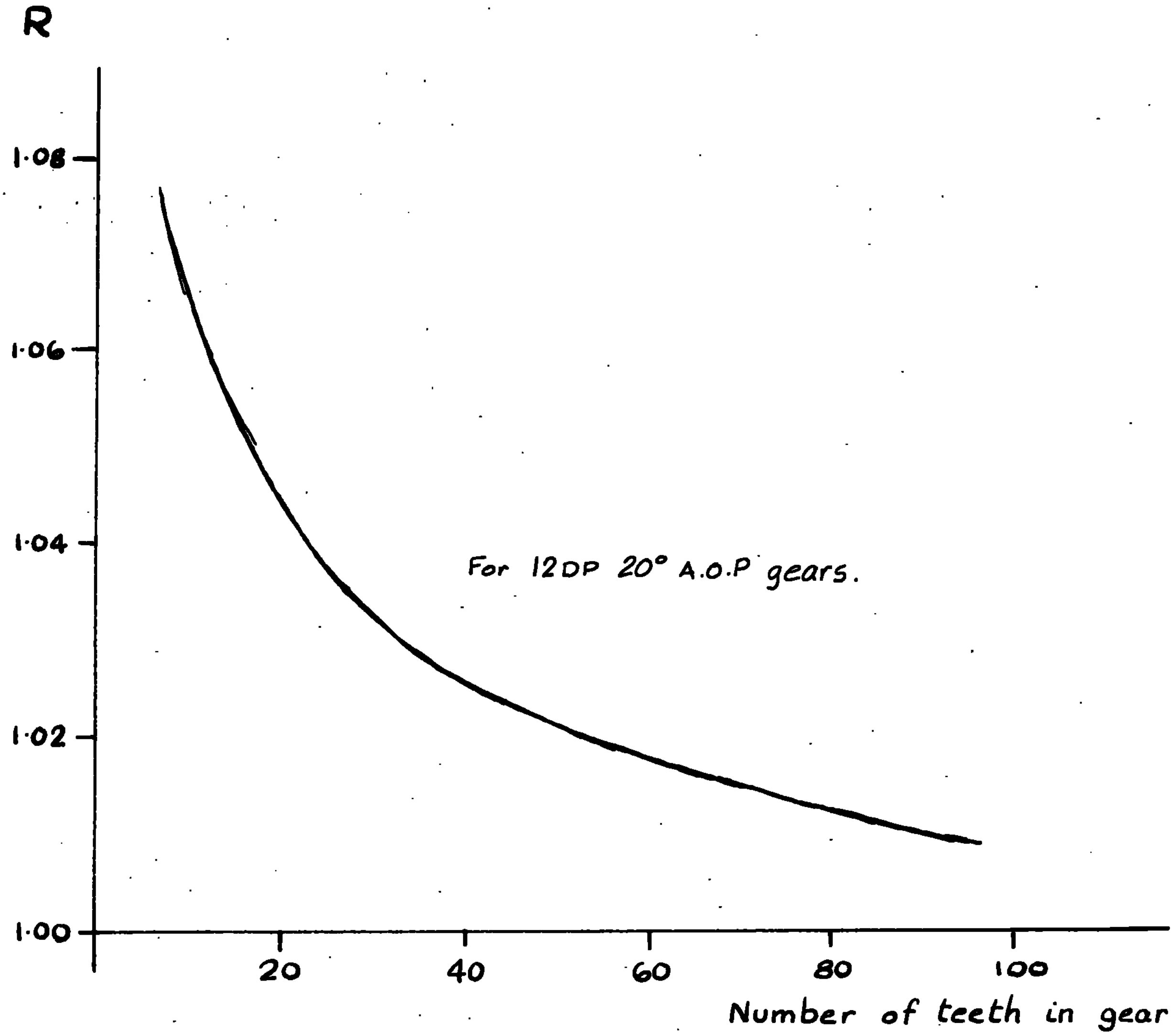

mm + in
0·003
0·050 0·002
0·001
0
0·001
0·050 0·002
0·003
−
Calculated cumulative pitch error
Direct measurement theory
Span gauging theory
Tooth N°
1 6 12 18 24 30 36 42 48
30 60 90 120 150 180 210 240 270 300 330 360
θ°
Theoretical pitch error curves
of the β flanks for a 48T 12DP (2·116mod)
20° AOP gearwheel.
Maximum negative eccentricity 0·0596mm (0·00235in)
opposite β flank pitch point of
tooth number 48

Fig 3.6b



$$R = \frac{\text{Pitch error given by direct measurement theory}}{\text{Pitch error given by span gauging theory}}$$

Fig 3.6 c.

CHAPTER 4.

Measurement of pitch - experimental

4.1. Pitch measurements were made on an eccentrically mounted master gear and the results compared with the theoretical equations given in Chapter 3. An eccentrically mounted master gear was chosen rather than an eccentrically cut gear because in the latter case it would be much more difficult to isolate the pitch error due to the eccentricity from that due to the cutting process.

4.2. The general procedure for the measurements made was as follows;

(i) initially the gear was mounted concentrically and the cumulative pitch error measured. (y_1).

(ii) using a special arbor allowing variable eccentricity settings to be made, the master gear was set eccentric and the cumulative pitch error of each tooth obtained (y_2)

(iii) then the cumulative pitch error graph (y) due to the eccentricity of the gear was determined from the relationship

$$y = y_2 - y_1$$

<u>Note:</u> The same datum tooth for measurements y_1, and y_2 was used and the actual measurements made at the same position along the tooth face width.

(iv) the equation of the best sine wave through the cumulative pitch error points was calculated using a least mean squares analysis (see Appendix I).

From this equation the magnitude of the gear eccentricity indicated by the measured cumulative pitch error was determined.

This result was then compared with the actual eccentricity of the gear mounting.

To determine the magnitude of the mounting eccentricity, the "run out" of the master gear setting ring was measured by a dial indicator at eighteen equally spaced intervals. The results plotted on a graph and the numerical value of the eccentricity obtained from the graph.

4.3. <u>Direct measurement of cumulative pitch error</u>.

Apparatus used; O.M.T. optical dividing head model OW 12, S.I.P. two dimensional measuring machine model 214B.

The master gear supported on its arbor was mounted between the optical dividing head collet and a rear supporting centre, along the longitudinal axis (x axis) of the measuring machine. A ball ended measuring stylus was set vertically above the centre of the gear at a height such that it contacted the tooth flank at the reference circle of the gear. The position of the stylus being monitered by the y axis of the measuring machine. Measurements were then made in accordance with section 4.2.

The optical dividing head was used for indexing the gear and the cumulative pitch error of each tooth was detected by measurements on the 'y' axis of the measuring machine. Before indexing the gear, the teeth were cleared from the measuring stylus by moving the measuring machine table, the gear indexed, and the table returned to its datum position in readiness for taking the next pitch reading. The 'x' axis measuring scale was used to ensure correct re-positioning of the table.

4.4. A typical set of results for a 48T 12DP 20° A.O.P. master gear are shown on Fig. 4.4a. Probable accuracy of these results:

Indexing accuracy, the dividing head had a maximum error between two settings 180° apart of 0.3 minutes of arc, the individual angular setting could be repeated to nominal ± one second of arc. One second of arc represents a linear error of 5×10^{-6} units at a radius of 1 unit.

Cumulative pitch error measurements could be repeated to ± 0.0005 mm (± 0.00002 in)

The probable error of an individual cumulative pitch error of an individual cumulative pitch error reading for the 48T 12DP gear is ± 0.0008 mm, and for the pitch error curve due to the eccentricity ± 0.0011 mm (± 0.00004 in)

But the maximum indexing error between two angular settings 180° apart expressed as a linear error at the reference circle is 0.0046 mm (0.00018 in).

4.5. <u>Span gauging of tooth pitch error</u>

Apparatus used;

Goulder involute testing machine, model I2,

Pitch measuring attachment.

The general method of approach was as detailed in section 4.2. But in this case the cumulative pitch error graph was built up from successive measurements of adjacent pitch error.

The master gear supported on its special arbor was set up between the vertical centres of the involute testing

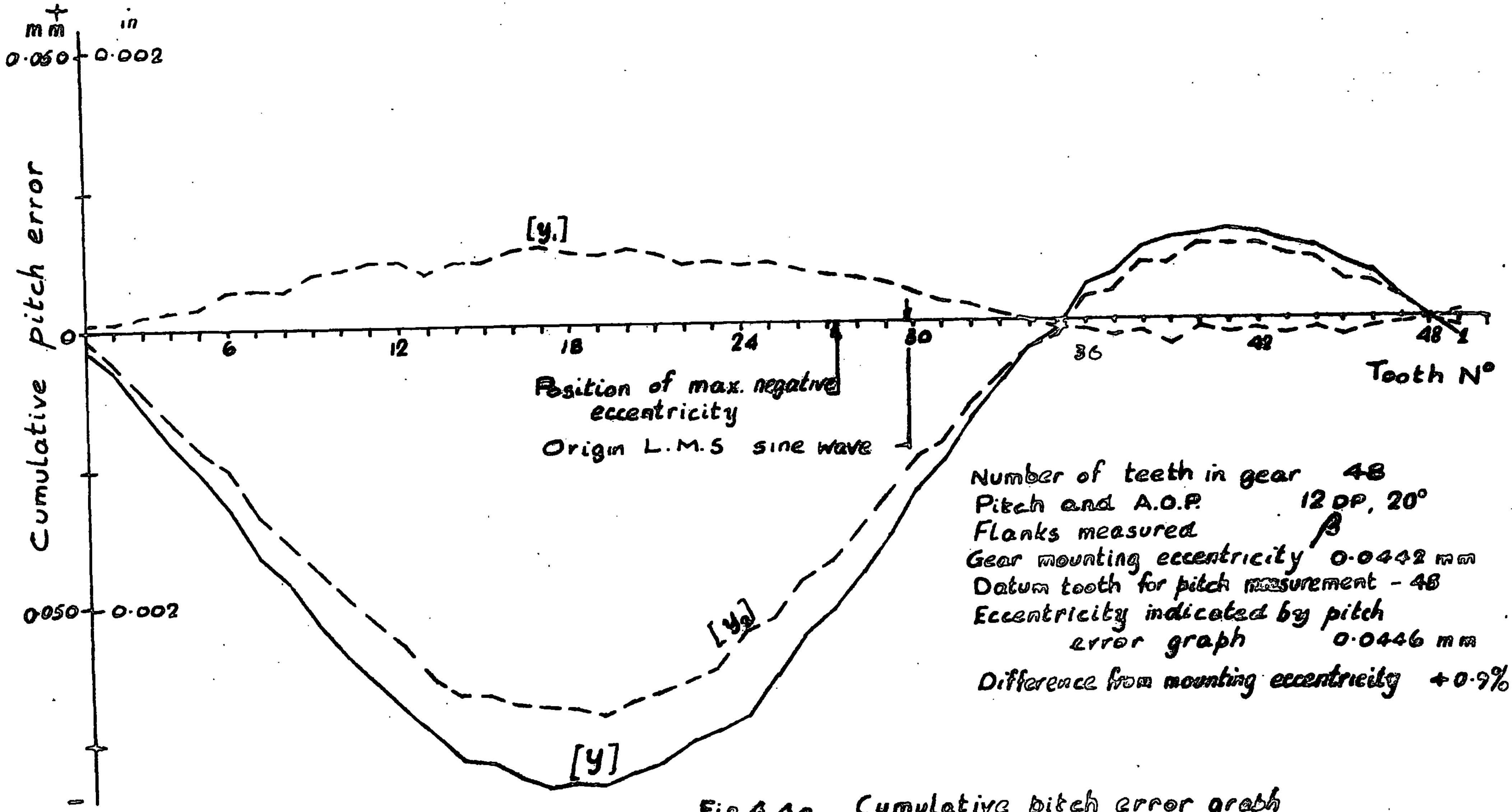


Fig 4.4a. Cumulative pitch error graph by direct measurement

machine and the pitch measuring attachment fitted to the vertical column of the machine's measuring slide. Care being taken to ensure that datum and measuring styli contacted the gear tooth flanks at the reference circle (i.e. positioned as shown in Fig. 2.3b). When indexing the gear between successive adjacent pitch measurements it was necessary to withdraw the measuring slide in order to clear the styli from the gear teeth. In order to do this the measuring slide was traversed by means of its lead screw, being re-set to the measuring datum by means of a dial indicator secured to the machine bed. The adoption of this method, rather than using the slide quick withdrawal mechanism ensured that all measurements were made at the same position relative to the centre of the gear.

When using this method of measurement the gear must be turned over to measure the opposite set of tooth flanks.

4.6. The method of building up the cumulative pitch error curve from the span gauging measurements is as follows:

If the teeth of the gear under inspection are numbered in a clockwise direction then when using the span pitch gauging attachment the measuring stylii contact the β flanks of adjacent teeth as shown in Fig. 4.6a.

The adjacent pitch error Δp obtained will be that of tooth 2 relative to tooth 1, $\Delta_2 p_1$

$$\Delta_2 p_1 = y_2 - y_1$$

$$y_2 = y_1 + \Delta_2 p_1$$

where (y = absolute pitch error)

The results are normally set out in tabular form, the data for measurements on a 10 tooth pinion is given below (β Flanks) ref. (4).

Span	Indicator reading i	Sum Σi	Nominal	Cumulative pitch error y
10-1	0	0	+0.6	-0.6
1-2	+2	+2	+1.2	+0.8
2-3	+3	+5	+1.8	+3.2
3-4	+1	+6	+2.4	+3.6
4-5	0	+6	+3.0	+3.0
5-6	+1	+7	+3.6	+3.4
6-7	-3	+4	+4.2	-0.2
7-8	-1	+3	+4.8	-1.8
8-9	+2	+5	+5.4	-0.4
9-10	+1	+6	+6.0	0

$\bar{i} = \frac{\Sigma i}{n} = +0.6$ this represents the nominal adjacent pitch of the gear.

An individual adjacent pitch error will be given by $i - \bar{i}$ and the cumulative pitch error of a particular tooth by

Σi - (nominal cumulative pitch for that tooth).

Illustrated graphically by Fig. 4.6b.

In order to measure the adjacent pitch error for the α flanks the gear under inspection must be turned over. Thus the measuring stylii will contact the tooth flank as shown by Fig. 4.6c.

When measuring the α flanks although the span gauging instrument is measuring tooth 1 relative to tooth 2, the same numerical value of adjacent pitch will be indicated no matter which tooth is taken as the datum.

Thus the method of determining the cumulative pitch error for the α tooth flanks will be exactly the same as that used for the β flanks.

4.7. Typical sets of results obtained using the span gauging technique of pitch measurement are shown by Figs. 4.7a and 4.7b. Fig. 4.7c illustrates the spread of the cumulative pitch error curves for five sets of measurements. Probable accuracy of the results; because of the method used the accuracy of the final curve will be a function of the number of teeth in the gear. For the eccentrically mounted master gear the probable inaccuracy in the cumulative pitch error will be due to the accumulation of the inaccuracies of

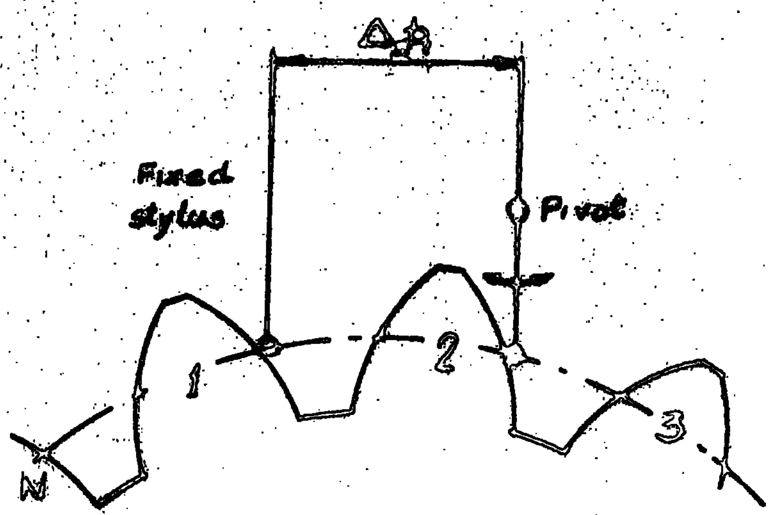


Fig 4.6a Span gauging β flanks.

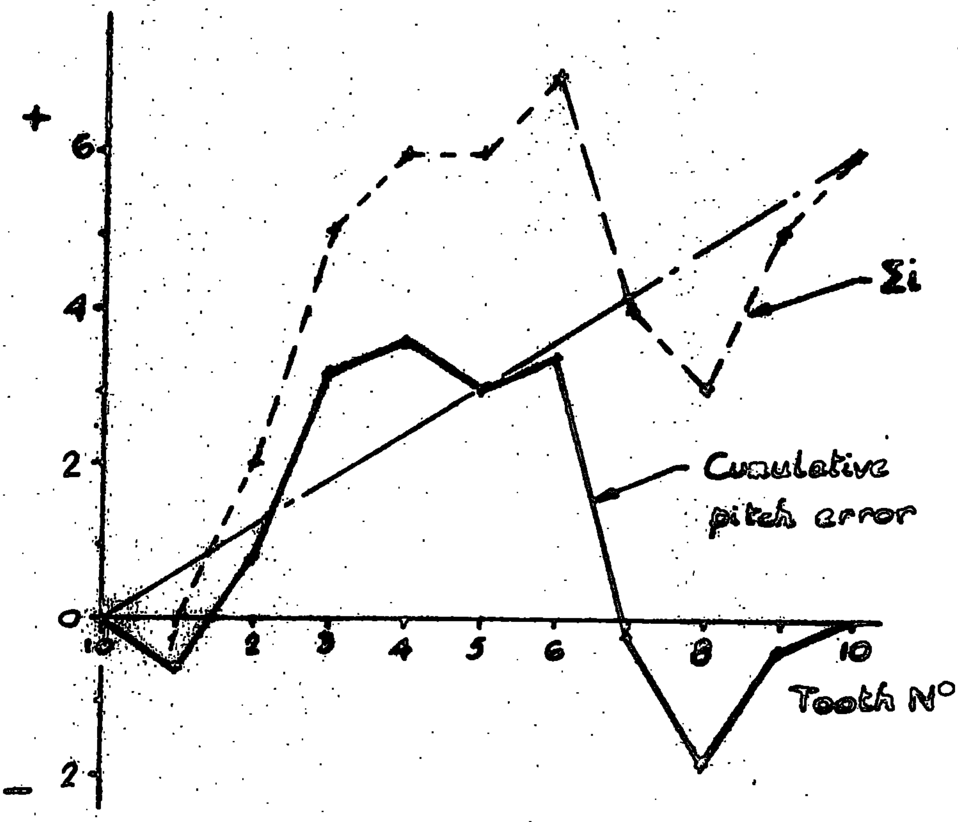


Fig 4.6b

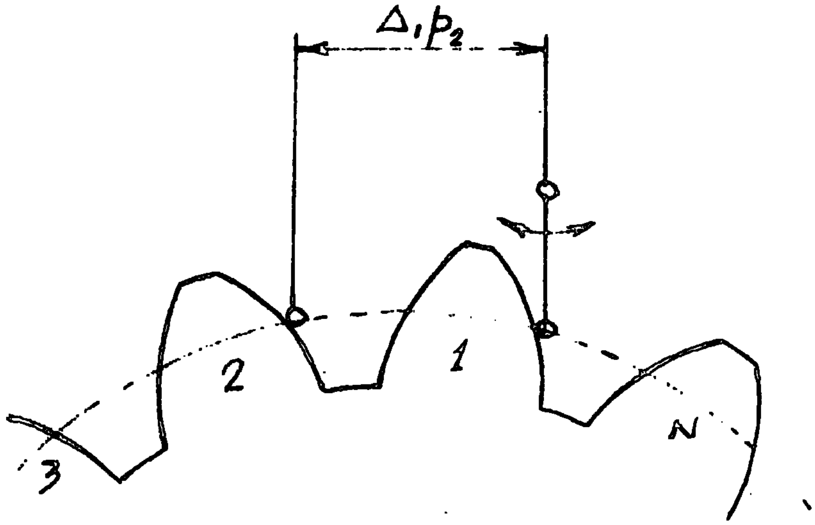


Fig 4.6c Span gauging
∝ flanks.

adjacent pitch measurements in one half the number of teeth in the gear.

$$(\text{Probable error in } y)^2 = \frac{T}{2} \times (\text{error in adjacent pitch measurement})^2$$

$$y = d(\Delta p_a) \cdot \sqrt{\frac{T}{2}}$$

The repeatability of the individual span measurements was $\pm$ 0.0005 mm.

Thus for a 48T gear

$$y = \pm 0.0005 \sqrt{\frac{48}{2}}$$

$$= \pm 0.0025 \text{ mm} \quad (\pm 0.000\ 1 \text{ in})$$

which agrees with the spread of the results indicated by Fig. 4.7c.

Since the cumulative pitch error curve due to mounting the gear eccentrically is built up from two sets of measurements, the estimated inaccuracy in the resulting maximum cumulative pitch will be $\pm$ 0.0035 mm.

4.8. The cumulative pitch error curve for a 36T 7DP 20° AOP gear which was deliberately cut eccentric is shown by Fig. 4.8a curve (i). Analysis of the curve indicated an eccentricity of 0.054 mm having a maximum negative value opposite the β flank of tooth number one. Using the special arbor the gear was "set up" for pitch testing in a manner such as to correct the above eccentricity. The resulting pitch error curve being as illustrated by Fig. 4.8a curve (ii).

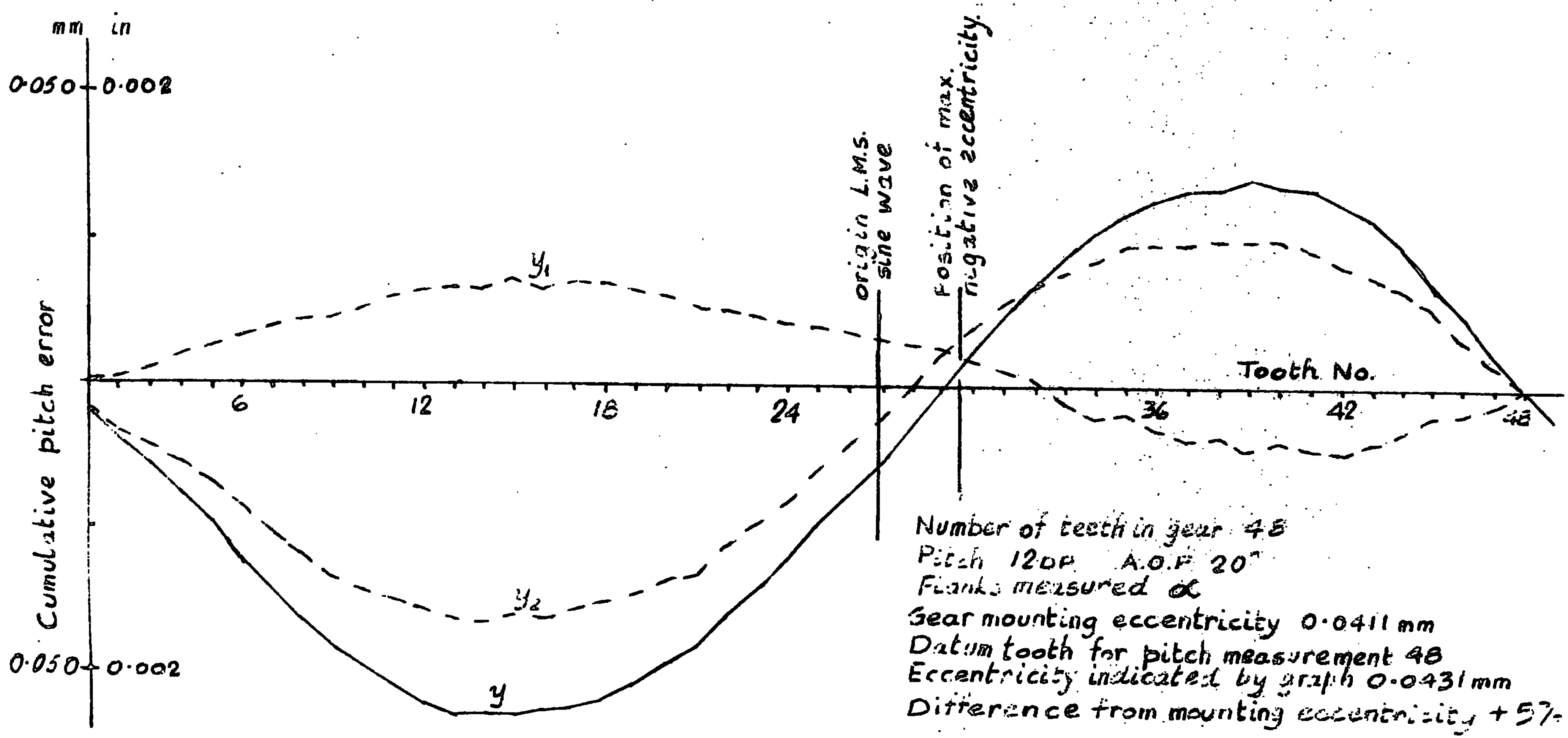


Fig 4.7a Cumulative pitch error graph by span gauging

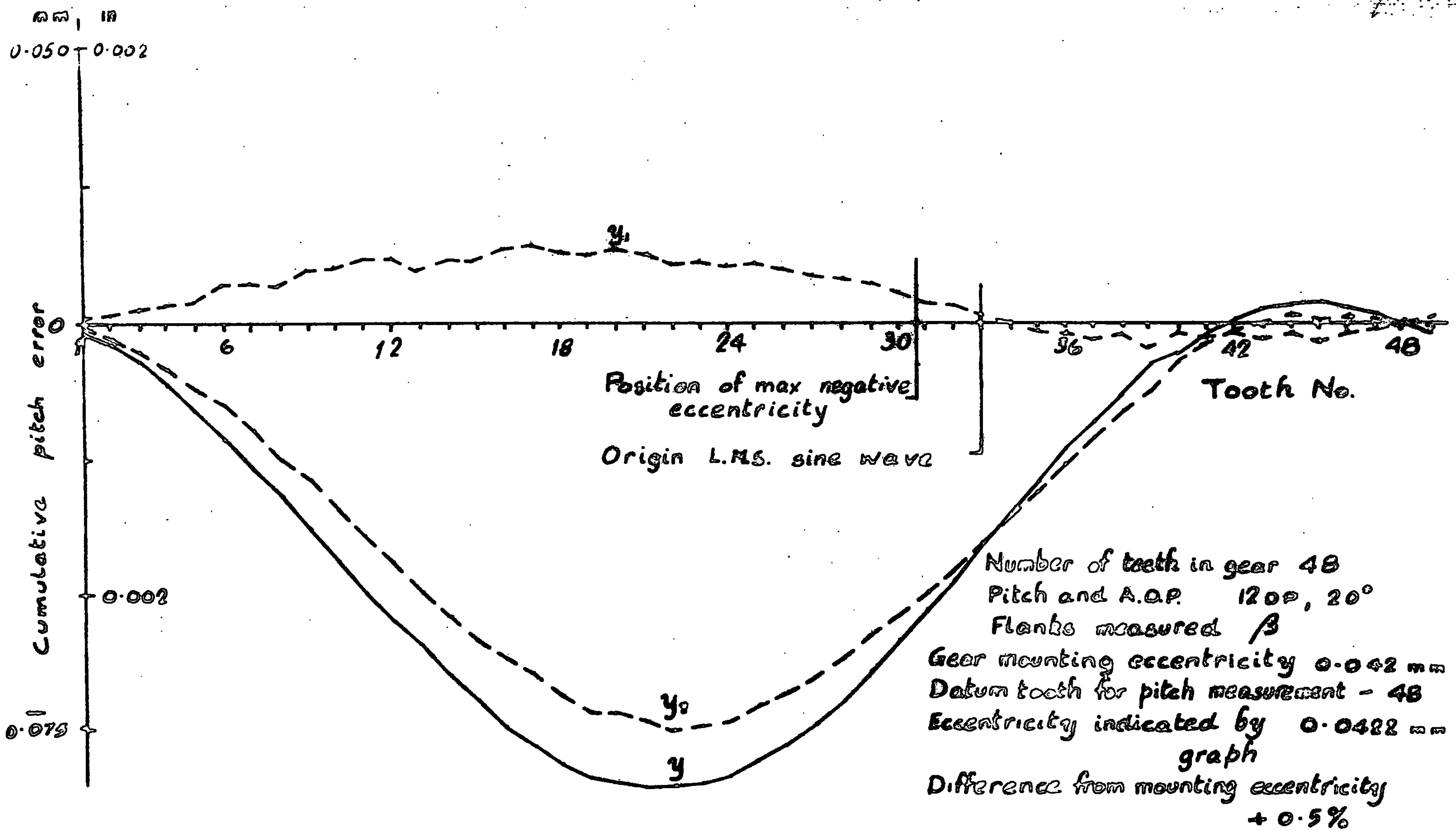


Fig 4.7b Cumulative pitch error graph by span gauging

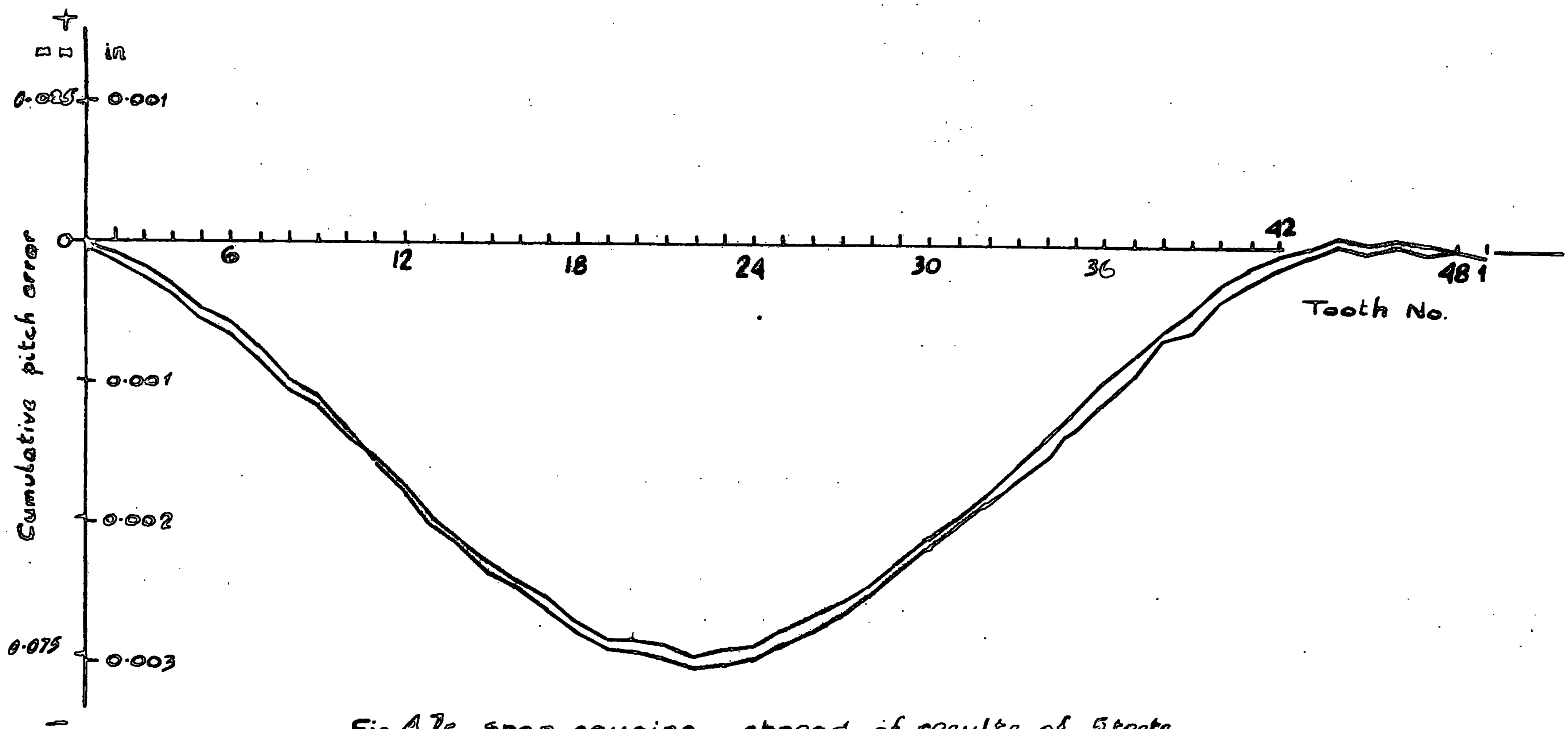


Fig 4.7e Span gauging, spread of results of 5 tests, measured eccentricity of gear 0·042 (0·00165 in).
Gear 48 T. 12 DP 20° AOR

Results after correction:

	A	B	C
Adjacent pitch error	0.011	0.019	0.014
Max. cumulative pitch error	0.036	0.046	0.038

all values in mm.

A = Values from Fig. 4.8a curve (ii)

B = Precision cut gears to BS 436: 1940, Class A 2.

C = BS 436: Part I: 1967, Grade 6.

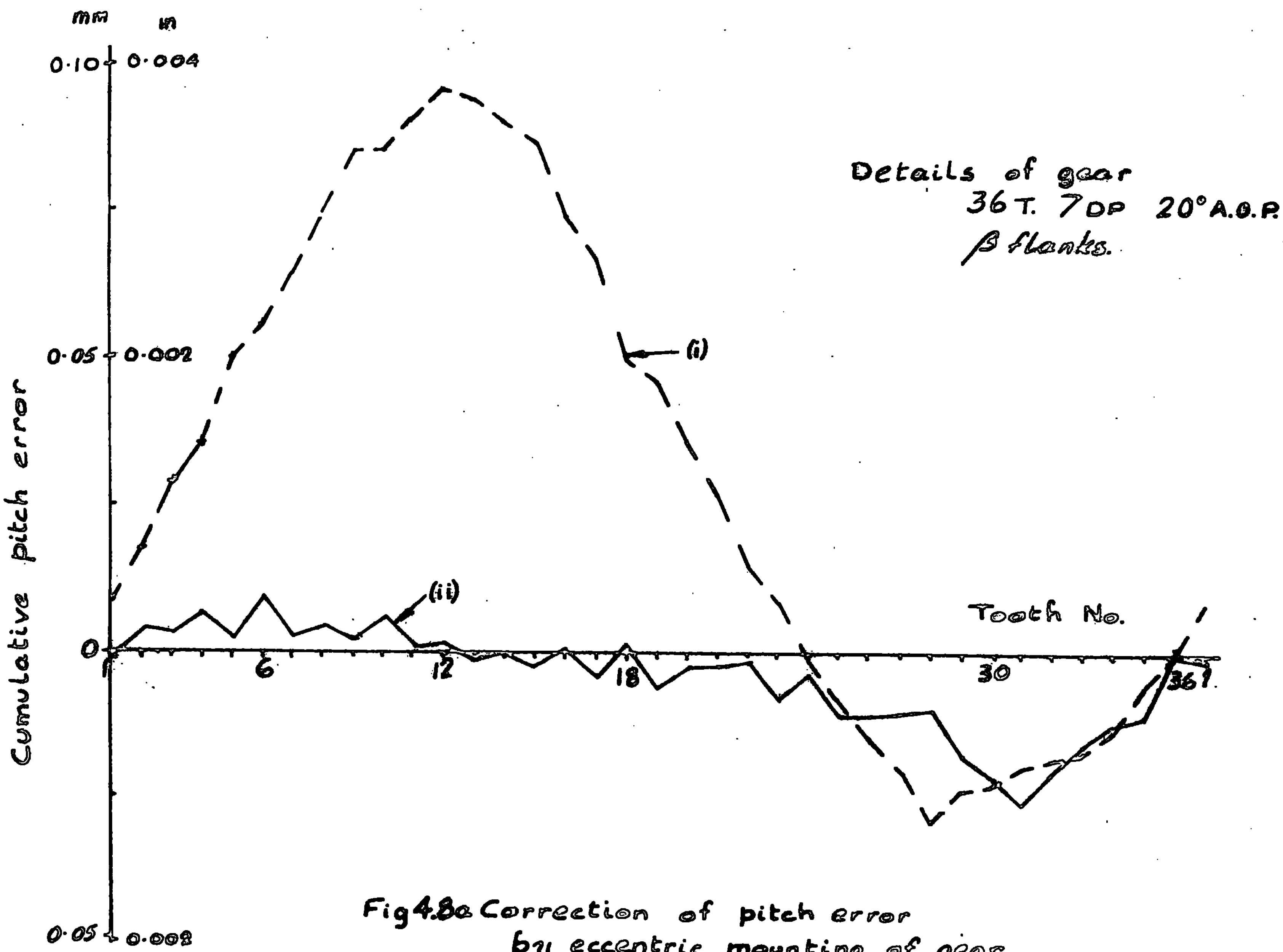


Fig 4.8a Correction of pitch error by eccentric mounting of gear

CHAPTER 5.

Dual flank meshing test - theory.

see also sections 2.9(i) and 2.9(ii).

5.1. When dual flank mesh testing using either a perfect master gear or rack, the radial displacement of the gear under inspection will be determined by tooth contact at α and β tooth flanks as shown in Fig. 5.1a.

Let X_α and X_β be the largest values of the composite tooth error on the α and β flanks measured along the path of contact. Because the gear under inspection is free to rotate about its axis as it is displaced radially these two errors can be assumed to be symetrically disposed between the two flanks

Composite tooth error = cumulative pitch error + tooth profile error

$$\text{effective error} = \frac{X_\alpha + X_\beta}{2} \qquad \text{—— 5.1a}$$

and this error produces a radial displacement of;

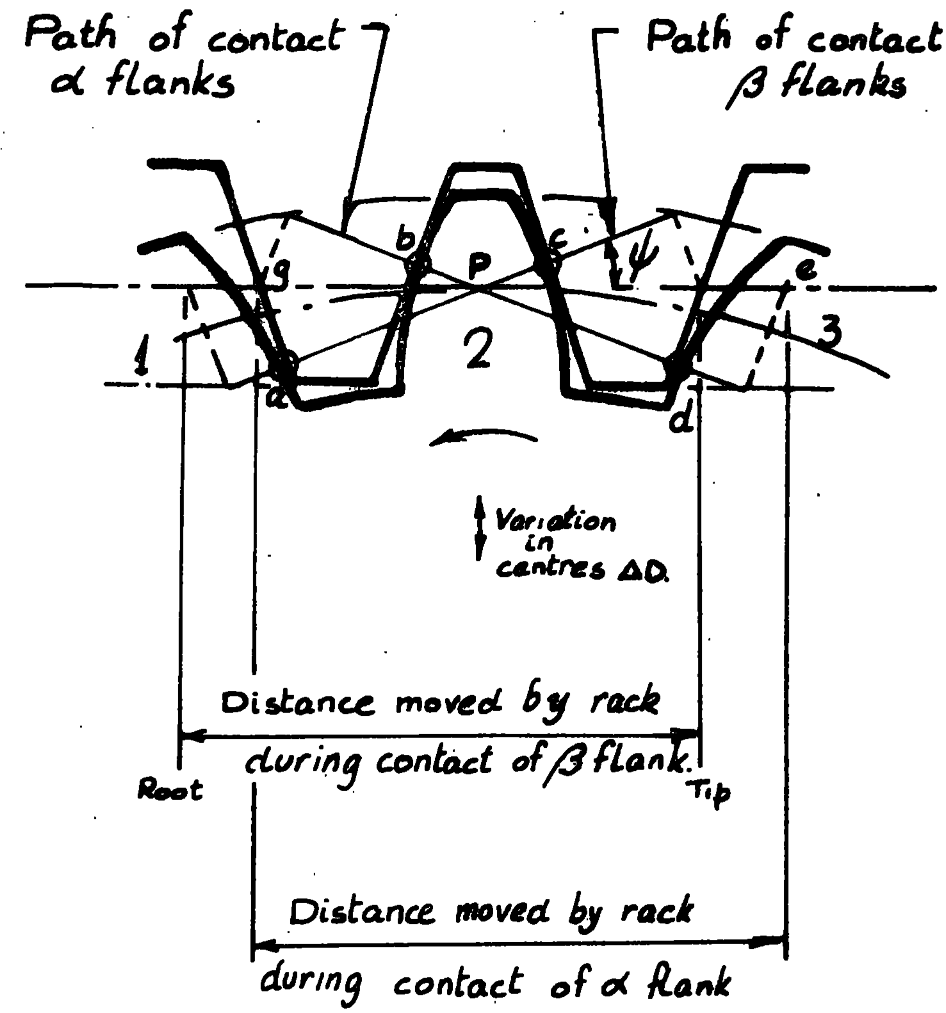
$$\Delta D = \frac{X_\alpha + X_\beta}{2.\ \sin\psi} \qquad \text{—— 5.1b.}$$

5.2. When single flank mesh testing with only the α flanks of the gear under inspection in contact with the master gear. The single flank or transmission error will be determined by the largest value of X_α of the teeth theoretically in contact.

Single flank error

$$\Delta S_\alpha = \frac{X_\alpha}{\cos\psi} \qquad \text{—— 5.2a}$$

measured as a linear displacent at the pitch circle of the gear.



The centre distance will be determined by contact at one of the following pairs of points acting in a manner similar to a wedge;

bc , ad, ab, or cd

Fig 5.1a   Dual flank rolling gear test

and for the β flanks

$$\Delta S_\beta = \frac{X_\beta}{\cos\psi} \qquad \text{———} \qquad 5.2b.$$

$$(\Delta S_\alpha + \Delta S_\beta)\cos\psi = X_\alpha + X_\beta$$

$$= 2.\Delta D.\ \sin\psi$$

$$\Delta D = \frac{\Delta S_\alpha + \Delta S_\beta}{2\tan\psi} \qquad \text{———} \qquad 5.2c.$$

5.3. If the master gear has composite tooth errors of such a magnitude that they can not be neglected, then the value X used in the above equations should be the algebraic sum of the errors of the master gear and the gear under inspection at the points of tooth contact.

5.4. The above theory neglects the small change in pressure angle with the variation in centre distance when dual flank mesh testing. This variation will have a negligible effect unless the gear under inspection has excessively thin teeth thus allowing a comparatively large difference between the nominal centre distance and the running centre distance when dual flank mesh testing (see appendix II).

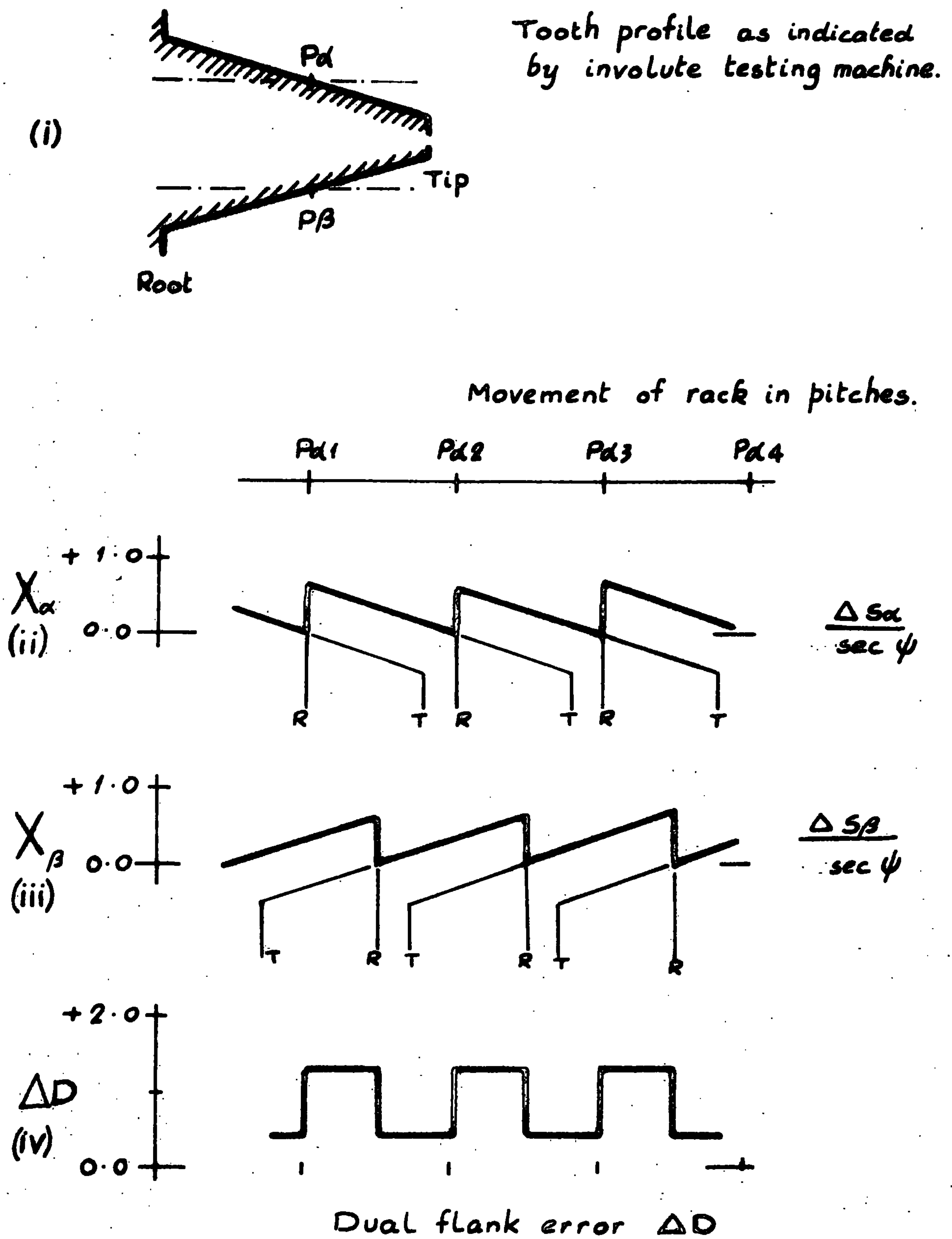
5.5. Figs. 5.5a to 5.51 show theoretical single and dual flank meshing test error curves for gears having various types of involute profile error. Fig. 5.5m illustrates the effect on Fig. 5.51 of the introduction of a tooth pitch error.

The method of constructing the graphs is as follows; it is assumed that the gear under inspection is in tight mesh with a perfect master rack Fig. 5.1a. With anti-clockwise rotation of the gear the tooth contact for any α gear tooth flank will commence at the root of the gear tooth and cease at the tip, during which period the rack moves a distance e - g. For the β flanks of the gear teeth the initial point of contact will be at the tip of the tooth and the final point of contact at the root. In order to orientate these two paths of tooth contact it is known that the movement of the rack between contact with the pitch points of the α and β flanks of the same tooth will be equal to one half the rack pitch.

It should be noted that on the tooth profile errors (i), illustrated by Fig. 5.5a to 5.5l the position marked root represents the point of root tooth contact and not the actual root of the teeth.

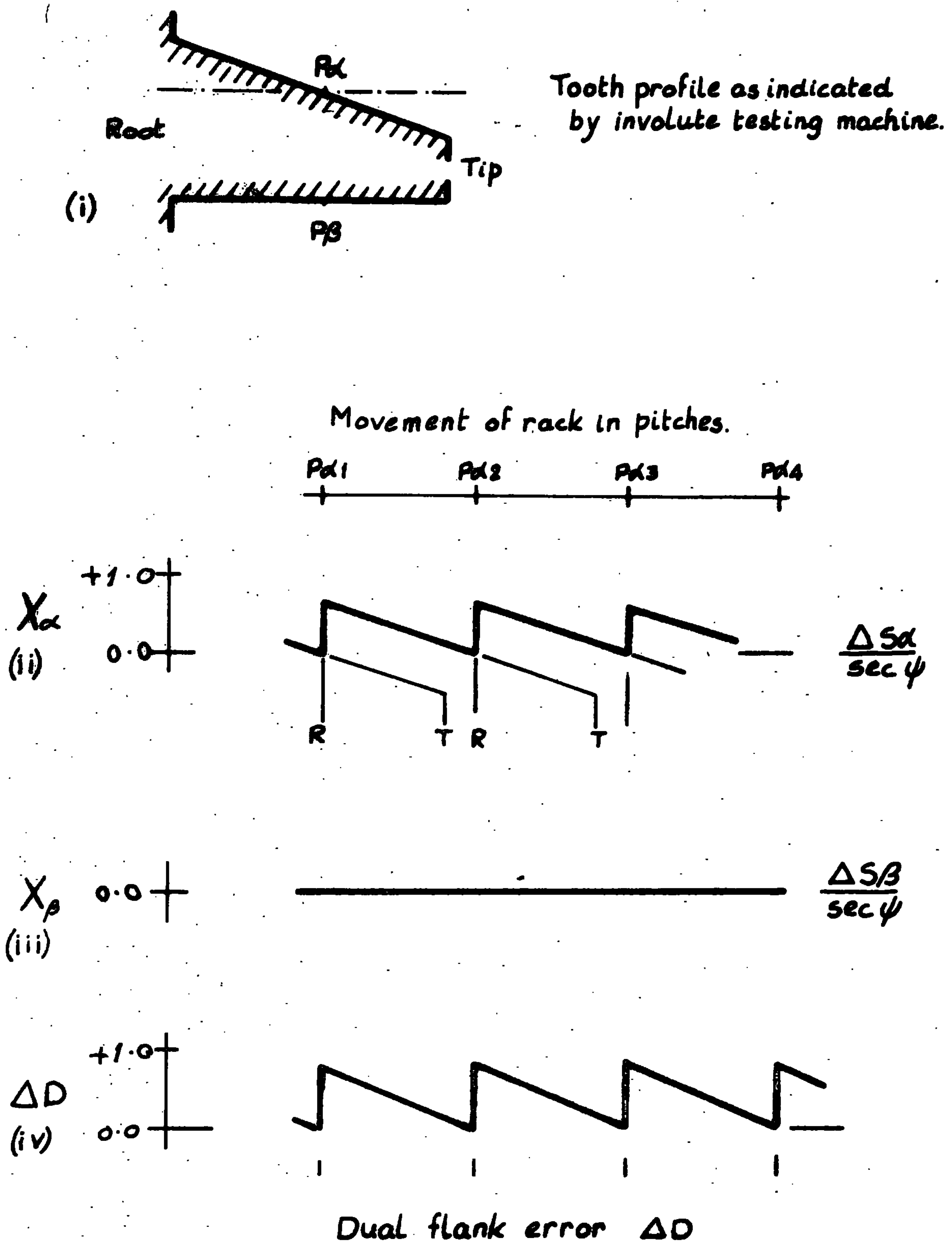
The graphs shown are based in a 20T 2DP 20° A.O.P. pinion (for the relevant calculations see Fig. 5.5n).

5.6. In practice a master gear is more commonly used than a master rack, the only effect this would have on the given theoretical curves is a reduction in the length of the path of contact.



Tooth profile as indicated by involute testing machine.
(i)
Pα
Pβ
Tip
Root
Movement of rack in pitches.
Pα1
Pα2
Pα3
Pα4
+1·0
0·0
X_α
(ii)
R
T
$\frac{\Delta S\alpha}{\sec\psi}$
X_β
(iii)
$\frac{\Delta S\beta}{\sec\psi}$
+2·0
ΔD
(iv)
Dual flank error ΔD

Fig. 5.5a



Pα
Root
Tip
(i)
Pβ
Tooth profile as indicated by involute testing machine.
Movement of rack in pitches.
Pα1
Pα2
Pα3
Pα4
X_α
(ii)
+1·0
0·0
R
T
R
T
$\frac{\Delta S\alpha}{\sec\psi}$
X_β
(iii)
0·0
$\frac{\Delta S\beta}{\sec\psi}$
ΔD
(iv)
+1·0
0·0
Dual flank error ΔD

Fig 5.5b

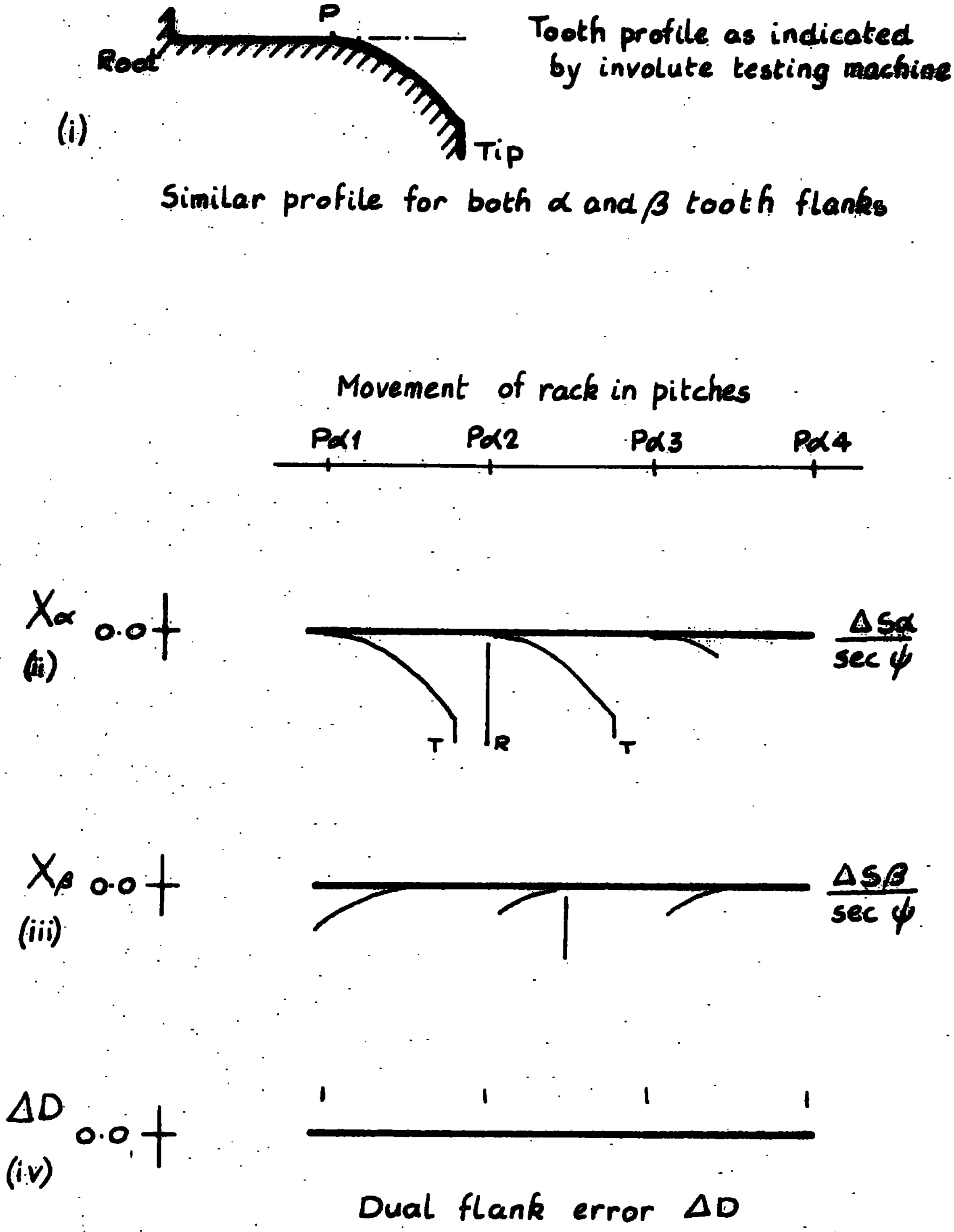
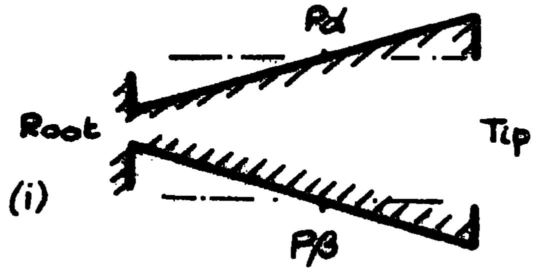

P
Root
Tip
Tooth profile as indicated by involute testing machine
(i)
Similar profile for both α and β tooth flanks
Movement of rack in pitches
Pα1
Pα2
Pα3
Pα4
Xα 0·0
(ii)
T
R
T
ΔSα / sec ψ
Xβ 0·0
(iii)
ΔSβ / sec ψ
ΔD 0·0
(iv)
Dual flank error ΔD

Fig 5.5c



Tooth profile as indicated by involute testing machine.

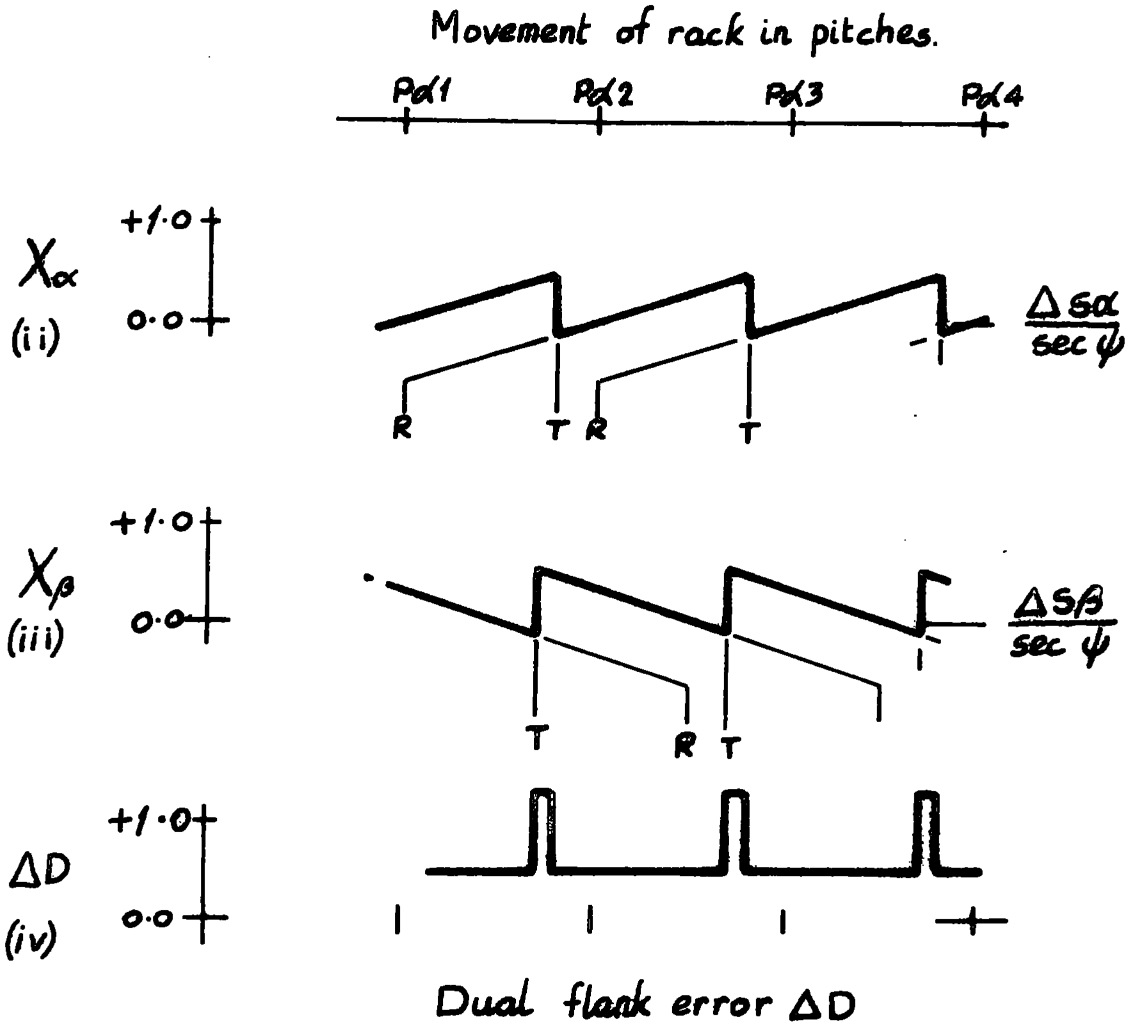
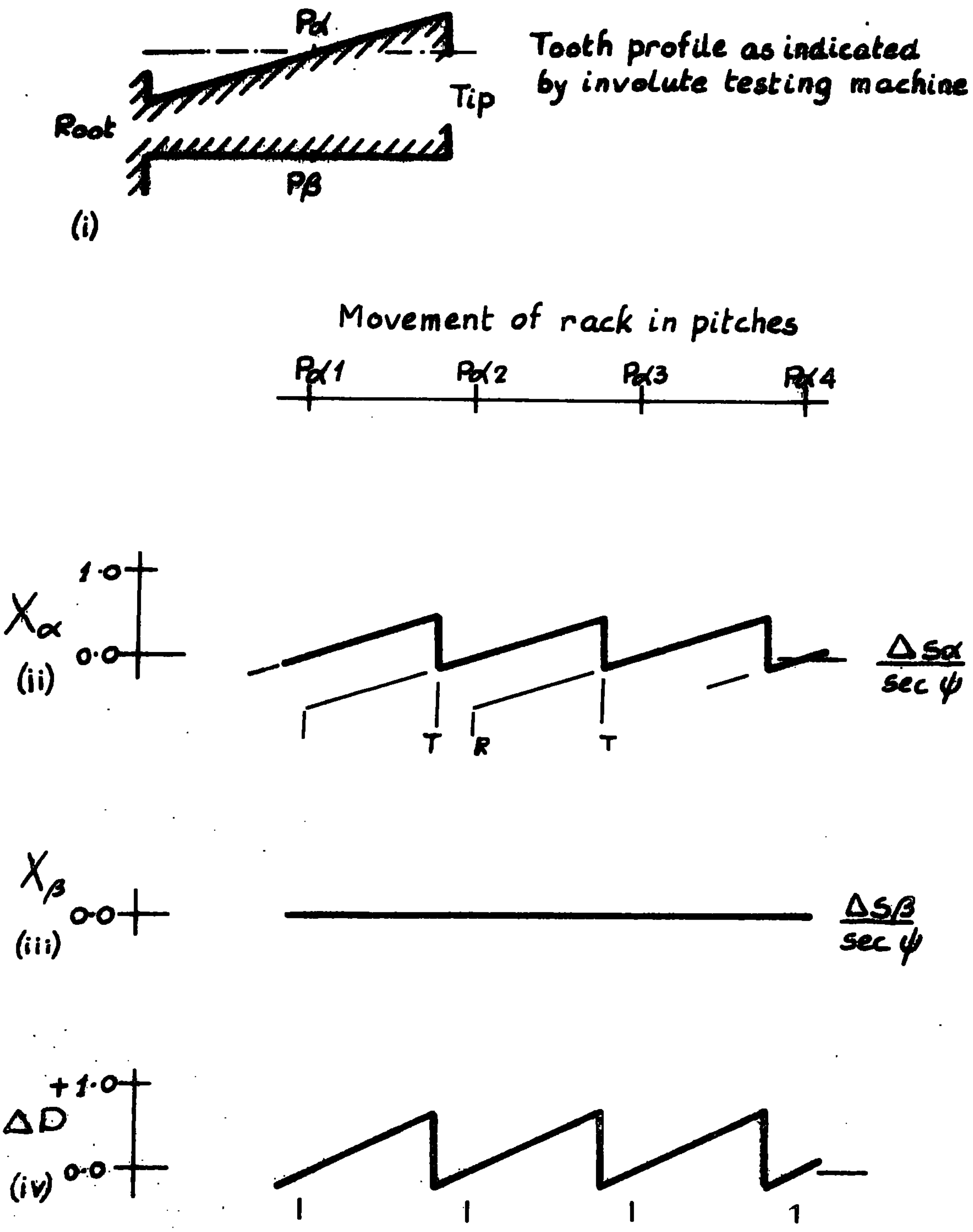


Fig 5.5d



Dual flank error ΔD

Fig 5.5e

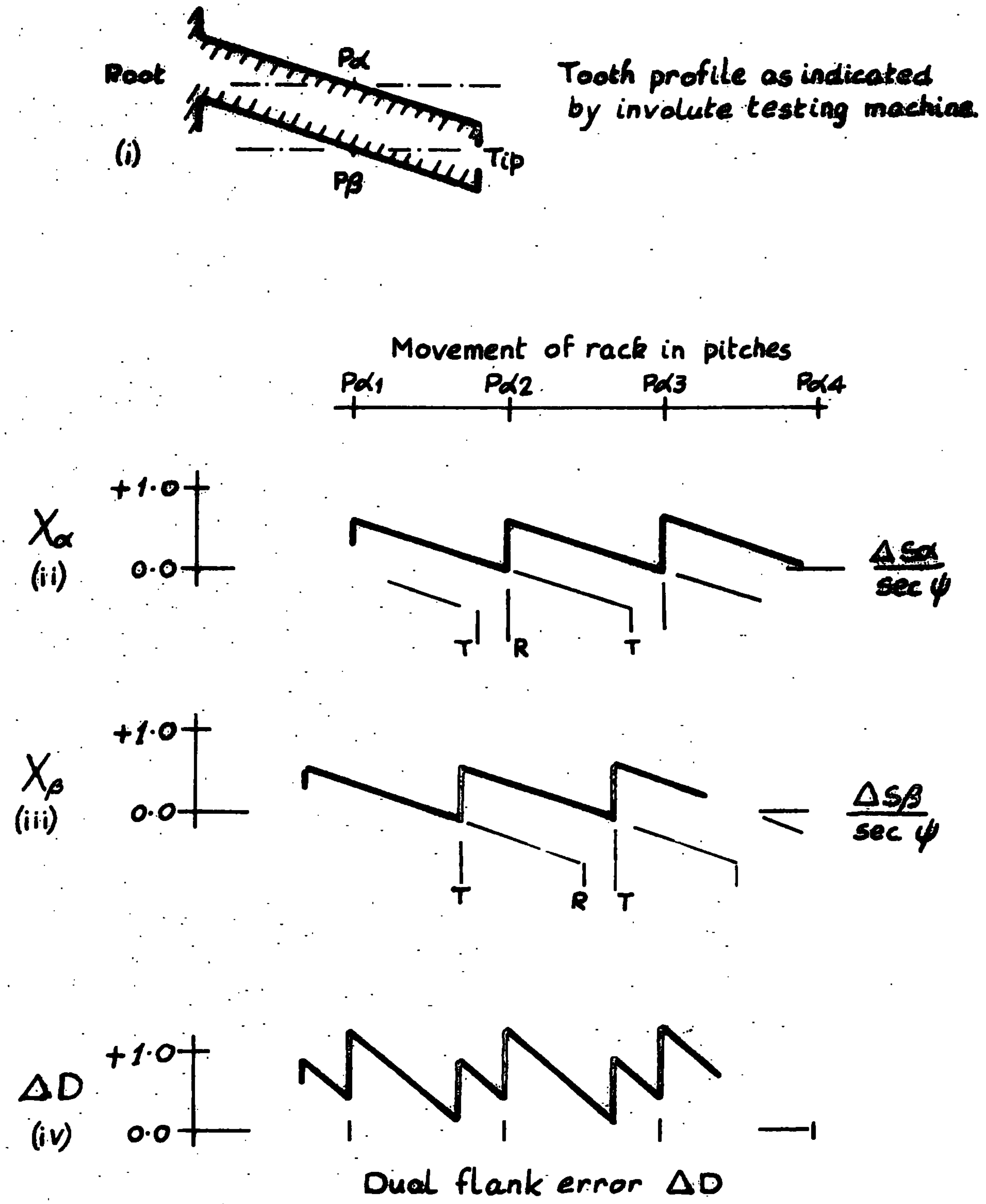

Root
Pα
Tooth profile as indicated by involute testing machine.
(i)
Pβ
Tip
Movement of rack in pitches
Pα1
Pα2
Pα3
Pα4
+1·0
X_α
(ii)
0·0
$\frac{\Delta S\alpha}{\sec \psi}$
T
R
T
+1·0
X_β
(iii)
0·0
$\frac{\Delta S\beta}{\sec \psi}$
T
R
T
+1·0
ΔD
(iv)
0·0
Dual flank error ΔD

Fig. 5.5f

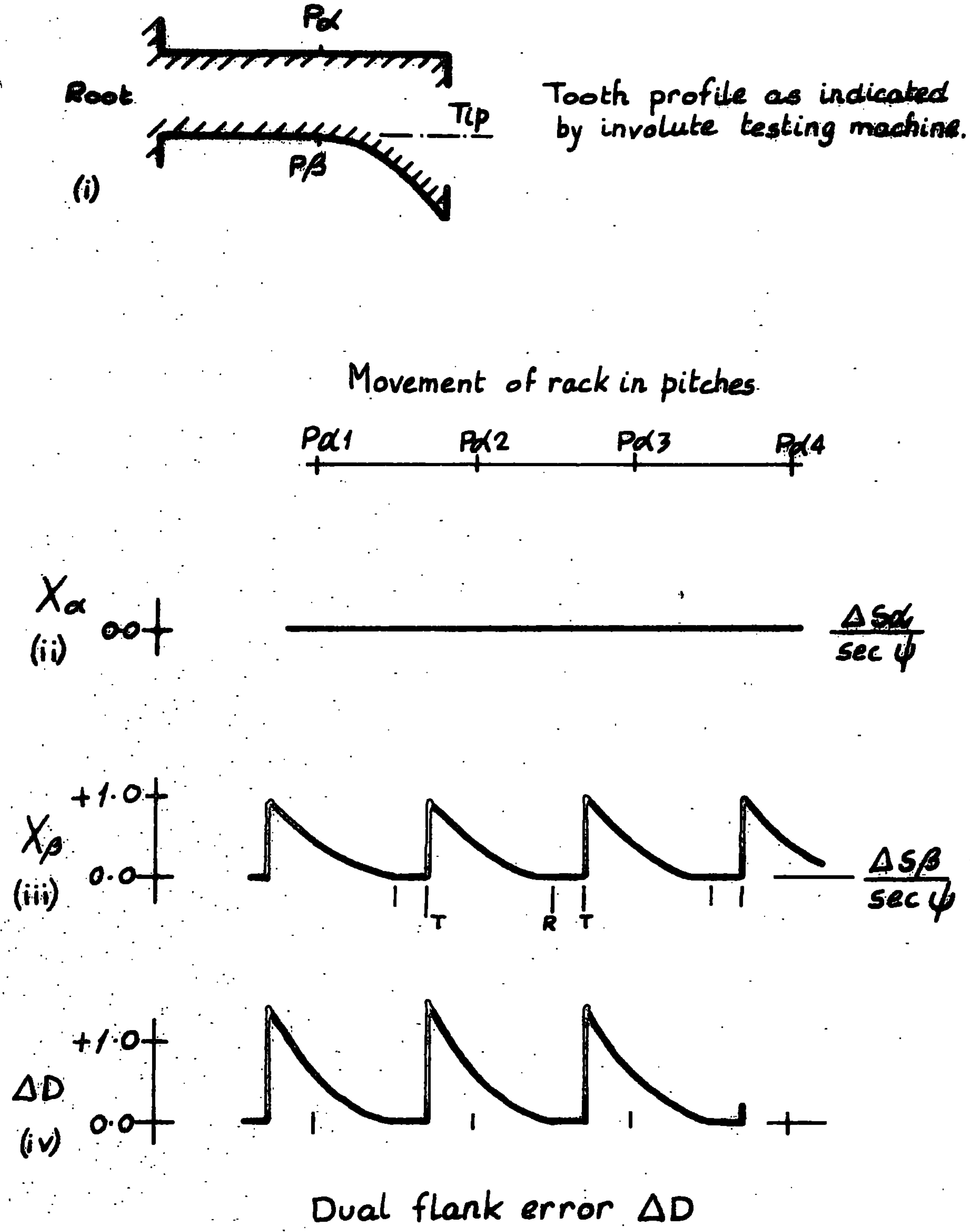

Pα
Root
Tip
Pβ
(i)
Tooth profile as indicated by involute testing machine.
Movement of rack in pitches
Pα1
Pα2
Pα3
Pα4
Xα
(ii)
0.0
ΔSα / sec ψ
+1.0
Xβ
(iii)
0.0
T
R
T
ΔSβ / sec ψ
+1.0
ΔD
(iv)
0.0
Dual flank error ΔD

Fig. 5.5g.

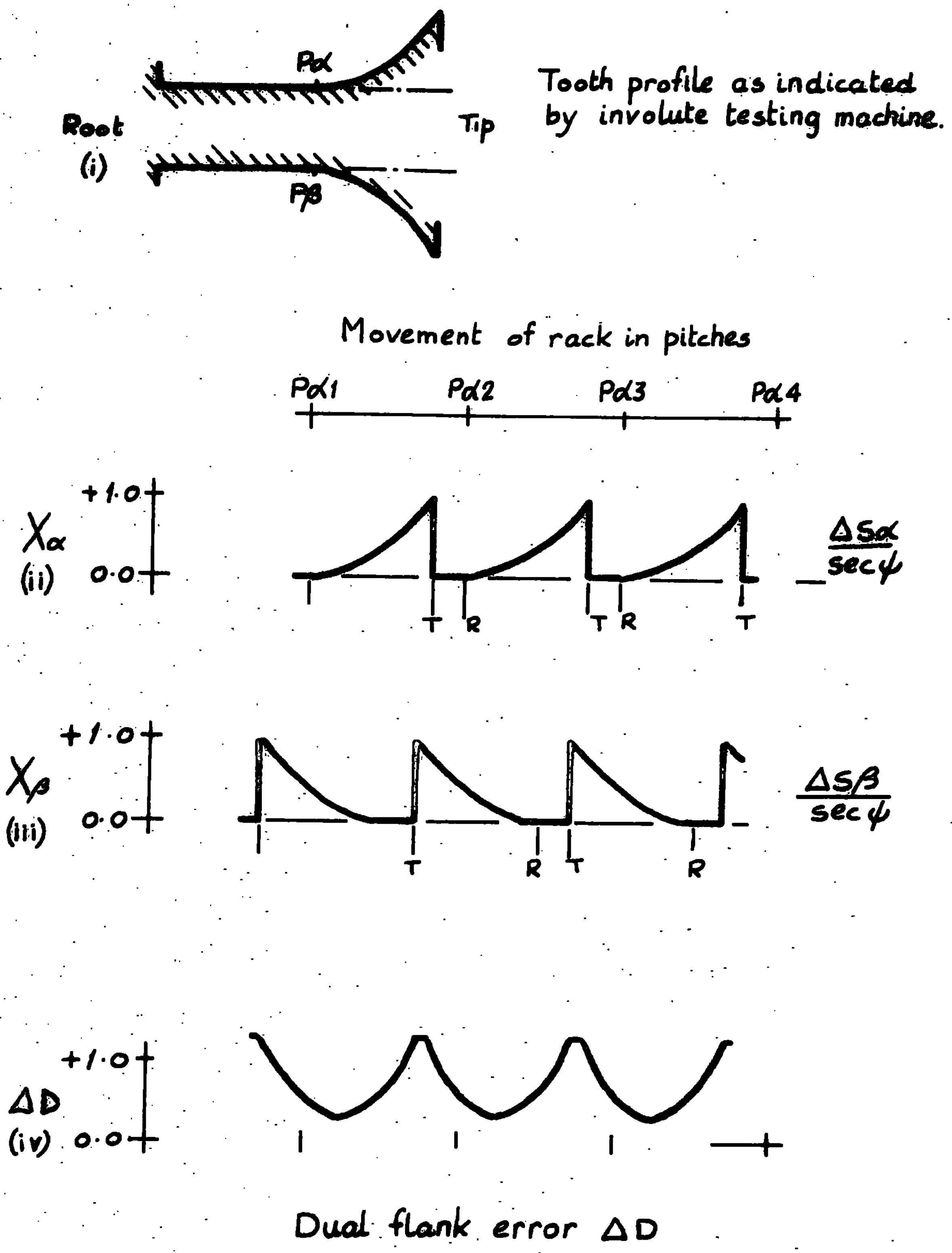
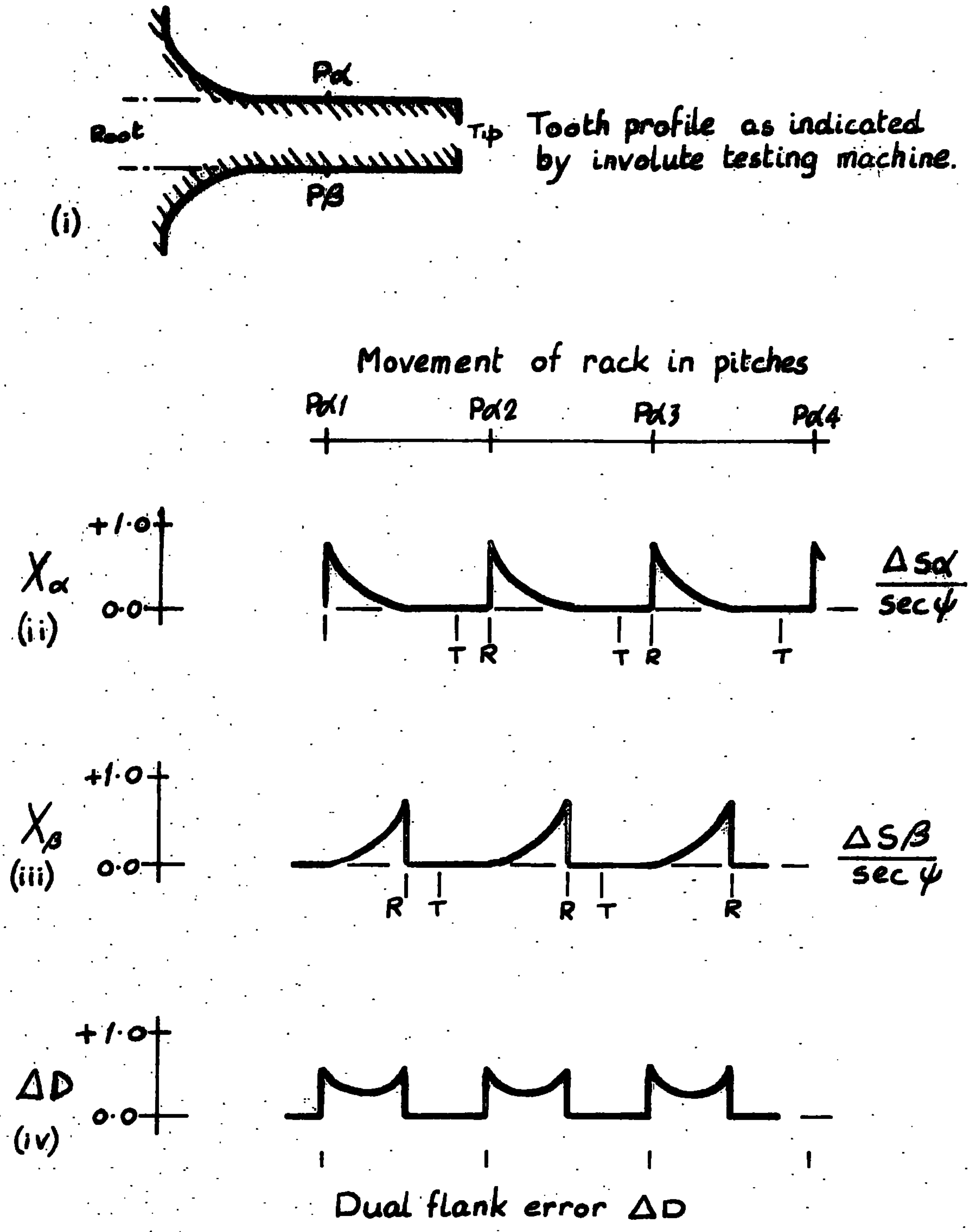

Root
(i)
Pα
Pβ
Tip
Tooth profile as indicated by involute testing machine.
Movement of rack in pitches
Pα1
Pα2
Pα3
Pα4
Xα
(ii)
+1.0
0.0
T
R
T
R
T
ΔSα / sec ψ
Xβ
(iii)
+1.0
0.0
T
R
T
R
ΔSβ / sec ψ
ΔD
(iv)
+1.0
0.0
Dual flank error ΔD

Fig. 5.5h



Pα
Root
Tip
Tooth profile as indicated by involute testing machine.
Pβ
(i)
Movement of rack in pitches
Pα1
Pα2
Pα3
Pα4
+1·0
X_α
0·0
(ii)
T R
T R
T
$\frac{\Delta S\alpha}{\sec\psi}$
+1·0
X_β
0·0
(iii)
R T
R T
R
$\frac{\Delta S\beta}{\sec\psi}$
+1·0
ΔD
0·0
(iv)
Dual flank error ΔD

Fig. 5.5j

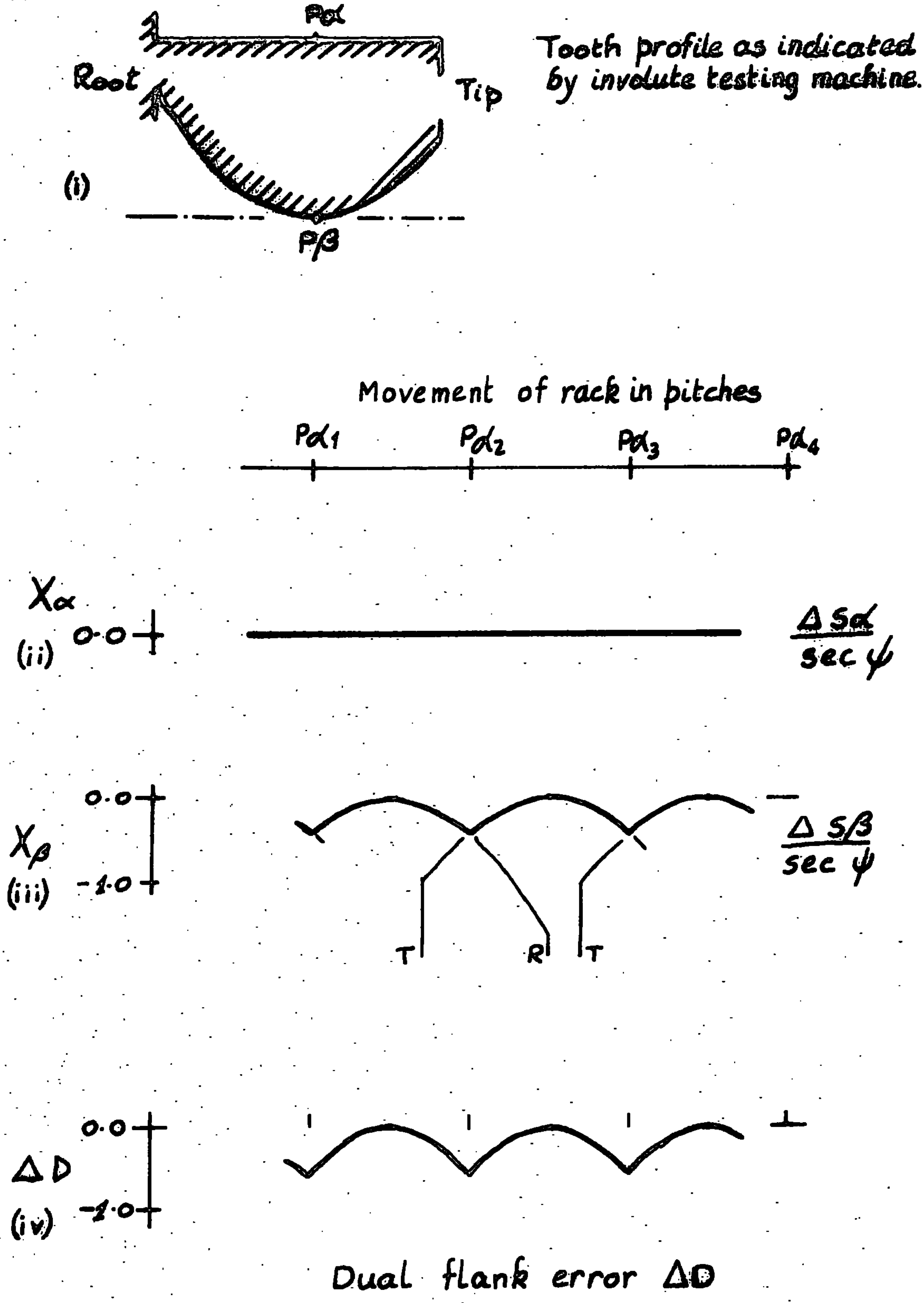

Tooth profile as indicated by involute testing machine.
Pα
Root
Tip
(i)
Pβ
Movement of rack in pitches
Pα1
Pα2
Pα3
Pα4
Xα
(ii)
0·0
ΔSα / sec ψ
Xβ
(iii)
0·0
-1·0
ΔSβ / sec ψ
T
R
T
ΔD
(iv)
0·0
-1·0
Dual flank error ΔD

Fig. 5.5k.

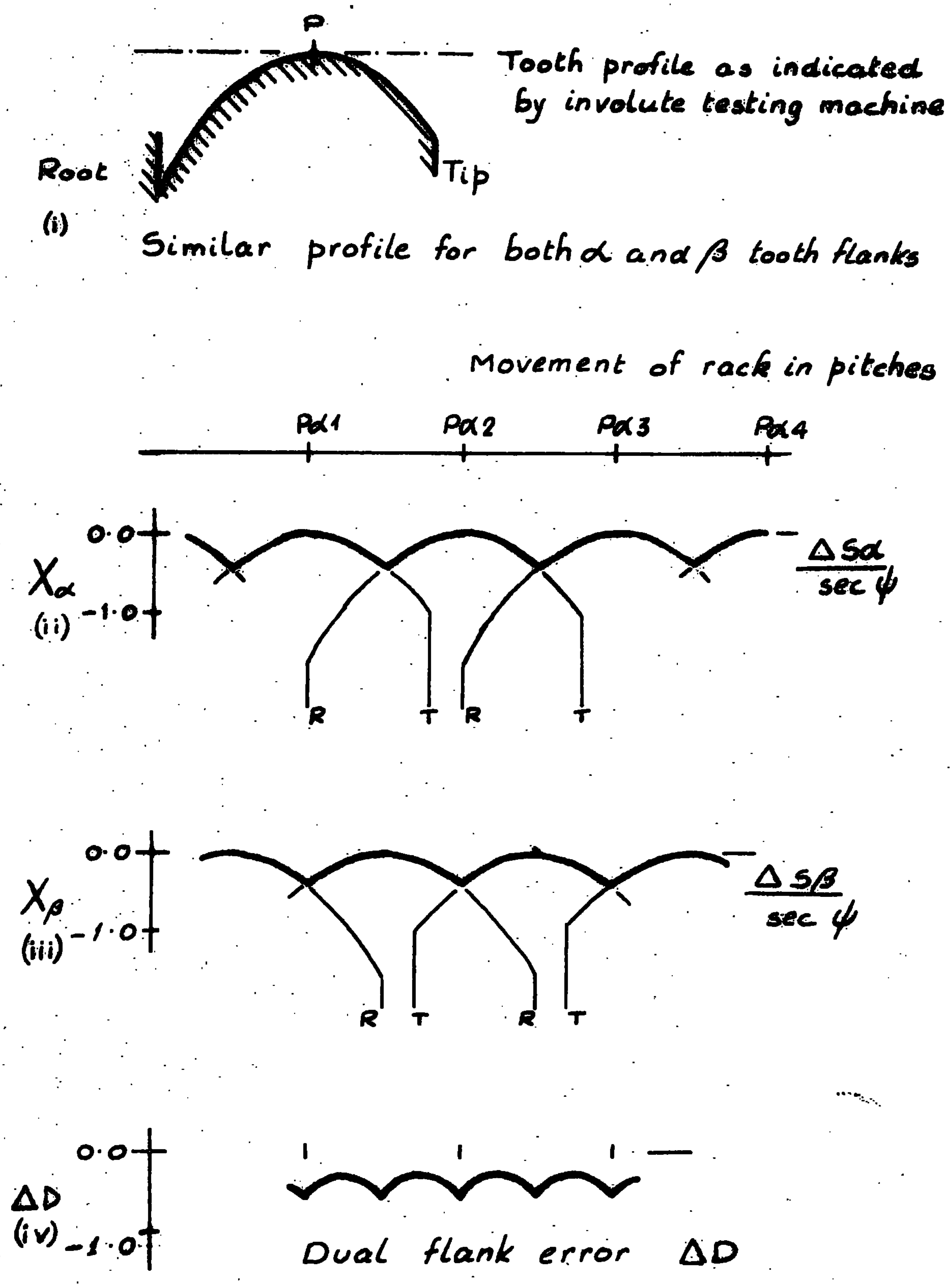
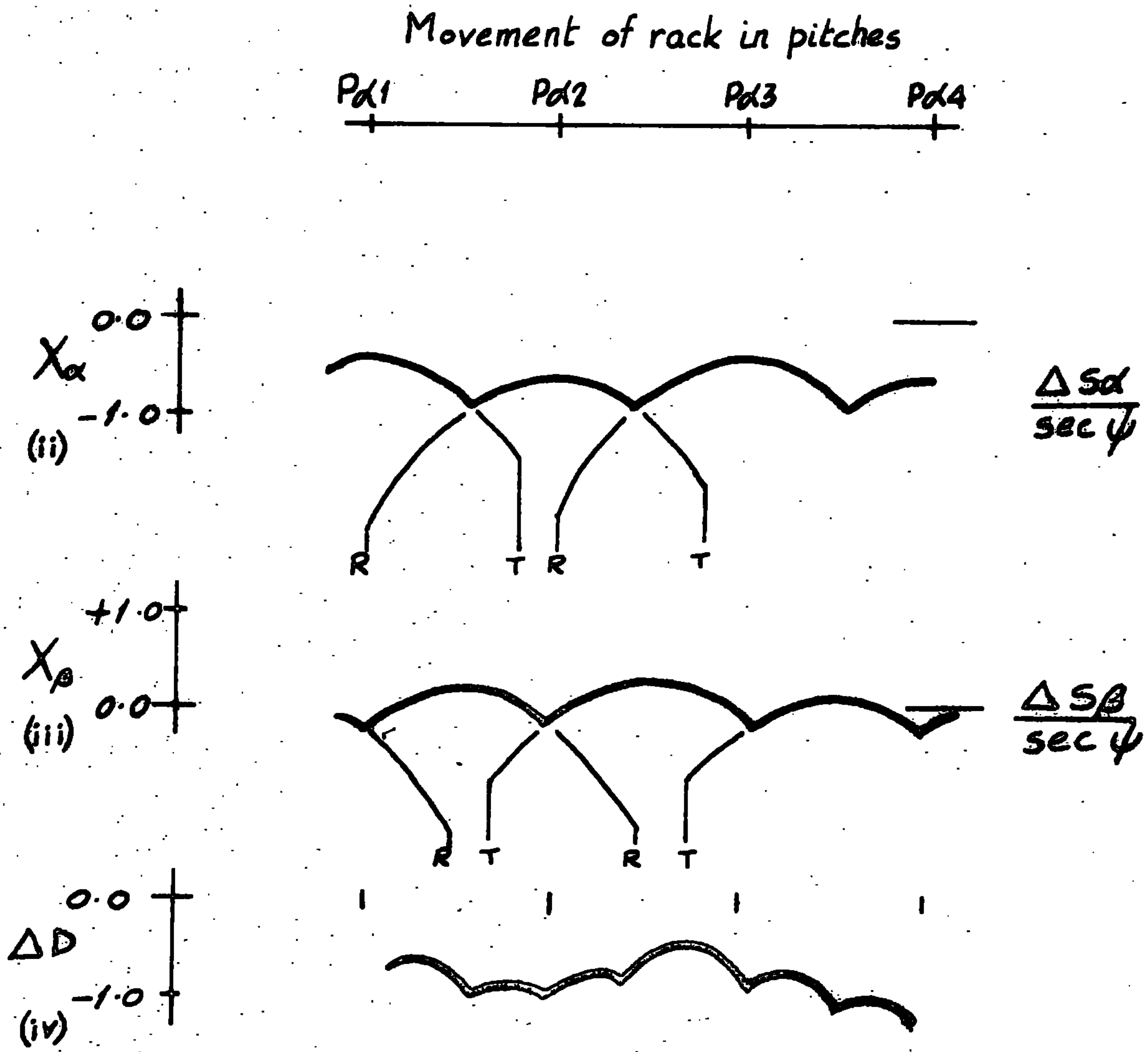

P
Tooth profile as indicated by involute testing machine
Root
Tip
(i)
Similar profile for both α and β tooth flanks
Movement of rack in pitches
Pα1
Pα2
Pα3
Pα4
0·0
Xα
(ii)
-1·0
ΔSα / sec ψ
R
T
R
T
0·0
Xβ
(iii)
-1·0
ΔSβ / sec ψ
R
T
R
T
0·0
ΔD
(iv)
-1·0
Dual flank error ΔD

Fig 5.5l.

(i) Tooth profiles as in fig. 5.5l but having pitch error on both α and β flanks.



Dual flank error ΔD

Fig. 5.5m

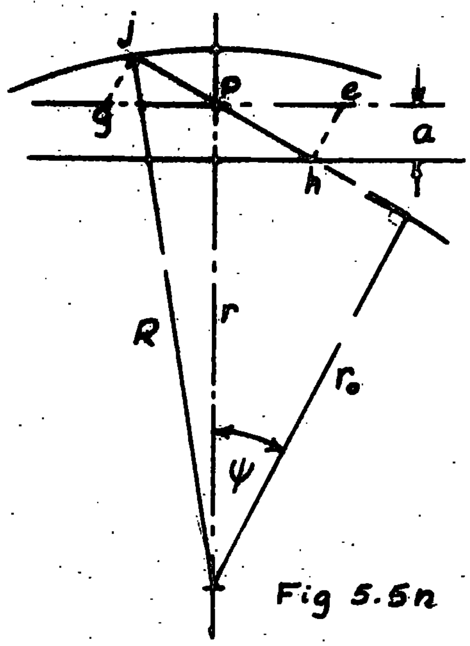


Fig 5.5n

Outside radius of gear

$R = 139.700\ mm$

Pitch circle radius

$r = 127.000\ mm$

Base circle radius

$r_o = 119.355\ mm$

$a = 12.700\ mm$

$\psi = 20°$

Length of path of contact

$$hj = (R^2 - r_o^2)^{\frac{1}{2}} - (r^2 - r_o^2)^{\frac{1}{2}} + a\,\mathrm{cosec}\,\psi$$

$$= (139.7^2 - 119.355^2)^{\frac{1}{2}} - (127^2 - 119.355^2)^{\frac{1}{2}} + 12.7\,\mathrm{cosec}\,20°$$

$$= 66.370\ mm$$

Movement of rack during full contact with a single tooth flank $= hj.\sec\psi$

i.e. $ge = 70.637\ mm$ $[1.77\,p_c]$

$Pg = 31.115\ mm$ $[0.78\,p_c]$

$Pe = 39.522\ mm$ $[0.99\,p_c]$

where p_c = Linear pitch of the mating rack $= 12.7\pi\ mm$.

CHAPTER 6.

Dual flank meshing test - experimental.

6.1. Measurements were made to determine the cumulative pitch and tooth profile errors of a pair of gears and the data so obtained used to construct a dual flank meshing test error curve using the theory derived in Chapter 5. This constructed curve was then compared with one obtained by direct measurement of the variation in centre distance when the two gears were rotated on a dual flank testing machine.

6.2. Tests were carried out using three pairs of gears:

(i) master gear and a reject master gear both having 48T, 12 DP, 20° A.O.P., face width of teeth 50 mm

(ii) master gear and machine cut gear having 36T, 7 DP 20° A.O.P., face widths; master gear 75 mm, cut gear 12.7 mm

(iii) master gear as in (ii) and a test gear having 36T, 7 DP but with a deliberate profile error.

The face width of the two 36T test gears was made narrow in order to minimise the effect of tooth alignment errors, pitch variation and profile variation across the tooth face. Cutting was carried out on a rack planing machine with a cutter having thin teeth which would normally be used for "roughing out" ground finished gears. The cutting process was controlled to give standard tooth thickness at the reference circle this resulting in a greater than standard root clearance. This extra clearance ensured that there would be no fouling between the fillet radius at the root of the test gear and the unrelieved tips of the master gear.

The test gear with deliberately exaggerated profile error was produced on a gear grinding machine using a formed grinding wheel. On the machine used, the shape of the grinding wheel profile is maintained by two diamond dressers, one for each side of the wheel. The path traced out by the diamonds being controlled from two master profiles through a reducing pantograph. Two master profiles made in the form of circular arcs were produced on a numerically controlled milling machine and these profiles used on the grinding machine in place of the normal involute masters. The radius of the arc being calculated so as to give excessive relief to the tips and roots of the test gear, and yet keep the profile error relative to the true involute within the capacity of the profile testing machine.

6.3. Cumulative pitch error curves for each of the gears used were determined by the span gauging technique as described in Chapter 4. In the case of the 48T wheels three sets of pitch measurements were obtained at positions equally spaced across the gear face, the mean value being used in the subsequent calculations. The datum for the cumulative pitch errors was tooth number 48 in the case of the 12 DP gears and tooth 36 for the 7 DP gears, for both α and β tooth flanks.

A Goulder I2 involute testing machine fitted with chart recording facilities was used to determine the tooth profile errors.

Dual flank meshing test errors were measured using a Goulder R2 rolling gear testing machine. The chart recorder fitted to this machine was not used as it was found more convenient to obtain the dual flank error curve on an incremental basis, in the following manner. The test gear was mounted on the spindle of the floating carriage of the testing machine, the master gear being assembled on the fixed spindle. The variation in centre distance (dual flank error) was then measured every half degree of arc rotation of the master gear. To obtain this indexing a precision square (from a set of angle slip gauges) was placed on the master gear and the angle turned through measured using an "angle-dekkor". This method was chosen because the length of the dual flank error trace produced by the chart recorder for one revolution of the gear pair was too short to enable accurate measurements to be made at small intervals of gear rotation.

Results obtained for the three pairs of gears described in section 6.2 are shown on:

Fig. 6.3a	48T, 12 DP, gear pair (6.2(i))
Fig. 6.3b	36T, 7 DP gear pair, test gear machine cut (6.2(ii))
Fig. 6.3c	36T 7 DP gear pair, test gear profile ground (6.2(iii))

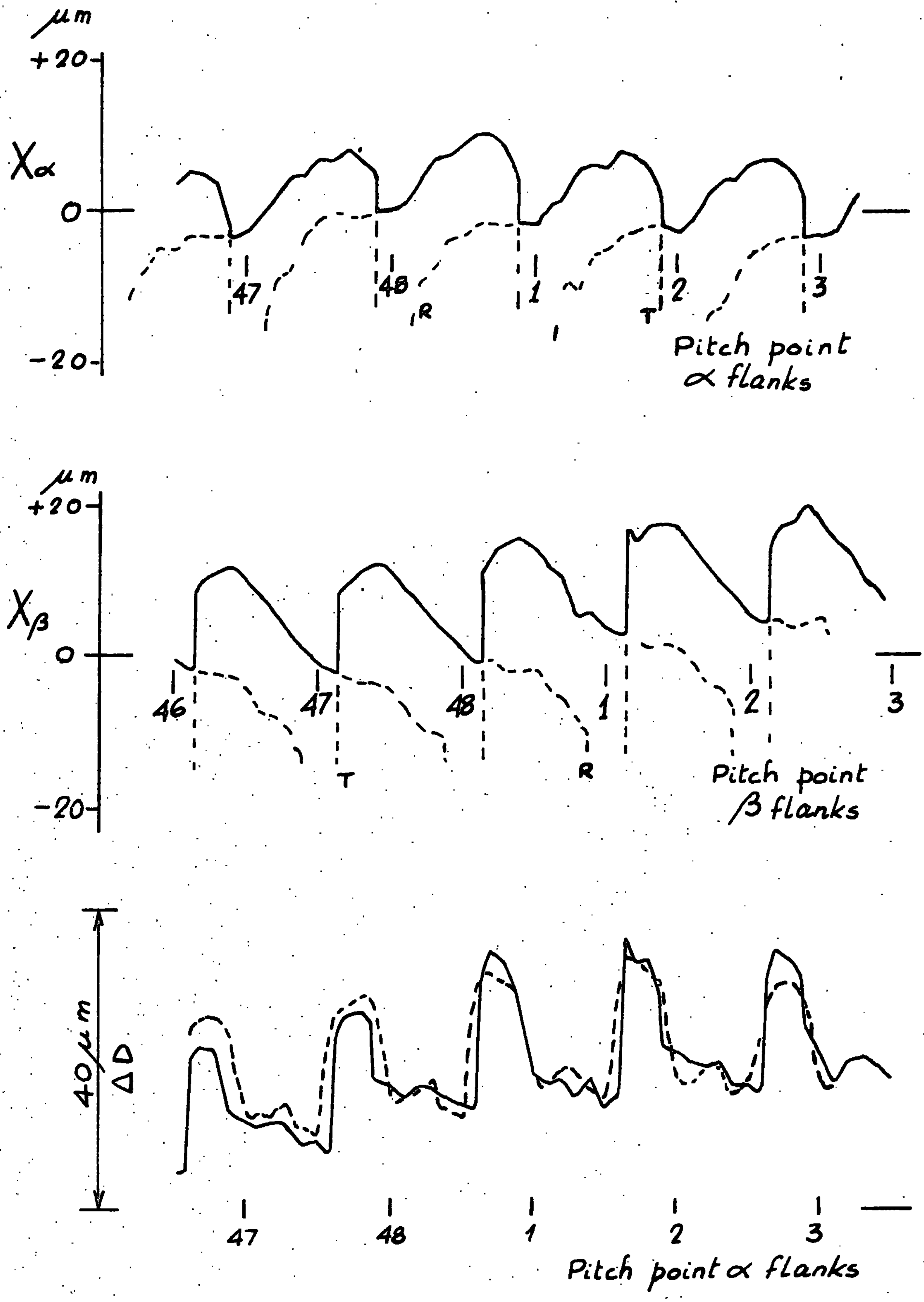
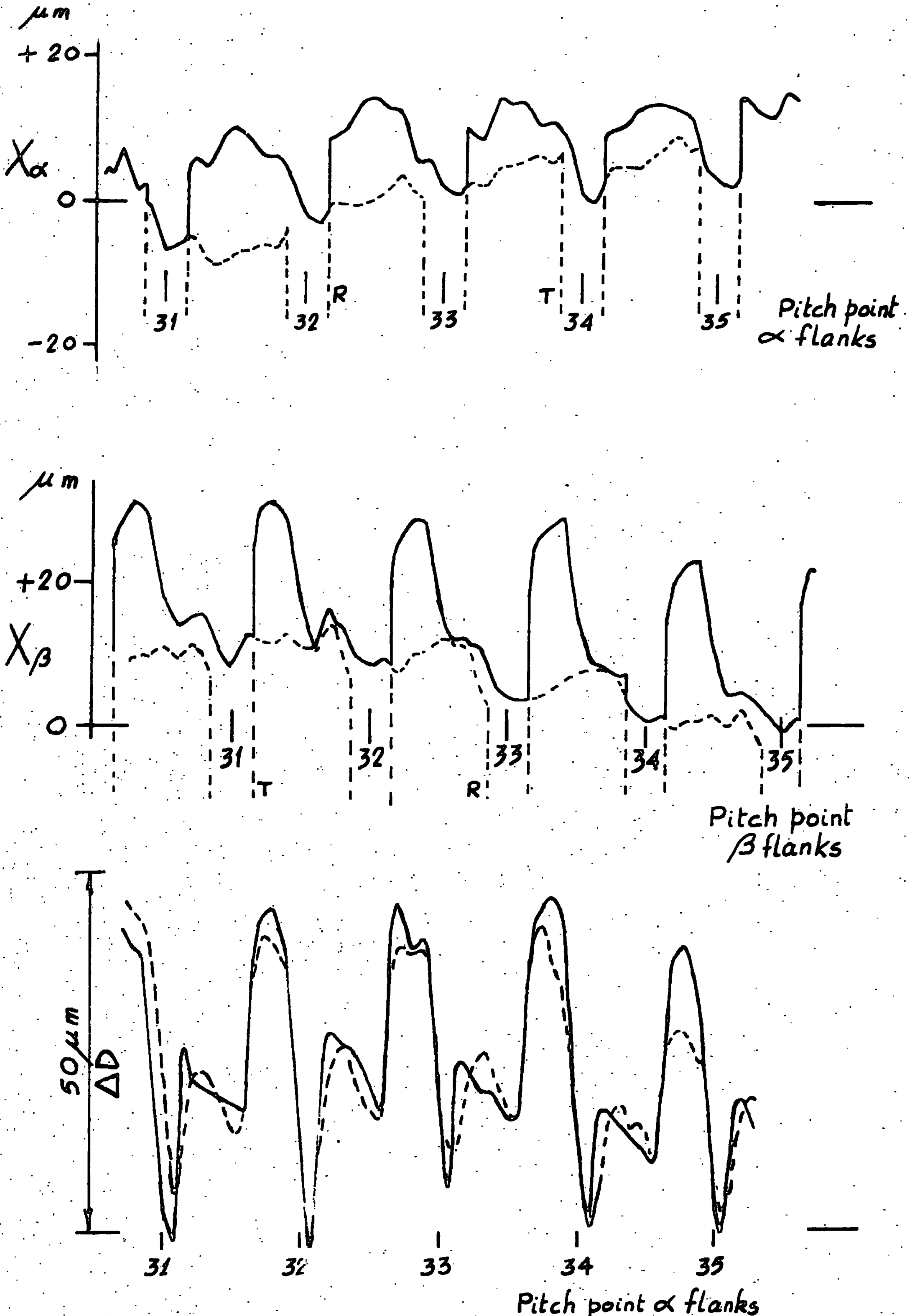


Fig. 6.3a Dual flank meshing test error curve
—— curve calculated from elementol errors.
- - - curve by direct measurement.



—— curve calculated from elemental errors
– – – curve by direct measurement

Fig. 6.3b Dual flank meshing test error curve.

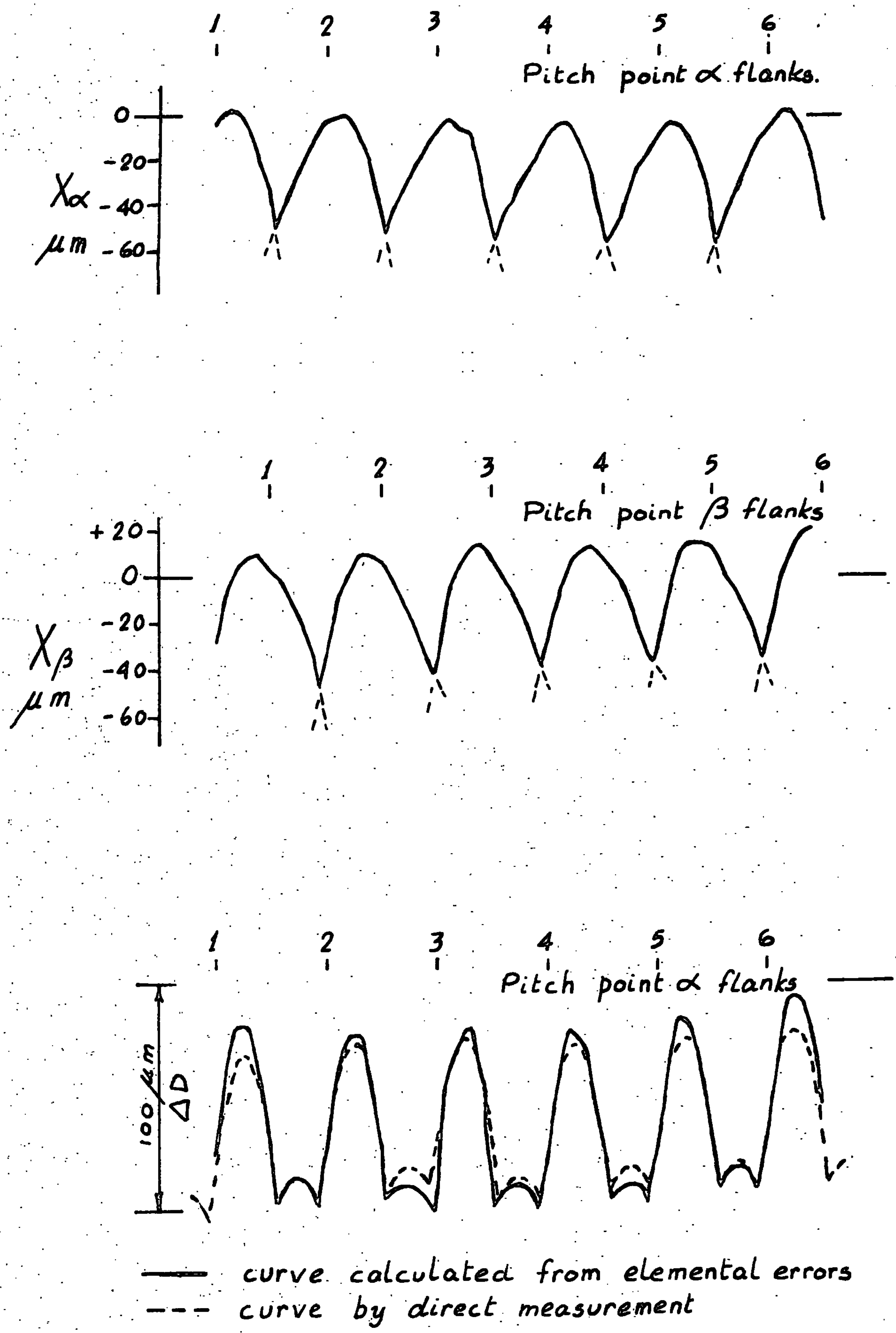


Fig 6.3c Dual flank meshing test error curve.

6.4. Specimen set of calculations for gear pair 6.2(ii).

Fig. 6.4a shows the tooth meshing order used during the dual flank testing. Two gears having the same number of teeth were used so that the dual flank error curve would repeat every revolution of the gears.

Fig. 6.4b

Length of roll when involute testing:-

For master gear

$$wy = (R_b^2 - R_o^2)^{\frac{1}{2}}$$

and for the test gear

$$xz = (r_b^2 - r_o^2)^{\frac{1}{2}}$$

$$wP = (R_f^2 - R_o^2)^{\frac{1}{2}}$$

$$zP = (r_f^2 - r_o^2)^{\frac{1}{2}}$$

Distance from tip to pitch point:

master gear $yP = wy - wP$

test gear $xP = xz - zP$

For the 36T 7DP 20° A.O.P. gear pair

base pitch $p_o = 10.71$ mm

$wy = xz = 31.37$ mm

$yP = xP = 9.04$ mm $= 0.844\, p_o$

A typical tooth profile trace for the β flanks of the test gear is shown by Fig. 6.4c., the magnification of the length of roll was determined by direct measurement (m = 3.69:1).

Thus the distance from the tooth tip to the pitch point measured on the profile trace

$$XP = m.\ xP$$

$$= 33.27 \text{ mm}$$

The equivalent length of one base pitch on the profile trace $= m.p_o$

$$= 39.52 \text{ mm}$$

Points $0.1p_o$, $0.2p_o$ etc. were "marked off" on the tooth profile trace and the profile error at each of these points determined using point P as the datum. These values were then corrected for the base pitch error of each particular tooth so giving the composite tooth error measured normal to the tooth flank at each point. This procedure was repeated for each tooth flank (α and β) for both the master and test gears. Knowing the meshing conditions, as detailed in Fig. 6.4a the master and test gear composite errors were added algebraically for each tooth contact point this giving the combined composite tooth error, i.e. the values of X_α and X_β (see section 5.1). These values being in effect the composite tooth errors of the test gear relative to a perfect master gear. The values of X_α and X_β were then plotted to a base of gear rotation and a dual flank error curve obtained using the theory described in chapter 5.

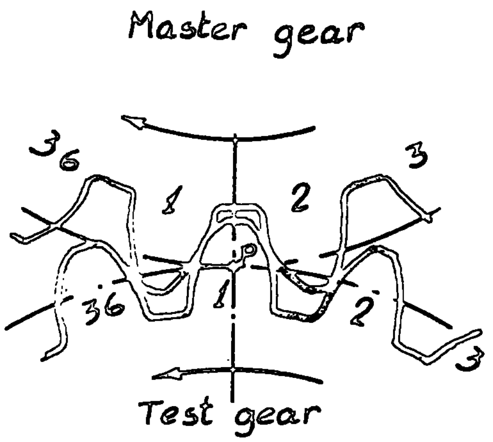


Fig 6.4a Tooth meshing order

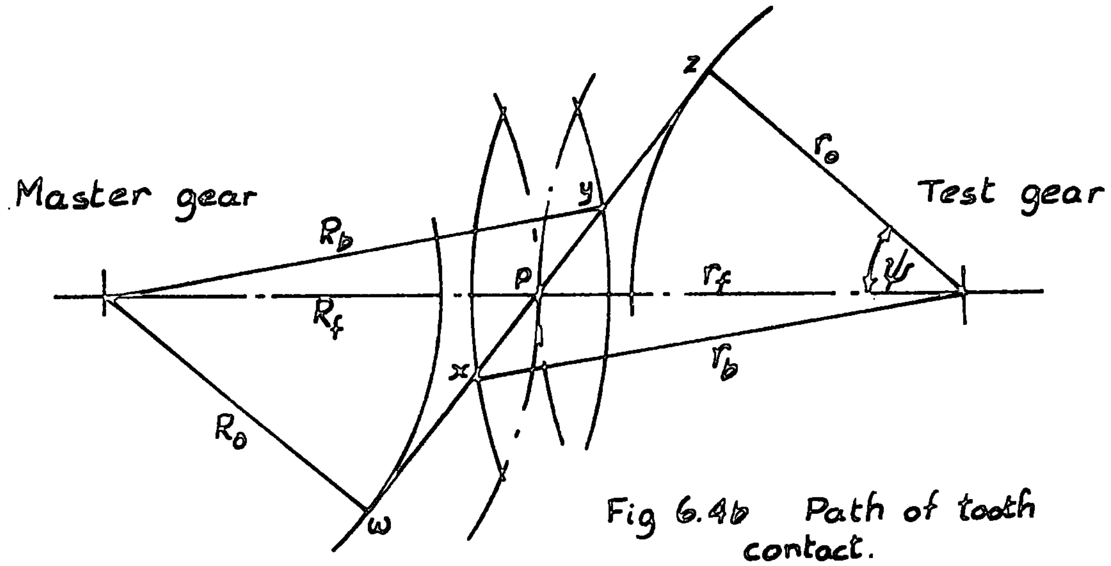


Fig 6.4b Path of tooth contact.

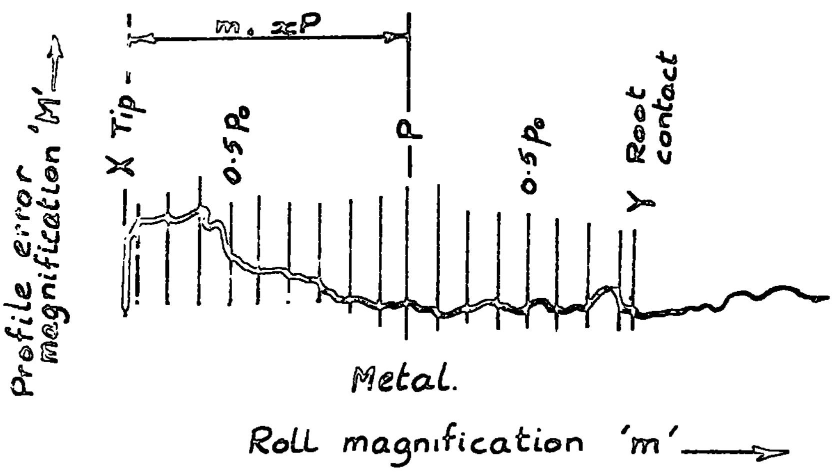


Fig 6.4c Tooth profile trace

6.5. Accuracy of measurements.

(i) Pitch measurement.

The repeatability of an individual span measurement was $\pm$ 0.0005 mm

This giving a probable error in the maximum cumulative pitch of:

for 48T gear $\pm$ 0.0025 mm

for 36T gear $\pm$ 0.0021 mm

(ii) Profile error

Gear	M	E
48T (master gear)	2000	0.5
48T (test gear)	1000	0.5
36T (master gear)	1000	0.5
36T (cut gear)	1000	0.5
36T (profile ground gear)	200	2.0

M = magnification of profile error trace

E = repeatability of profile error trace μm

(iii) Dual flank error

The repeatability of the points in the case of the measured dual flank error curve was $\pm$ 0.002 mm. In calculating the dual flank error from the elemental errors the inaccuracies in the individual measurements build up to give a probable error in the resulting curve of $\pm$ 0.007 mm. Apart from isolated points the agreement between the two curves is better than is indicated by the above estimates.

In some cases due to the characteristics of the tooth profile error the tip point on the tooth profile trace was by no means as pronounced as is shown on Fig. 6.4c. A small error in the estimation of the tooth tip position would produce a slight out of phasing of the composite tooth error curve (X curve) for the particular tooth involved leading to an error in the calculated dual flank curve (ΔD).

CHAPTER 7.

Conclusions and discussion

7.1. Since the dual flank error curve is dependent upon the errors in tooth pitch and tooth profile of the gears in mesh, it was necessary to investigate the accuracy of measurement of these two tooth elements as a preliminary stage to the project.

7.2. Tooth profile measurement

The accuracy of the profile measurement is a function of; the alignment of the involute testing machine, the diameter of the base circle disc used and the repeatability of the trace produced by the chart recorder. The diameter of the base disc was controlled during the manufacturing process and the other two items checked before any experimental results were taken. Although it was found to be unnecessary in this case, it is possible to correct the results for small errors in base circle disc diameter.

7.3. Pitch measurement

Direct experimental comparison of the two methods of pitch measurement used was not possible due to the difficulty in setting the same gear eccentricity in terms of magnitude and phase, relative to the teeth, on the two measuring machines used.

The results obtained from the pitch measurement of an eccentrically mounted master gear produced a better overall degree of accuracy than is indicated by calculation of the probable inaccuracy of the maximum cumulative pitch error. This was expected since the least mean squares analysis involves every point of the pitch error curve.

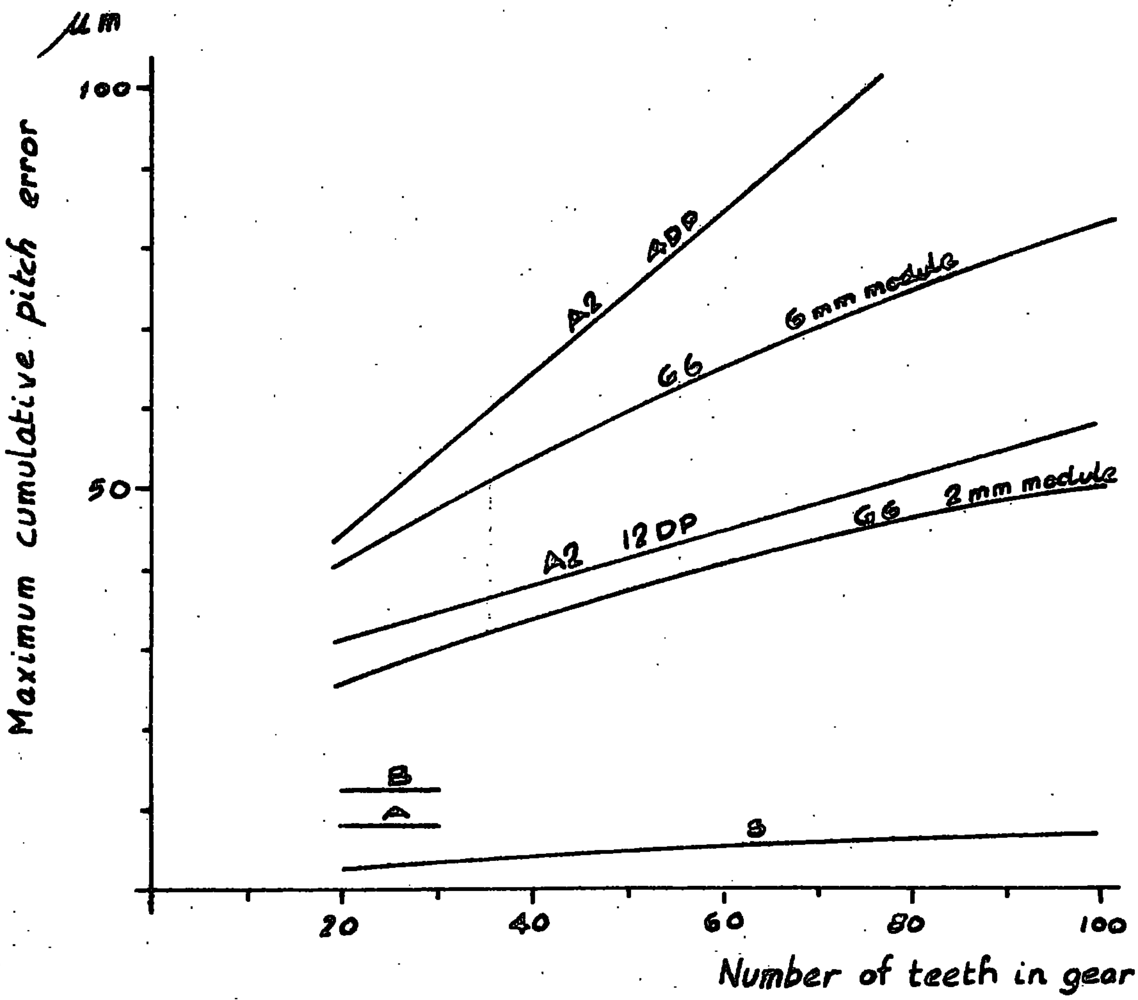
As stated in a previous chapter the accuracy of the direct method of measurement of cumulative pitch error is directly proportional to the accuracy of the indexing arrangement, whilst in the case of the span gauging technique the resulting accuracy is directly proportional to the square root of the number of teeth in the gear. It would seem therefore that the direct method is best when the number of teeth in the gear is large. But in opposition to this is the increasing diameter of the gear which in turn necessitates very accurate dividing arrangements to minimise the effect of indexing errors. Equipment for the direct measurement of pitch error can be perhaps twenty times more expensive than span pitch gauging apparatus. When the gear under inspection by the span gauging method has a large number of teeth it is possible to prevent the uncertainty in the pitch error increasing to excessive values by using a slightly different technique from that described in chapters 3 and 4. The teeth of the gear under inspection are subdivided into a number of spans each containing the same number of teeth, the number chosen being divisible into the total number of teeth in the gear. As a first setting the errors in the spans are determined and then at a second setting the single span pitch errors measured. These single pitch errors are then interpolated into the graph of span errors so producing the overall pitch error of the gear.

Fig. 7.3a illustrates the probable inaccuracy in the measurement of maximum cumulative pitch error by the single pitch span gauging method used in this project in relation to the allowable tolerances on pitch quoted in various British Standard Specifications. This indicates that the method is acceptable for precision cut and ground gears to grades G6 and A2, the inaccuracy of measurement being of the order of 10% of the allowable tolerance; but not so satisfactory for the measurement of master gears. Although it was found that the repeatability of measurements was better than 1 μm when measuring the master gears, this was most probably due to the higher quality of surface finish of the teeth.

7.4. Dual flank mesh testing - experimental results

(see Fig. 7.4a for definitions of errors).

The cumulative pitch error measurements required for the dual flank mesh testing experiments were obtained using the single span gauging technique, the 36T 7 DP gears being too large for the capacity of the S.I.P. measuring machine. This particular size of gear wheel was chosen because of the availability of the master gear, and the special rack cutter referred to in section 6.2. Also this size of gear pair when in mesh gives a workable length to the active involute profile trace produced by the involute testing machine (due to the length of the path of tooth contact).



S = probable error in measured maximum cumulative pitch error when using span gauging technique (accuracy of span measurements 1 µm).

A & B = allowable tolerance for master gears of tooth pitch 2 to 10 mm module, BS 3696 : 1963

G6 = allowable tolerance, grade 6, gears, BS 436 part 2: 1970

A2 = allowable tolerance, precision cut gears, BS 436 : 1940

Fig 7.3a Comparision of the accuracy of span pitch gauging with allowable pitch tolerance

Comparison of the calculated and measured dual flank error curves shown in Fig. 6.3a, 6.3b and 6.3c., indicate agreement well within the estimated probable inaccuracy of the measurements. In each case the form or pattern of the curves agrees with the theory of chapter 5.

(i) 48T 12 DP pair, reject master test gear, dual flank results Fig. 6.3a. The profile curves of the test gear show excess material at the tips of both the α and β tooth flanks, indicating a pressure angle of generation which was less than 20°. The theoretical curve for this particular form of error is illustrated by Fig. 5.5d.

(ii) 36T 7 DP pair, rack generated test gear, dual flank results Fig. 6.3c. The profile curves of the test gear indicate excess material at the root of the α flanks and tip of the β flanks, such a condition could occur if the rack cutter was not set square to the gear blank. Fig. 5.5f shows the theoretical curve for a similar condition.

(iii) 36T 7 DP pair, profile ground test gear, dual flank results 6.3c. In this case the profile error high point is slightly off set from the pitch point towards the tooth tip thus the pattern is not directly comparable with the theoretical curve Fig. 5.31.

The results produced give a good indication of the optimum possible with the present generation of gear testing equipment.

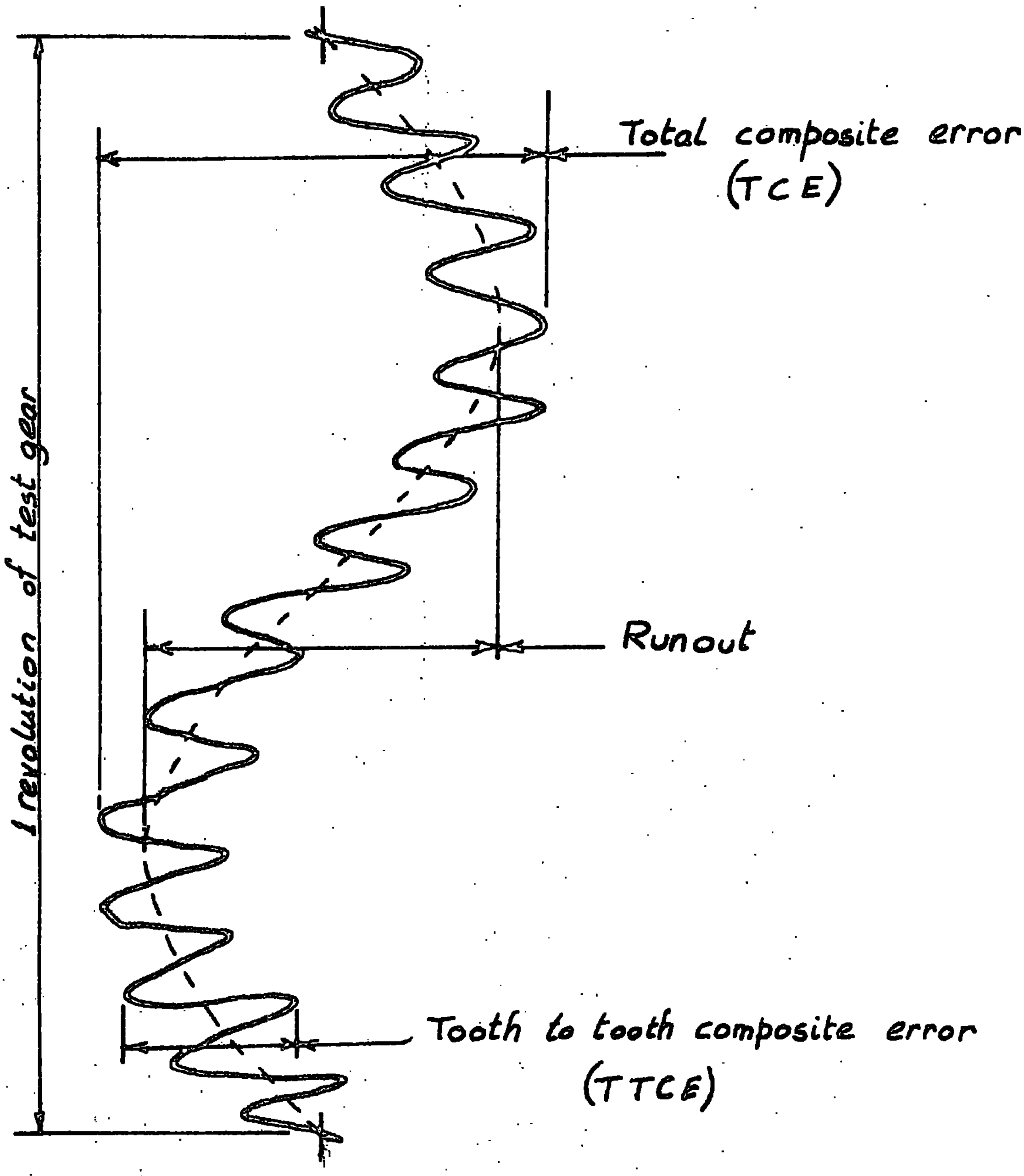


Fig 7.4a Dual flank meshing test, definition of terms

7.5. Dual flank testing - cause of errors detected

The errors indicated by the test namely; tooth to tooth composite error (TTCE), run out, and total composite error (TCE) can be ascribed to one or more of the following;

(i) incorrect setting of the gear blank,

(ii) incorrect setting of the cutter,

(iii) inaccuracies in the cutter,

(iv) variation of centre distance between blank and cutter occurring during the cutting process.

The test will not reveal the presence of periodic or cumulative pitch errors in the drive mechanism of the work spindle of the gear cutting machine. Consider a gear produced on a gear shaping machine, the shaper cutter having the same number of teeth as the mater gear to be used for the inspection of the finished gear. Any variation in the velocity of the blank due to errors in the work table drive will not affect the depth of tooth cut by the shaper cutter, but the gear cut will have pitch and profile errors. On inspecting the finished gear on a dual flank testing machine no errors will be indicated because the test does not record errors in angular displacements of the gear. If the same gear was checked on a single flank rolling gear tester the error would be clearly shown as this machine measures angular displacements.

7.6. Dual flank testing - practice

In practice the errors in the master gear may not be known, in which case the master and test gear should be rolled together for a number of revolutions and the average value of the errors computed. If the master and test gears have the same number of teeth or the tooth numbers are multiples of each other the gear should be remated in positions 90° apart after each complete revolution to obtain averaging of the master gear error.

For maximum benefit from the test the length of the path of contact between the master gear and the test gear should be equal to or greater than that occurring between the test gear and its mate. This will ensure the inspection of the less accurate root area of the tooth. Interference in the root of the test gear tooth can occur when a master gear (which normally has no tip relief) is meshed with a gear produced on a gear shaper, the master gear having many more teeth than the gear shaper cutter.

7.7. Dual flank test for quality control

(i) At the very least the test provides a check on the consistency of product quality, in that initially each individual tooth element of a production gear may be inspected and then a dual flank test carried out against a master gear. Subsequent production gears need then only to be dual flank tested against the same master gear and the resulting traces compared for compatibility with the trace from the initial gear.

(ii) The theoretical traces shown in chapter 5 and the traces produced in the experimental section would seem to indicate that some diagnosis of errors is possible; certain profile errors produce characteristic shapes to the TTCE undulations of the dual flank error graph. This may require some experience since the pattern will tend to change in appearance with different magnifications along the two axes. But a catalogue of typical curves could be prepared to which an inspector could refer.

Not all gear tooth errors produce variations in the tight meshing centre distance when dual flank testing, see section 7.5, also teeth having excessive tip relief show no variation in the dual flank graph, Fig. 5.5c, due to tooth contact overlap. This latter error would not be shown in a single flank meshing test if the test was carried out at the standard running centre distance for the master and product gears.

(iii) Simple limits may be set for the permitted variation of centre distance when dual flank testing by considering the allowable variation in backlash between master and product gear if they were meshed at the standard centre distance. Using involute geometry it is possible to calculate the two extremes of the tight meshing centre distance of the product and master gear, due to allowable variation in product gear tooth thickness to give the required backlash. This rather simplified approach based on calculations involving perfect involutes makes no attempt to account for variations in tooth pitch or profile errors.

(iv) The TCE is equal to the sum of the runout and the TTCE. In practice the runout is usually approximately sinusoidal in form and can be attributed to an eccentric setting error of the blank on the gear cutting machine or a cutting error which produces the same effect relative to the blank. It is therefore possible to calculate a value for this eccentricity and hence the runout from the maximum permitted cumulative pitch error.

Maximum cumulative pitch error

$$\Delta p_{cm} = 2.e.\sec\psi_f \qquad \text{from 3.3a or 3.3b}$$

$$\text{runout} = 2e = \Delta p_{cm} \cdot \cos\psi_f$$

This runout due to pitch errors would be the TCE if there were no additional profile errors other than those caused by the eccentric setting of the blank. The profile errors due to the cutting process will form the TTCE component of the dual flank error. BS 436 Part 2 makes the following recommendation in respect of profile errors, "In most gear applications positive departures from the design profile should not occur outside the central third of the working depth. The permissible positive departure within this central third shall not exceed one third of the profile tolerance".

One point of tooth contact b or c of Fig. 5.1a will always be within one quarter of a base pitch measured along the profile trace of the pitch point, see Fig. 5.5a to m. The TTCE can be determined by considering the involute profile error within a band of width equal to one base pitch, measured on the profile trace about the pitch point.

$$\text{TTCE} = \frac{2}{3} \cdot \Delta i \cdot \operatorname{cosec} \psi_f$$

where Δi = allowable profile tolerance
the $\frac{2}{3}$ fraction is used as being the
probable fraction of the error within
prescribed limits

$$\text{TCE} = \Delta p_{cm} \cdot \cos \psi_f + \frac{2}{3} \cdot \Delta i \cdot \operatorname{cosec} \psi_f$$

In order to comply with backlash requirements the TCE should lie within the two extreme limits described in 7.7(iii).

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NOTATIONS

A O P	pressure angle
D P	diametral pitch
T	number of teeth in gear
T C E	total composite error when dual flank testing
T T C E	tooth to tooth composite error when dual flank testing
X	composite tooth error
a	tooth addendum
d_b	blank diameter, outside diameter
d_f	reference circle diameter
do	base circle diameter
e	eccentricity
g	arc tooth thickness
g_f	arc tooth thickness at the reference circle
mod	module
p	circular pitch
p_o	base pitch
r_b	blank radius
r_f	reference circle radius
r_o	base circle radius
$\left.\begin{matrix}\alpha\\ \beta\end{matrix}\right\}$	tooth flank indentification
ψ	pressure angle
ΔD	variation in centre distance when dual flank testing
ΔS	single flank mesh testing error
Δp_a	adjacent pitch error
Δp_{cm}	maximum cumulative pitch error
Δi	involute profile error

Appendix I.

The best sine wave through the points (θ_1, y_1), (θ_2, y_2) etc. is given by the equation,

$$Y = a\sin(\theta - \phi)$$

and the error between the predicted value and the actual value is,

$$y - Y = y - a\sin(\theta - \phi)$$

the sum of the squares of all such errors is

$$E = \Sigma[y - a\sin(\theta - \phi)]^2$$

and this is to be a minimum.

$$\frac{\partial E}{\partial a} = -2\Sigma[y - a\sin(\theta-\phi)]\sin(\theta-\phi) = 0$$

$$\therefore \Sigma y\sin(\theta-\phi) - a\Sigma\sin^2(\theta-\phi) = 0 \quad \text{—— } 1$$

$$\frac{\partial E}{\partial \phi} = 2a\Sigma[y - a\sin(\theta-\phi)]\cos(\theta-\phi) = 0$$

$$\therefore \Sigma y\cos(\theta-\phi) - a\Sigma\sin(\theta-\phi)\cos(\theta-\phi) = 0 \quad \text{—— } 2$$

when the summation is over 2π radians

$$\Sigma\sin(\theta-\phi)\cos(\theta-\phi) = 0$$

and in this case equation 2 becomes

$$\cos\phi\,\Sigma y\cos\theta + \sin\phi\,\Sigma y\sin\theta = 0$$

$$\Sigma y\cos\theta + \tan\phi\,\Sigma y\sin\theta = 0$$

$$\therefore \tan\phi = -\frac{\Sigma y\cos\theta}{\Sigma y\sin\theta} \quad \text{—— } 3$$

from equation 1

$$a = \frac{\Sigma y\sin(\theta-\phi)}{\Sigma\sin^2(\theta-\phi)}$$

when the summation is over 2π radians

$$\Sigma\sin^2(\theta-\phi) = \frac{N}{2}$$

where N = number of readings

$$\therefore \quad a = \frac{\cos\phi\,\Sigma y \sin\theta - \sin\phi\,\Sigma y\cos\theta}{\frac{N}{2}} \quad \text{——— } 4$$

which can be solved numerically once the value of ϕ has been determined from equation 3.

from 3

$$-\Sigma y \cos\theta = \tan\phi\,\Sigma y \sin\theta$$

$$\therefore \quad a = \frac{2(\cos\phi + \sin\phi\tan\phi)\,\Sigma y \sin\theta}{N}$$

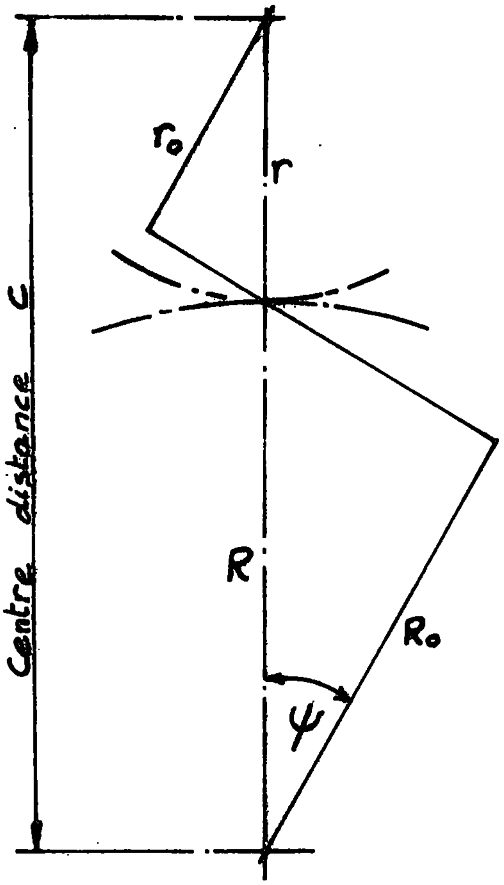
$$= \frac{2\left(\frac{\cos^2\phi + \sin^2\phi}{\cos\phi}\right)\Sigma y \sin\theta}{N}$$

$$= \frac{2\,\Sigma y \sin\theta}{N\cos\phi}$$

$$\therefore \quad a = \frac{2\sec\phi.\,\Sigma y \sin\theta}{N} \quad \text{——— } 5.$$

Appendix II

Effect of varying meshing centre distance of a pair of gears on the pressure angle.

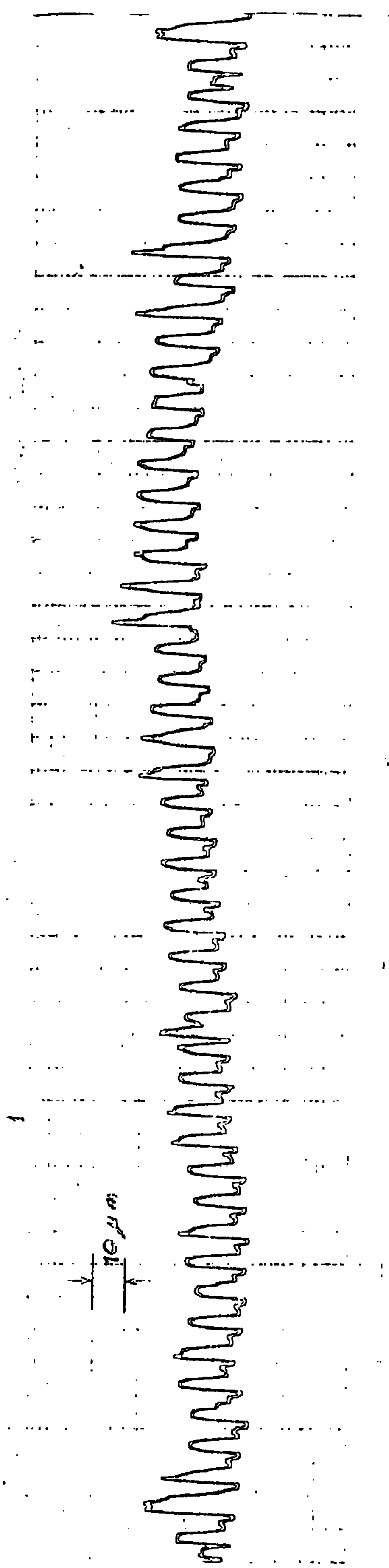


$$C = R + r$$
$$= \frac{R_o + r_o}{\cos\psi}$$
$$\therefore \cos\psi = \frac{R_o + r_o}{C}$$
$$-\sin\psi . d\psi = -\frac{R_o + r_o}{C^2} . dC.$$
$$d\psi = \frac{(R_o + r_o)}{\sin\psi . C^2} . dC$$
$$d\psi = \frac{\cot\psi}{C} . dC$$

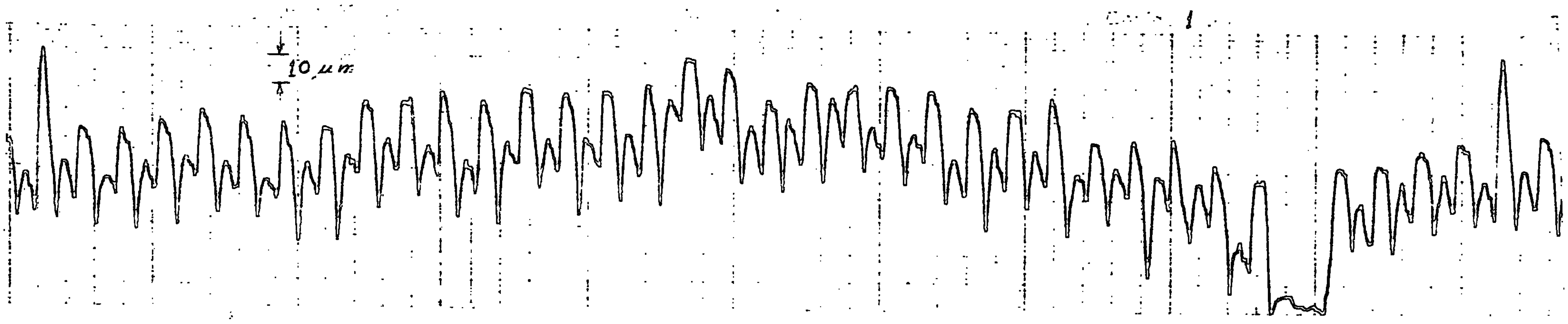
For a pair of 20° pressure angle gears having a nominal centre distance $C = 125$ mm
Let $\Delta C = -0{\cdot}25$ mm.

then $d\psi = -\frac{2{\cdot}7475 \times 0{\cdot}25}{125} = -0{\cdot}0055$ radians
$(0{\cdot}31^\circ)$

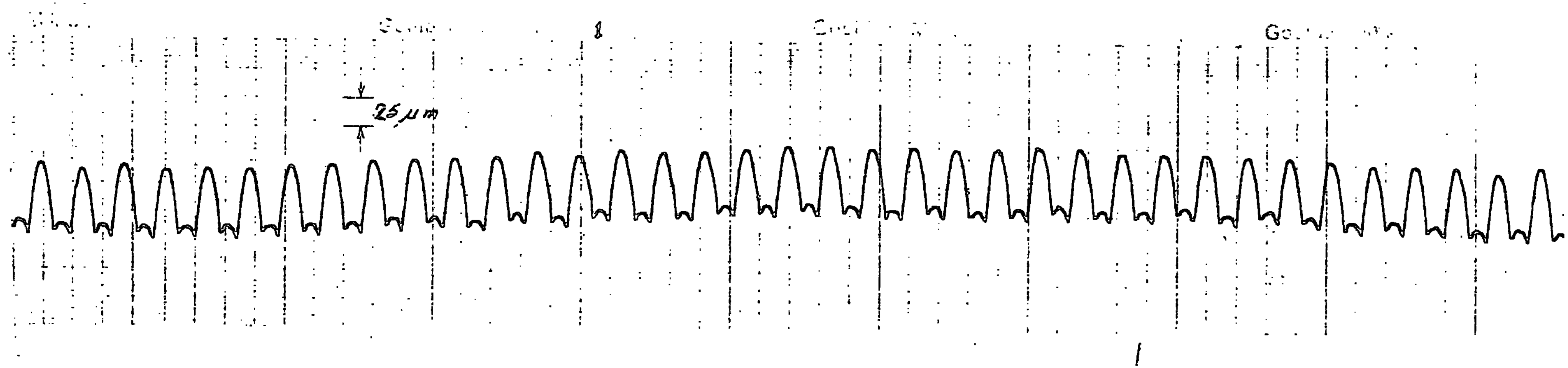
when dual flank mesh testing the effect of this change in the value of ψ, is to produce an error of 1·5% in ΔD, the actual value being larger than that indicated by theory.



Dual flank error - 48T 12DP 20° A.O.P reject master gear.
6.2(i)



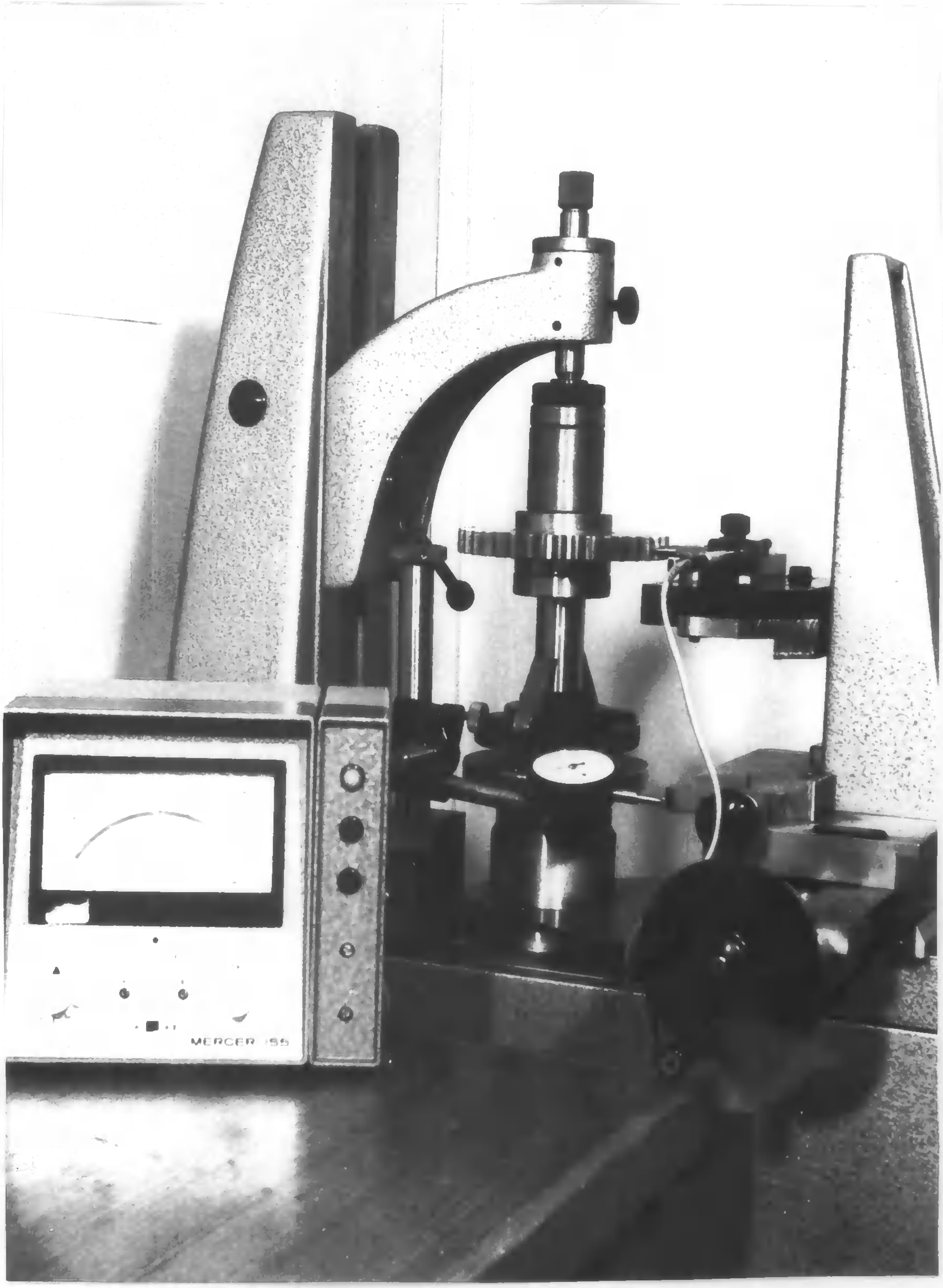
Dual flank error - 36T 7DP 20° AOP, machine cut gear. 6.2(ii)



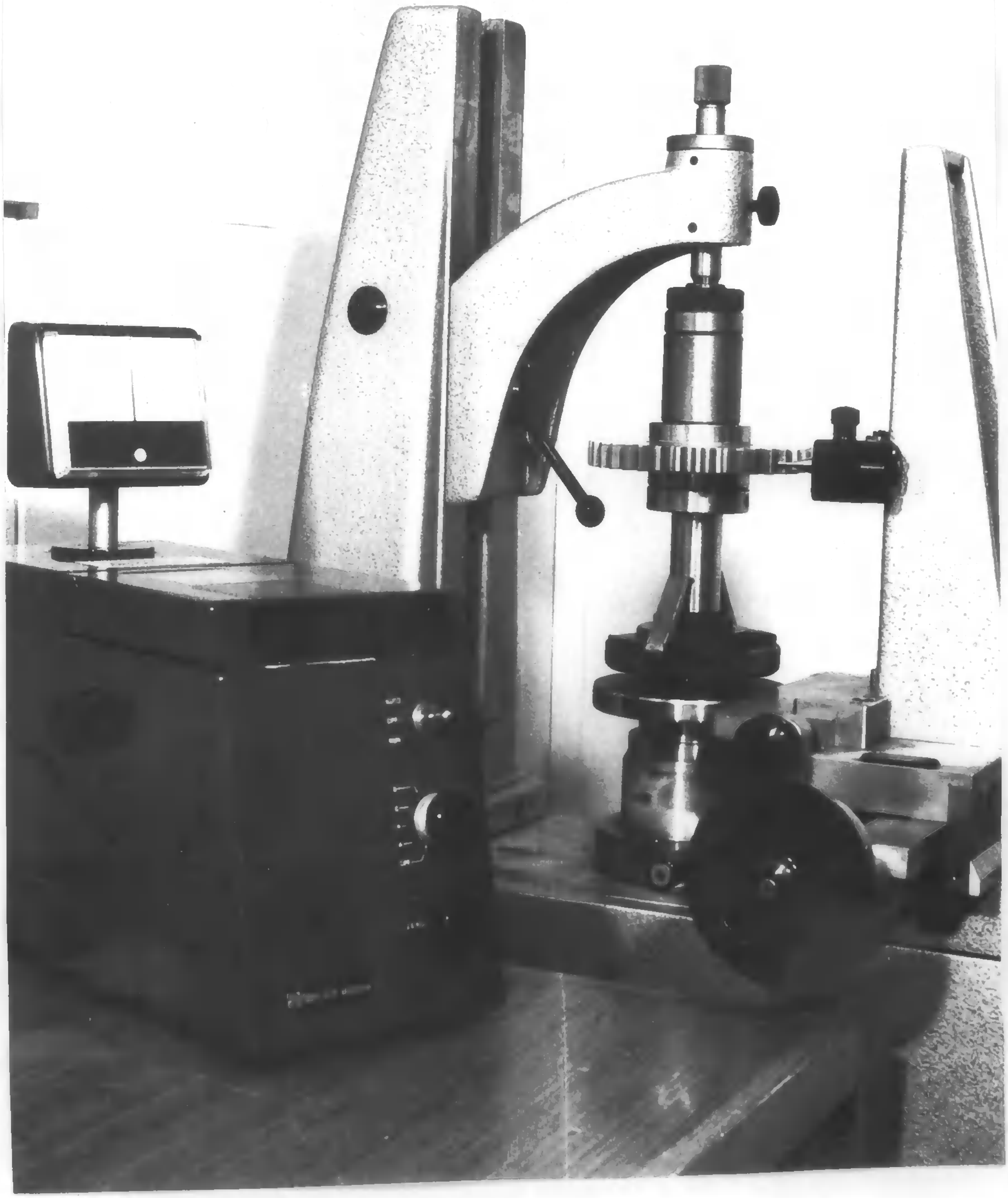
Dual flank error - 36T 7DP 20° AOP ground gear
6.2(ii)

Appendix IV

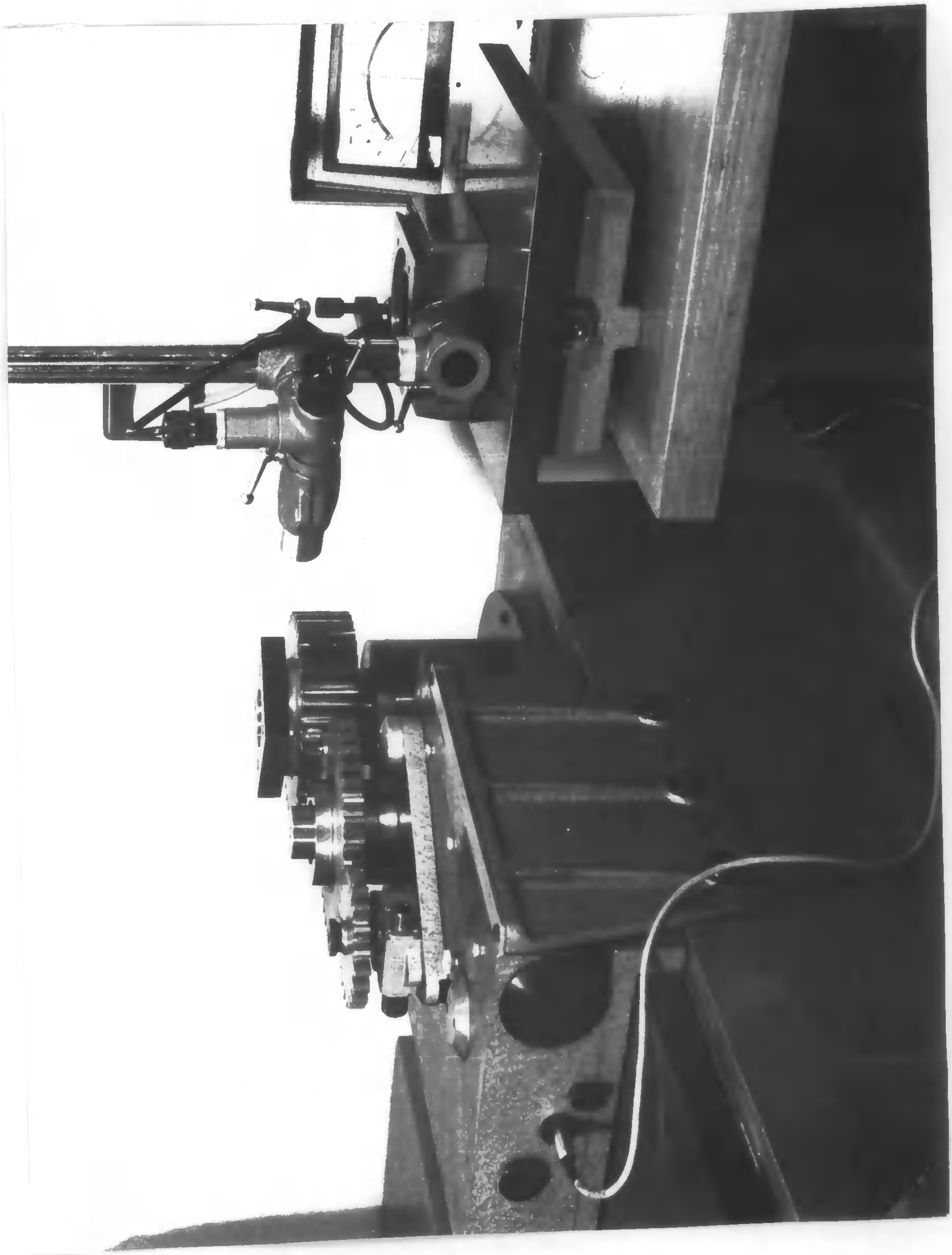
Direct measurement of tooth pitch error.



Span gauging of tooth pitch error.



Involute profile measurement.



Dual flank mesh testing.